

1. Show that if  $U$  and  $V$  are unitary operators acting on separate Hilbert spaces  $\mathcal{H}_1$  and  $\mathcal{H}_2$  respectively, then  $U \otimes V$  acting on the product space  $\mathcal{H}_1 \otimes \mathcal{H}_2$  is also a unitary operator.
2. Consider the unitary operation  $U$  resulting from applying the Hadamard gate to each of  $n$  qubits. Describe  $U$  by giving a formula for its  $(i, j)^{th}$  entry (where  $i$  and  $j$  are the row and column indices of the matrix).
3. Consider two quantum systems each with two qubits, where the first is in the state  $1/2|00\rangle + i/2|10\rangle + 1/\sqrt{2}|11\rangle$ , and the second is in the state  $1/\sqrt{3}|00\rangle + i/\sqrt{3}|10\rangle - 1/\sqrt{3}|01\rangle$ . What is the state of the composite system? If we apply a CNOT to the first system and the transformation  $H \otimes Z$  to the second, what is the new state of the system. Write out the matrix corresponding to the unitary transformation you applied to the second system (writing this out is a bit of work, so do it in more than one step).
4. Suppose that Alice has two qubits in an entangled state  $|\psi\rangle \in \mathcal{C}^2 \otimes \mathcal{C}^2$ . If she teleports each of her two qubits to Bob using the teleportation protocol presented in lecture, can Bob faithfully reconstruct the (entangled) state of Alice's two qubits? Justify your answer.
5. Suppose Alice and Bob share the entangled state  $\alpha|00\rangle + \beta|11\rangle$ . Alice wants Bob to have the state  $\alpha|0\rangle + \beta|1\rangle$ . Alice proposes to do this by applying a Hadamard rotation to her qubit then making a measurement, after which Bob will just have to perform a single qubit unitary. What is the result of the Hadamard rotation on Alice's qubit? What is the basis in which Alice should measure and what is the final one qubit unitary that Bob needs to perform?
6. Reconcile the no-cloning theorem with the copying action of the CNOT gate. i.e. if the control qubit is  $b \in \{0, 1\}$  and the target qubit is 0, then the CNOT gate copies the control qubit into the target.