1. This problem demonstrates another instance of backward sign propagation, in the context of errors. Suppose you have a qubit in state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and that wo kinds of unitary errors can occur; Bit flip errors will transform $|\psi\rangle$ to $\alpha|1\rangle + \beta|0\rangle$ and phase errors will send $|\psi\rangle$ to $\alpha|0\rangle - \beta|1\rangle$. Note that the unitary operators for these transformations are the same as our gates $X$ and $Z$ - it is just when these processes happen without our controlling or wanting them to happen that they will be classified as errors. Now suppose we have a two qubit state $|\psi\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and that a phase error acts on qubit 2. Show that when a CNOT gate is performed on these qubits, the phase error will be transferred also to qubit 1.

2. Consider the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$. What is the state that results from applying the 2-qubit Hadamard transform to $|\psi\rangle$?

   Now regard $\psi$ as a superposition over numbers modulo 4 (i.e. it is a superposition of 0 and 2 written in binary notation). What is the state that results from applying the Fourier transform modulo 4 to $\psi$?

3. Consider a superposition on $n$ bit strings $|\psi\rangle = \frac{1}{\sqrt{M}} \sum_{x: u \cdot x = 0} |x\rangle$ where $u$ is a fixed non-zero $n$ bit string and $u \cdot x = u_1 x_1 + u_2 x_2 \ldots u_n x_n (mod\ 2)$. What is $M$? (hint: how many different $n$-bit strings $x$ exist? If $u$ is not all zeros, then for what fraction of the possible strings $x$ do we have $u \cdot x = 0$? Hint to the hint: Remember that $u \cdot x$ can only take the values 0 or 1).

   Apply the n-bit Hadamard transform to $|\psi\rangle$ and let the result be $|\phi\rangle = \sum_x \alpha_x |x\rangle$ (the somewhat cryptic subscript on the summation sign means "sum over all strings $x$ for which $u \cdot x = 0$"). What is $\alpha_{00\ldots0}$? What is $|\phi\rangle$? (hint: remember that for $n$-bit strings $i$ and $j$, we have $H^{\otimes n}|i\rangle = 2^{-n/2} \sum_j (-1)^{i \cdot j} |j\rangle$)