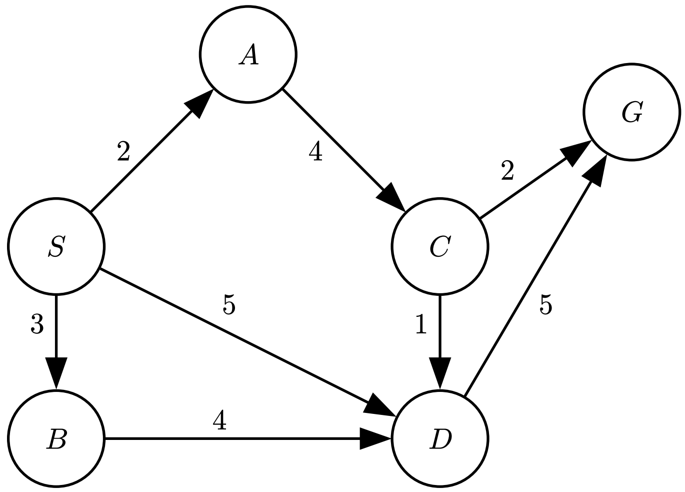
Q1 *Search Algorithms in Action* **(3 points)**



For each of the following graph search strategies, work out the order in which states are expanded, as well as the path returned by graph search.

In all cases, assume ties resolve in such a way that states with earlier alphabetical order are expanded first.

Remember that in graph search, a state is expanded only once.

Q1.1 (1 point) Depth-first search.

States expanded:

Path returned:

Q1.2 (1 point) Breadth-first search.

States expanded:

Path returned:

Q1.3 (1 point) Uniform-cost search.

States expanded:

Path returned:

Q2 ***Search and Heuristics*** **(5 points)**

Imagine a car-like agent wishes to exit a maze like the one shown below:

1. The agent is directional and at all times faces some direction $d \in (N, S, E, W)$.
2. With a single action, the agent can *either* move forward at an adjustable velocity v *or* turn. The turning actions are *left* and *right*, which change the agent's direction by 90 degrees. Turning is only permitted when the velocity is zero (and leaves it at zero).
3. The moving actions are *fast* and *slow*. *Fast* increments the velocity by 1 and *slow* decrements the velocity by 1; in both cases the agent then moves a number of squares equal to its NEW adjusted velocity (see example below).
4. A consequence of this formulation is that it is **impossible** for the agent to move in the same nonzero velocity for two consecutive timesteps.
5. Any action that would result in a collision with a wall or reduce v below 0/above a maximum speed V_{max} is illegal.
6. The agent's goal is to find a plan which parks it (stationary) on the exit square using as few actions (time steps) as possible.

As an example:

- If, at timestep t, the agent's current velocity is 2, by taking the *fast* action, the agent first increases the velocity to 3 and move 3 squares forward, such that at timestep $t + 1$ the agent's current velocity will be 3 and will be 3 squares away from where it was at timestep t.
- If instead the agent takes the *slow* action, it first decreases its velocity to 1 and then moves 1 square forward, such that at timestep $t + 1$ the agent's current velocity will be 1 and will be 1 square away from where it was at timestep t.
- If, with an instantaneous velocity of 1 at timestep $t + 1$, it takes the slow action again, the agent's current velocity will become 0, and it will not move at timestep $t + 1$. Then at timestep $t + 2$, it will be free to turn if it wishes.
- Note that the agent could not have turned at timestep $t + 1$ when it had a current velocity of 1, because it has to be stationary to turn.

Q2.1 (1 point) If the grid is M by N , what is the size of the state space? Justify your answer. You should assume that all configurations are reachable from the start state.

Q2.2 (1 point) Is the Manhattan distance from the agent's location to the exit's location admissible? Why or why not?

Q2.3 (1 point) State and justify a non-trivial admissible heuristic for this problem which is not the Manhattan distance to the exit.

Q2.4 (1 point) If we used an inadmissible heuristic in A* graph search, would the search be complete? Would it be optimal?

(Question 2 continued...)

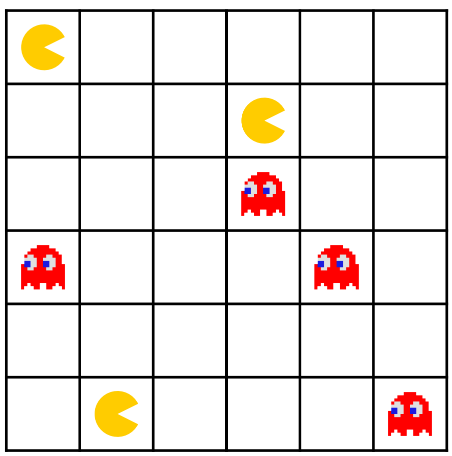
Q2.5 (1 point) Give a possible advantage that an inadmissible heuristic might have over an admissible one.

Q3 *Pacfriends Unite* **(9 points)**

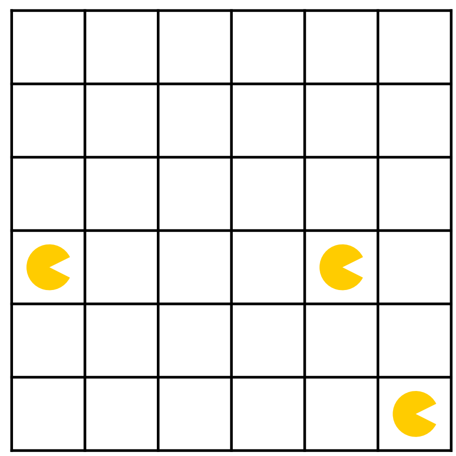
Pacman and his Pacfriends have decided to combine forces and go on the offensive, and are now chasing ghosts instead! In a grid of size M by N, Pacman and $P-1$ of his Pacfriends are moving around to collectively eliminate **all** of the ghosts in the grid by stepping on the same square as each of them. Moving onto the same square as a ghost will eliminate it from the grid, and move the Pacman into that square.

Every turn, Pacman and his Pacfriends may choose one of the following four actions: *left*, *right*, *up*, *down*, but may not collide with each other. In other words, any action that would result in two or more Pacmen occupying the same square will result in no movement for either Pacman or the Pacfriends. Additionally, Pacman and his Pacfriends are **indistinguishable** from each other. There are a total of G ghosts, which are indistinguishable from each other, and cannot move.

Treating this as a search problem, we consider each configuration of the grid to be a state, and the goal state to be the configuration where **all** of the ghosts have been eliminated from the board. Below is an example starting state, as well as an example goal state:



Possible Start State



Possible Goal State

Assume each of the following subparts are **independent** from each other. **Also assume that regardless of how many Pacmen move in one turn, the total cost of moving is still 1.**

For the next four subparts, suppose that Pacman has no Pacfriends, so $P = 1$.

Q3.1 (1 point) What is the size of the minimal state space representation given this condition? Recall that $P = 1$.

- (A) MN
- (B) MNG
- (C) $(MN)^G$
- (D) $(MN)^{G+1}$
- (E) 2^{MN}
- (F) 2^{MN+G}
- (G) $G(2)^{MN}$
- (H) $MN(2)^G$

For each of the following heuristics, select whether the heuristic is admissible or inadmissible. Recall that $P = 1$.

Q3.2 (1 point) $h(n) =$ the sum of the Manhattan distances from Pacman to every ghost.

Ⓐ Admissible　　Ⓑ Inadmissible

Q3.3 (1 point) $h(n) =$ the number of ghosts times the maximum Manhattan distance between Pacman and any of the ghosts.

Ⓐ Admissible　　Ⓑ Inadmissible

Q3.4 (1 point) $h(n) =$ the number of remaining ghosts.

Ⓐ Admissible　　Ⓑ Inadmissible

Suppose that Pacman has exactly one less Pacfriend than there are number of ghosts. Therefore, $P = G$. Recall that Pacman and his Pacfriends are indistinguishable from each other.

Q3.5 (1 point) What is the size of the minimal state space representation given this condition? Recall that $P = G$.

Ⓐ MNP

Ⓑ $MNGP$

Ⓒ $(MN)^G$

Ⓓ $(MN)^{(G+P)}$

Ⓔ $(MN)^P \cdot 2^G$

Ⓕ $(MN)^G \cdot P$

Ⓖ $(MN)^{G+1}$

Ⓗ $(MN)^{(G+1)P}$

Ⓘ $\binom{MN}{P}$

Ⓙ $\binom{MN}{P} \cdot 2^G$

Ⓚ $\binom{MN}{P} \cdot (MN)^G$

Ⓛ $\binom{MN}{P} \cdot \binom{MN}{G}$

Ⓜ 2^{MN}

Ⓝ 2^{MN+G+P}

Ⓞ $GP(2)^{MN}$

For each of the following heuristics, select whether the heuristic is admissible or inadmissible. Recall that $P = G$.

Q3.6 (1 point) $h(n) =$ the largest of the Manhattan distances between each Pacman and its closest ghost.

Ⓐ Admissible　　Ⓑ Inadmissible

Q3.7 (1 point) $h(n) =$ the smallest of the Manhattan distances between each Pacman and its closest ghost.

Ⓐ Admissible　　Ⓑ Inadmissible

Q3.8 (1 point) $h(n) =$ the number of remaining ghosts.

Ⓐ Admissible　　Ⓑ Inadmissible

Q3.9 (1 point) $h(n) =$ number of remaining ghosts / P.

Ⓐ Admissible　　Ⓑ Inadmissible