

Due: Wednesday 3/12 at 11:59pm.

Policy: Can be solved in groups (acknowledge collaborators) but must be submitted individually.

Make sure to show all your work and justify your answers.

Note: This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.

Note: Leave the self-assessment sections blank for the original submission of your homework. After the homework deadline passes, we will release the solutions. At that time, you will review the solutions, self-assess your initial response, and complete the self-assessment sections below. The deadline for the self-assessment is 1 week after the original submission deadline.

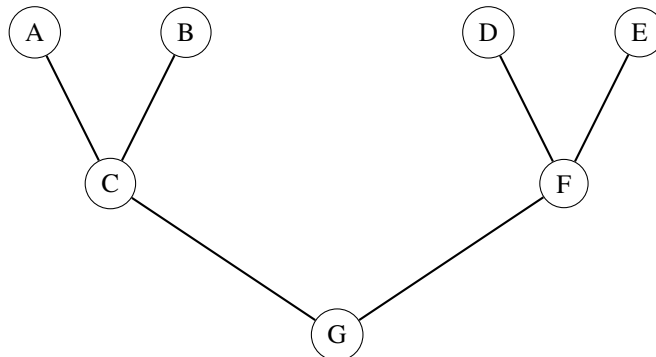
Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

Q1. [9 pts] Tracing Contacts

There is a rumor circulating that a past CS 188 head TA is submitting fake pet pics while disguising as a student. You are trying to retrace the origins of this rumor but do not know who spread the story to whom. However, you know the contacts between each pair of people and are able to create the network below.

- (a) Assume that the direction of each edge is uniformly random and that the direction of any two edges is independent.

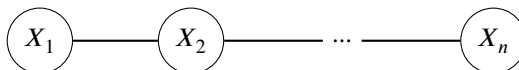


- (i) [1 pt] What is the probability that a member of the group ABC spread the rumor to group DEF through G? (Hint: Don't overthink this question. What must the direction of certain edges be in the graph?)

- (ii) [2 pts] What is the probability that $B \perp F$ is guaranteed via the independence relations encoded by the Bayes Net?

- (iii) [2 pts] If we add an edge between nodes B and D, what is the probability that the above Bayes Net is undefined?

- (b) In another class, you recover the following chain of contacts. Continue assuming that the direction of each edge is uniformly random and that the direction of any two edges is independent.



- (i) [1 pt] What is the probability that $X_1 \perp X_n$ is guaranteed for $n = 3$?

- (ii) [1 pt] What is the probability that $X_1 \perp X_n$ for any $n \geq 3$?

- | | | | |
|---|---|---|---|
| ☐ $\frac{n}{4} - \frac{1}{2}$ | ☐ $\frac{2^{n-1}-1}{2^{n-1}}$ | ☐ $\frac{1}{2^{n-2}}$ | ☐ $\frac{2^{n-1}-1}{2^{n-1}} - \frac{1}{2}$ |
| ☐ $\frac{1}{2^{n-2}} - \frac{1}{4}$ | ☐ $\frac{1}{2^{n-1}}$ | ☐ $\frac{2^{n-1}-n}{2^{n-1}}$ | ☐ None of the above |

- (iii) [2 pts] Conditioned on $X_1 \perp X_n$ guaranteed true, what is the probability of the $X_1 - X_2$ edge pointing left?

- | | | | |
|---|---|---|---|
| ☐ $\frac{1}{2}$ | ☐ $\frac{2^{n-2}-(n-1)}{2^{n-1}}$ | ☐ $\frac{2^{n-2}-1}{2^{n-1}-n}$ | ☐ $\frac{2^{n-2}-(n-1)}{2^{n-1}-n}$ |
| ☐ $\frac{2^{n-2}-1}{2^{n-1}}$ | ☐ $\frac{1}{4}$ | ☐ $\frac{2^{n-2}}{2^{n-1}-n}$ | ☐ None of the above |

Q1(a-b) Self-Assessment - leave this section blank for your original submission. We will release the solutions to this problem after the deadline for this assignment has passed. After reviewing the solutions for this problem, assess your initial response by checking one of the following options:

- ☐ I fully solved the problem correctly, including fully correct logic and sufficient work (if applicable).
- ☐ I got part or all of the question incorrect.

If you selected the second option, explain the mistake(s) you made and why your initial reasoning was incorrect (do not re-iterate the solution. Instead, reflect on the errors in your original submission). Approximately 2-3 sentences for *each* incorrect sub-question.