Quantum computing with trapped ions

Hartmut Häffner

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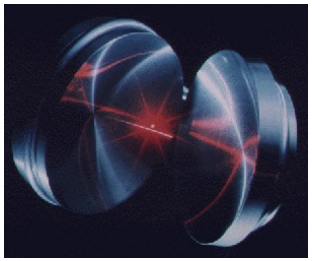
Innsbruck, Austria

- Basics of ion trap quantum computing
- Measuring a density matrix
- Quantum gates
- Deutsch Algorithm

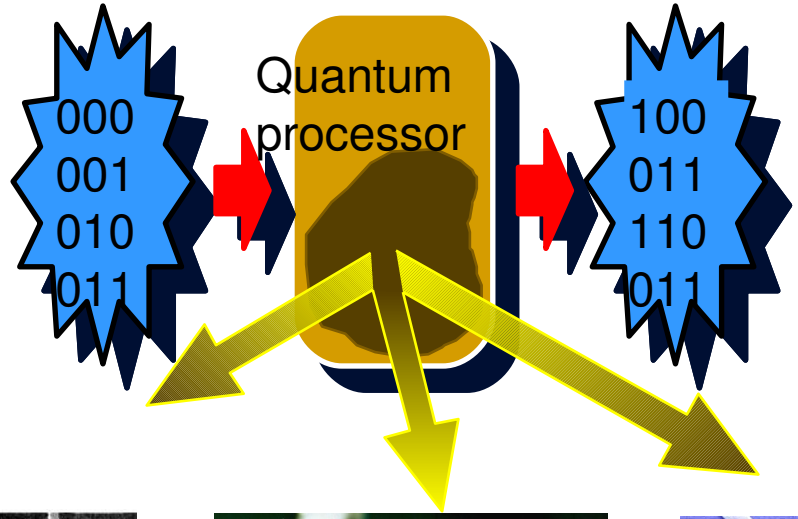
€ FWF SFB
SCALA QGATES
Industrie Tirol
IQI GmbH
FWF bm:bwk
$ IARPA

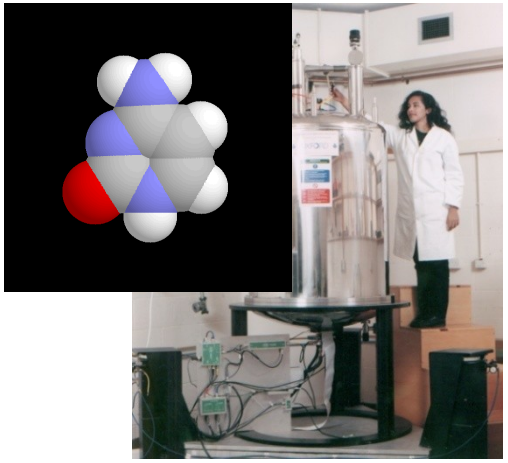
Berkeley, Nov 25th 2008

Which technology ?

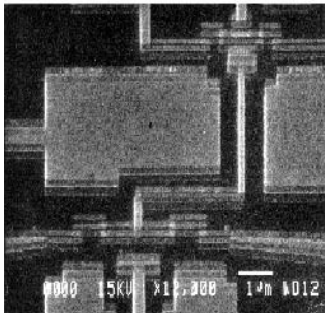


Cavity QED

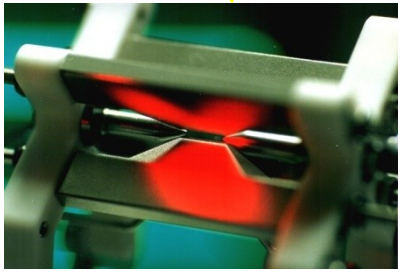


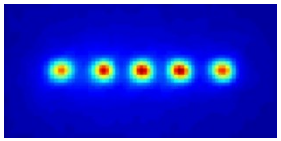


NMR

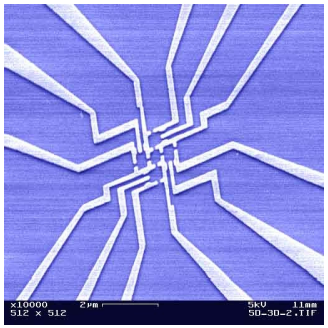


Superconducting qubits





Trapped ions

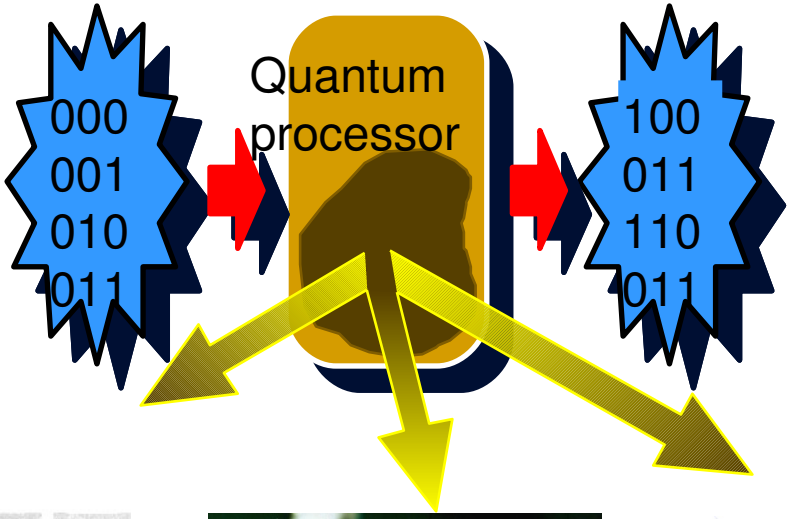


Quantum dots

© A. Ekert

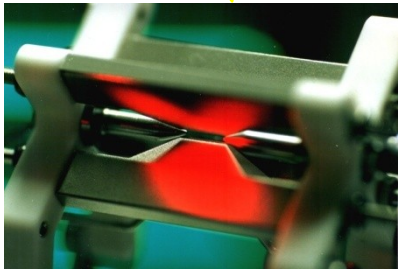
Which technology ?

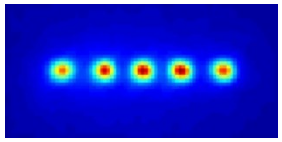
Cavity QED



NMR

Superconducting qubits





Trapped ions

Quantum dots

© A. Ekert

Why trapped ions ?

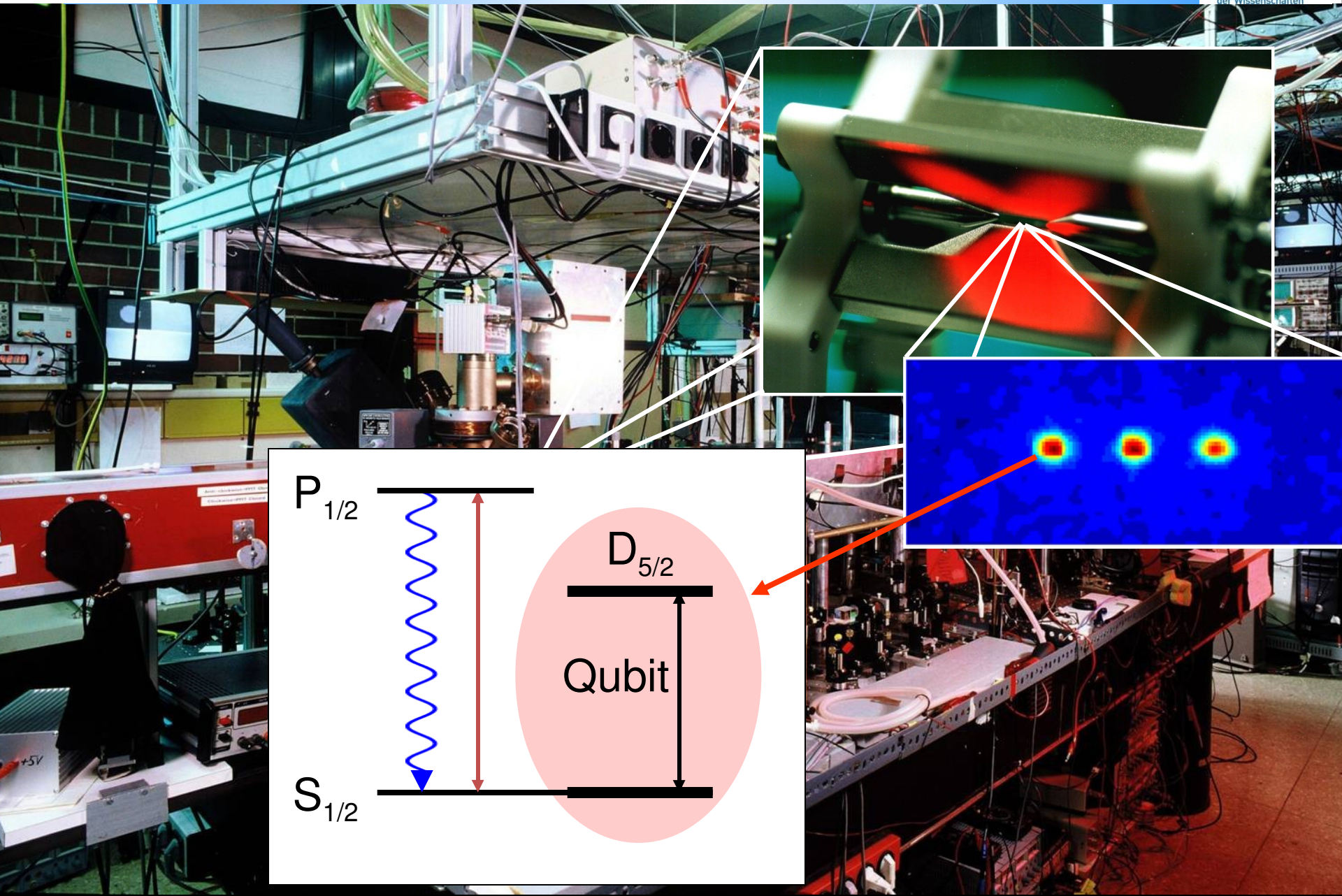
Good things about ion traps:

- Ions are excellent quantum memories; single qubit coherence times > 10 minutes have been demonstrated (Boulder 1991)
- Ions can be controlled very well
- Many ideas to scale ion traps

Bad things about ion traps:

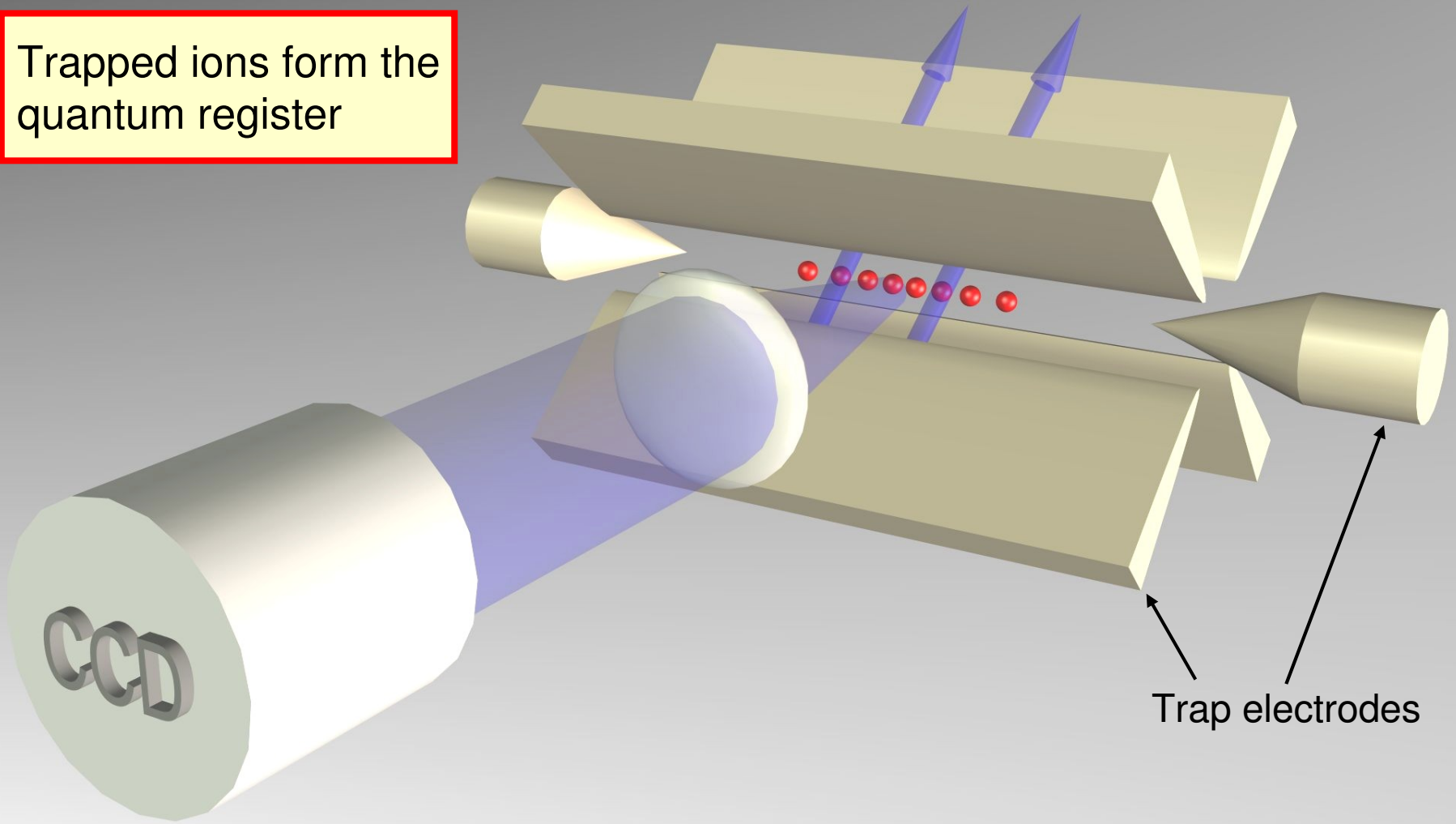
- Slow (~1 MHz)
- Technically demanding

The hardware



Innsbruck quantum processor

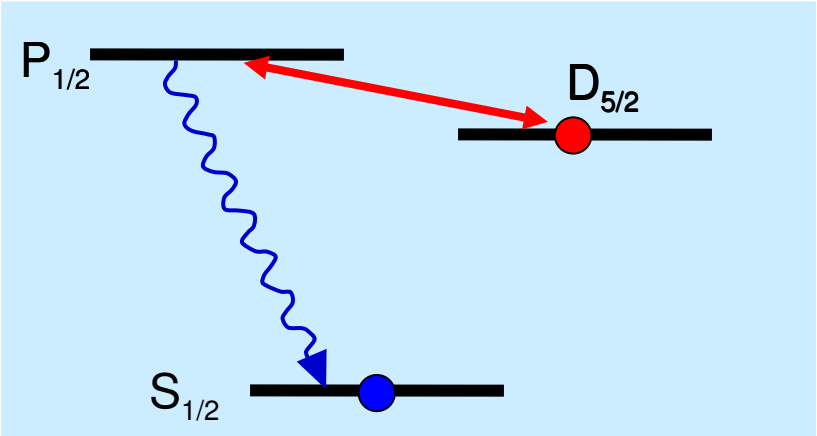
Trapped ions form the quantum register



DiVincenzo criteria

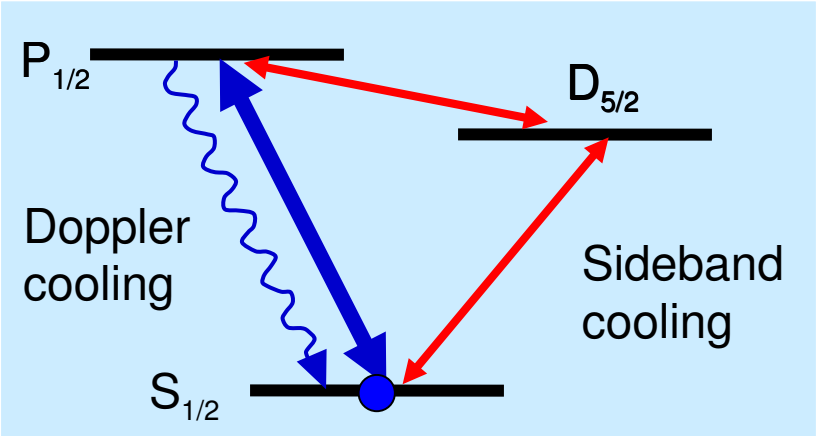
- Scalable physical system, well characterized qubits
- Ability to initialize the state of the qubits
- Long relevant coherence times, much longer than gate operation time
- “Universal” set of quantum gates
- Qubit-specific measurement capability

Experimental procedure



1. Initialization in a pure quantum state

Experimental procedure



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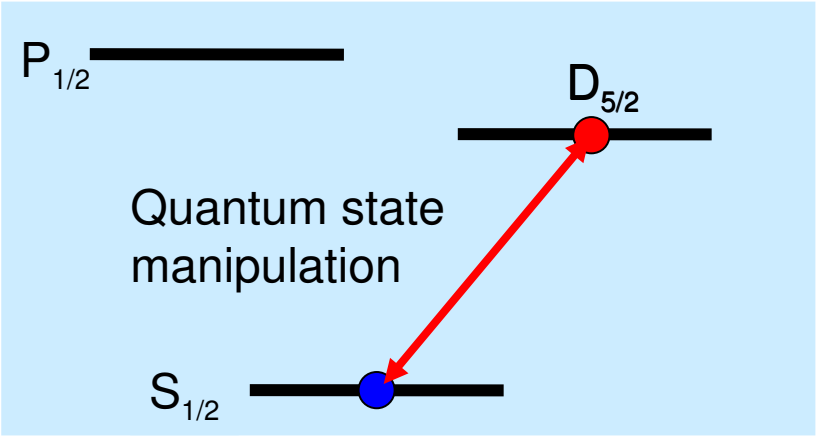
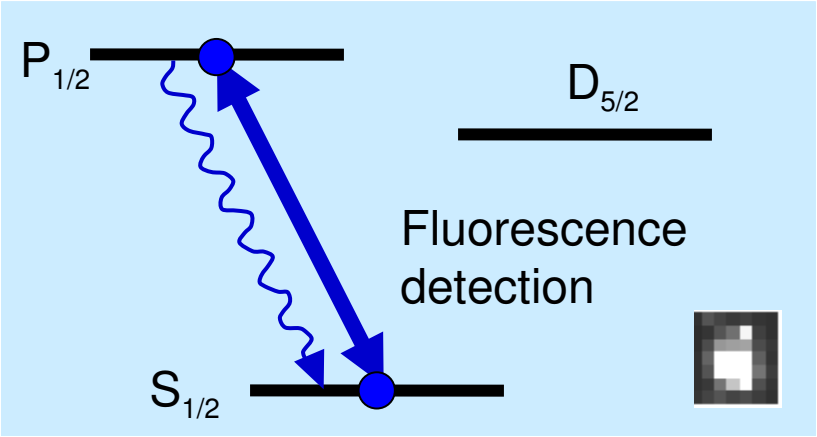


1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition

Experimental procedure



1. Initialization in a pure quantum state:

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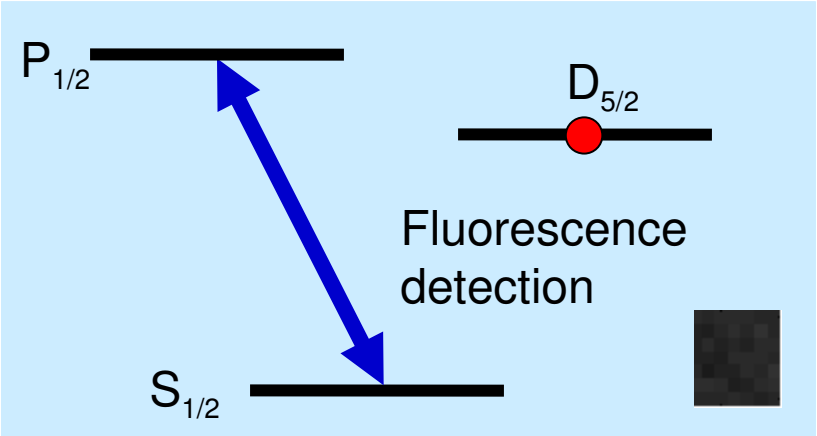
3. Quantum state measurement by fluorescence detection

Experimental procedure



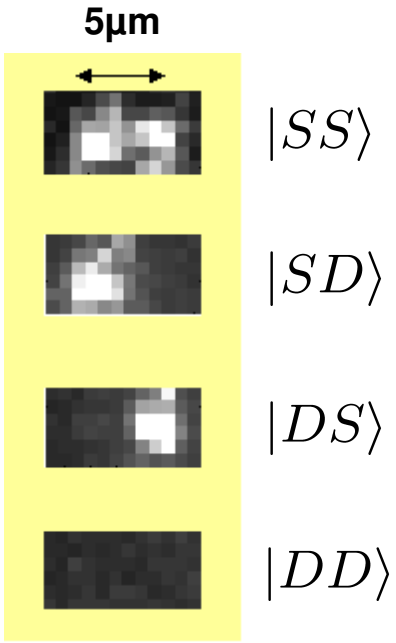
1. Initialization in a pure quantum state:

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

Two ions:

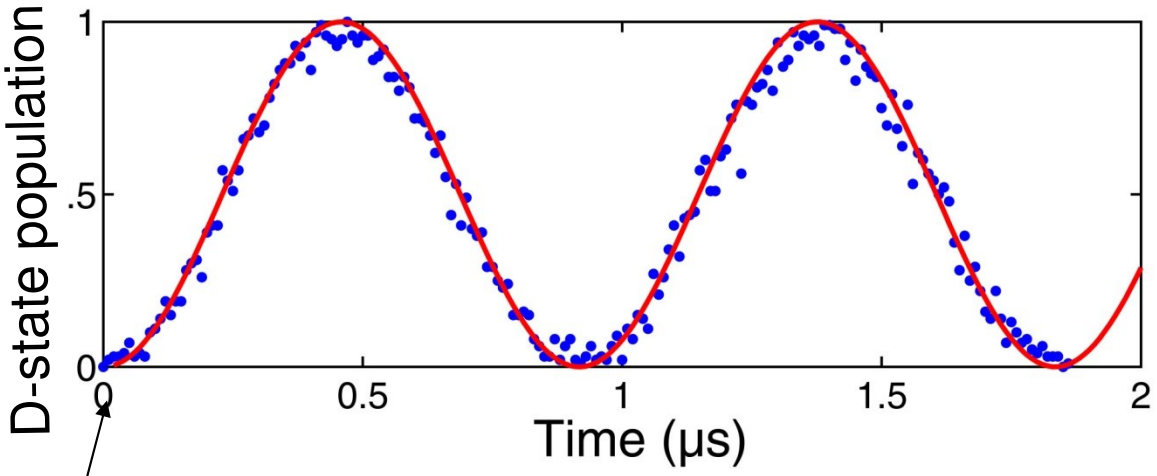
Spatially resolved detection with CCD camera



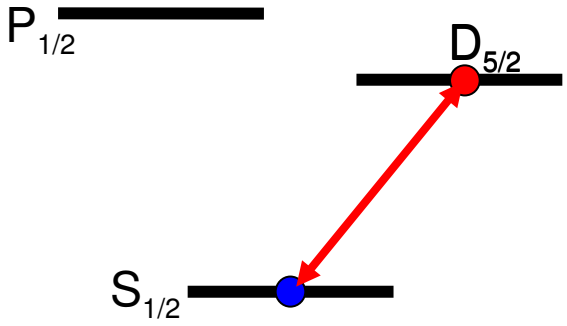
50 experiments / s

Repeat experiments 100-200 times

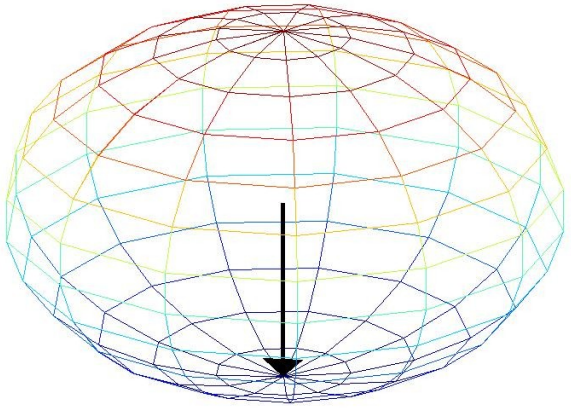
Rabi oscillations



$|S\rangle$



$|D\rangle$

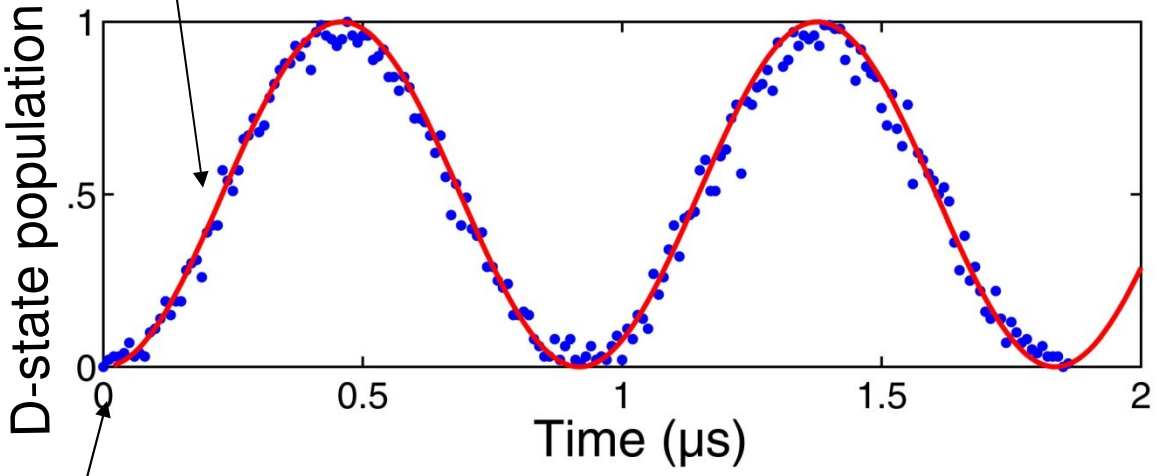


$|S\rangle$

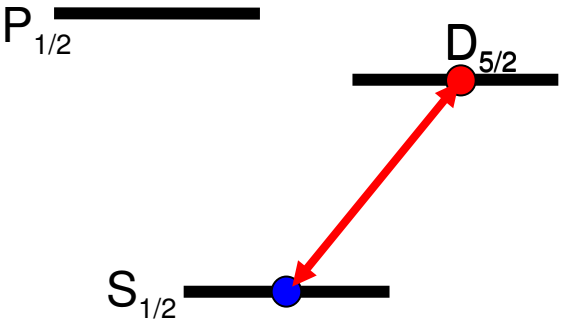
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

Rabi oscillations

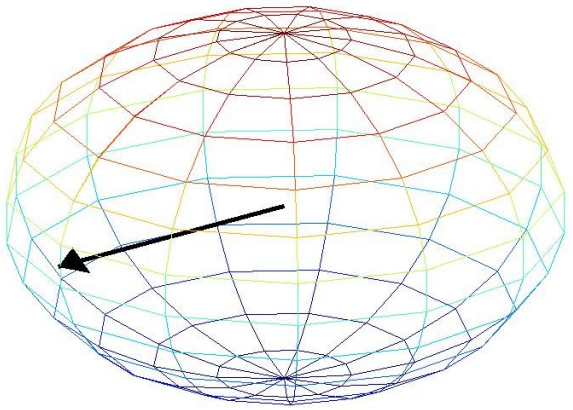
$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$



$|S\rangle$



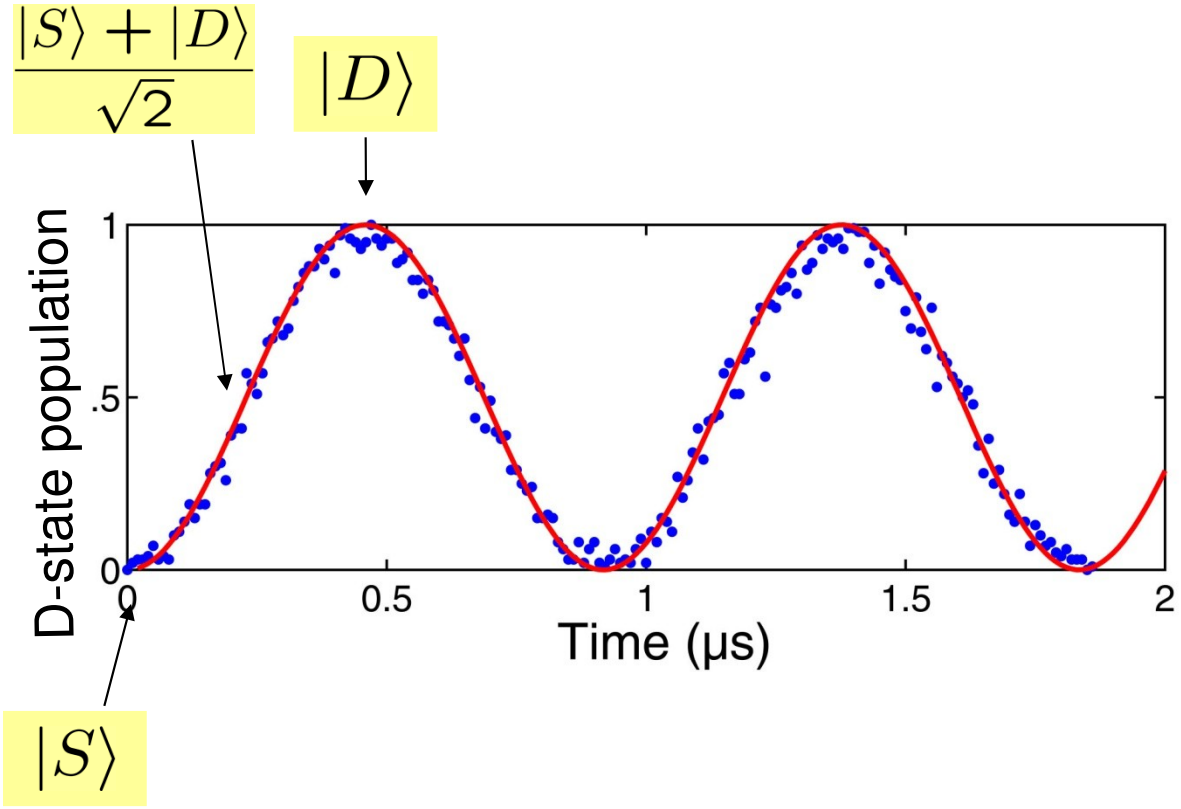
$|D\rangle$

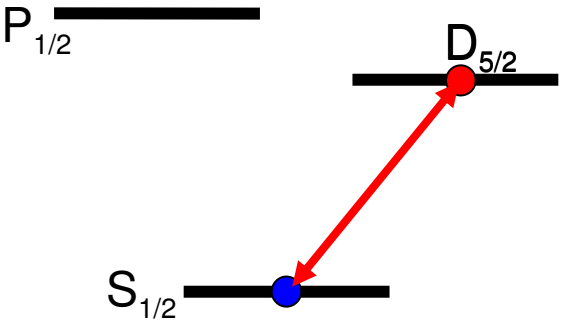


$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$

$|S\rangle$

Rabi oscillations





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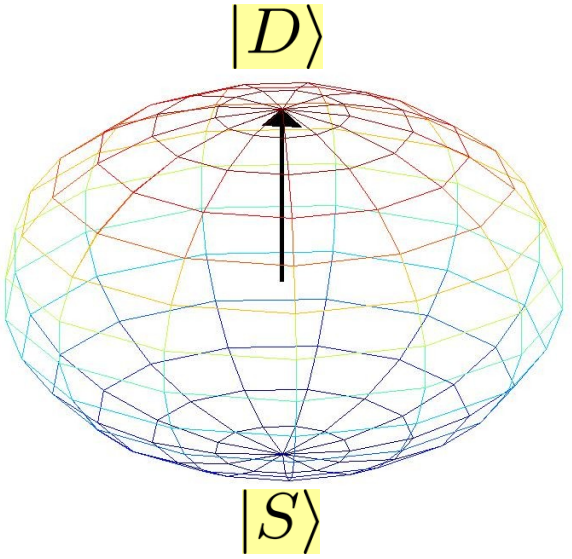
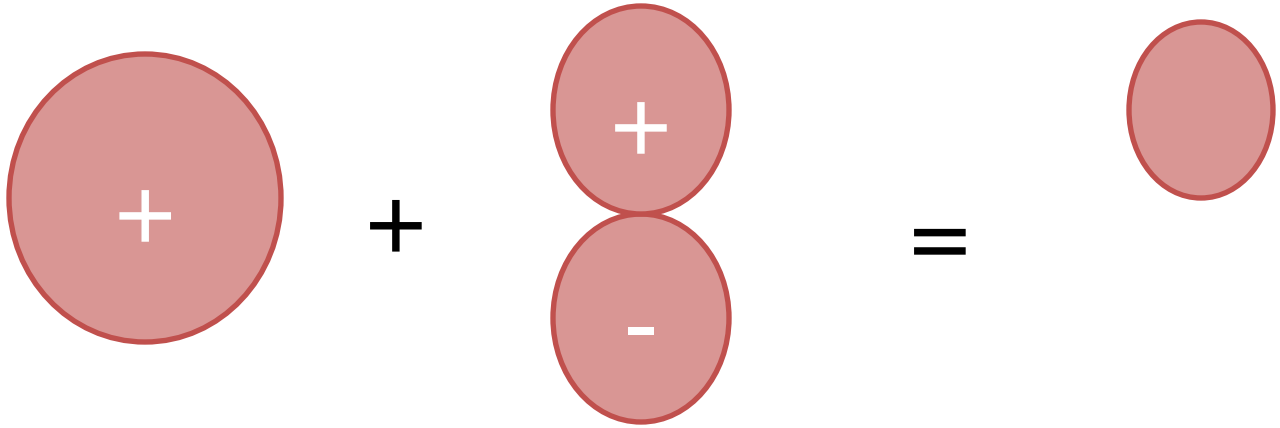
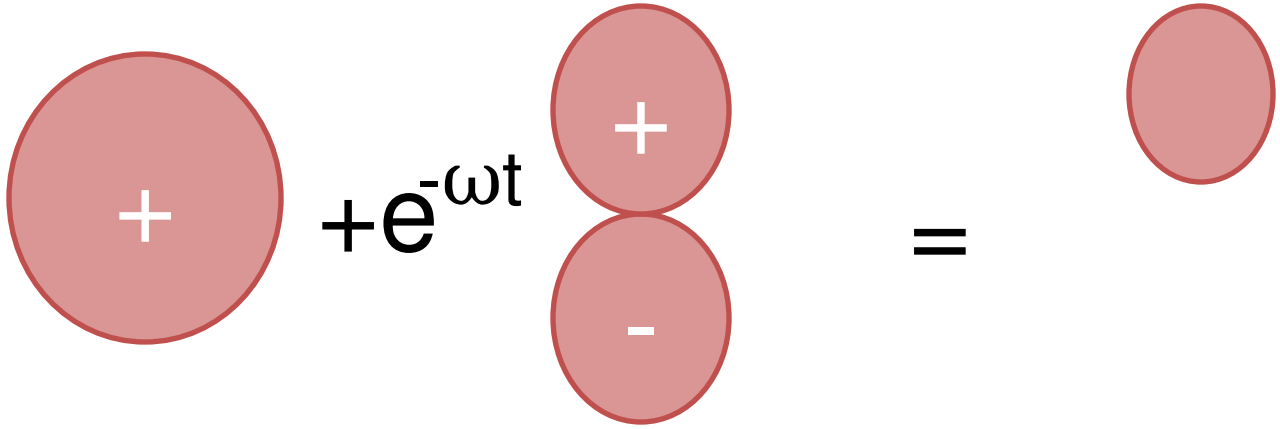
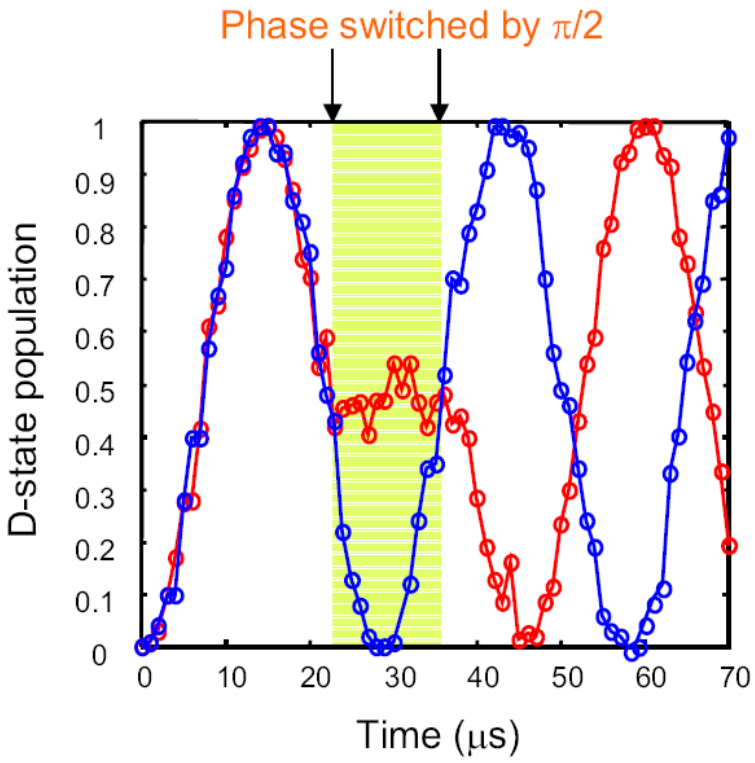


The phase ...

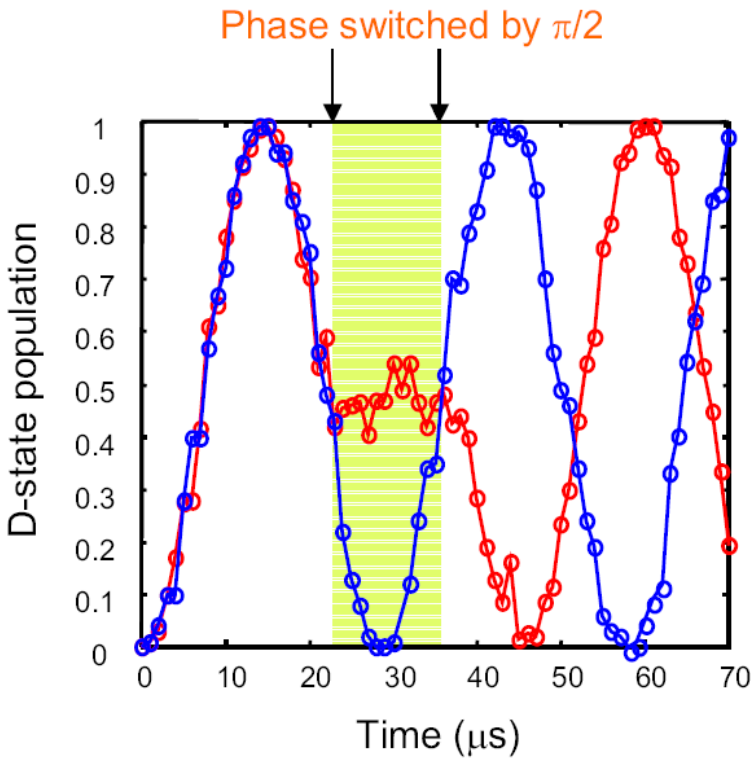




The phase ...


Phase switched by π/2
D-state population
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
Time (μs)

The phase ...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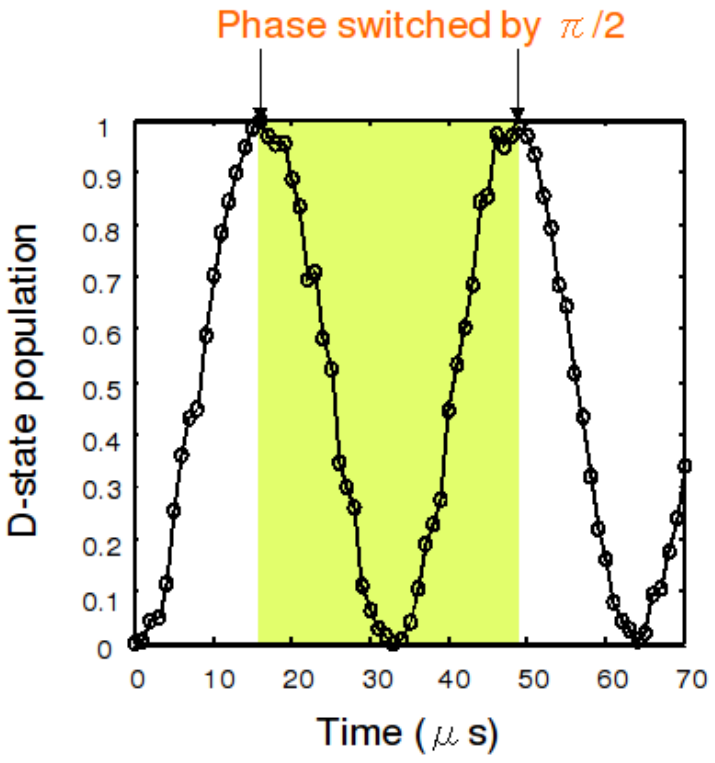
70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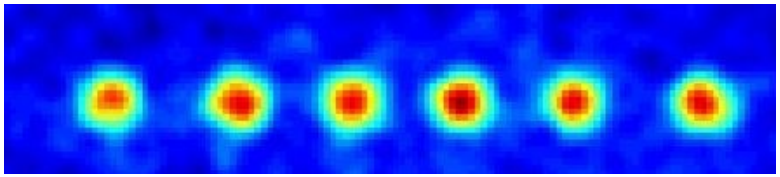
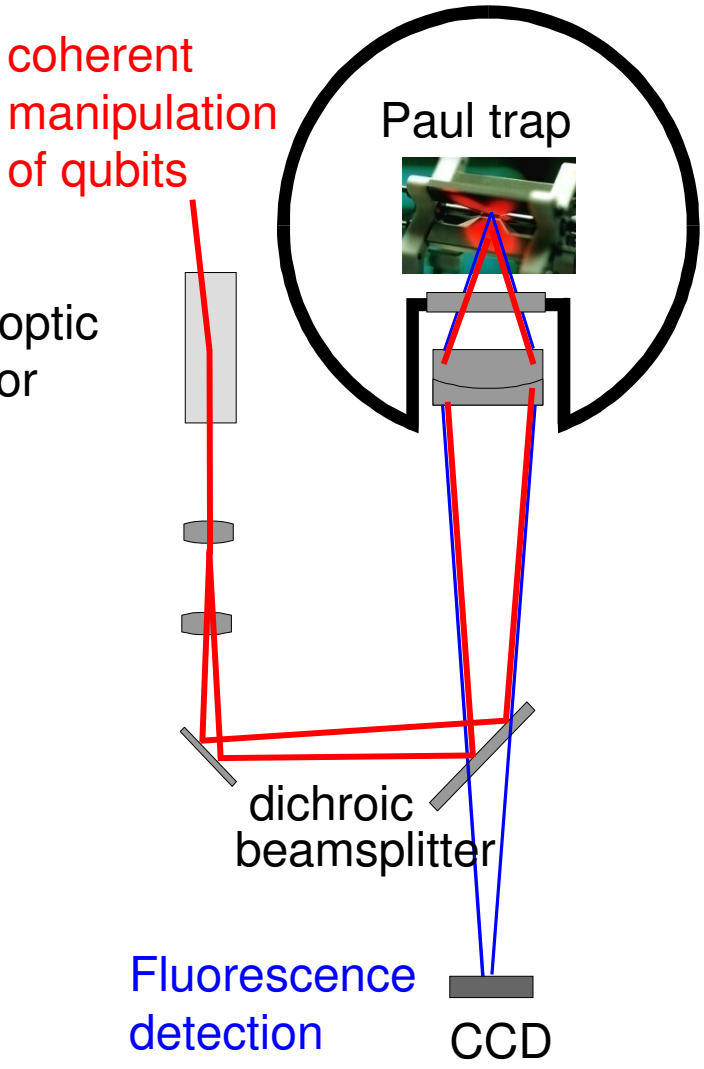
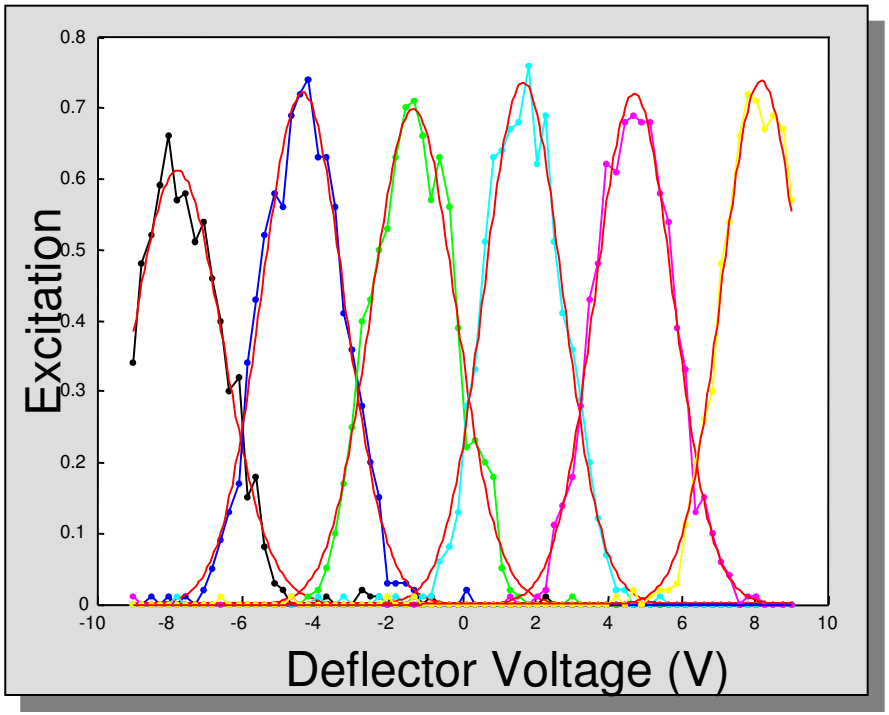
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70

Addressing single qubits







- inter ion distance: ~ 4 µm
- addressing waist: ~ 2 µm
- < 0.1% intensity on neighbouring ions

Decoherence mechanisms

Memory errors:

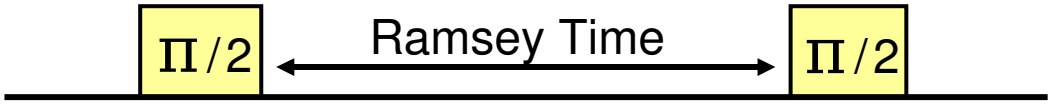
- Bit-flips
- Dephasing

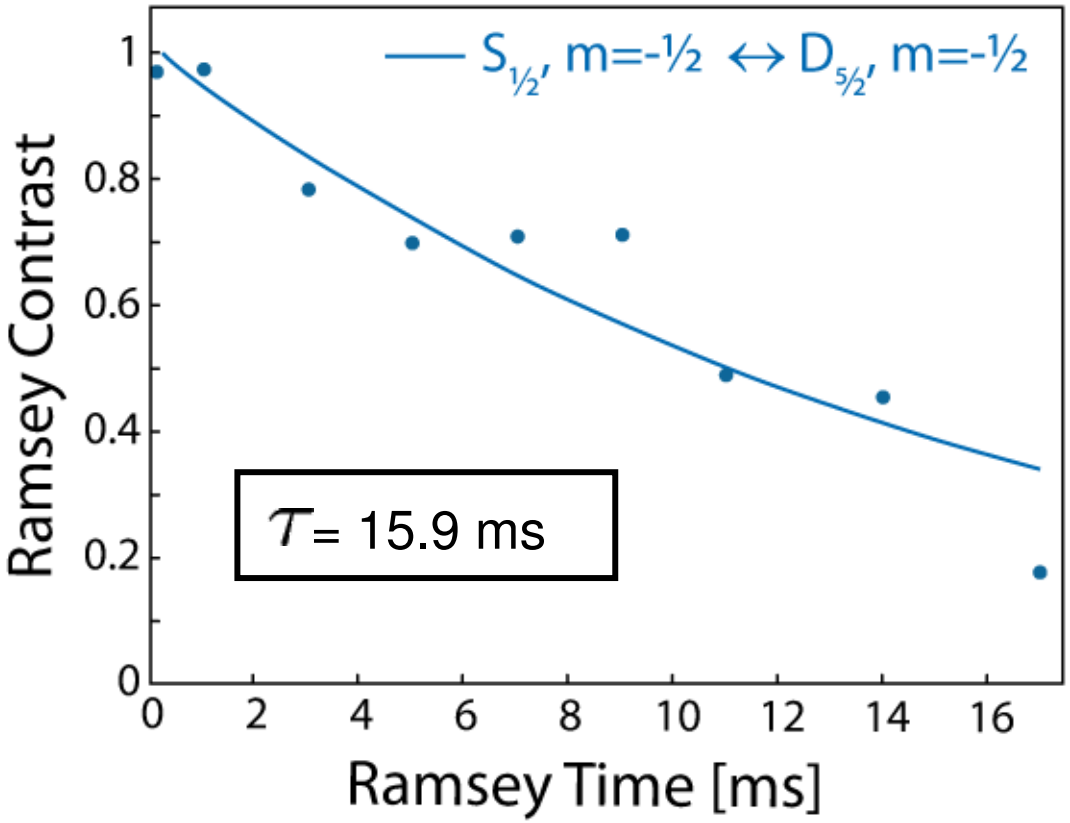
Operationial errors

- technical imperfections …

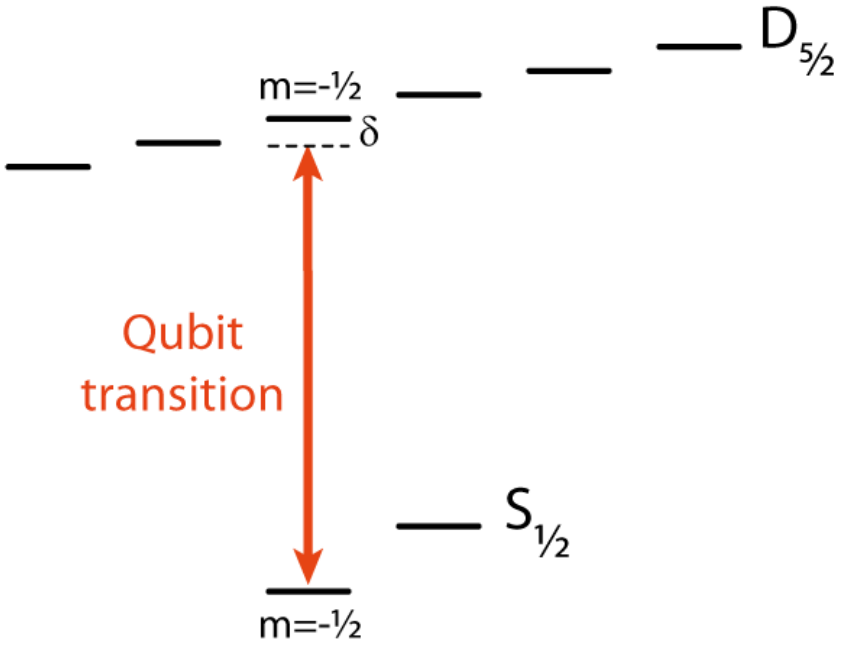
Dephasing of qubits

Ramsey Experiment



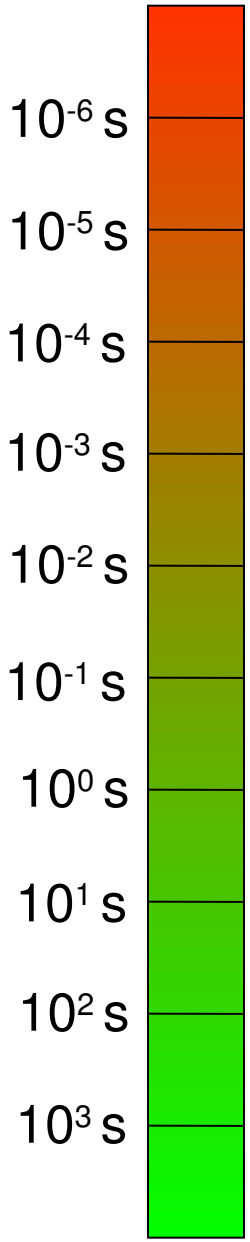


Zeeman shift



Detuning causes a linear
$\Rightarrow$ drift of the phase ϕ with
$e^{i\phi} = e^{i\delta t}$

Realized time scales



Single qubit gates

Two qubit gates (Geometric phase gates)

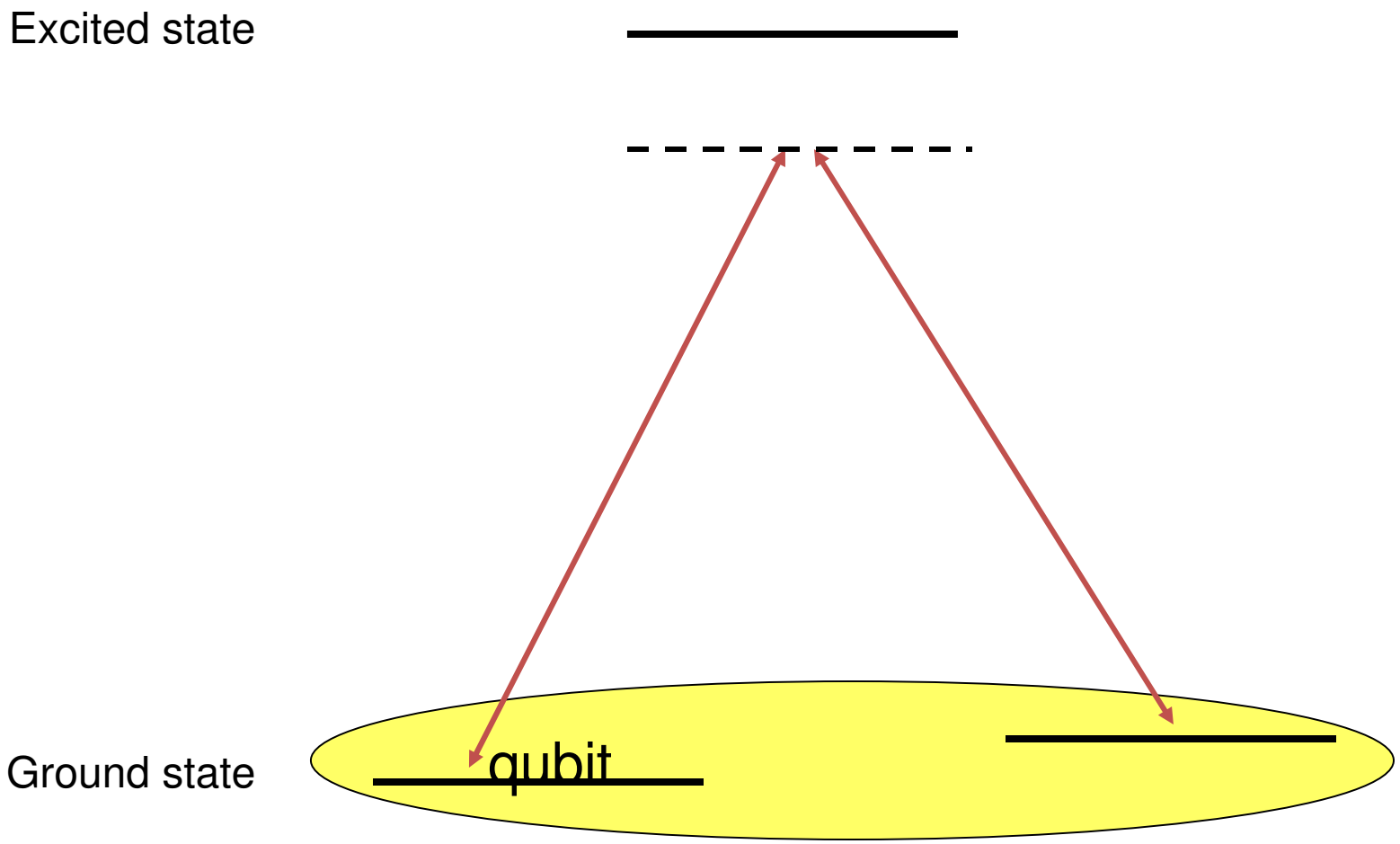
Two qubit gates (Cirac-Zoller approach)

Single qubit coherence (magnetic field sensitive)

Coherence of the motion

Long lived qubits

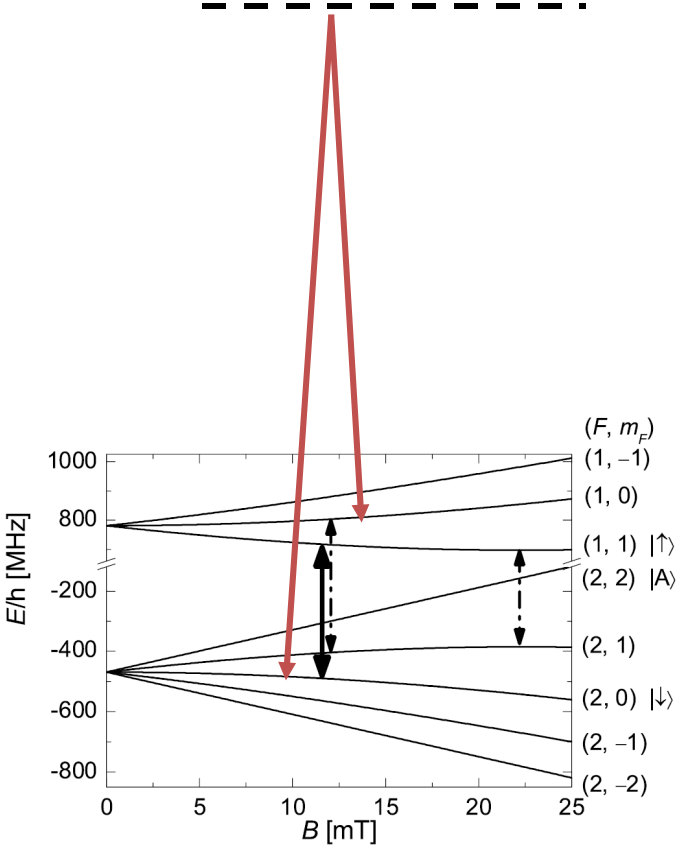
Raman transitions:



Long lived qubits

Raman transitions:

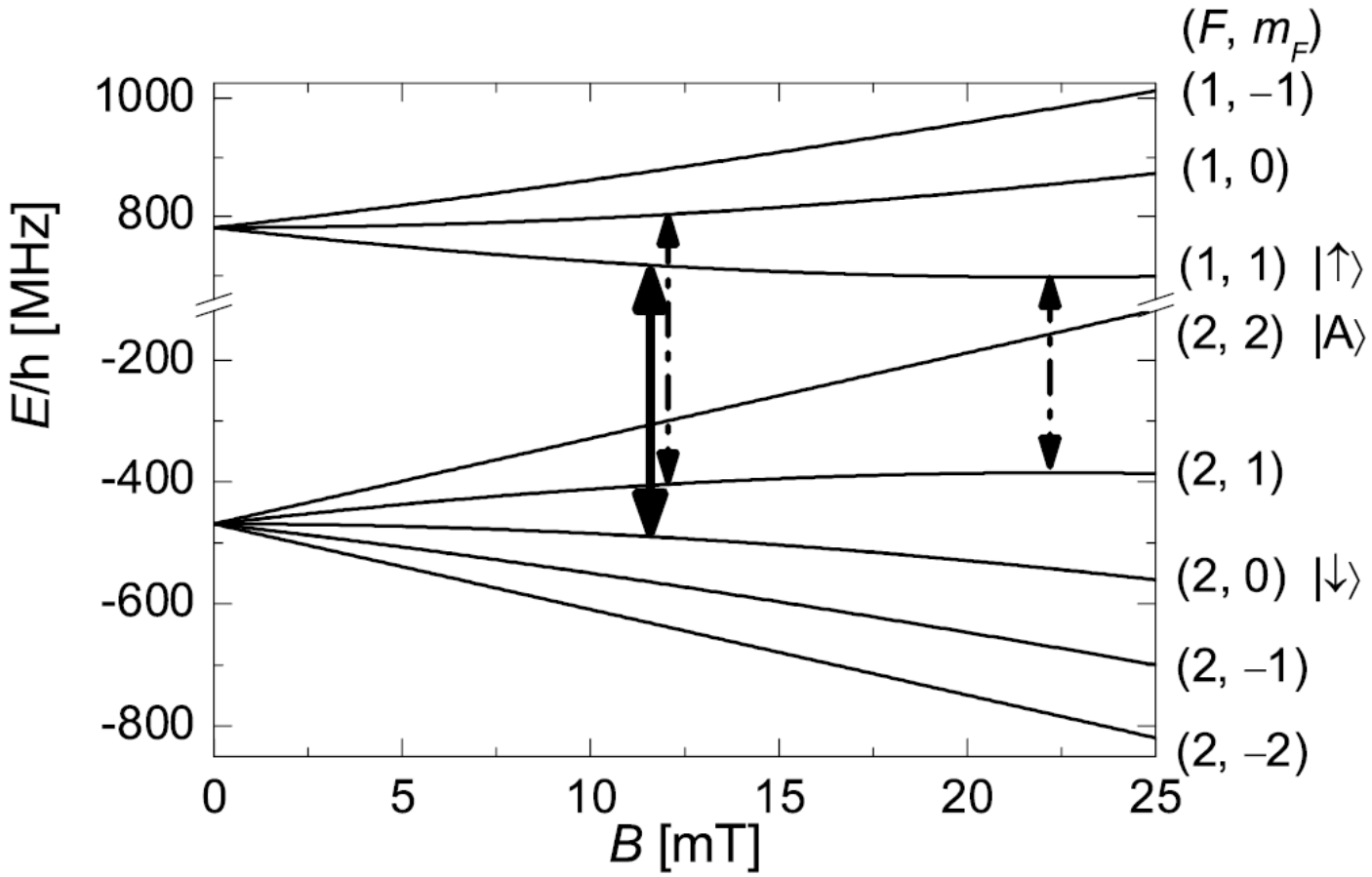
Excited state



Ground state

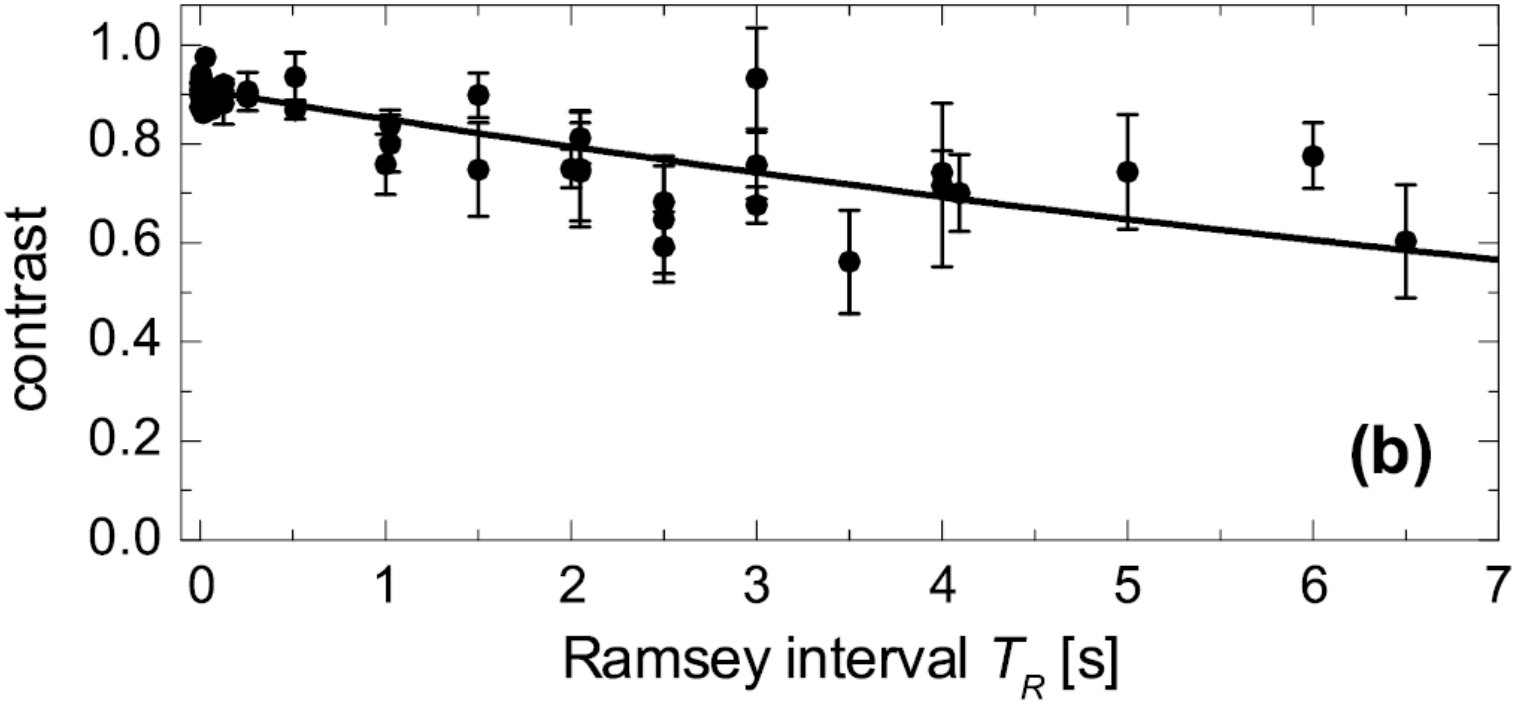
Long lived qubits

Level scheme of $^{9}Be^{+}$:



From: C. Langer *et al.*, PRL **95**, 060502 (2005), NIST

Long lived qubits



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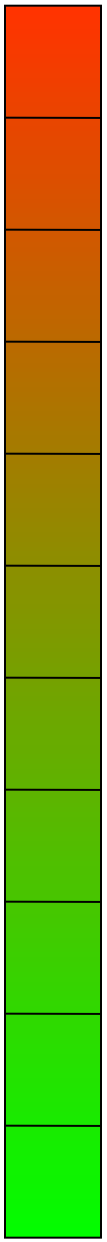
Realized time scales



10^{-6} s

10^{-5} s

10^{-4} s

10^{-3} s

10^{-2} s

10^{-1} s

10^{0} s

10^{1} s

10^{2} s

10^{3} s

Single qubit gates

Two qubit gates (Geometric phase gates)

Two qubit gates (Cirac-Zoller approach)

Single qubit coherence (magnetic field sensitive)

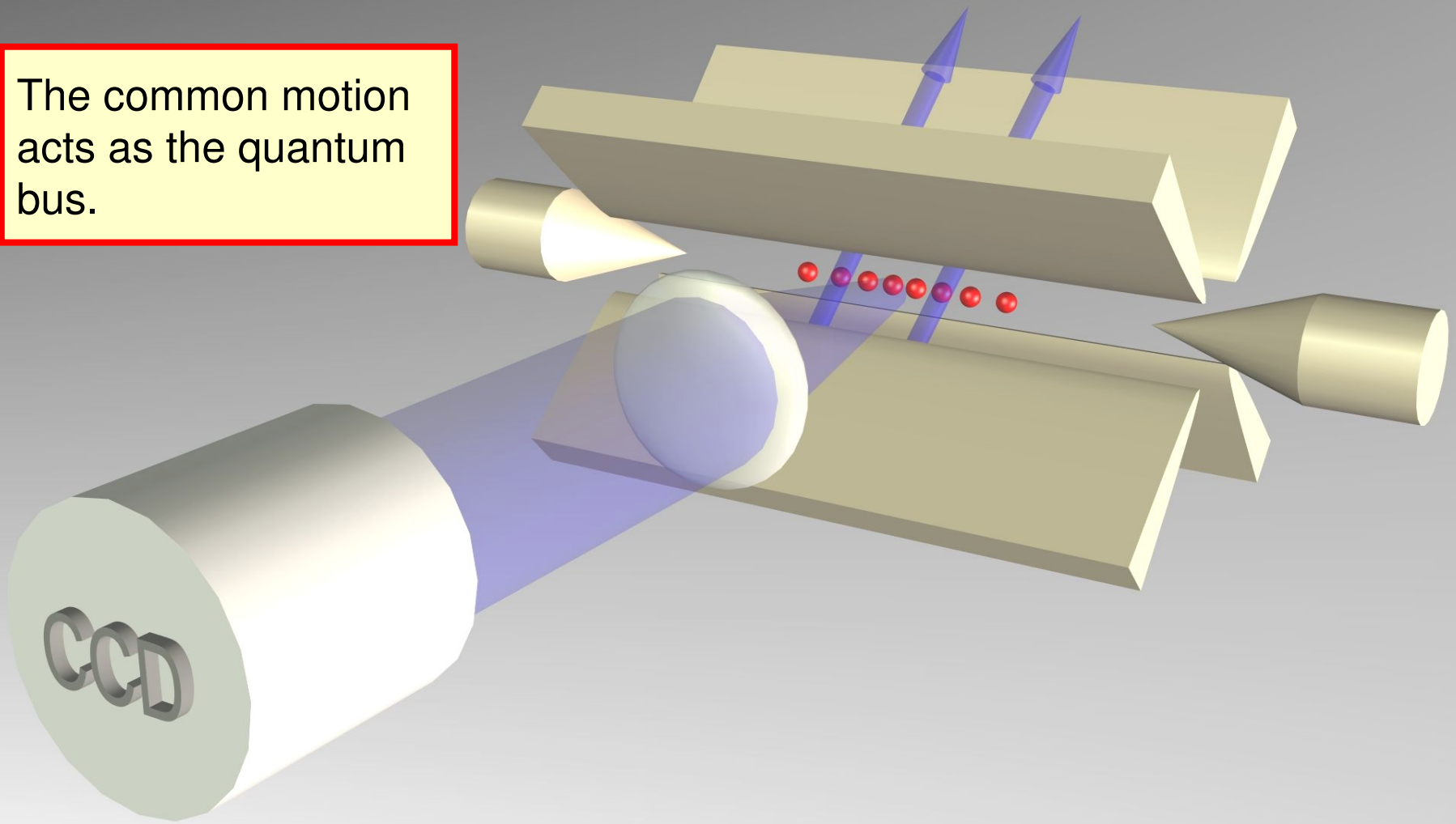
Coherence of the motion

Single qubit coherence (magnetic field insensitive)

Single qubit coherence (magnetic field insensitive + RF drive)

Having the qubits interact

The common motion acts as the quantum bus.



Having the qubits interact

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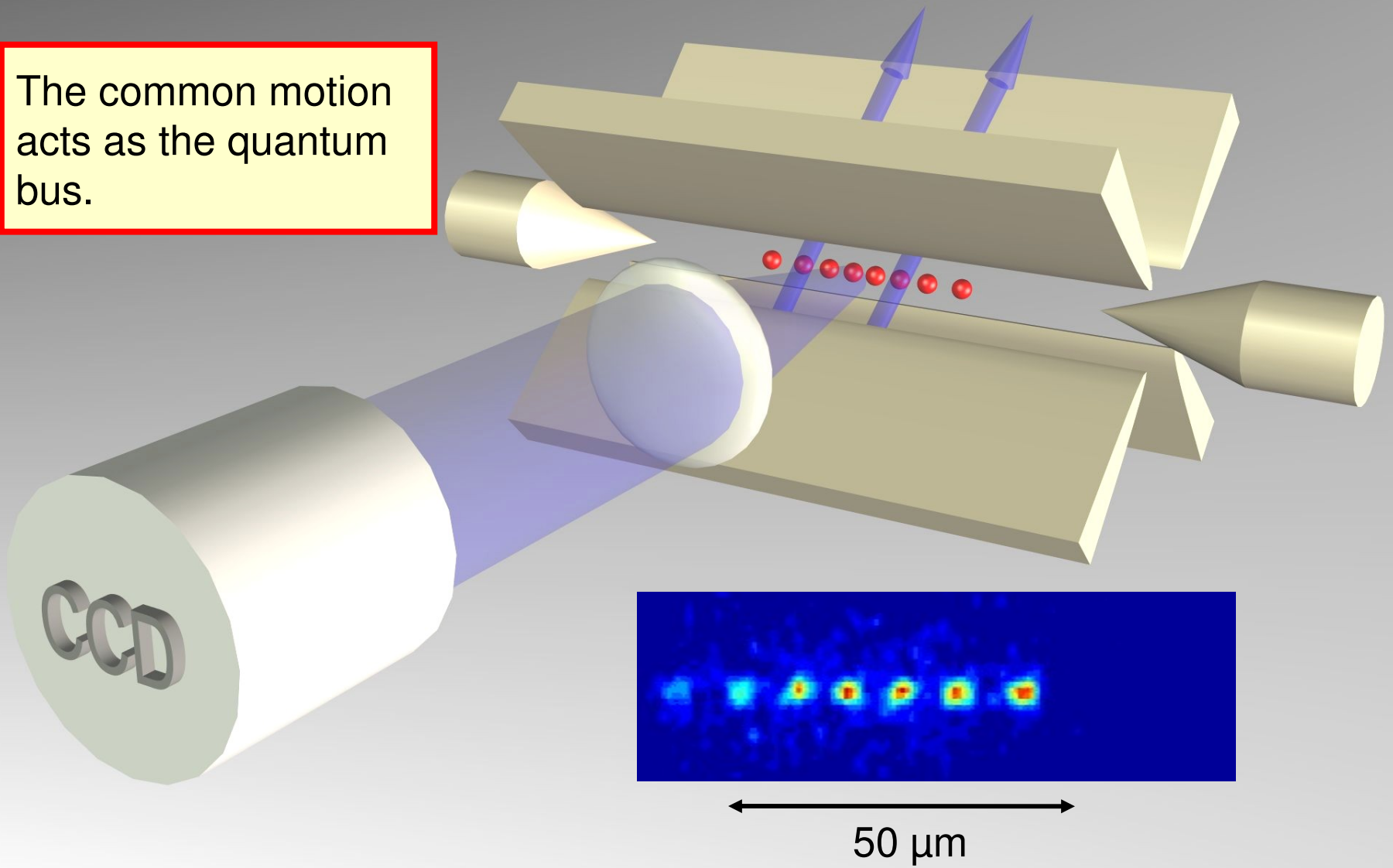
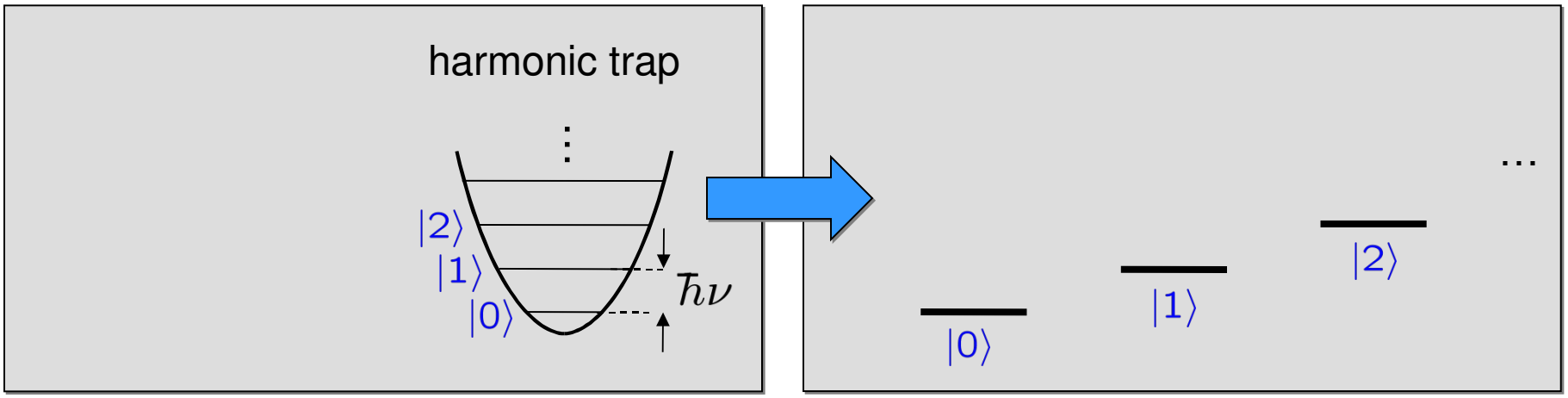
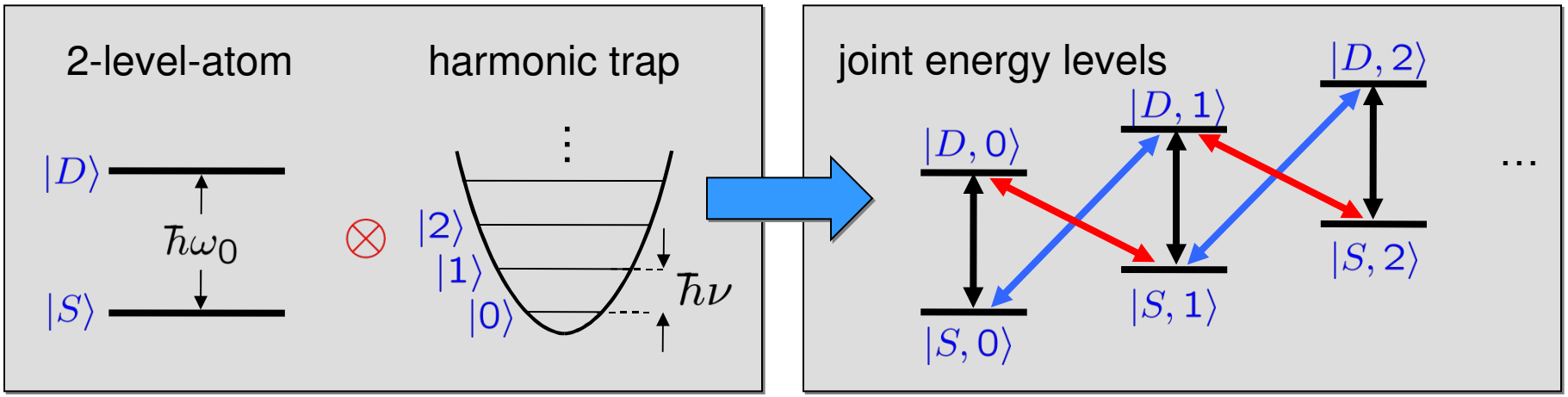
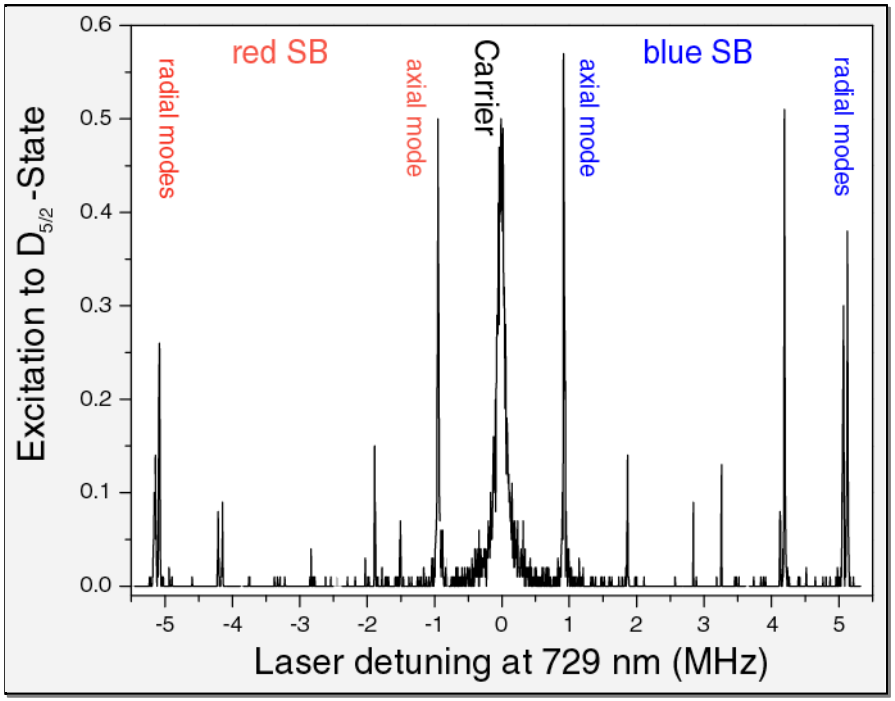


Ion motion

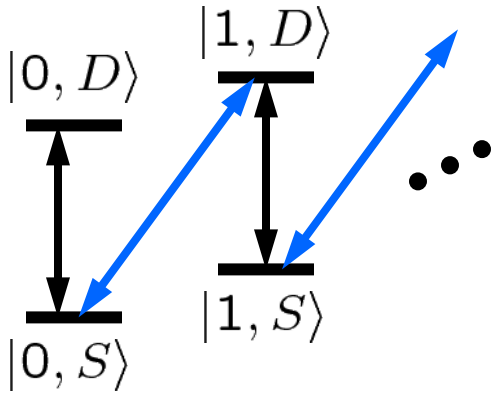

harmonic trap
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$
$|0\rangle$
$|1\rangle$
$|2\rangle$
...

Ion motion

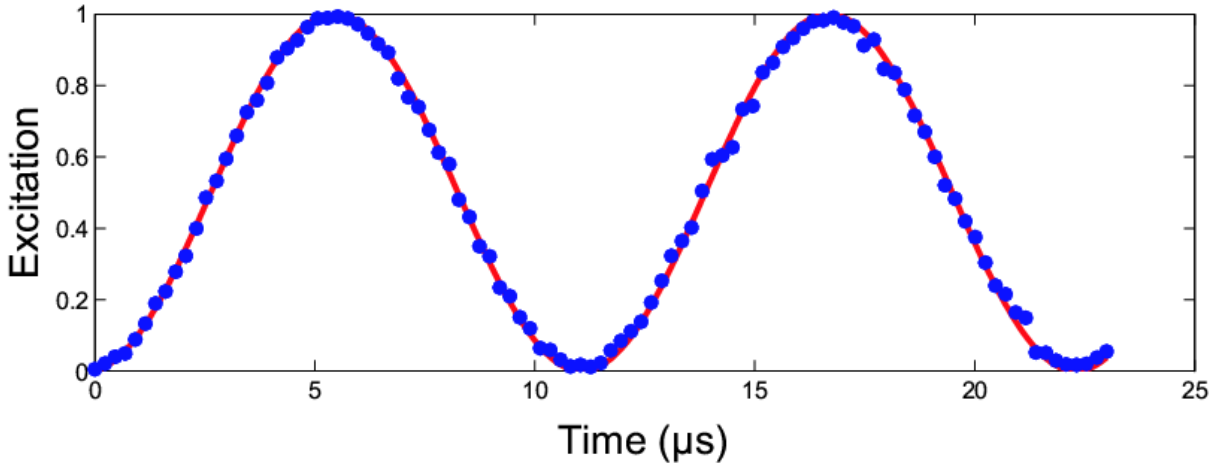

2-level-atom
harmonic trap
joint energy levels
$|D\rangle$
$|S\rangle$
$\hbar\omega_0$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...


Excitation to $D_{5/2}$-State
0.0
0.1
0.2
0.3
0.4
0.5
0.6
radial modes
red SB
axial mode
Carrier
axial mode
blue SB
radial modes
-5
-4
-3
-2
-1
0
1
2
3
4
5
Laser detuning at 729 nm (MHz)

Coherent manipulation

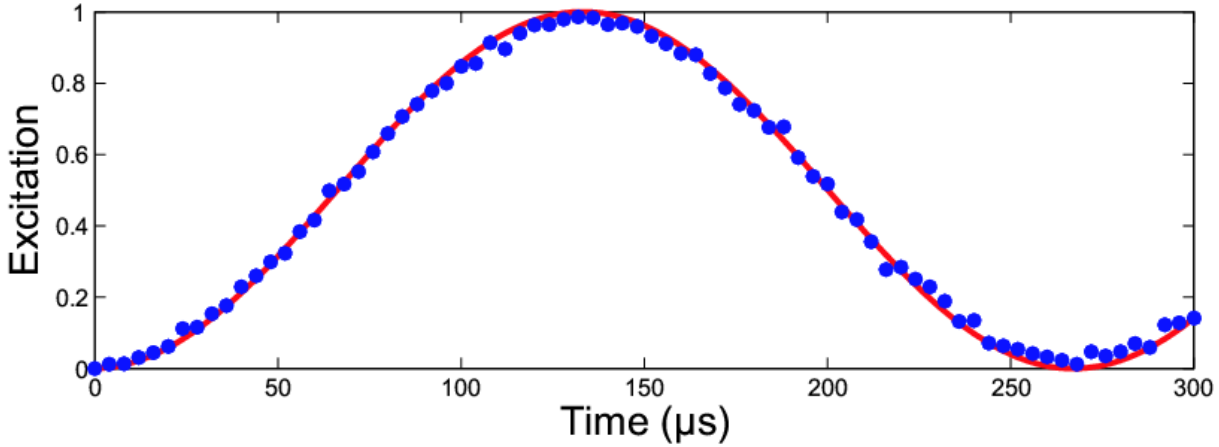


carrier



carrier and sideband
Rabi oscillations
with Rabi frequencies

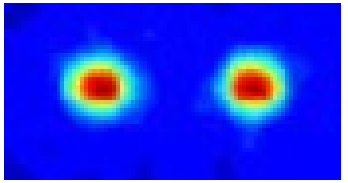
$\Omega, \quad \eta\Omega$

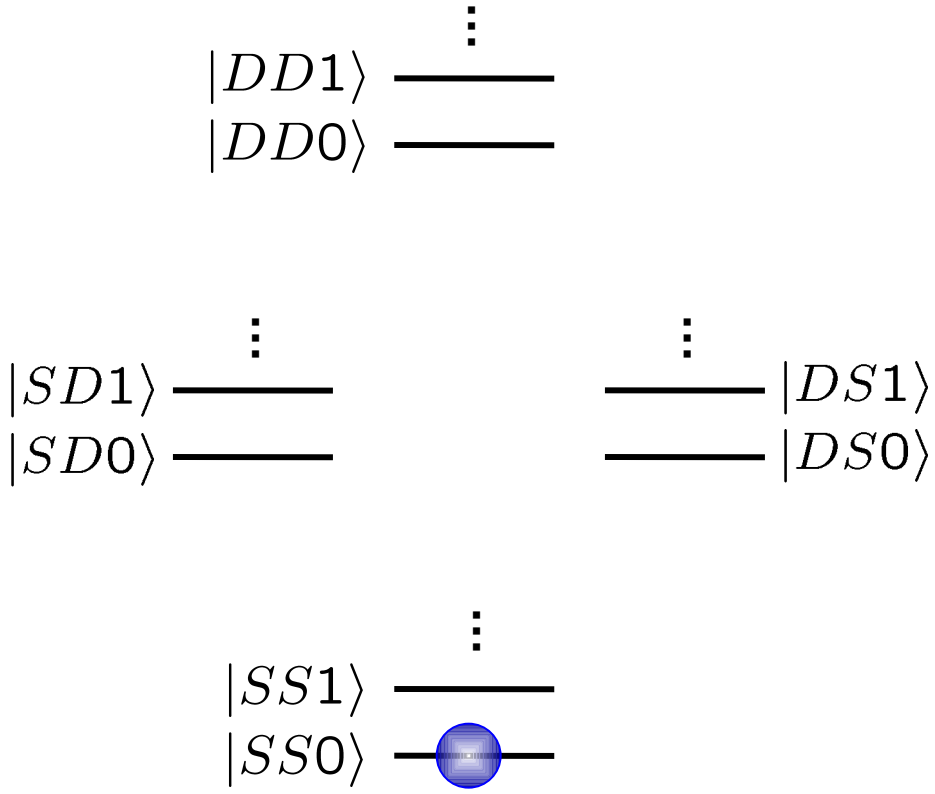


$\eta = kx_0$ Lamb-Dicke parameter

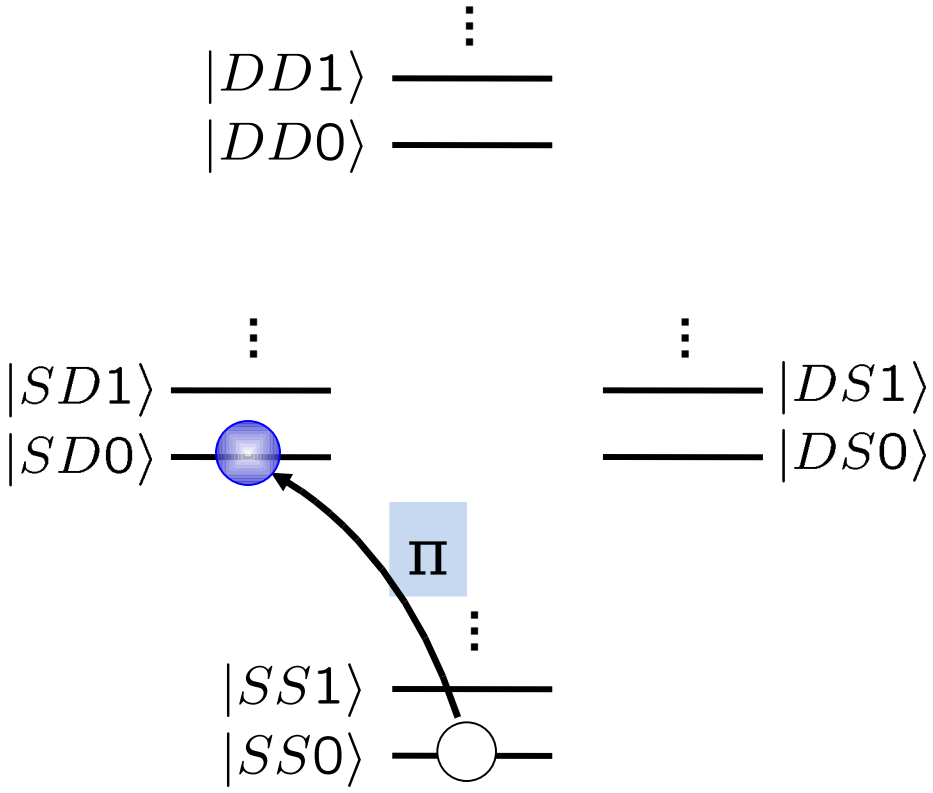
- Introduction to quantum information
- Entangled states
- Teleportation
- Scaling of ion trap quantum computers
- Wiring up trapped ions

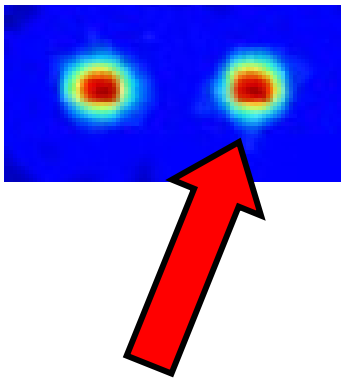
Generation of Bell states



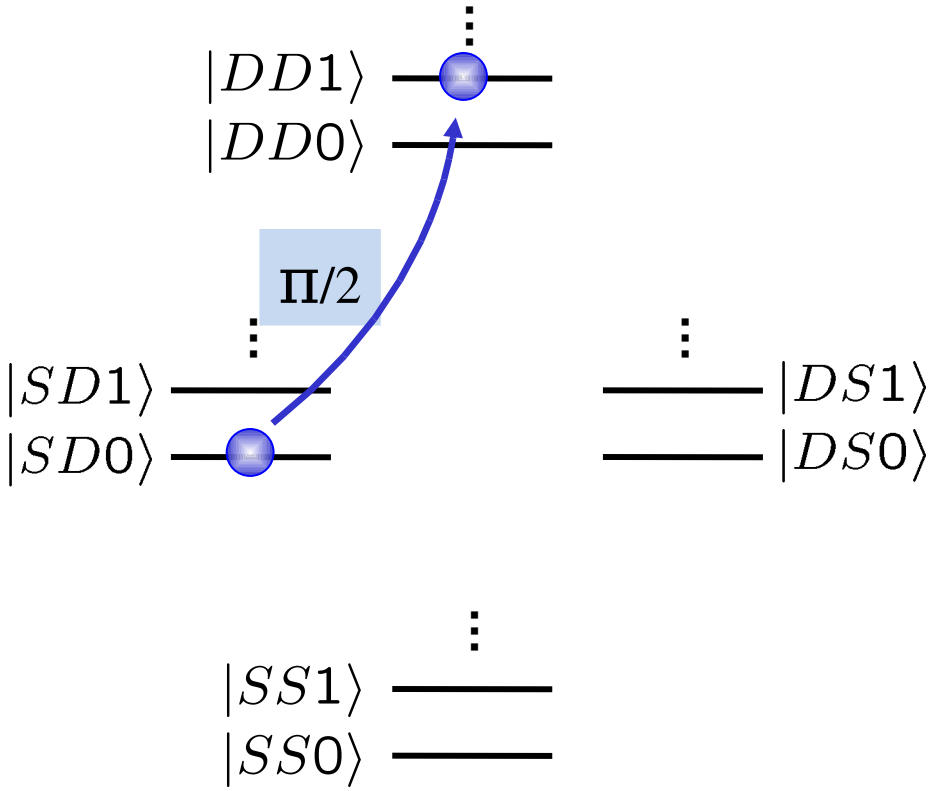


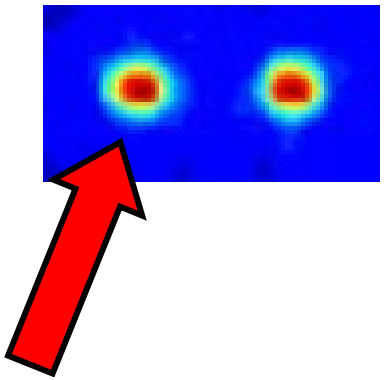
Generation of Bell states



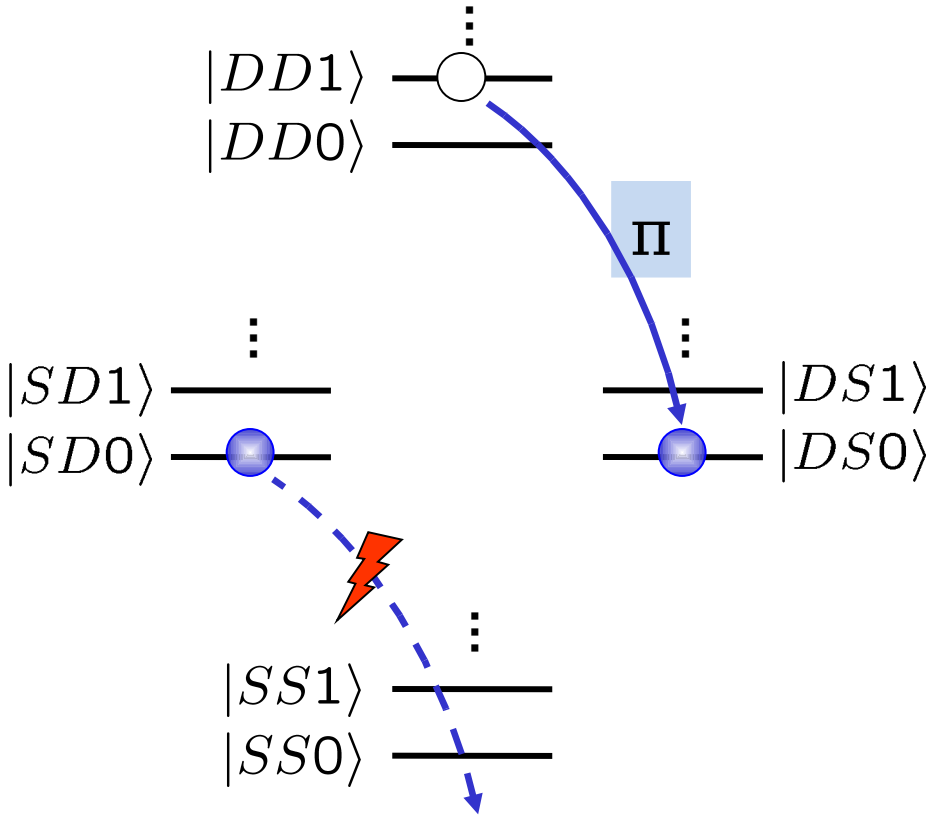


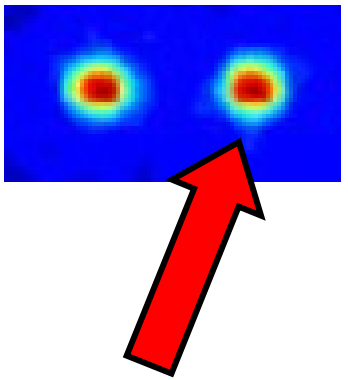
Generation of Bell states



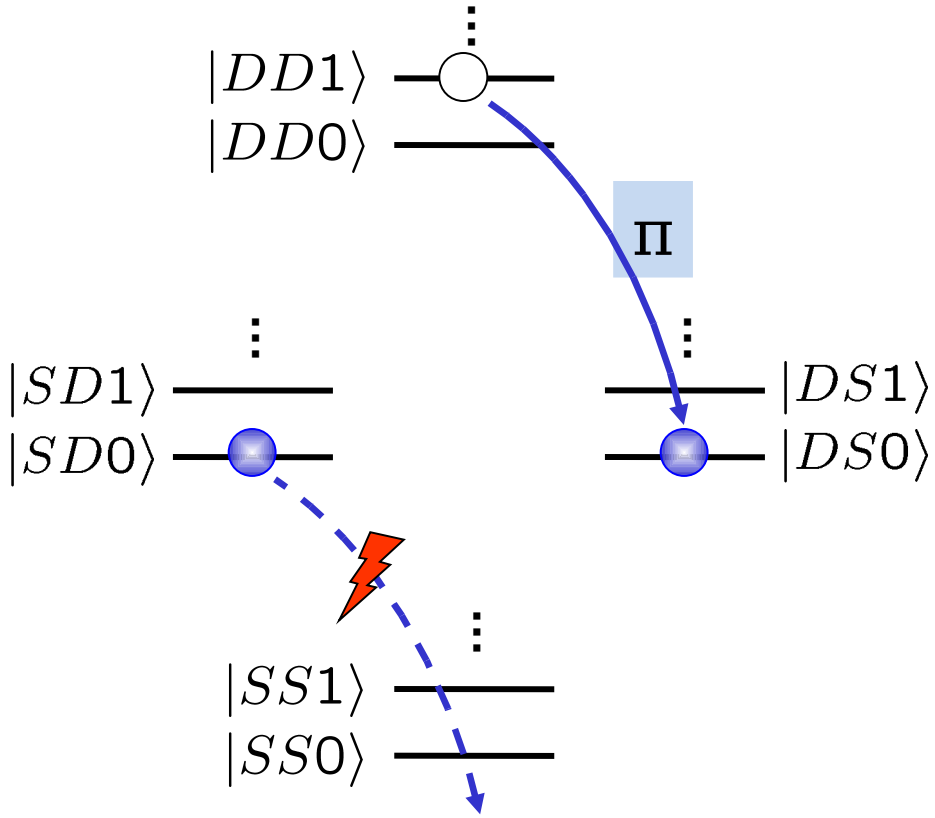


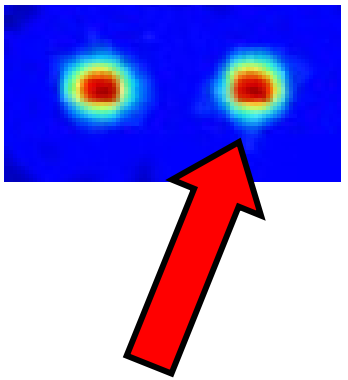
Generation of Bell states





Generation of Bell states





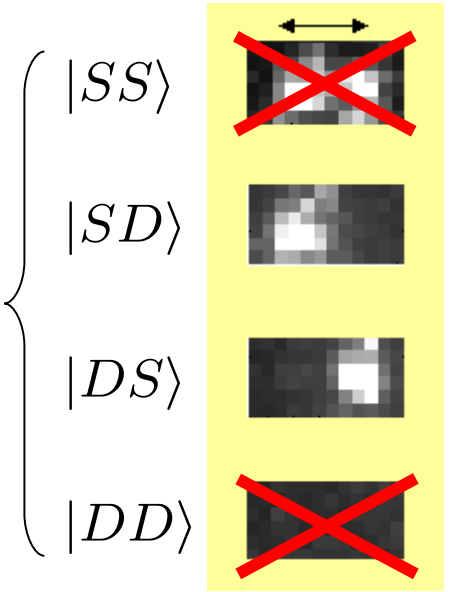
Bell states with atoms

- $^{9}Be^{+}$: NIST (fidelity: 97 %)
- $^{40}Ca^{+}$: Oxford (83%)
- $^{111}Cd^{+}$: Ann Arbor (79%)
- $^{25}Mg^{+}$: Munich
- $^{40}Ca^{+}$: Innsbruck (99%)

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

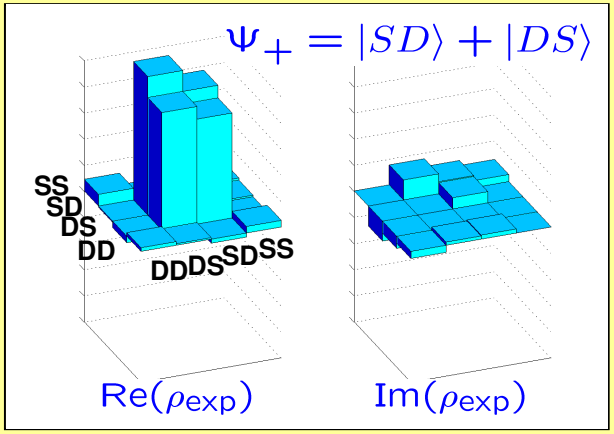
Fluorescence detection with CCD camera:



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

➡ Measurement of the density matrix:

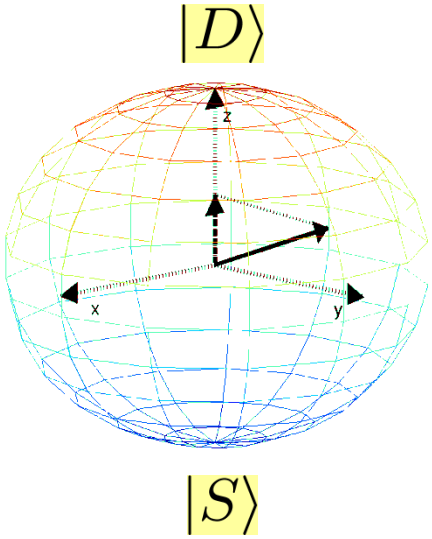


Measuring a density matrix

A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$



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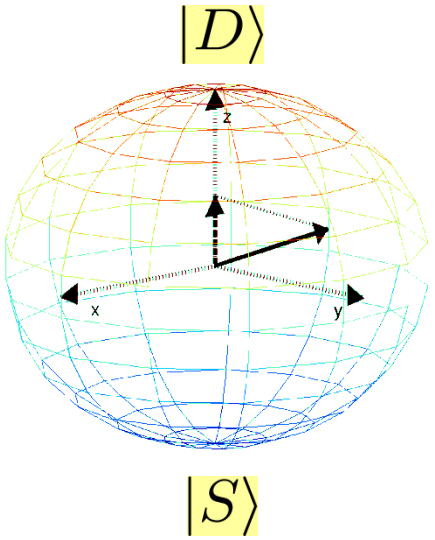
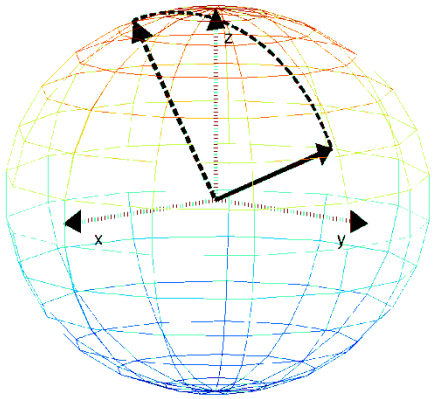


Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.



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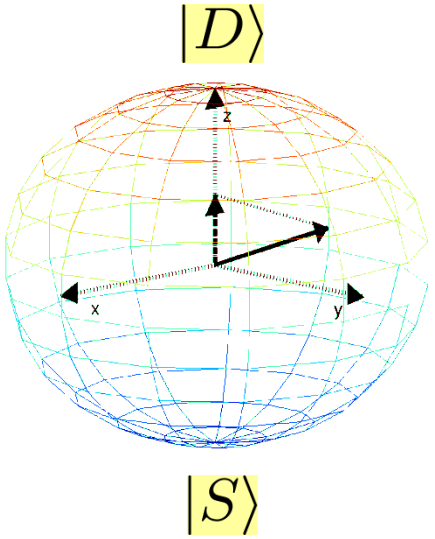
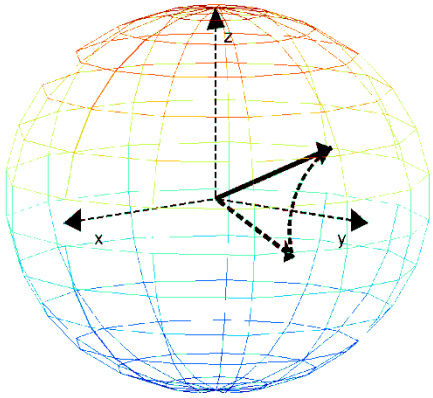
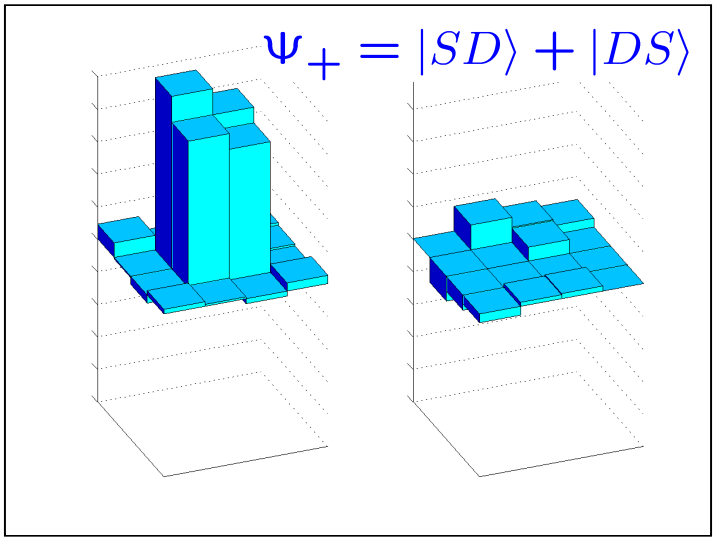


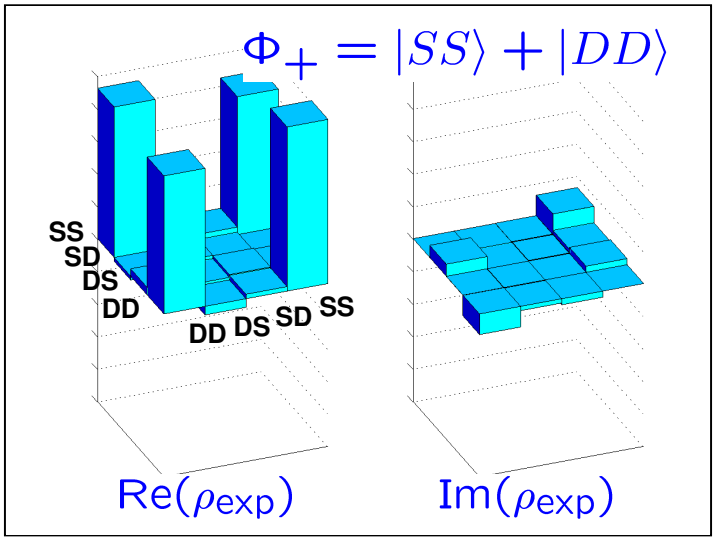
Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

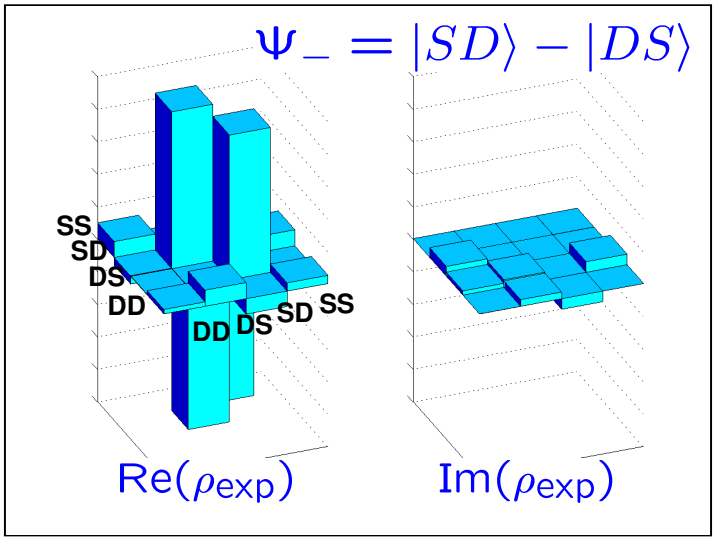
=> coherences of the density ma

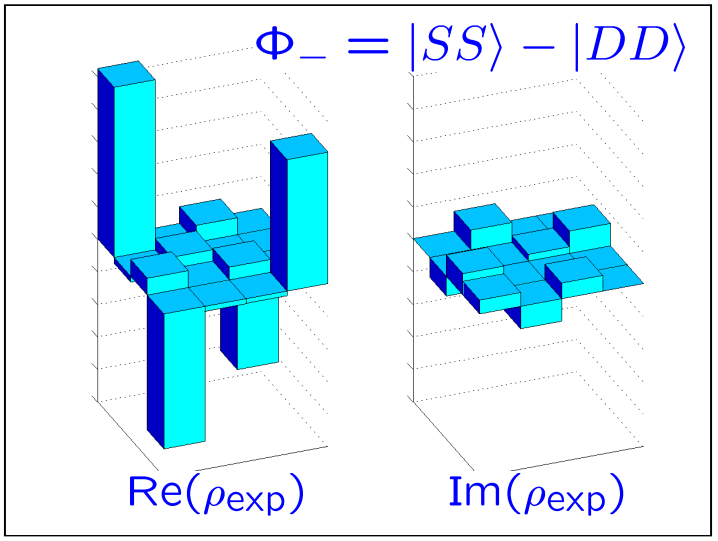


Decoherence properties of qubits



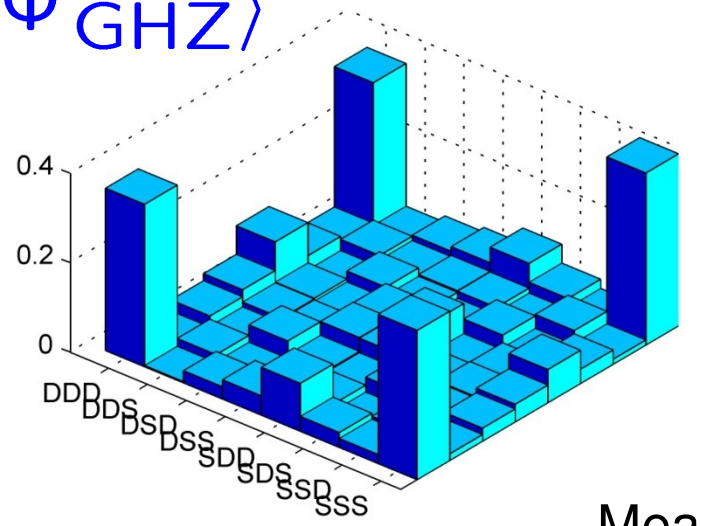




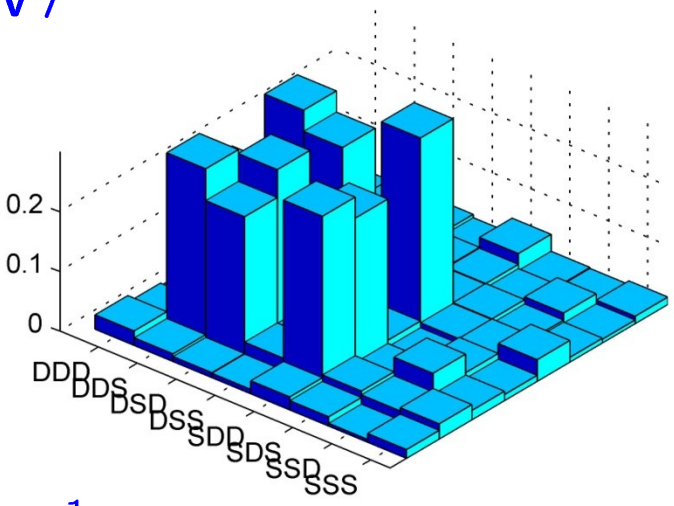


A “real” thought experiment

$|\Psi_{GHZ}\rangle$



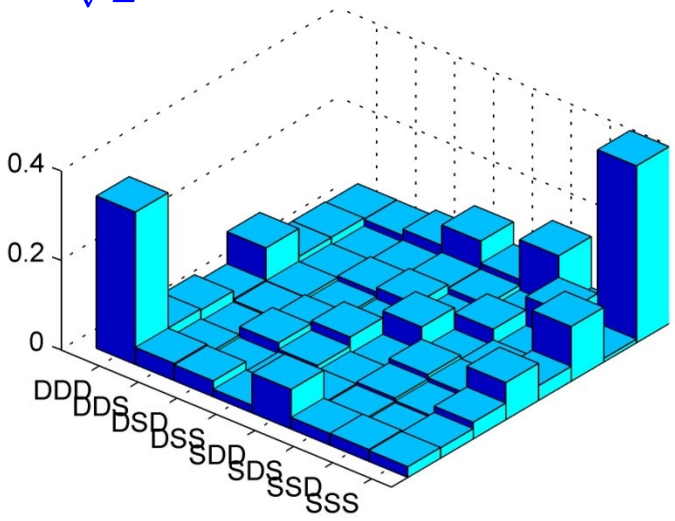
$|\Psi_{W}\rangle$

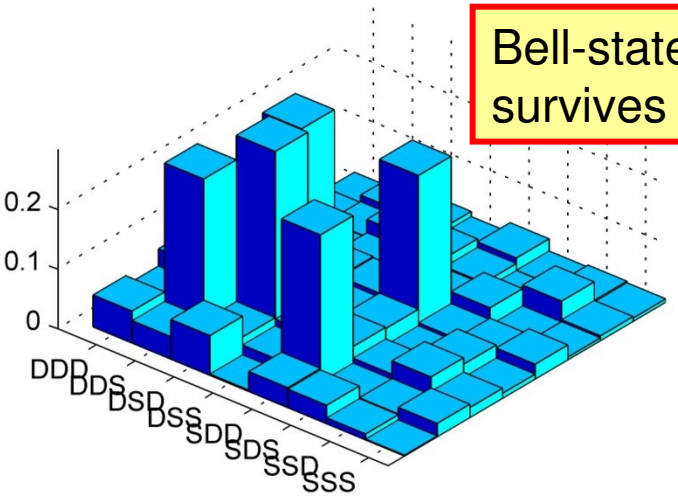


$\frac{1}{\sqrt{2}}(|SSS\rangle + |DDD\rangle)$

Measurement of the center ion

$\frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$



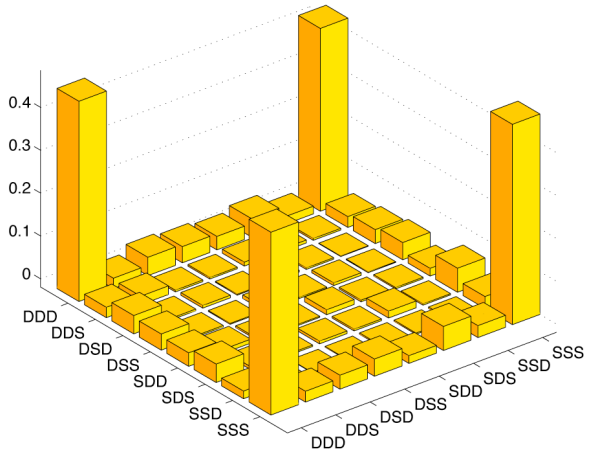


Bell-state survives

Roos et al., *Science* **304**, 1478 (200

Generalized Bell states

$$(|SSS\rangle + |DDD\rangle)/\sqrt{2}$$



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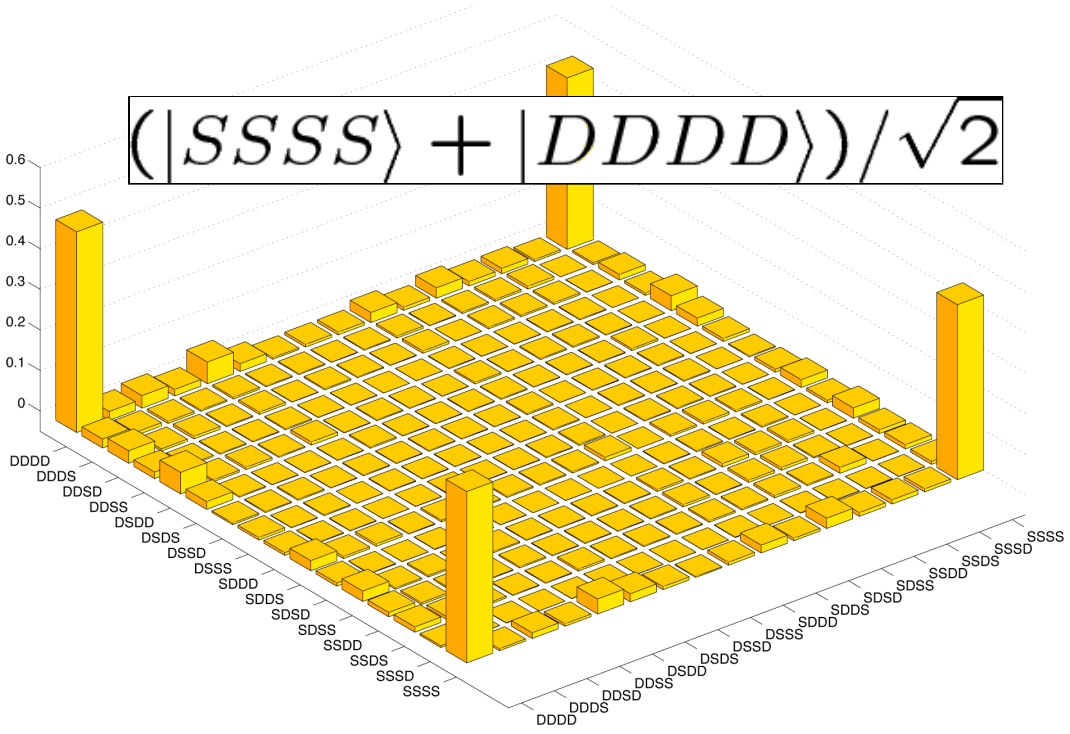
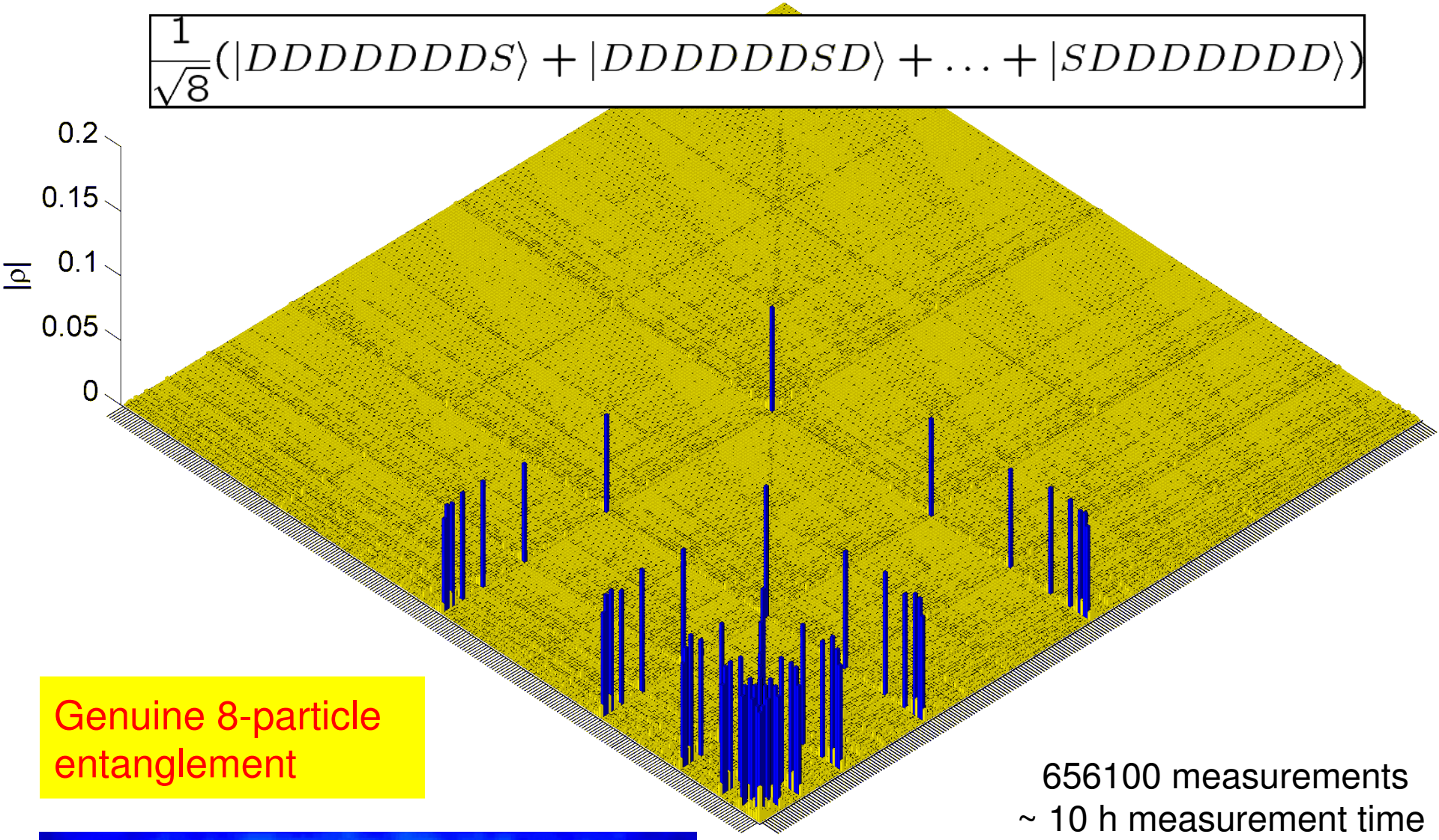
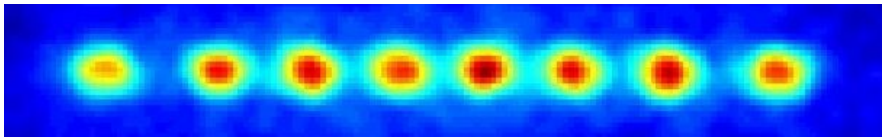


Generalized Bell states

$$\frac{1}{\sqrt{8}}(|DDDDDDDS\rangle + |DDDDDDSD\rangle + \ldots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement



656100 measurements
~ 10 h measurement time

Häffner et al., Nature **438**, 643 (2005)

Quantum gates …

Having the qubits interact

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

...allows the realization of a ***universal*** quantum computer !

$$|D\rangle|D\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|S\rangle \rightarrow |S\rangle|D\rangle$$

control target

Having the qubits interact

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$$|S\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|S\rangle \rightarrow |S\rangle|D\rangle$$

control target

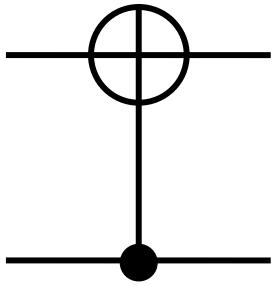
Most popular gates:

- Cirac-Zoller gate (Schmidt-Kaler et al., Nature **422,** 408 (2003)).
- Geometric phase gate (Leibfried et al., Nature **422,** 412 (2003)).
- Mølmer-Sørensen gate (Sackett et al., Nature **404**, 256 (2000)).

A controlled-NOT operation

Target bit

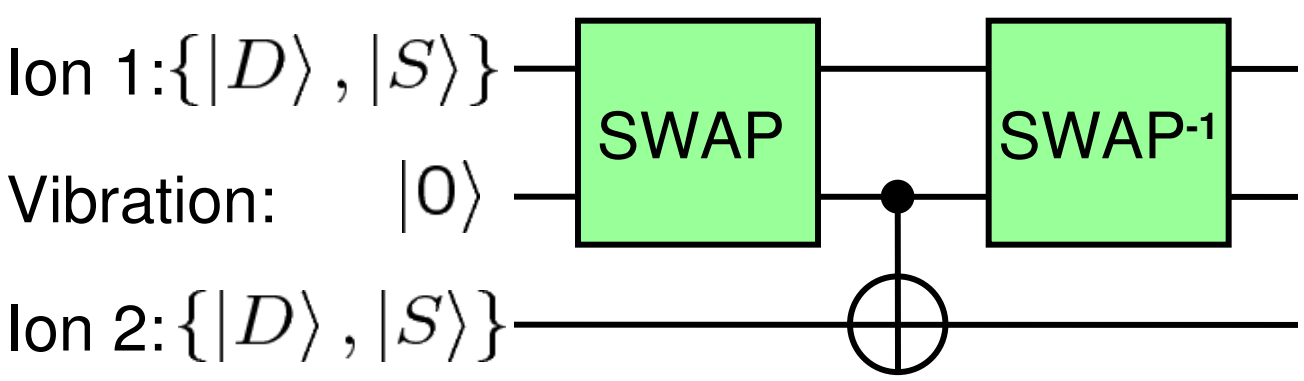
Control bit



$$
\begin{aligned}
|0\rangle|0\rangle &\rightarrow |0\rangle|0\rangle \\
|0\rangle|1\rangle &\rightarrow |0\rangle|1\rangle \\
|1\rangle|0\rangle &\rightarrow |1\rangle|1\rangle \\
|1\rangle|1\rangle &\rightarrow |1\rangle|0\rangle
\end{aligned}
$$

Target

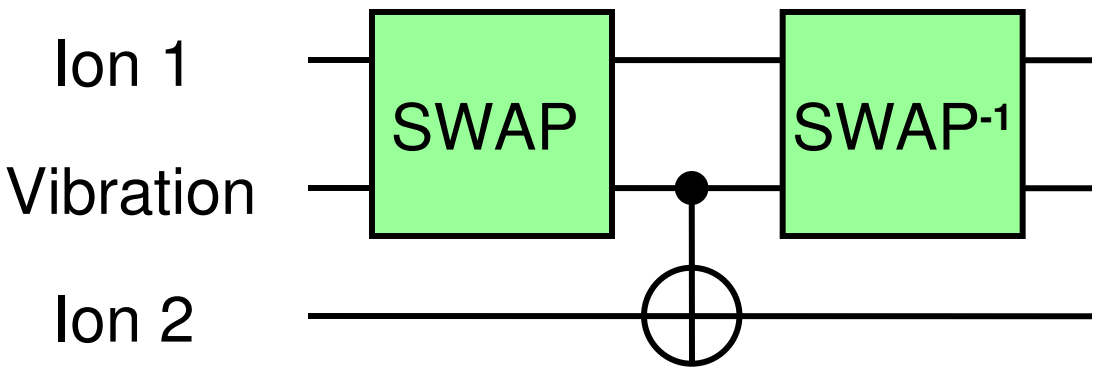
A controlled-NOT operation



Control qubit

Target qubit

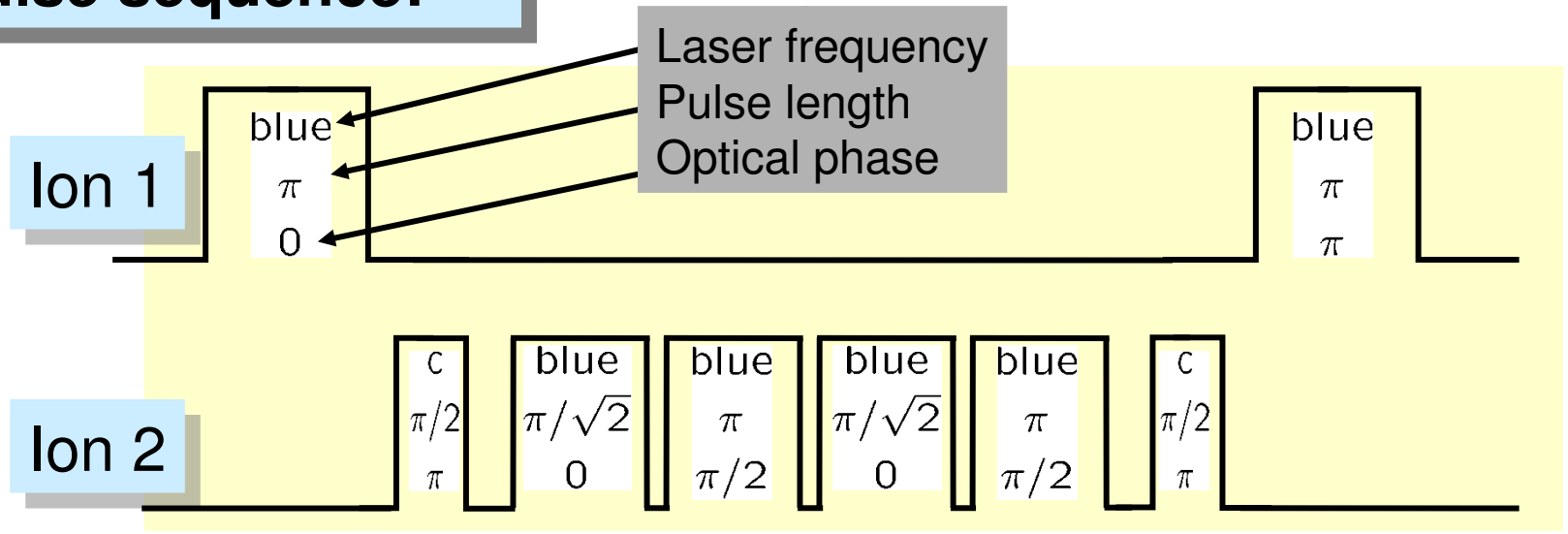
A controlled-NOT operation



Control qubit

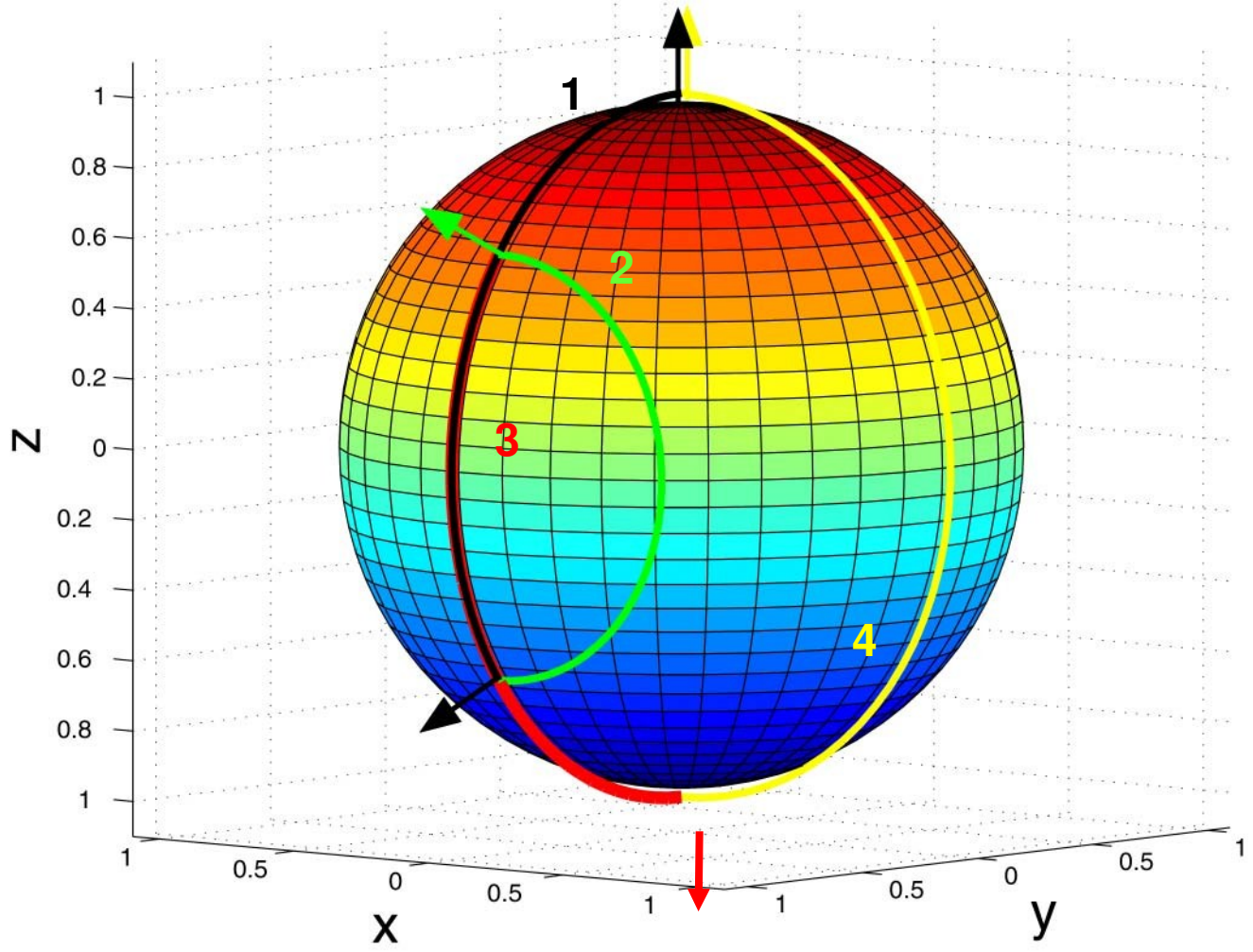
Target qubit

Pulse sequence:



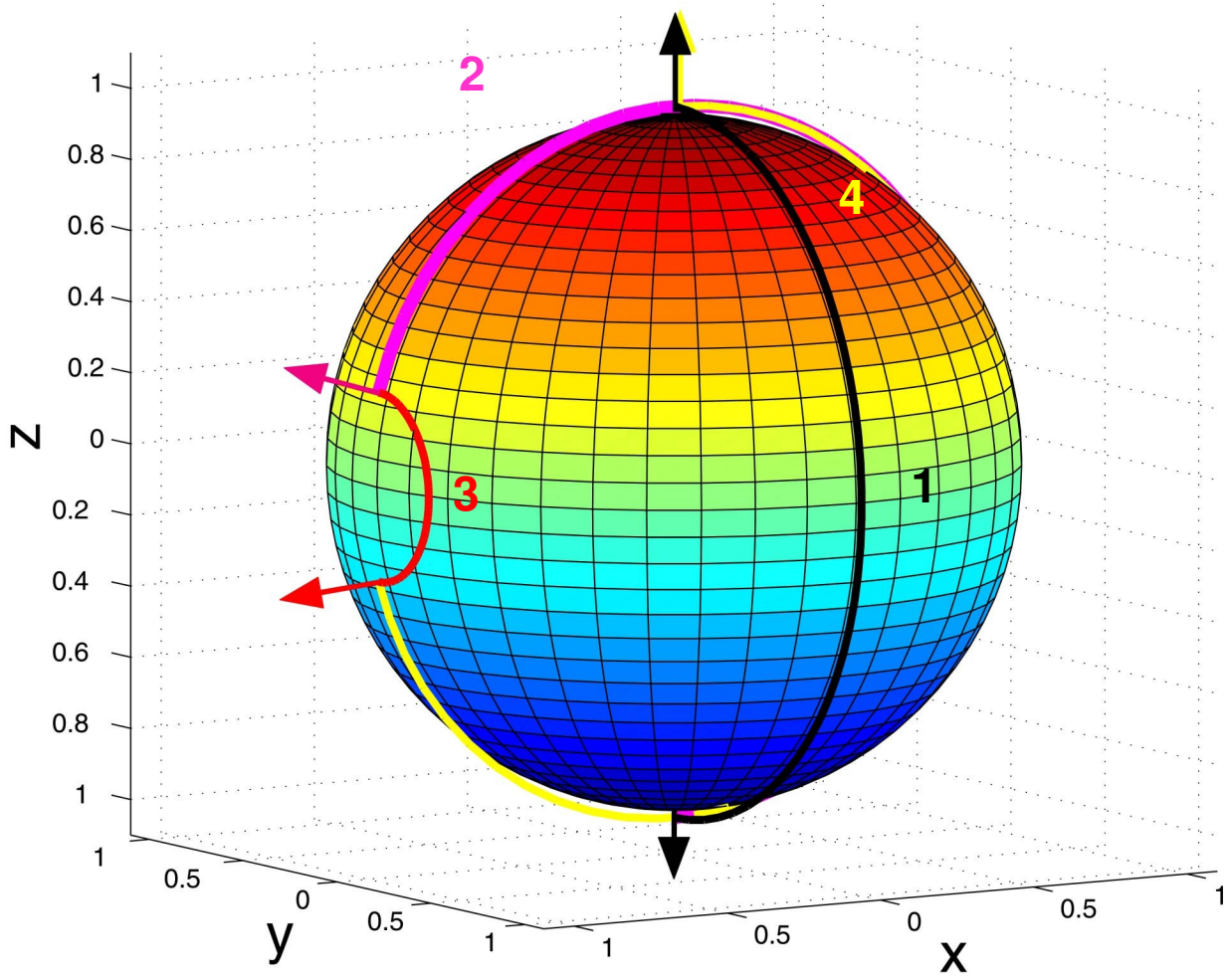
Composite phase gate (2π rotation)

$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$

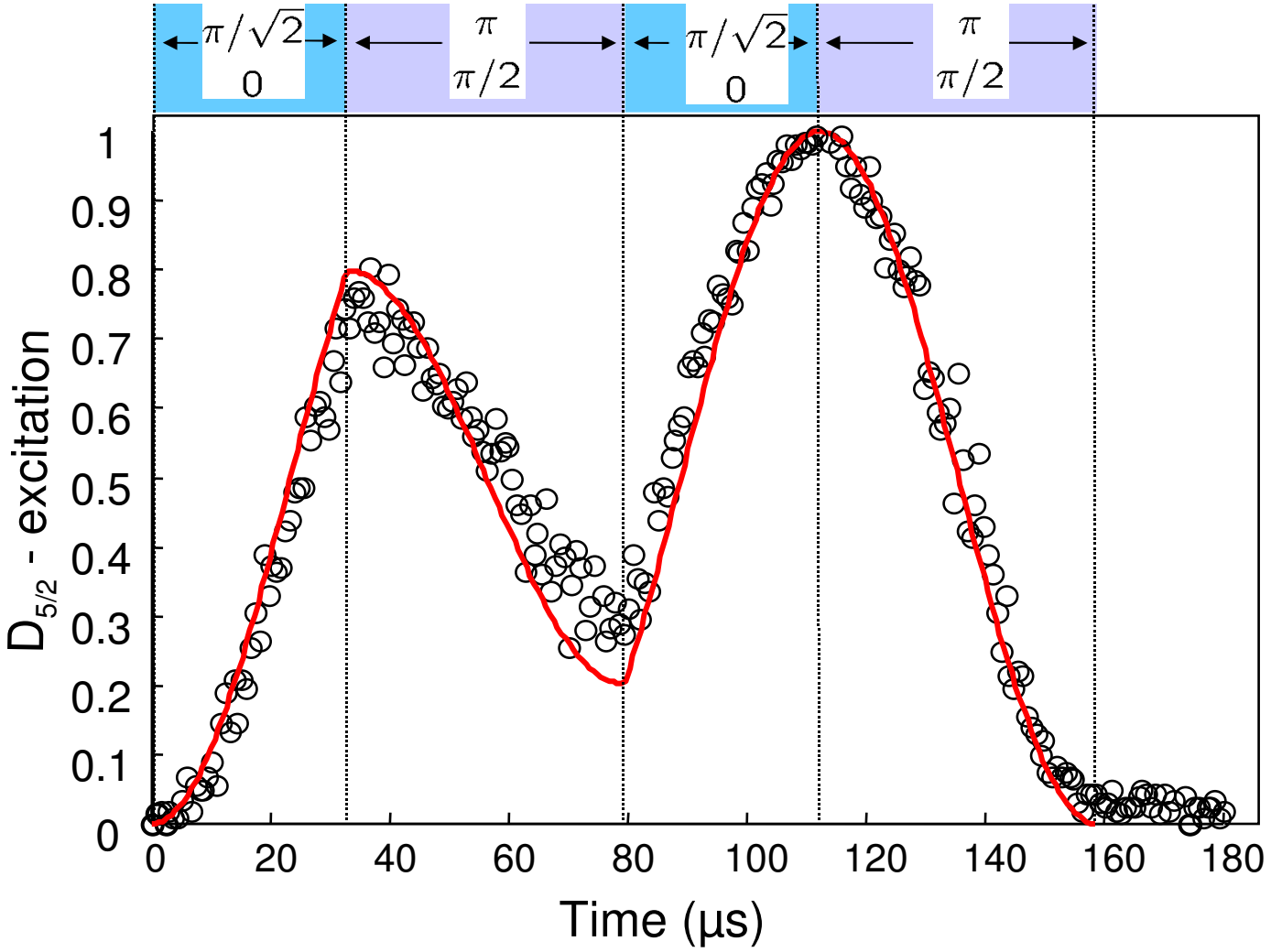


Action on |S,1> - |D,2>

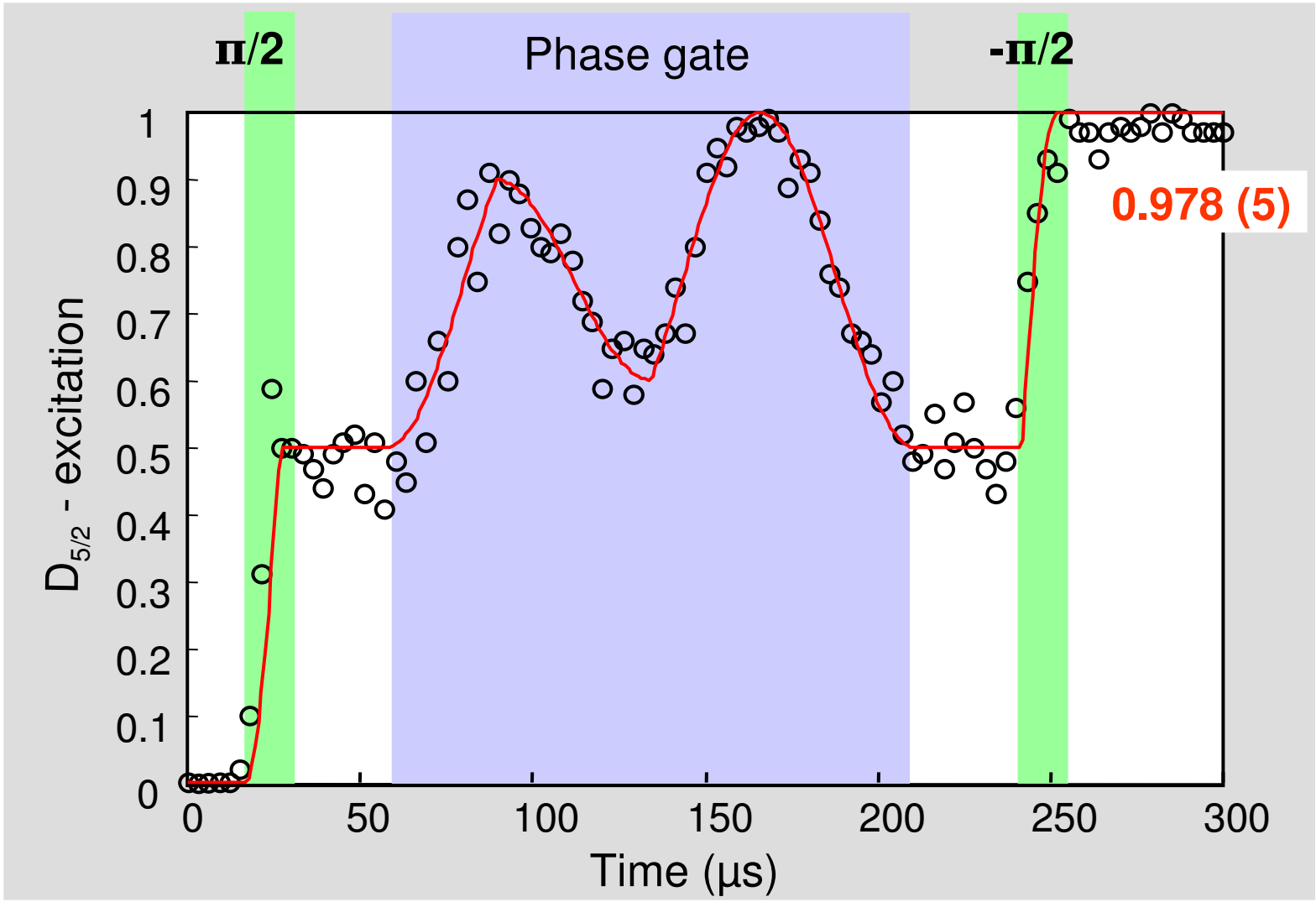
$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$



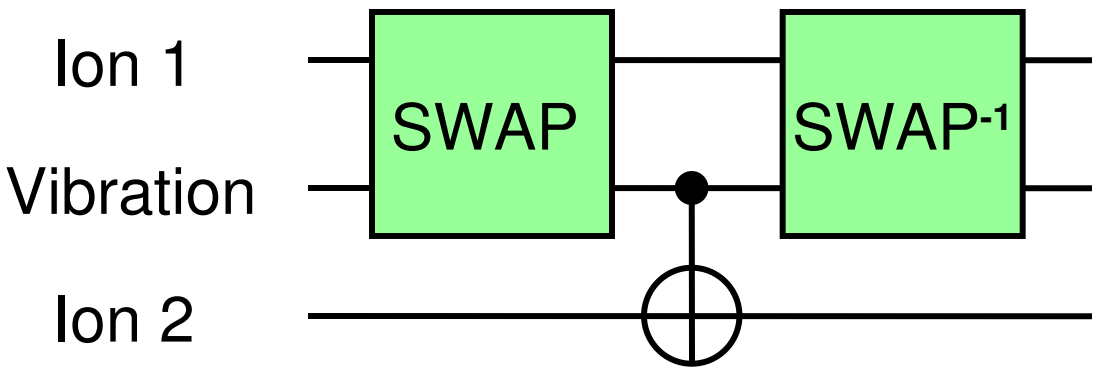
Single ion composite phase gate



Single ion composite CNOT gate



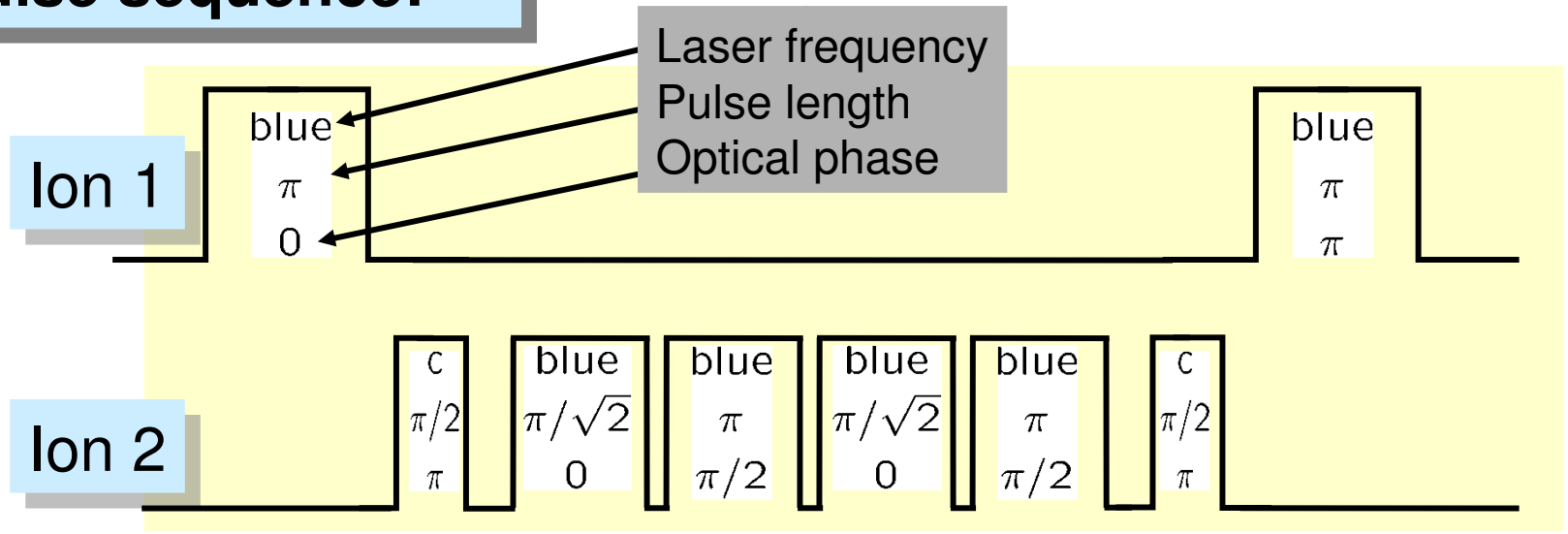
A controlled-NOT operation



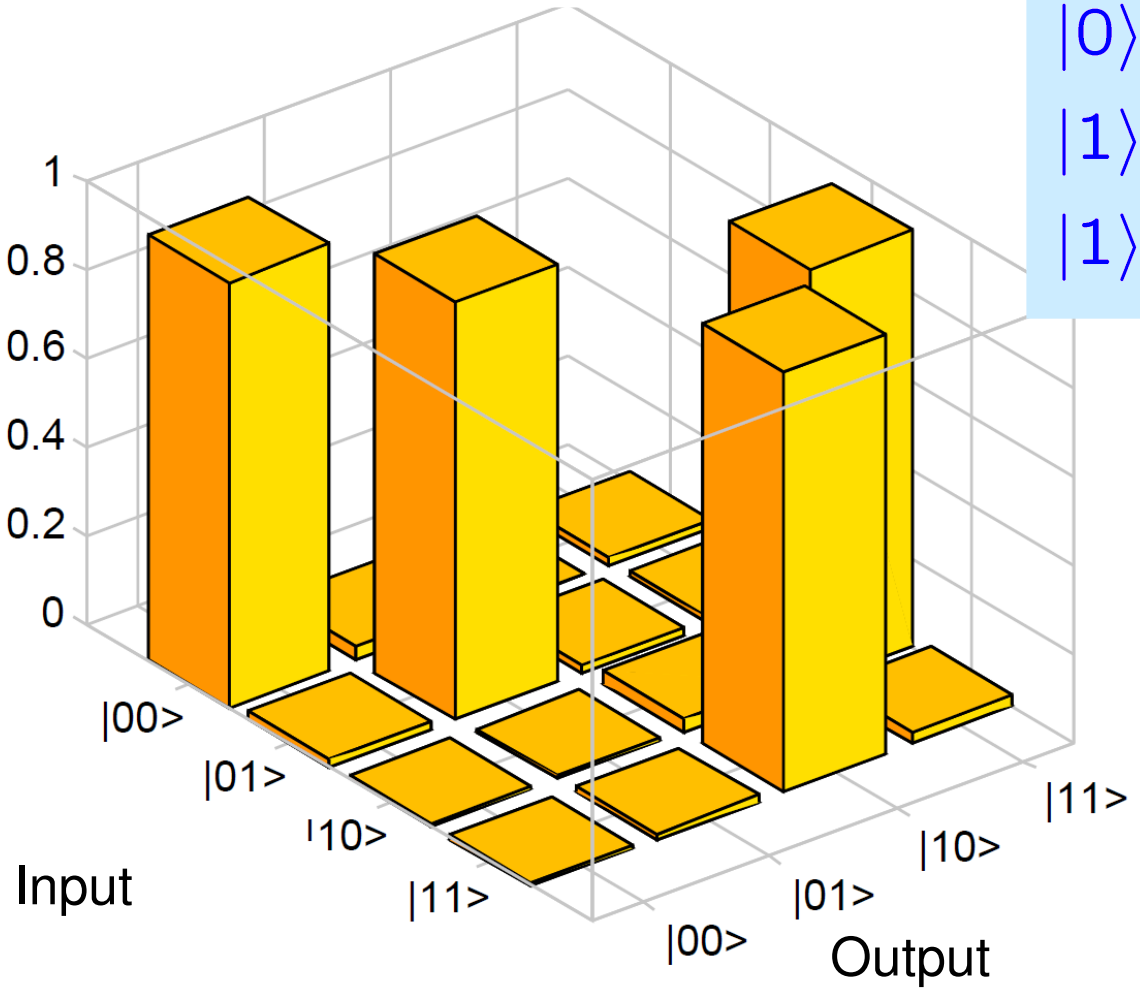
Control qubit

Target qubit

Pulse sequence:



Truth table of the CNOT



$$
\begin{aligned}
|0\rangle|0\rangle &\rightarrow |0\rangle|0\rangle \\
|0\rangle|1\rangle &\rightarrow |0\rangle|1\rangle \\
|1\rangle|0\rangle &\rightarrow |1\rangle|1\rangle \\
|1\rangle|1\rangle &\rightarrow |1\rangle|0\rangle
\end{aligned}
$$

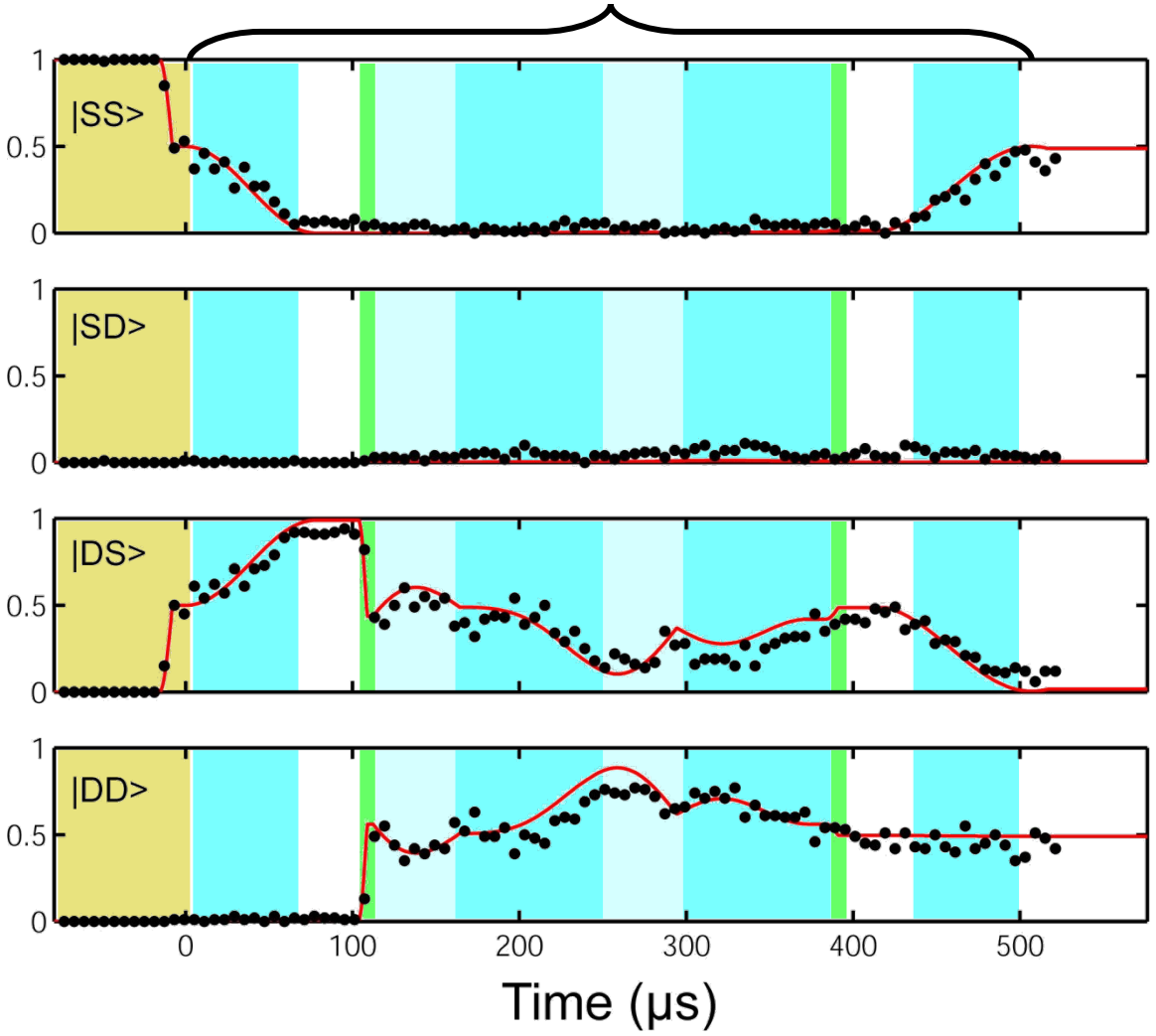
Using a CNOT to create a Bell state

$$|SS\rangle \rightarrow |S+D\rangle|S\rangle \longrightarrow |SS\rangle + |DD\rangle$$

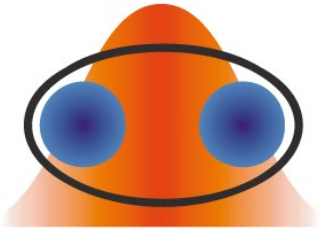
prepare

CNOT

output



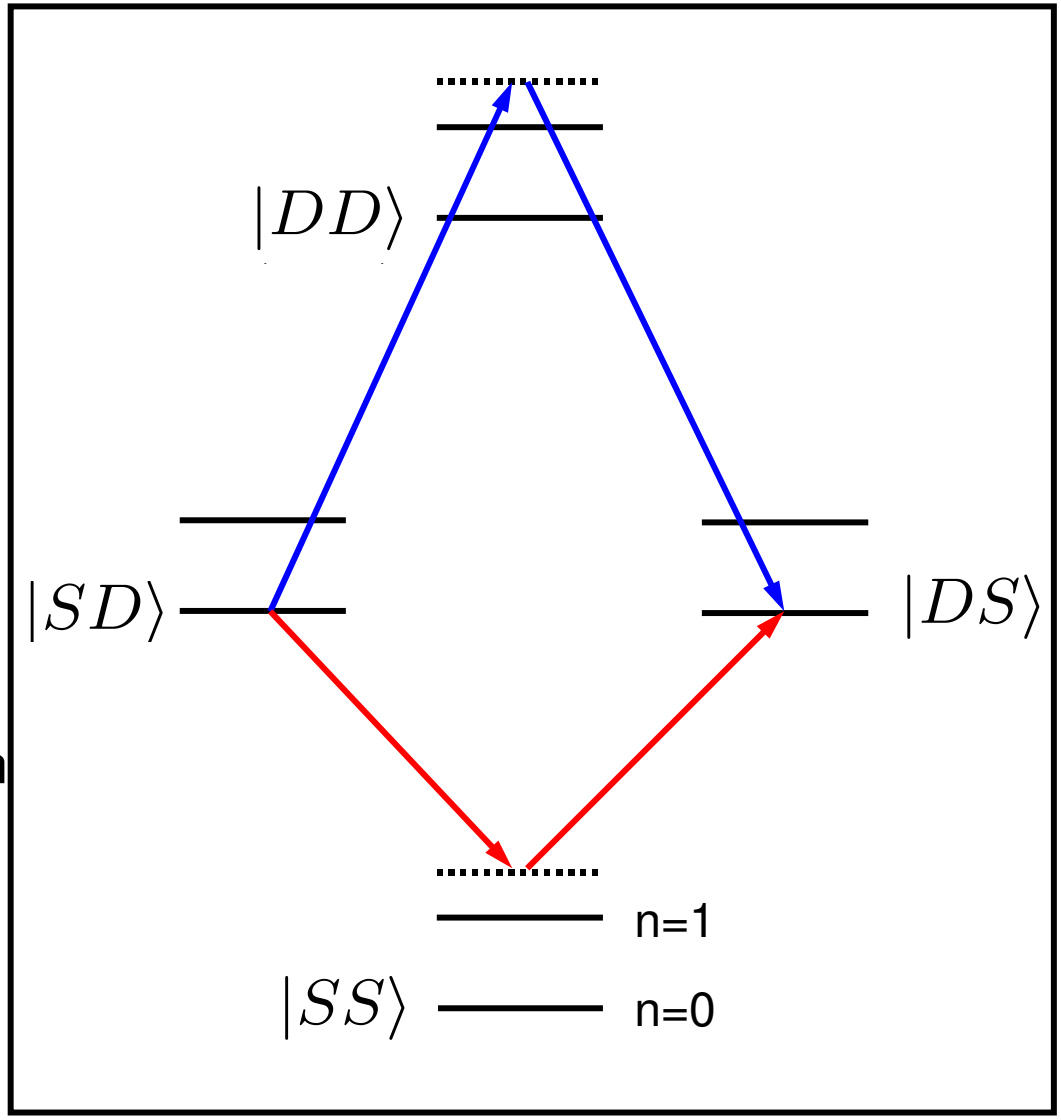
Mølmer-Sørensen gate



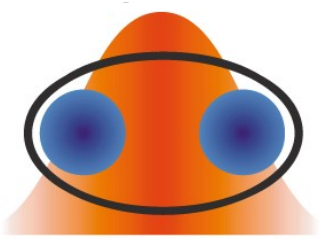
Raman transitions between

$$|SD\rangle \quad \Leftrightarrow \quad |DS\rangle$$

Interaction of two ions via common motion.



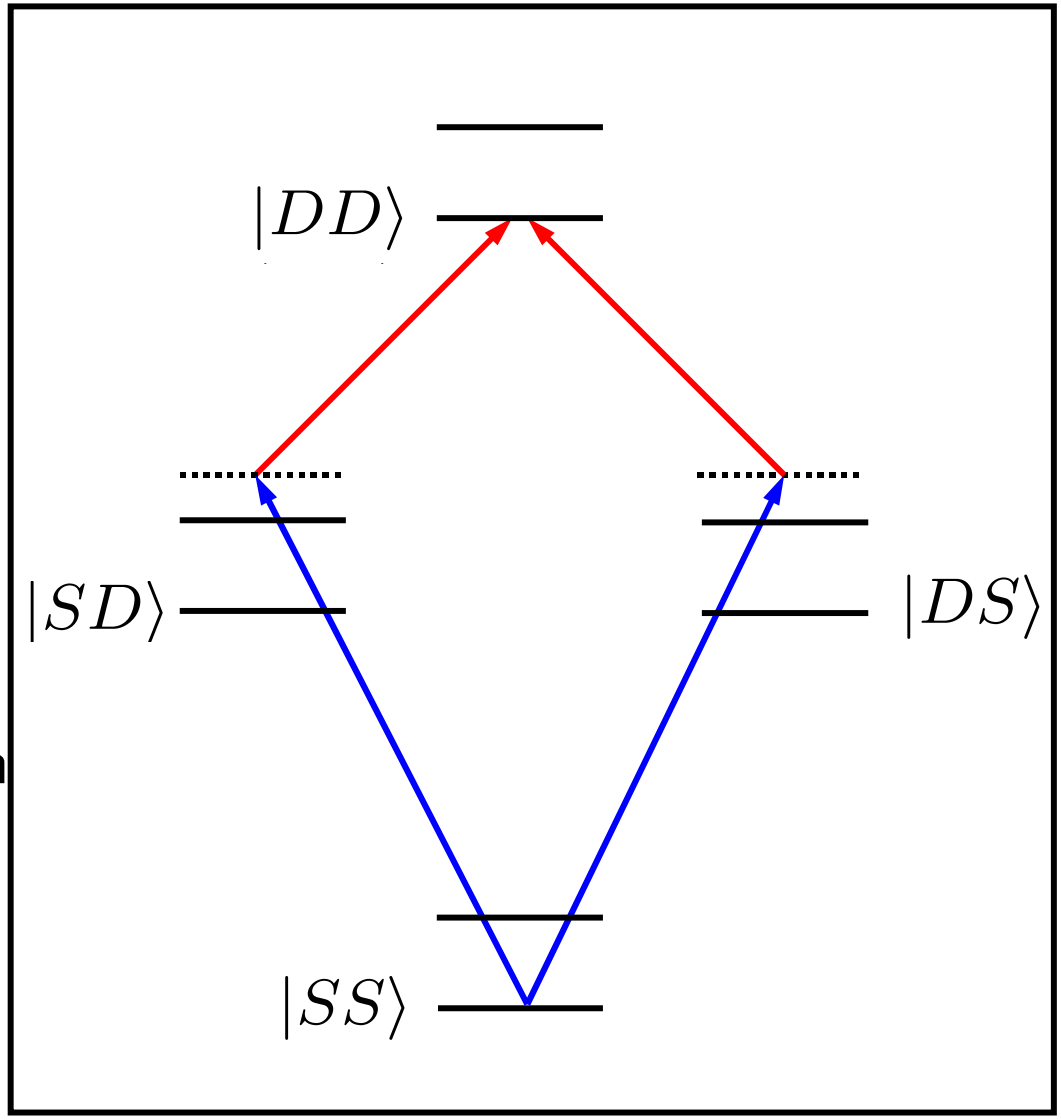
Mølmer-Sørensen gate



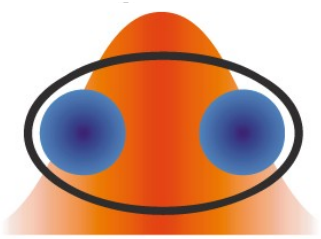
Raman transitions between

$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

Interaction of two ions via common motion.



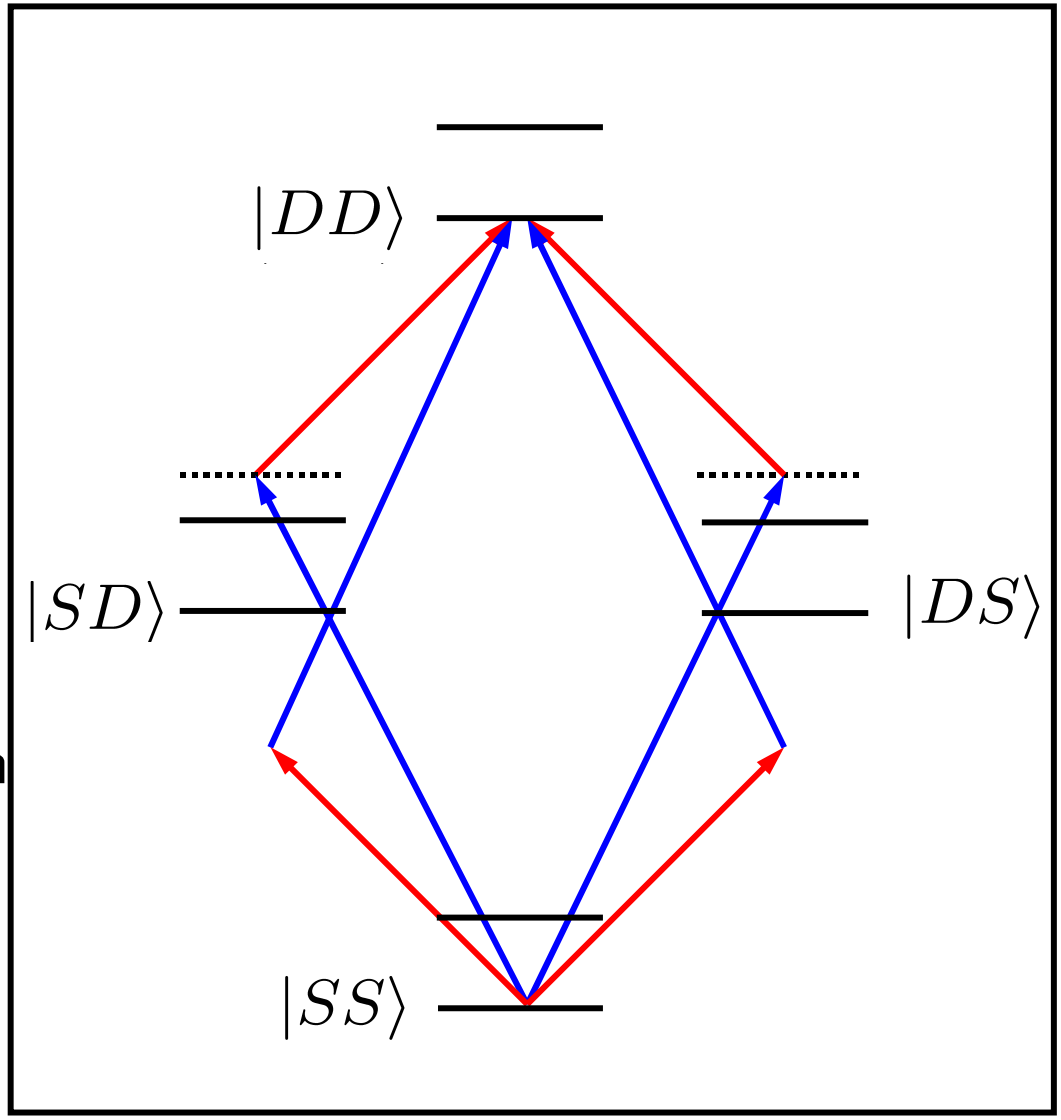
Mølmer-Sørensen gate



Raman transitions between

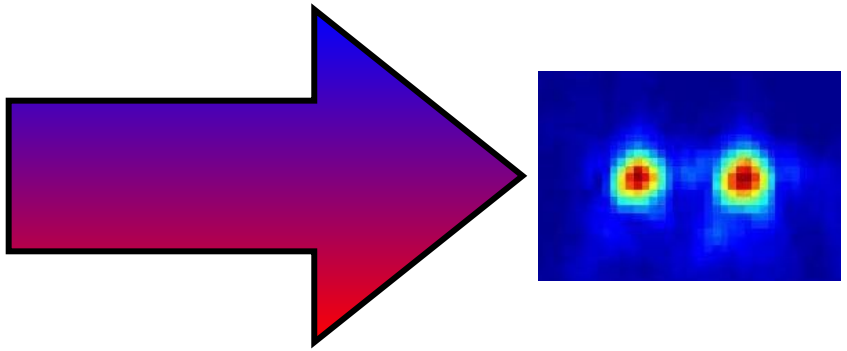
$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

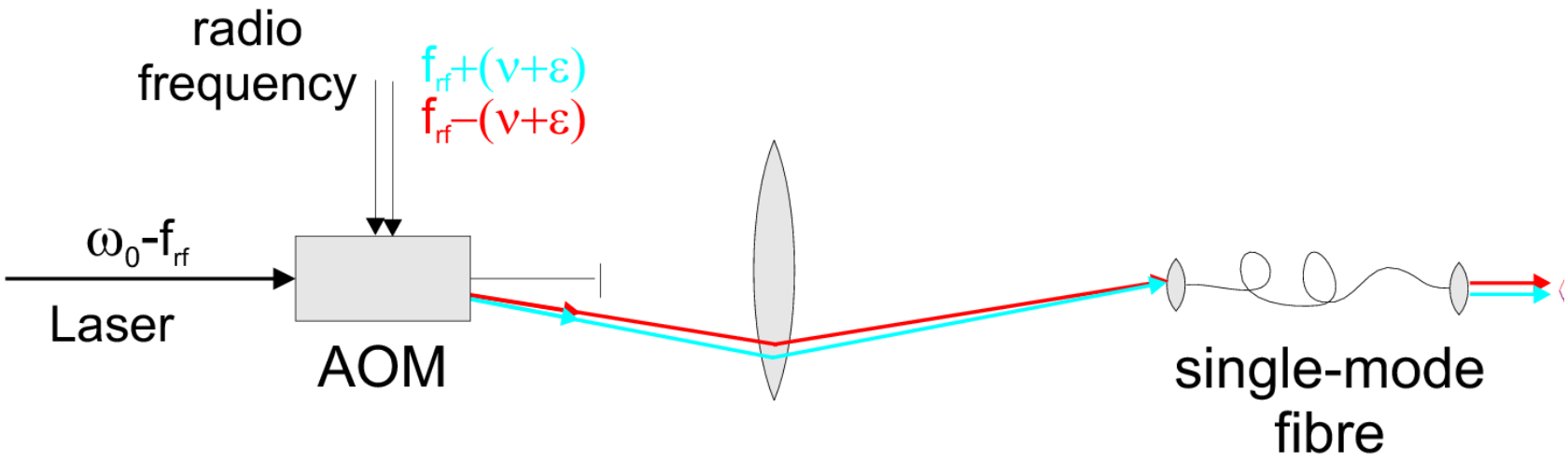
Interaction of two ions via common motion.

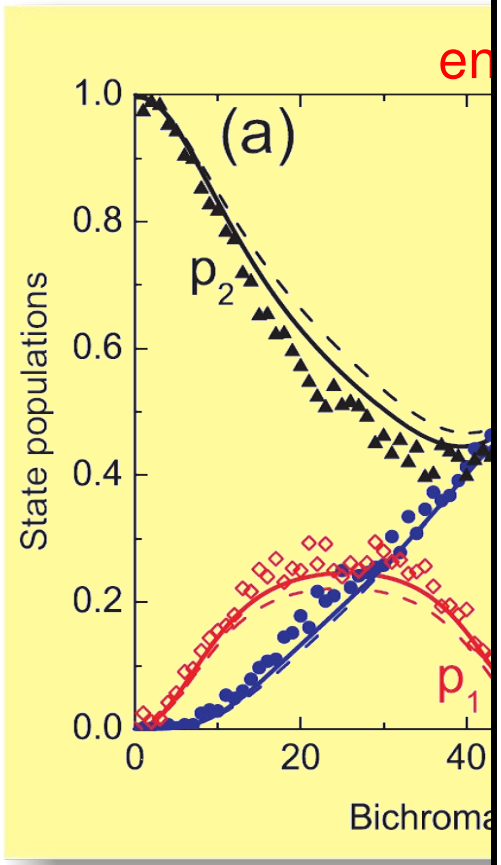


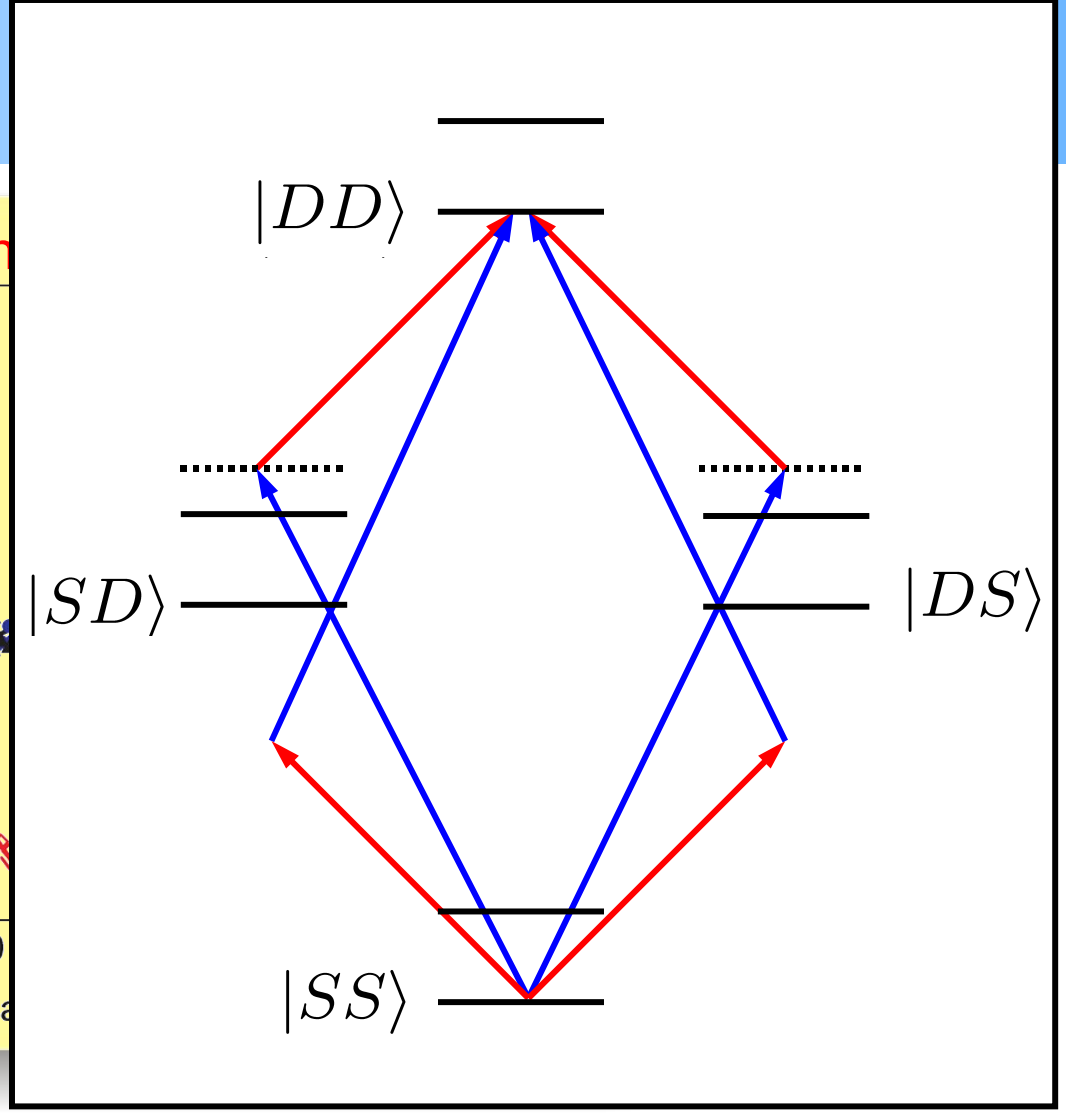
Technical realization

bicromatic beam
applied to both ions





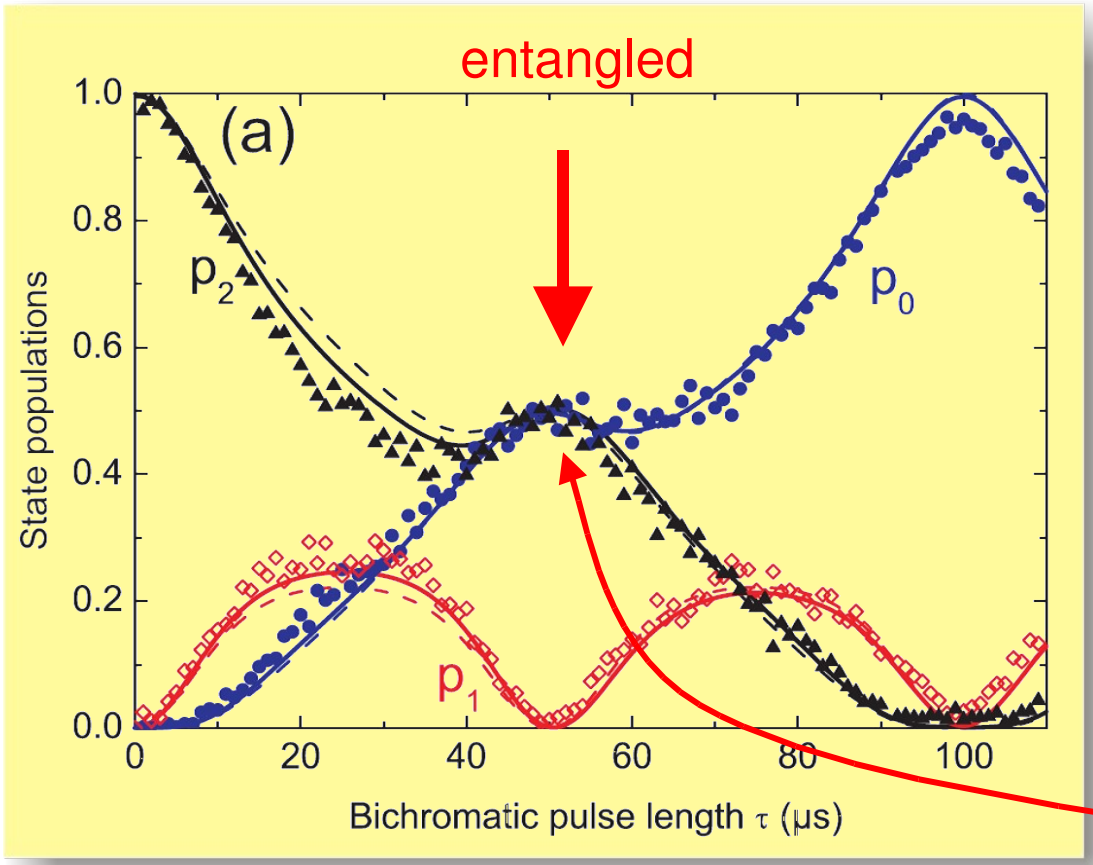




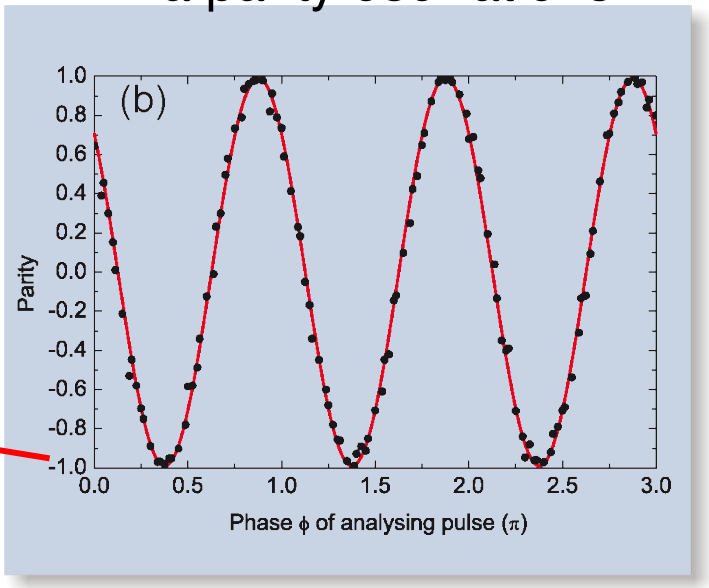
J. Benhelm et al., Nature Physics **4**, 463 (2008)

Theory: C. Roos, NJP **10,** 013002 (2008)

Entangling ions



measure entanglement via parity oscillations



gate duration $51\mu s$

average fidelity: 99.3 (2) %

J. Benhelm et al., Nature Physics **4**, 463 (2008)

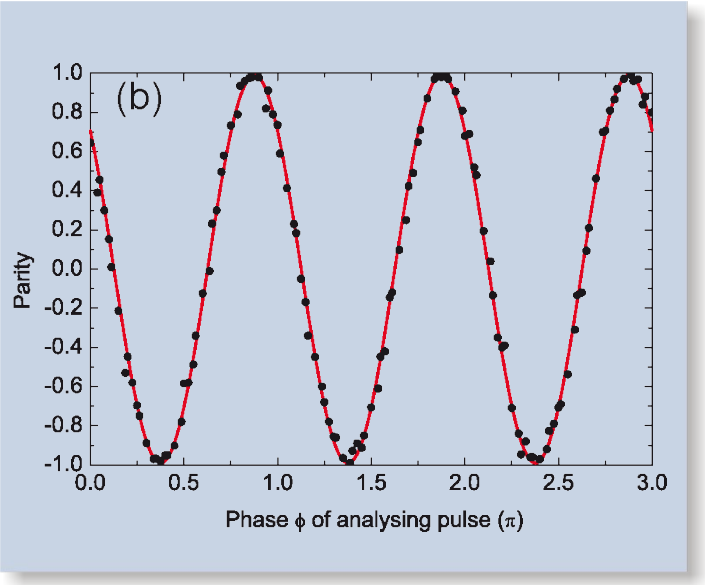
Theory: C. Roos, NJP **10,** 013002 (2008)

$$|00\rangle + |11\rangle \xrightarrow{R_2^C(\pi/2,\varphi),R_1^C(\pi/2,\varphi)}$$

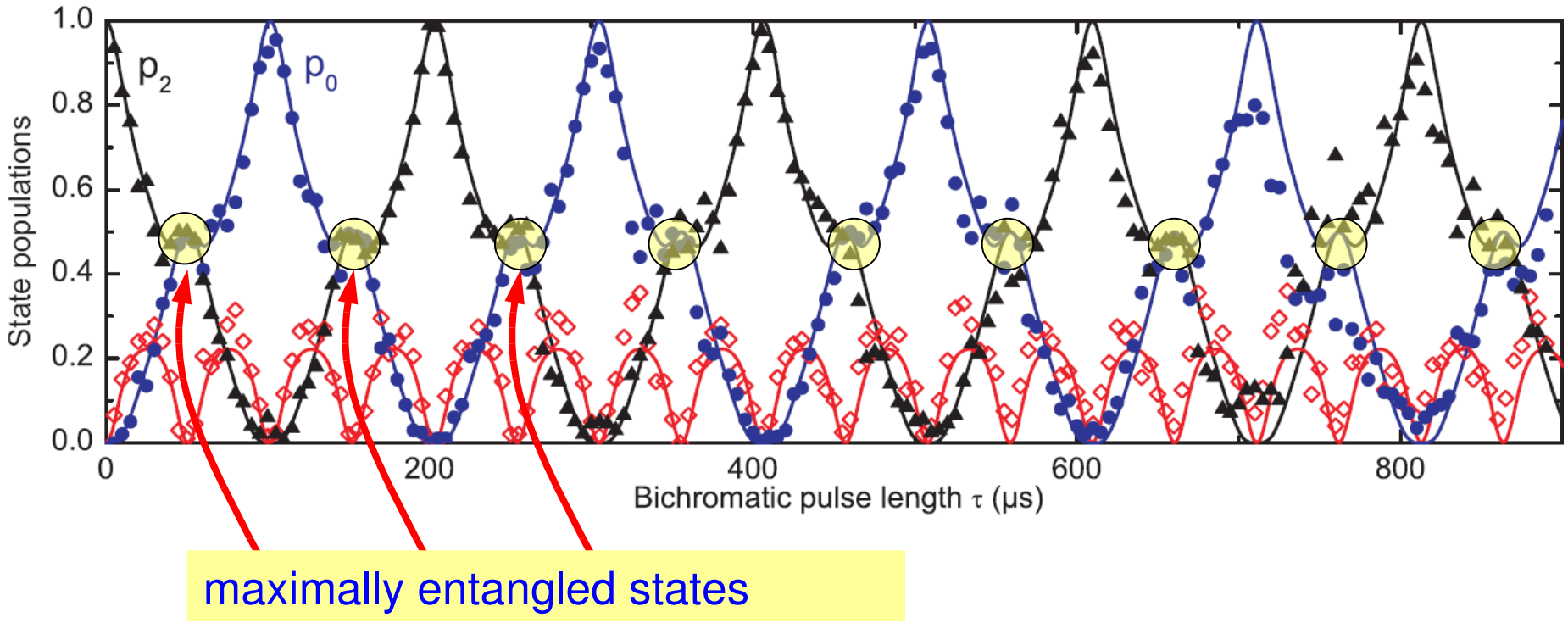
$$\left(|0\rangle + ie^{i\varphi}|1\rangle\right)\left(|0\rangle + ie^{i\varphi}|1\rangle\right) + \left(|1\rangle + ie^{-i\varphi}|0\rangle\right)\left(|1\rangle + ie^{-i\varphi}|0\rangle\right)$$

$$= (1 - e^{-2i\varphi})|00\rangle + ie^{i\varphi}(1 + e^{-2i\varphi})|01\rangle$$

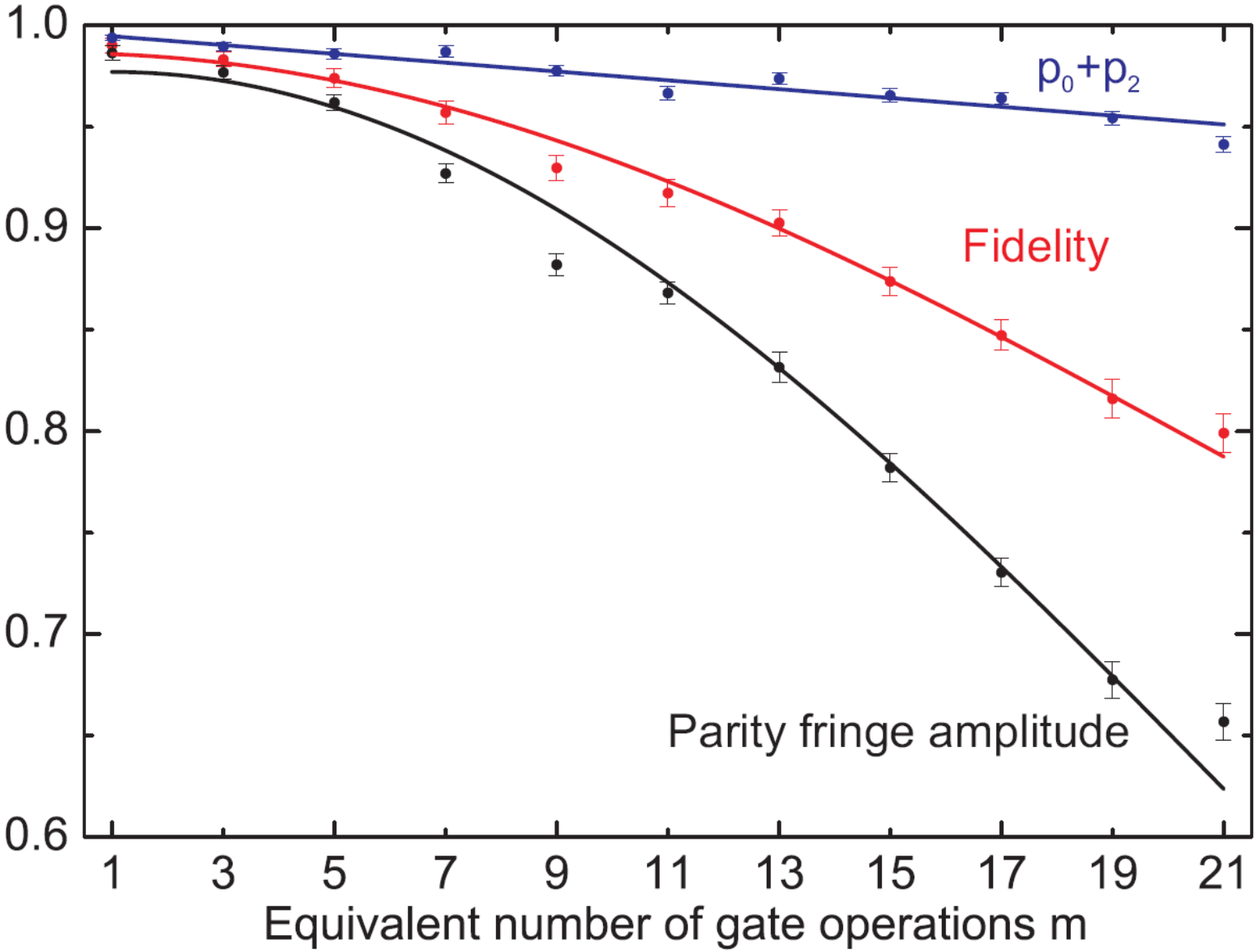
$$+ ie^{i\varphi}(1 + e^{-2i\varphi})|10\rangle + (1 - e^{-2i\varphi})|11\rangle ,$$



Gate concatenation



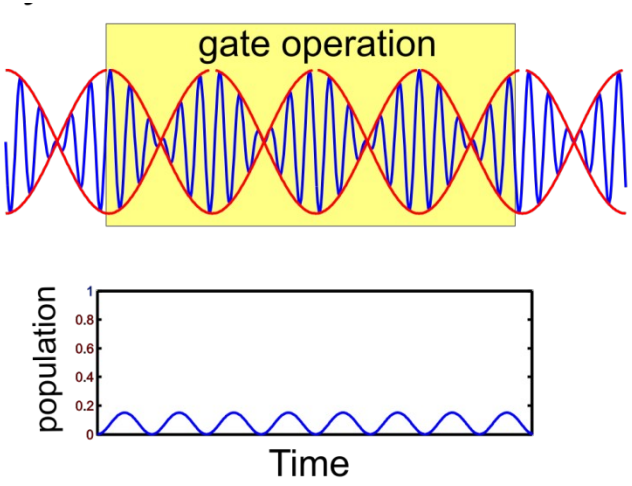
Gate performance

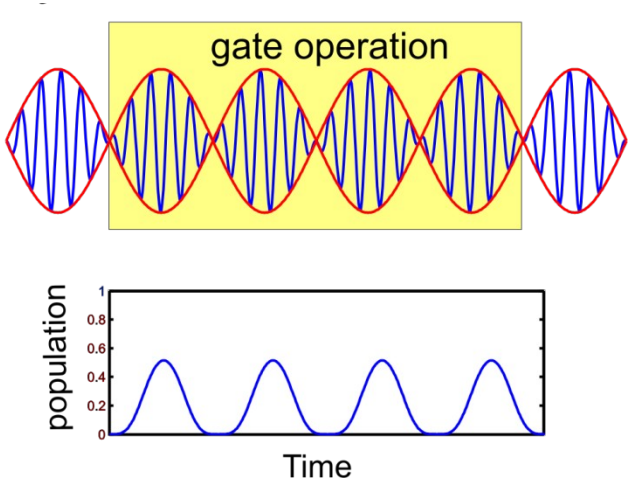


Optimization of the gate

The MS gate operation is optimized by

- pulse shaping, minimizes off-resonant excitation
- phase relation of amplitude modulated dual-frequency beam





C. Roos, NJP **10,** 013002 (2008).

Scaling of this approach?

Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes
- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings

DiVincenzo criteria

- Scalable physical system, well characterized qubits ✓ / ?
- Ability to initialize the state of the qubits ✓
- Long relevant coherence times, much longer than gate operation time ✓
- "Universal" set of quantum gates ✓
- Qubit-specific measurement capability

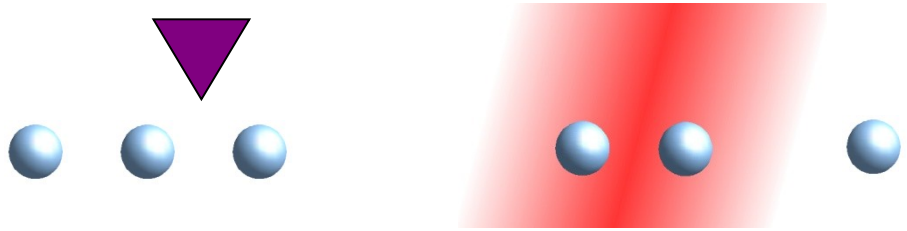
Often neglected:

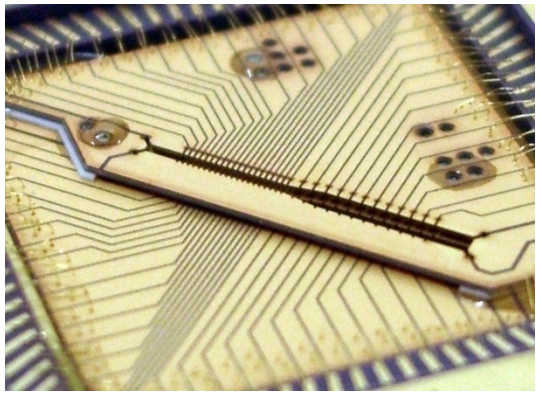
- exceptional fidelity of operations
- low error rate also for large quantum systems
- all requirements have to met at the same time

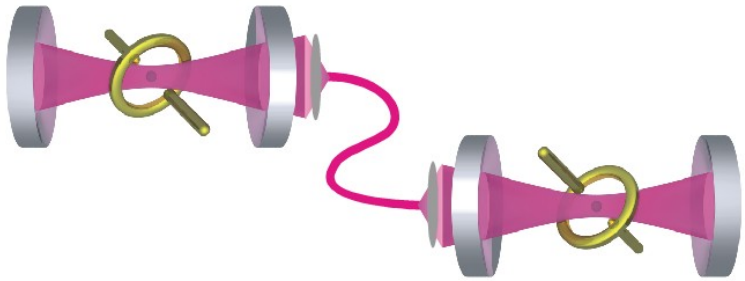
Scaling of ion trap quantum computers

Its easy to have thousands of coherent qubits …
but hard to control their interaction

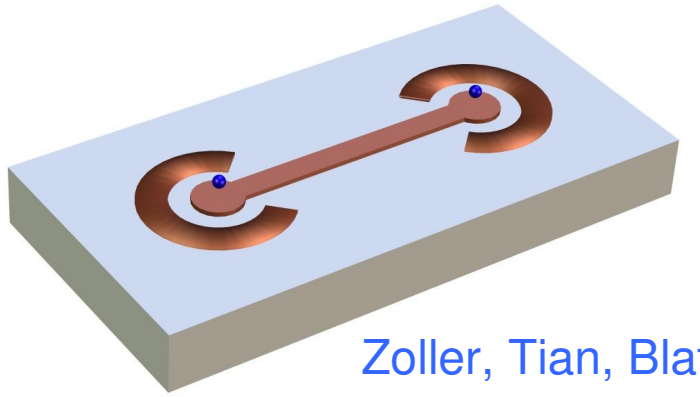
Kielpinski, Monroe, Wineland





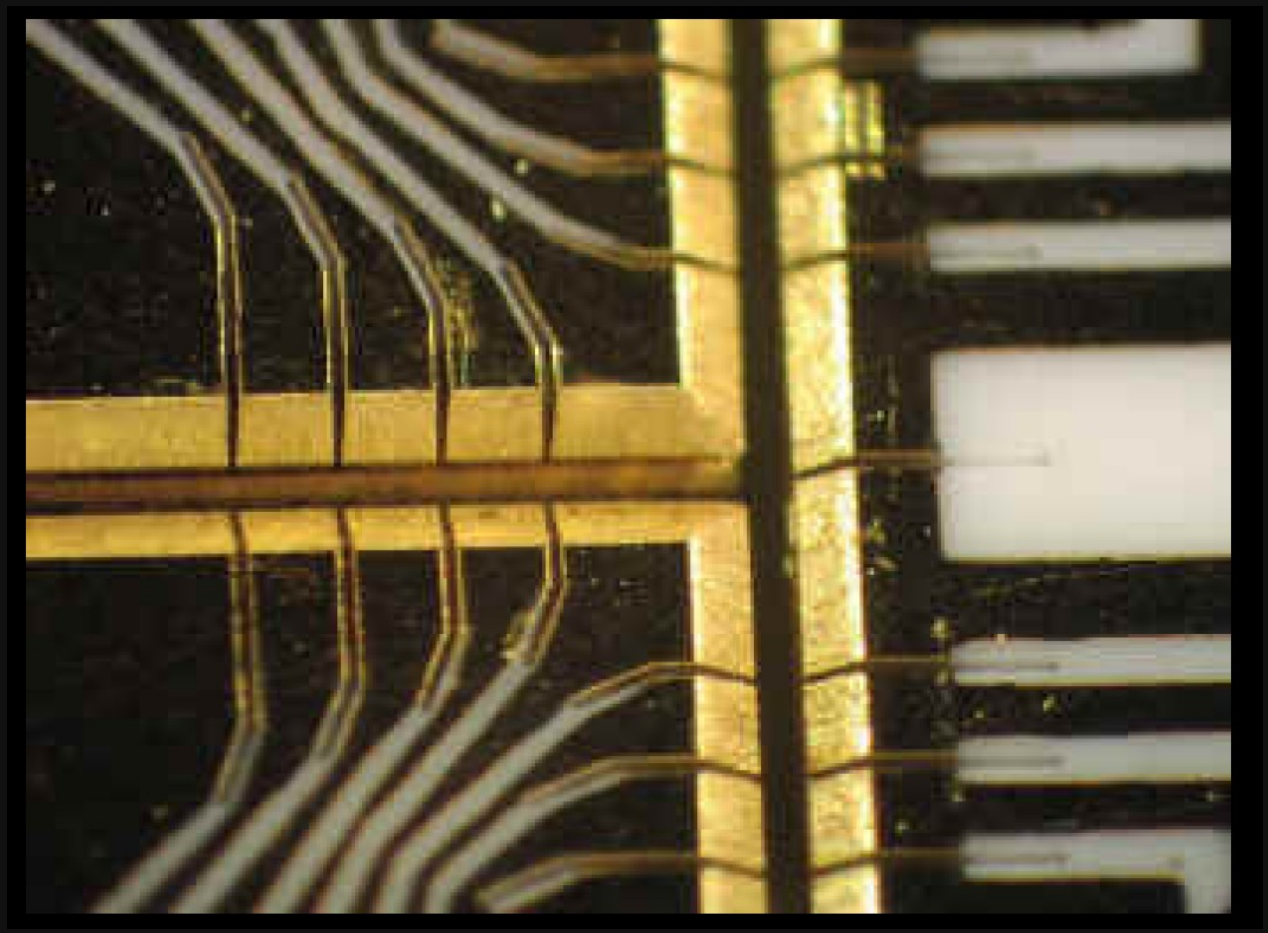


Cirac, Zoller, Kimble, Mabuchi



Zoller, Tian, Blatt

The Michigan T trap



An implementation of the Deutsch-algoritm …

Deutsch‘s problem: Introduction

Decide which class the coin is:

False (equal sides) or ***Fair***

Front

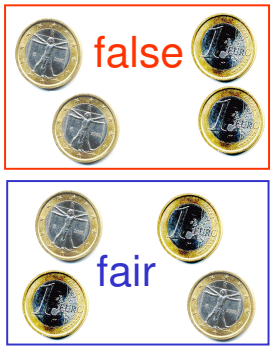
Back

A single measurement does NOT give the right answer

Deutsch‘s problem: Mathematical formulation

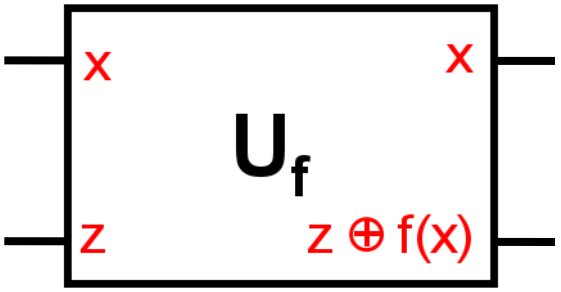
4 possible coins are representend by 4 functions

	Constant		Balanced	
	Case 1	Case 2	Case 3	Case 4
$f(0)$	0	1	0	1
$f(1)$	0	1	1	0



Deutsch‘s problem: Mathematical formulation

4 possible coins are representend by 4 functions

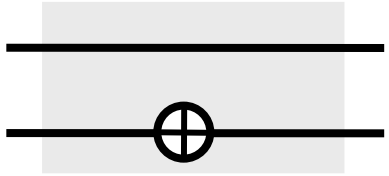
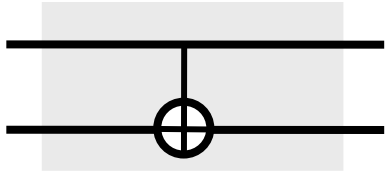
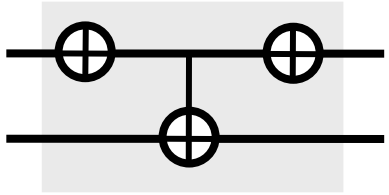


	Constant		Bal	
	Case 1	Case 2	Case 3	
f(0)	0	1	0	1
f(1)	0	1	1	0
z ⊕ *f*(*x*)	ID	NOT	CNOT	Z-CNOT

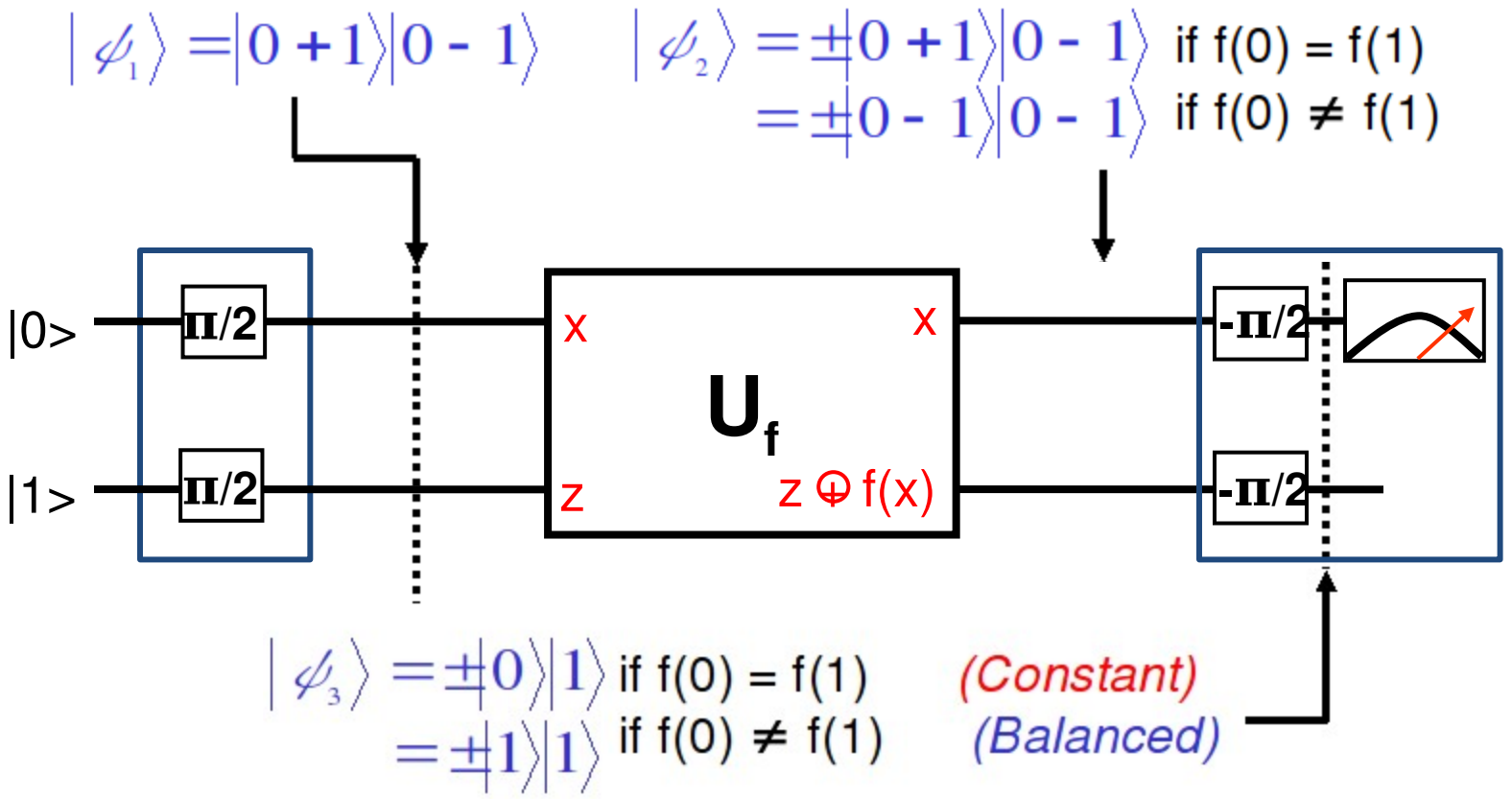
$$U_{fn}\,|x,z\rangle = |x, f_n(x) \oplus z\rangle$$

Physically reversible process
realized by a unitary transformation

Deutsch Jozsa quantum circuit

Case	Logic	Quantum circuit	Matrix U_{fn}
f_1	*ID*		1000 0100 0010 0001
f_2	*NOT*		0100 1000 0001 0010
f_3	*CNOT*		1000 0100 0001 0010
f_4	*Z-CNOT*		0100 1000 0010 0001

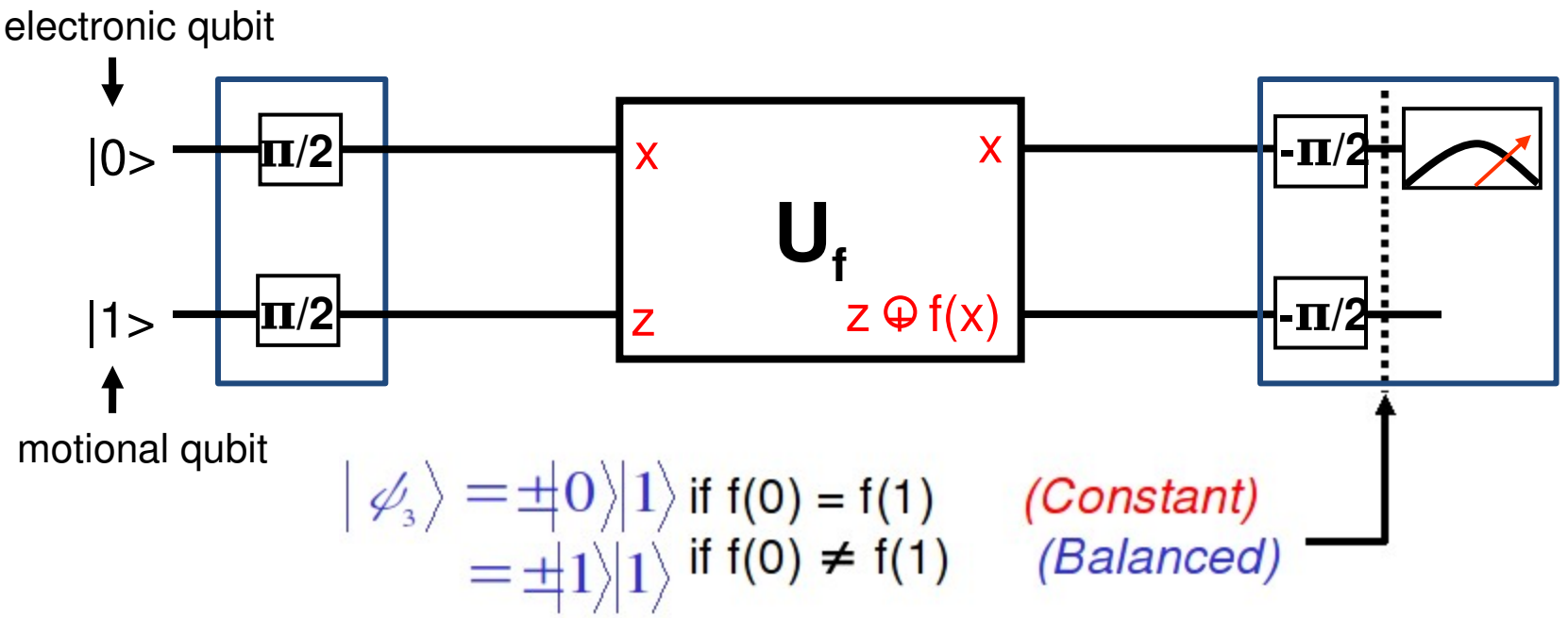
Deutsch Jozsa quantum circuit



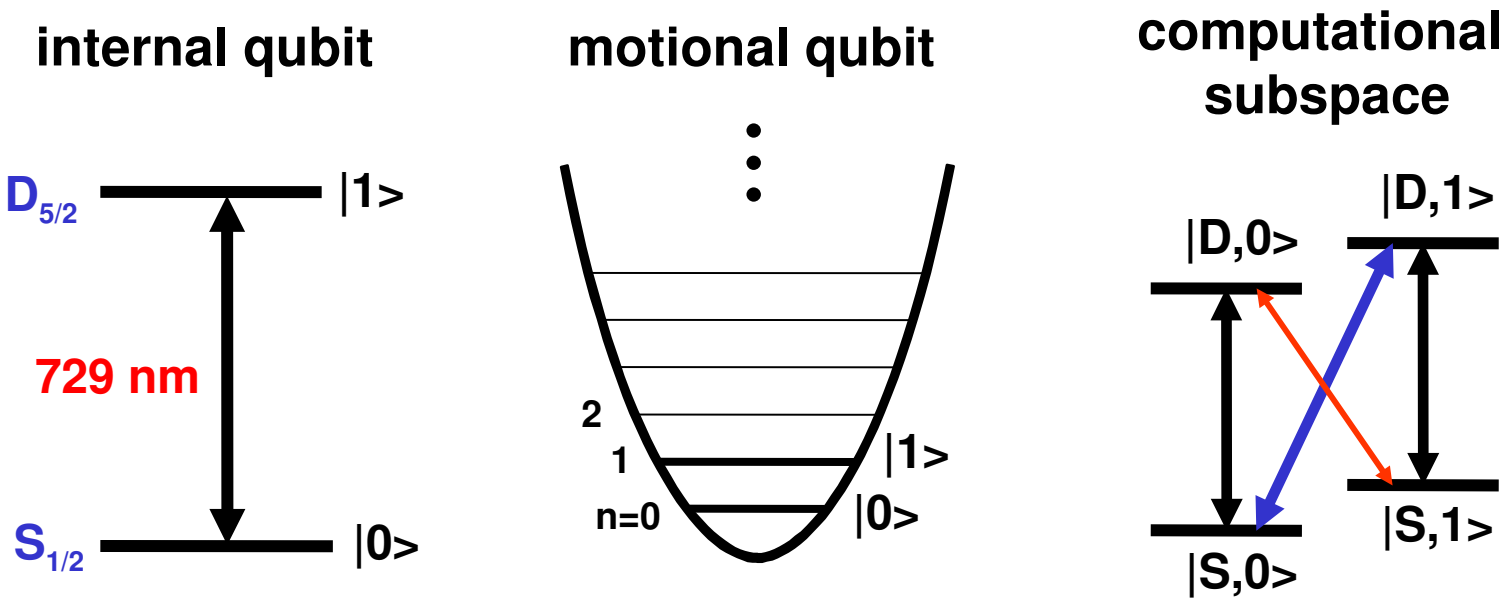
Quantum analysis gives the right answer after a single measurement!

- *D. Deutsch, R. Josza, Proc. R. Soc. London A439, 553 (1992)*
- *M. Nielsen, I. Chuang, QC and QI, Cambridge (2000)*

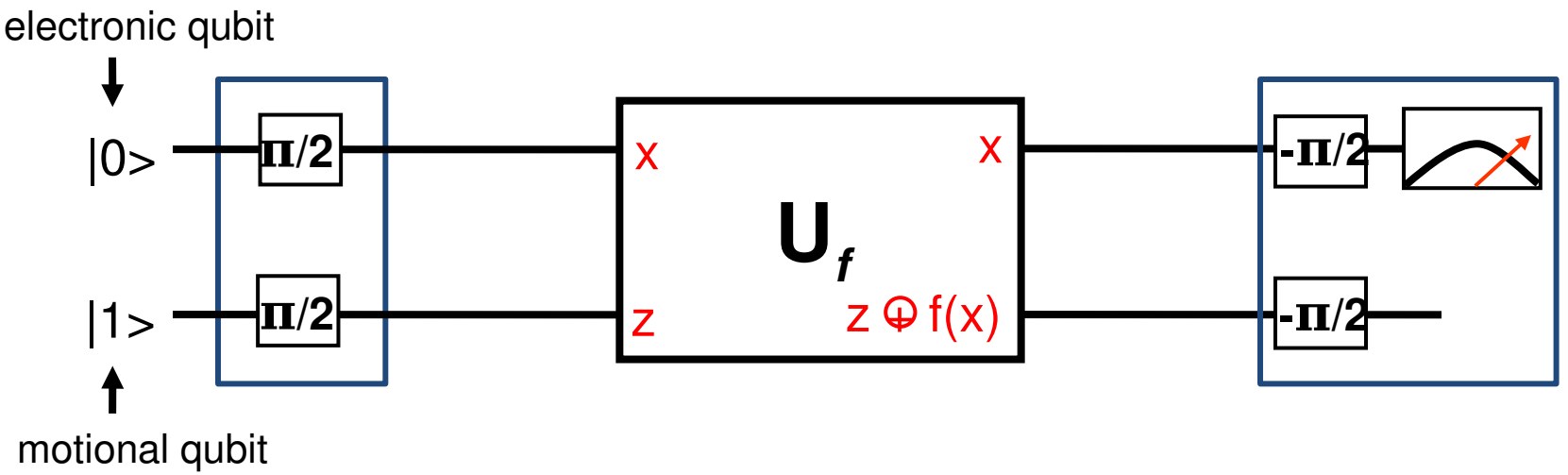
No information in the second qubit



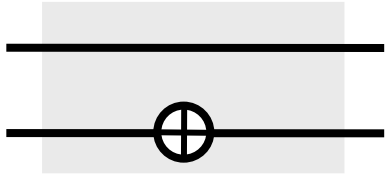
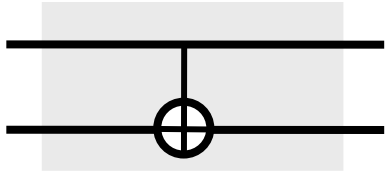
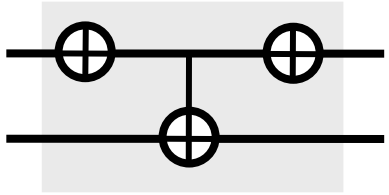
Qubits in $^{40}Ca^{+}$



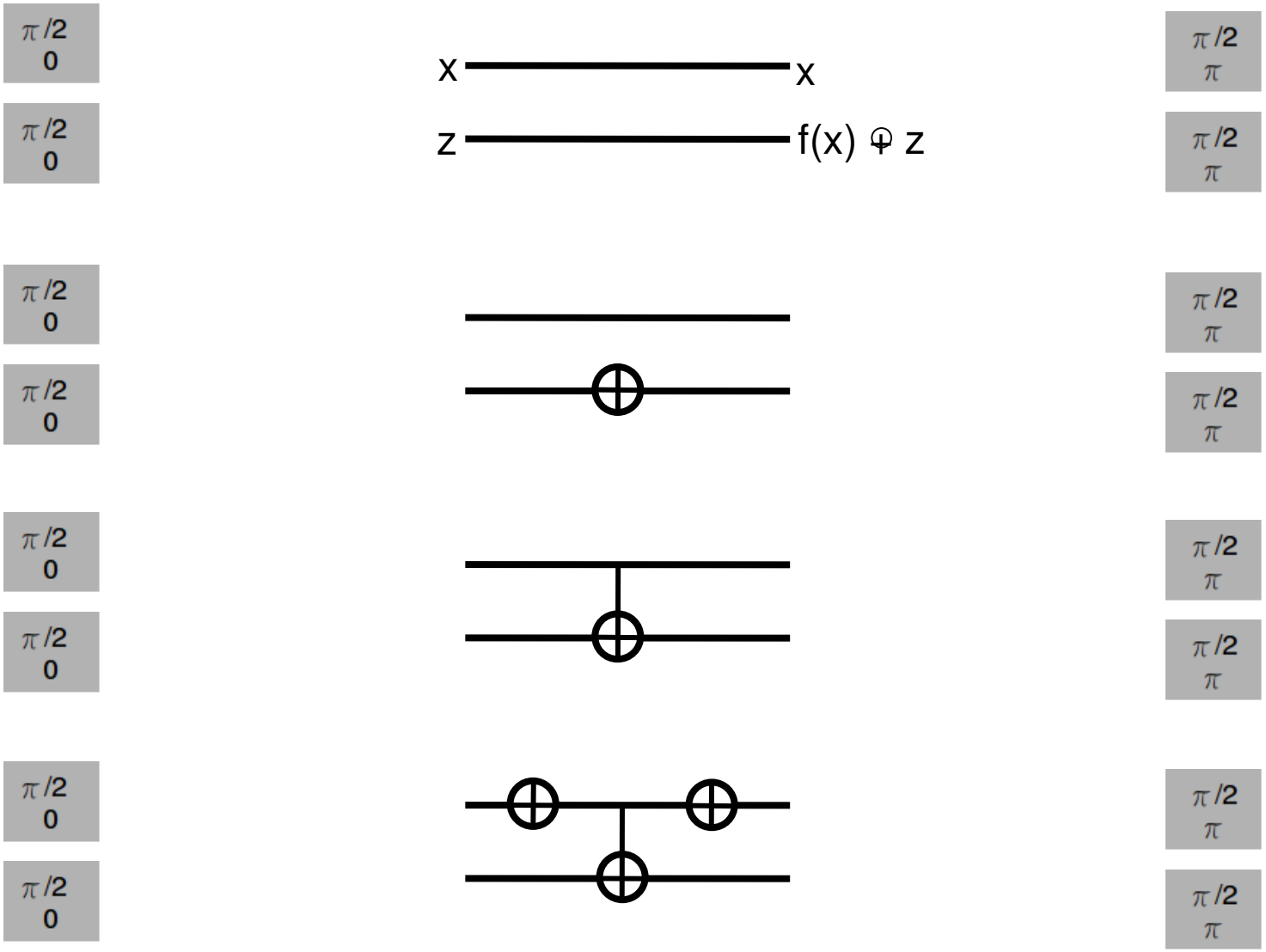
No information in the second qubit



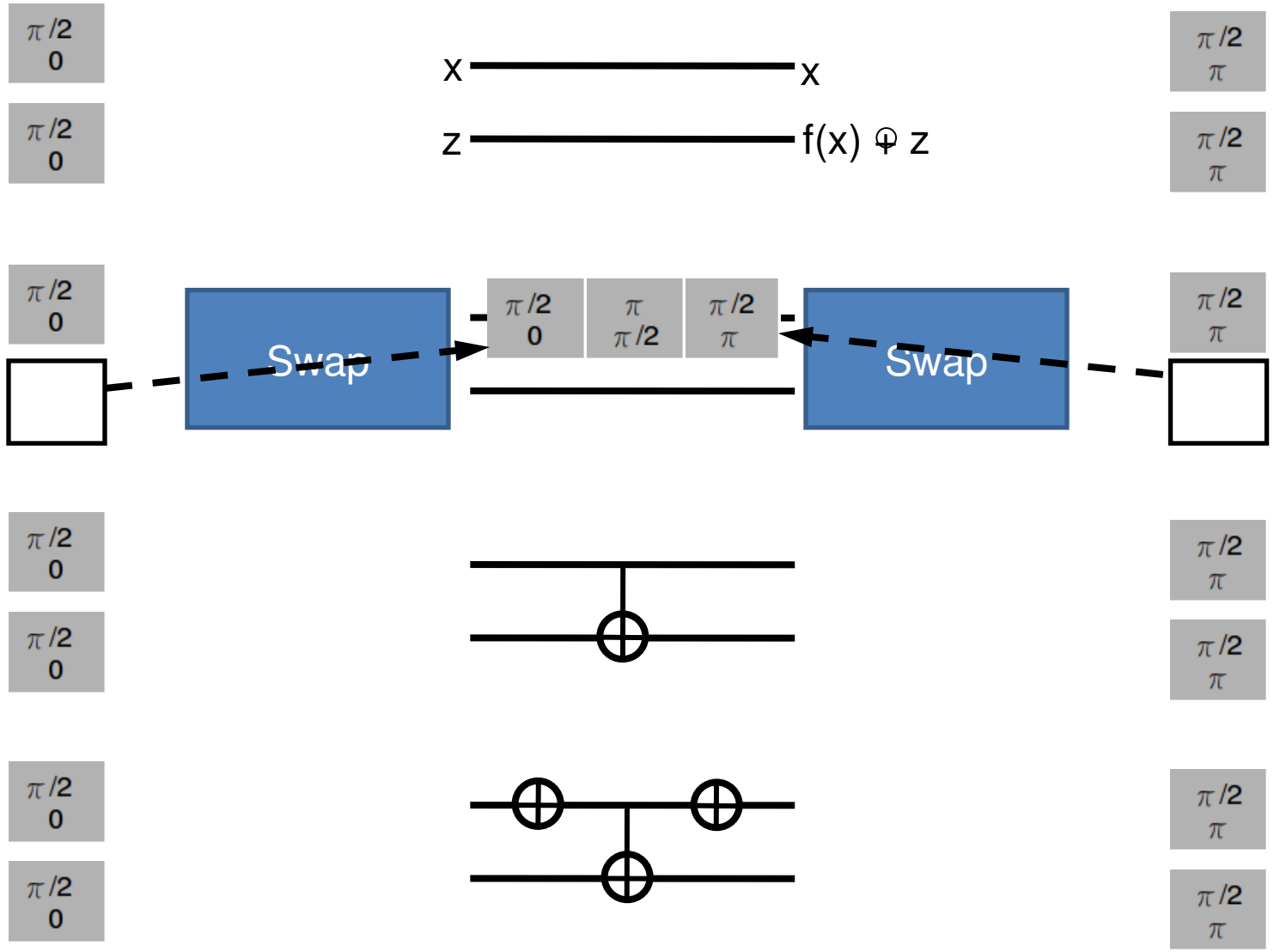
Deutsch Jozsa quantum circuit

Case	Logic	Quantum circuit	Matrix U_{fn}
f_1	*ID*		1000 0100 0010 0001
f_2	*NOT*		0100 1000 0001 0010
f_3	*CNOT*		1000 0100 0001 0010
f_4	*Z-CNOT*		0100 1000 0010 0001

Deutsch Jozsa: Realization

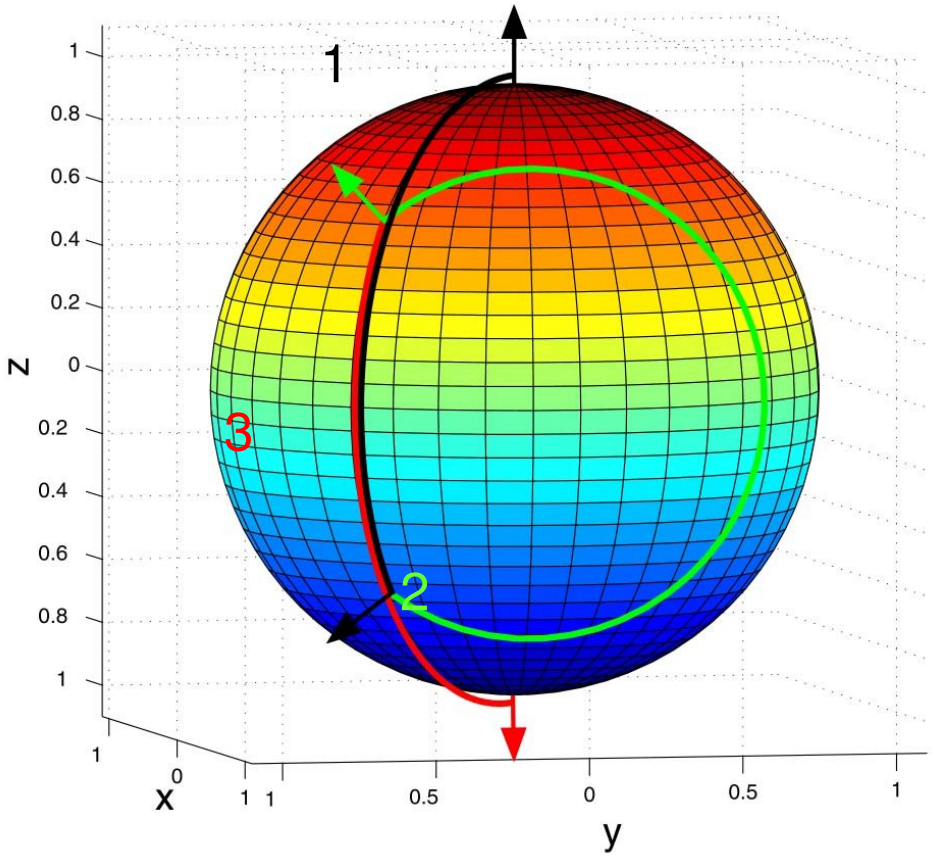


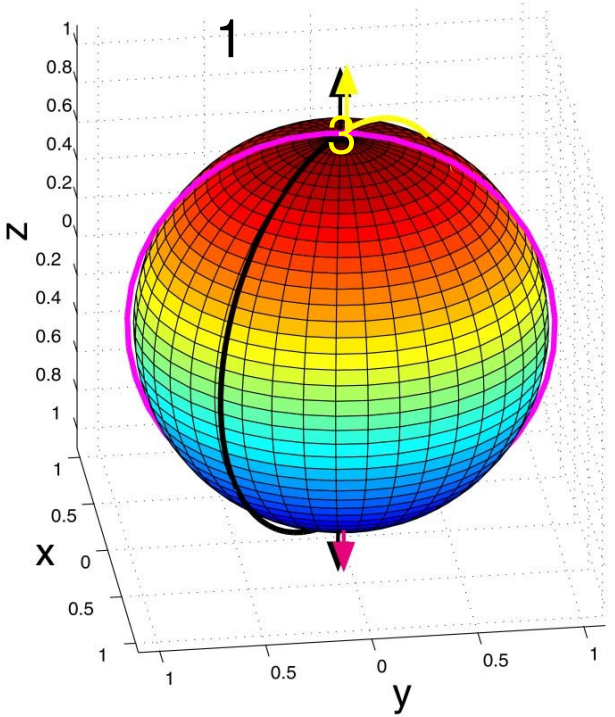
Deutsch Jozsa: Realization



3-step composite SWAP operation

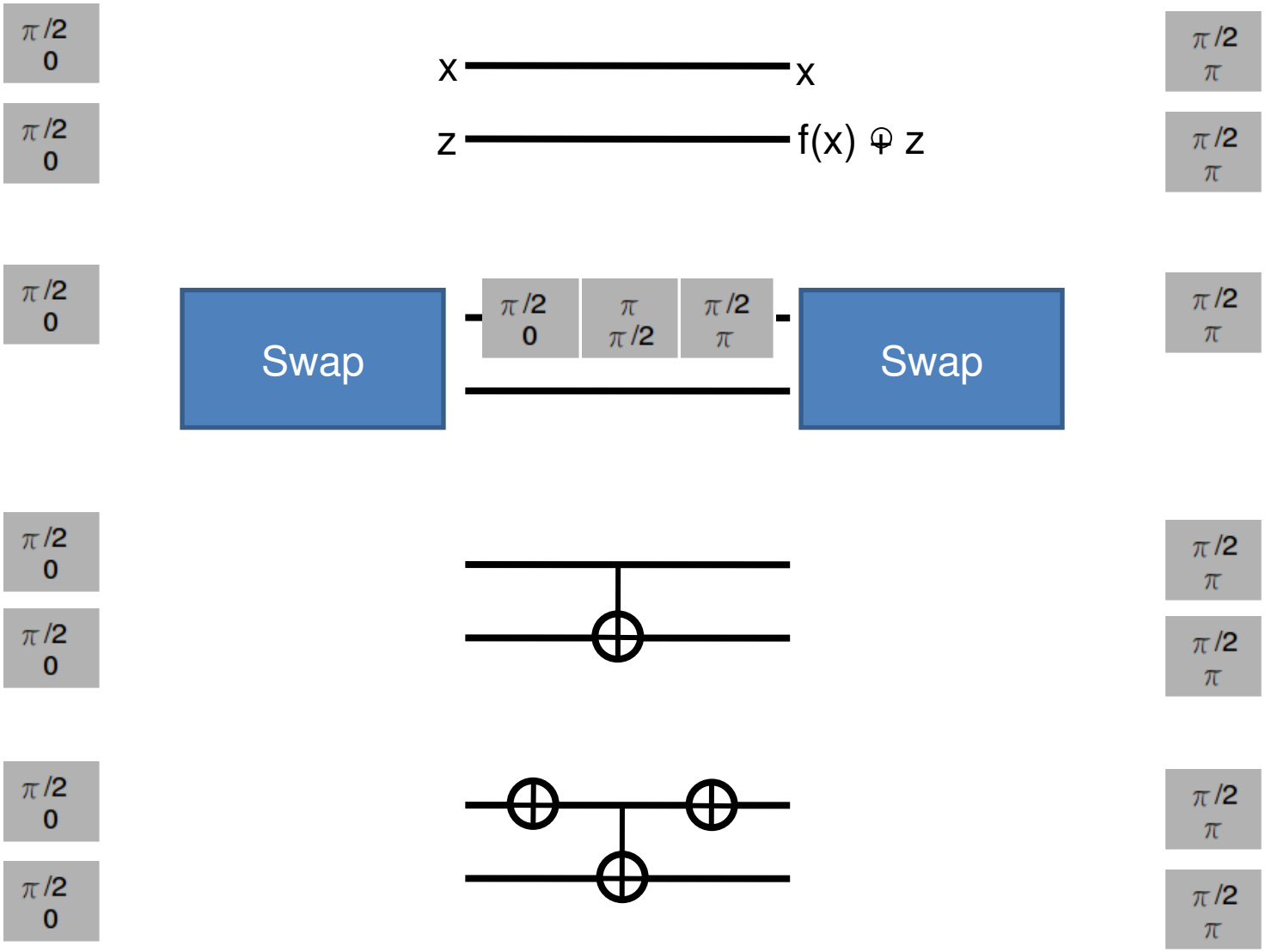
$$R^{+}\left(\frac{\pi}{\sqrt{2}},\pi\right)\,R^{+}\left(\frac{2\pi}{\sqrt{2}},\pi+\varphi_{\mathrm{swap}}\right)\,R^{+}\left(\frac{\pi}{\sqrt{2}},\pi\right)$$



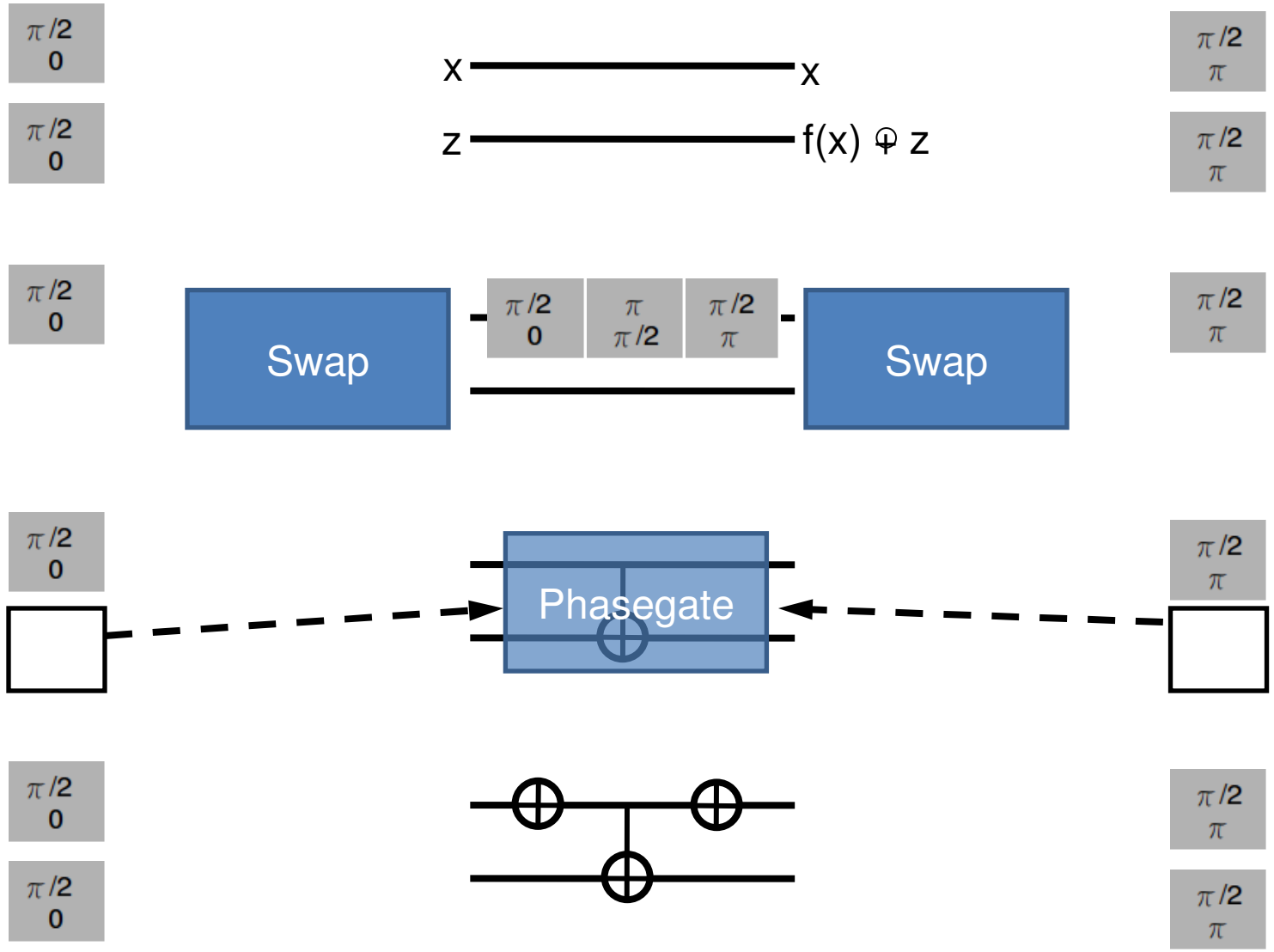


I. Chuang et al., Innsbruck (2002)

Deutsch Jozsa: Realization

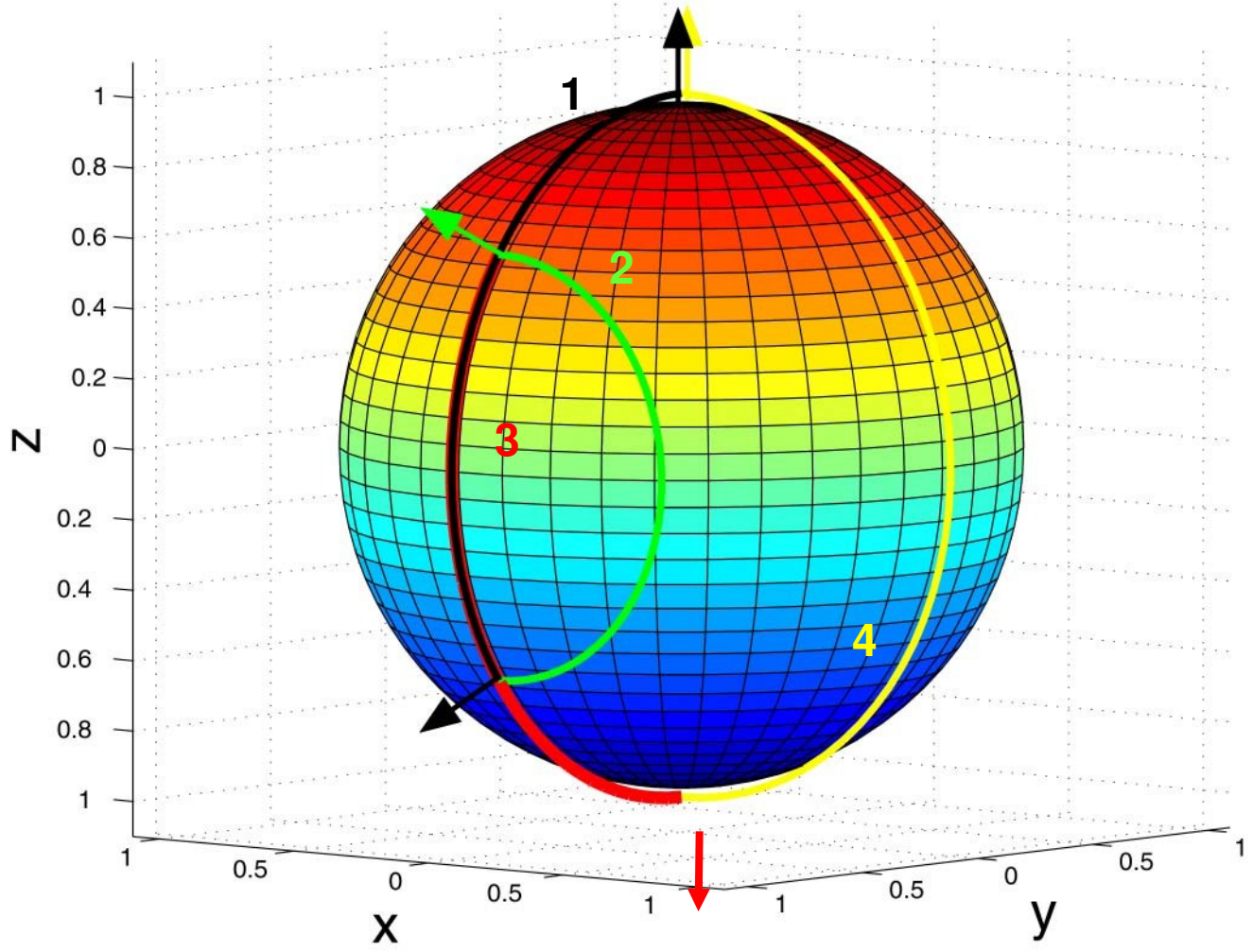


Deutsch Jozsa: Realization



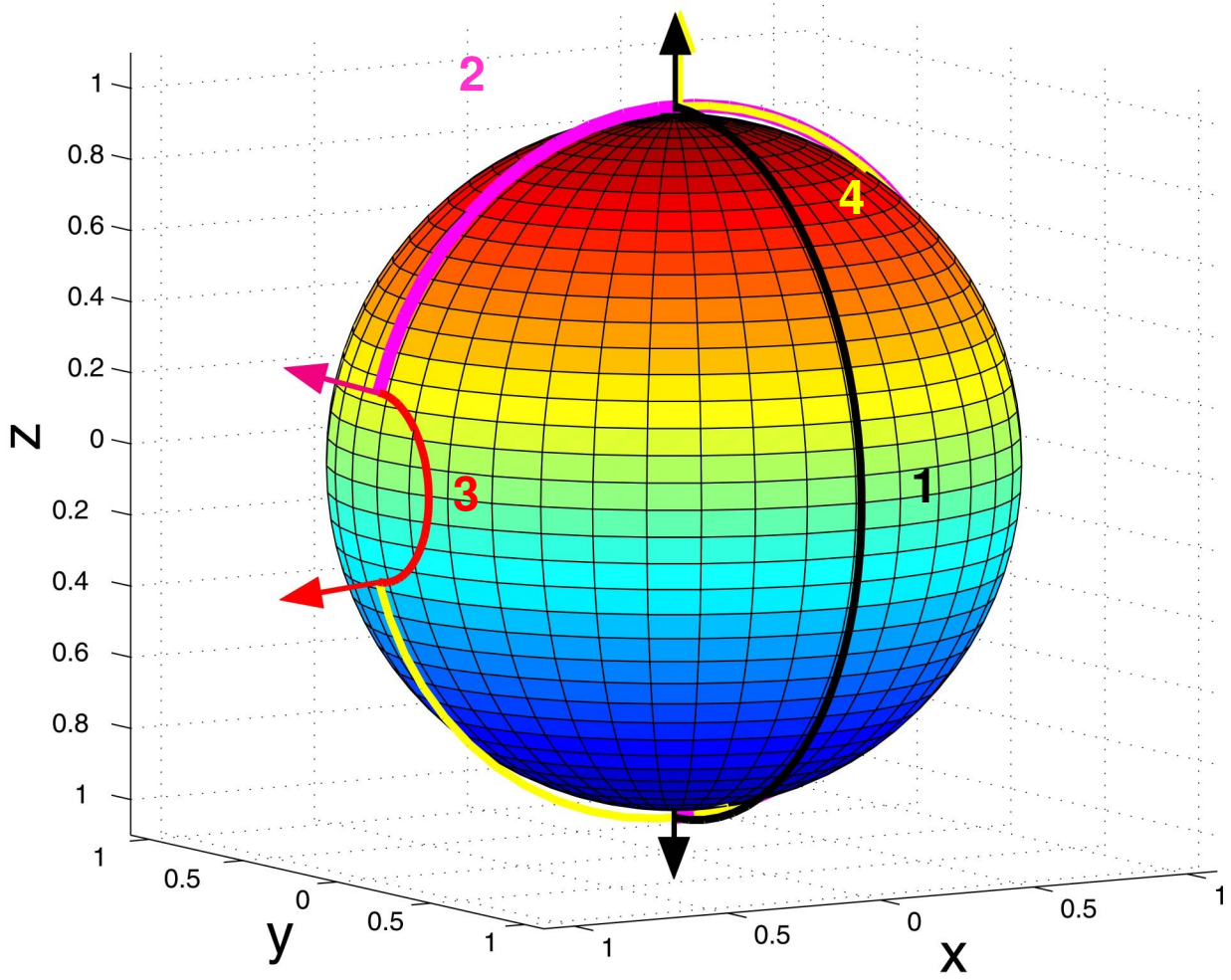
Composite phase gate (2π rotation)

$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$

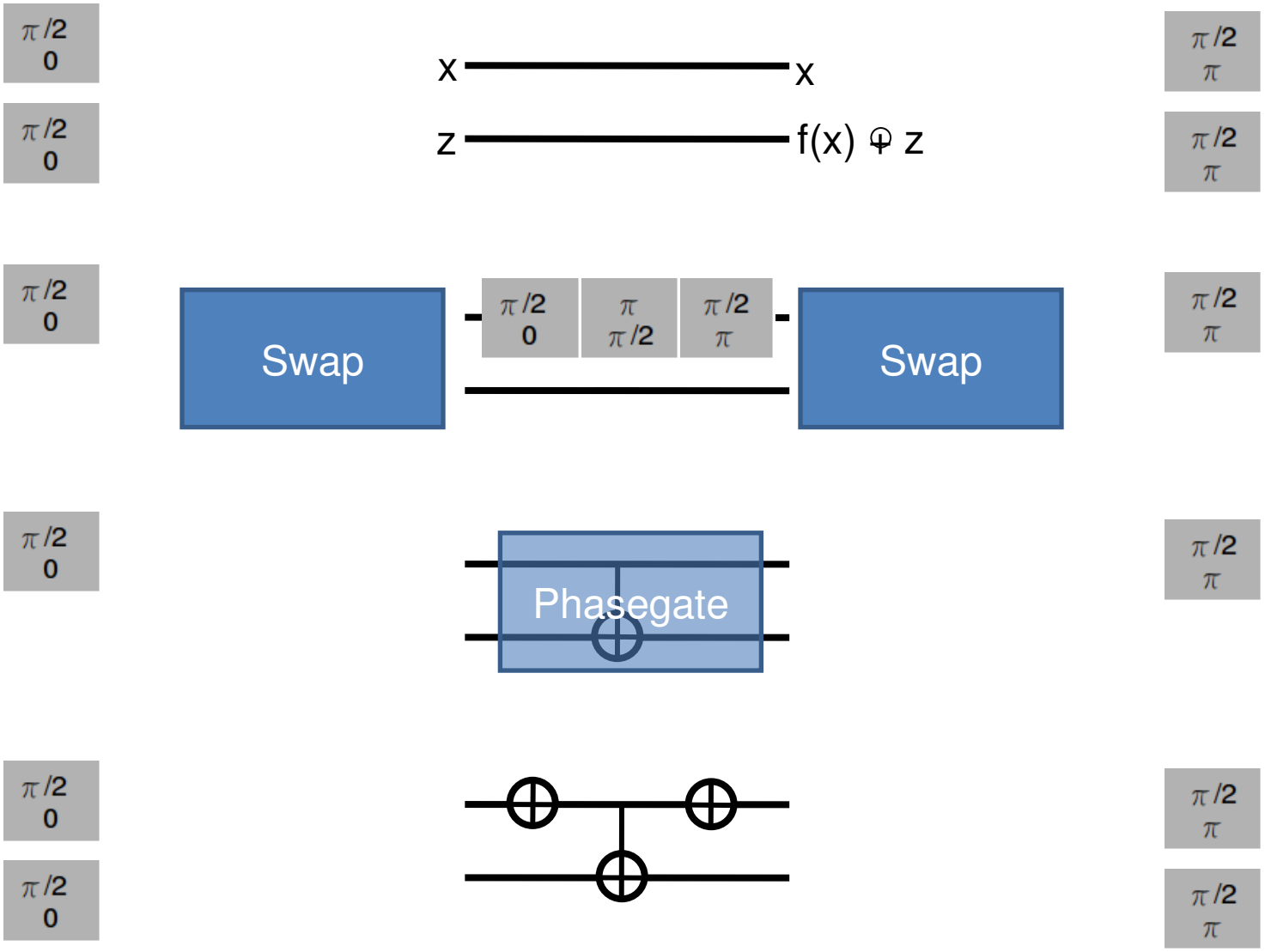


Action on |S,1> - |D,2>

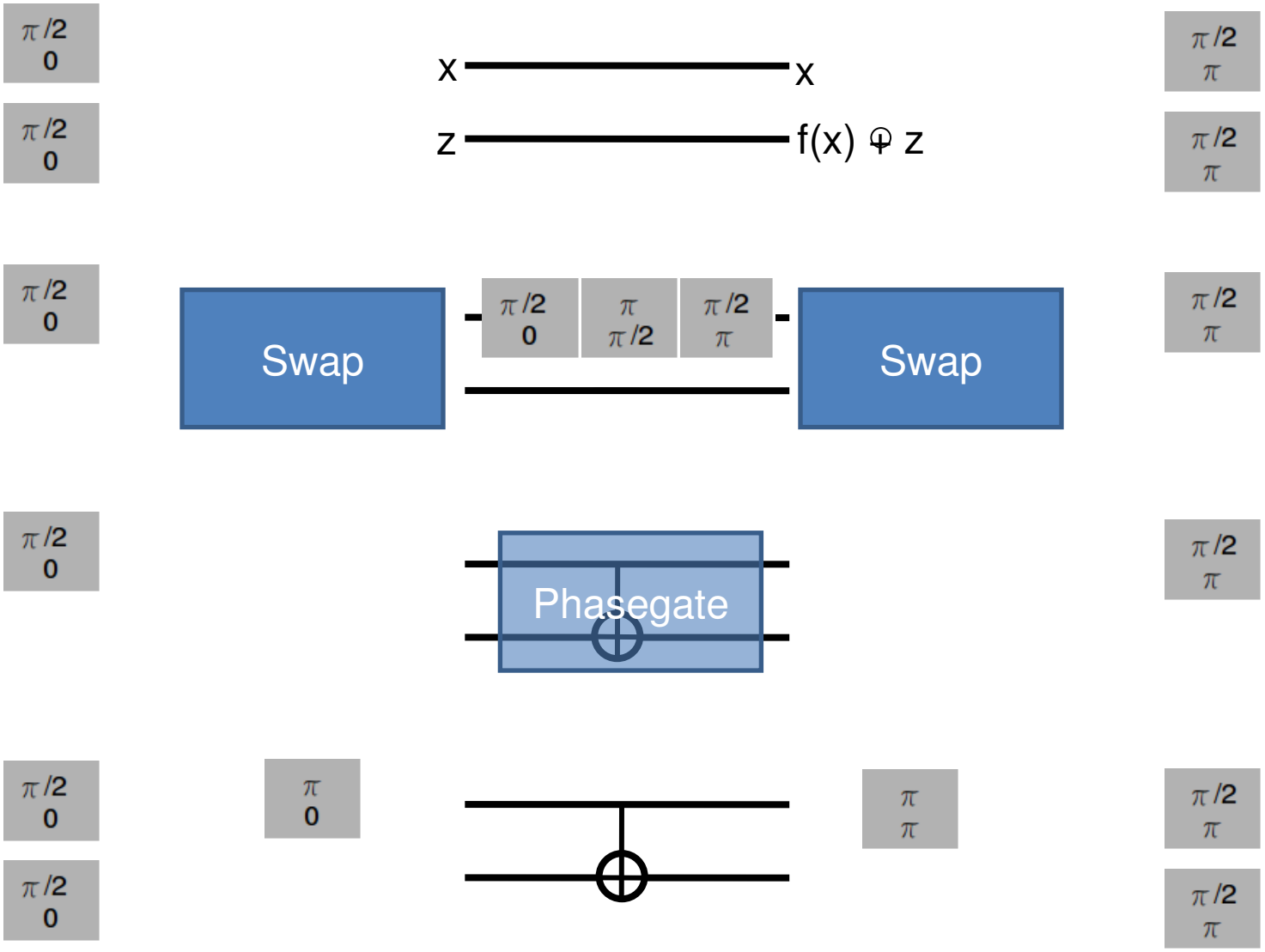
$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$



Deutsch Jozsa: Realization



Deutsch Jozsa: Realization



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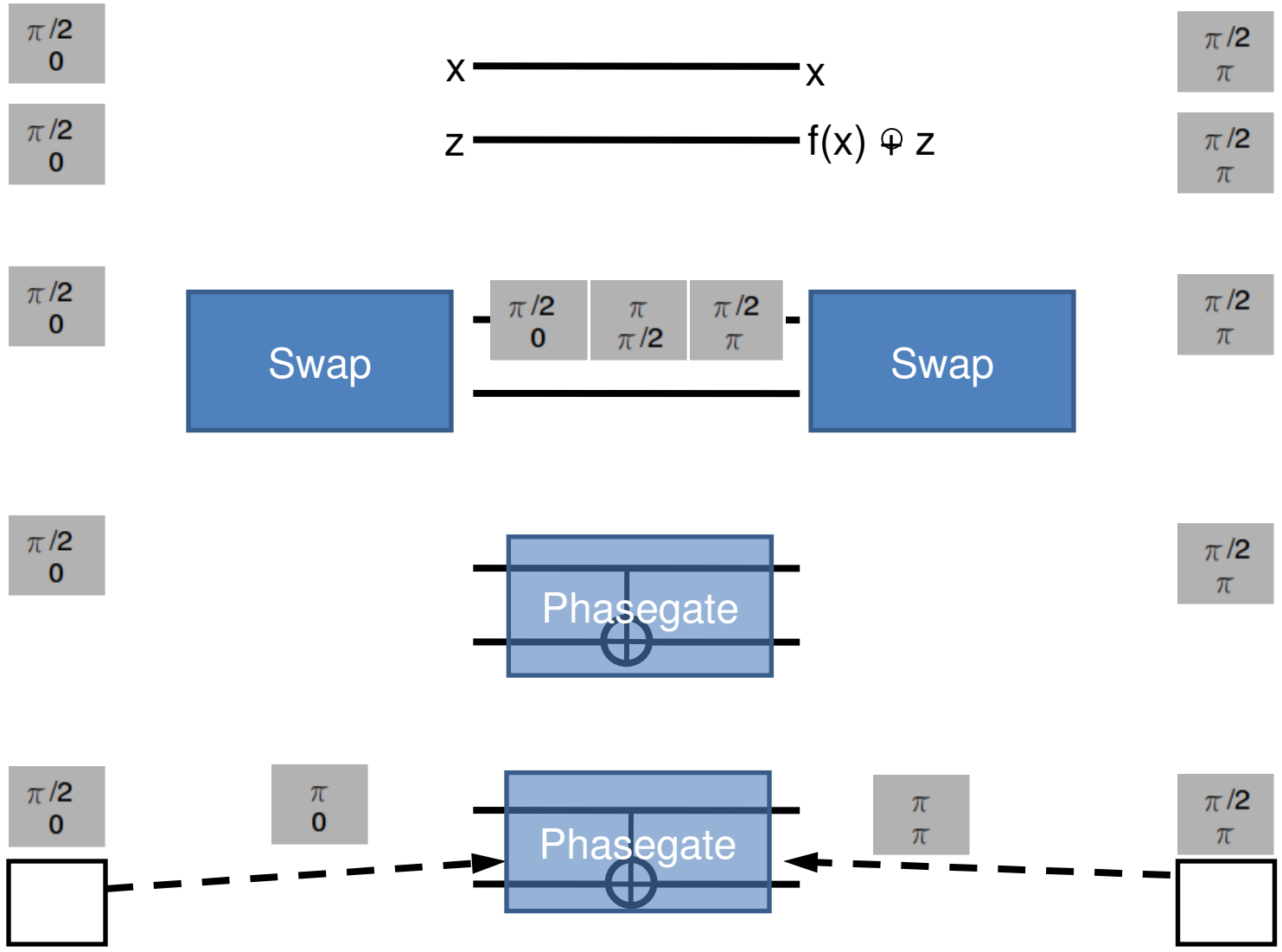

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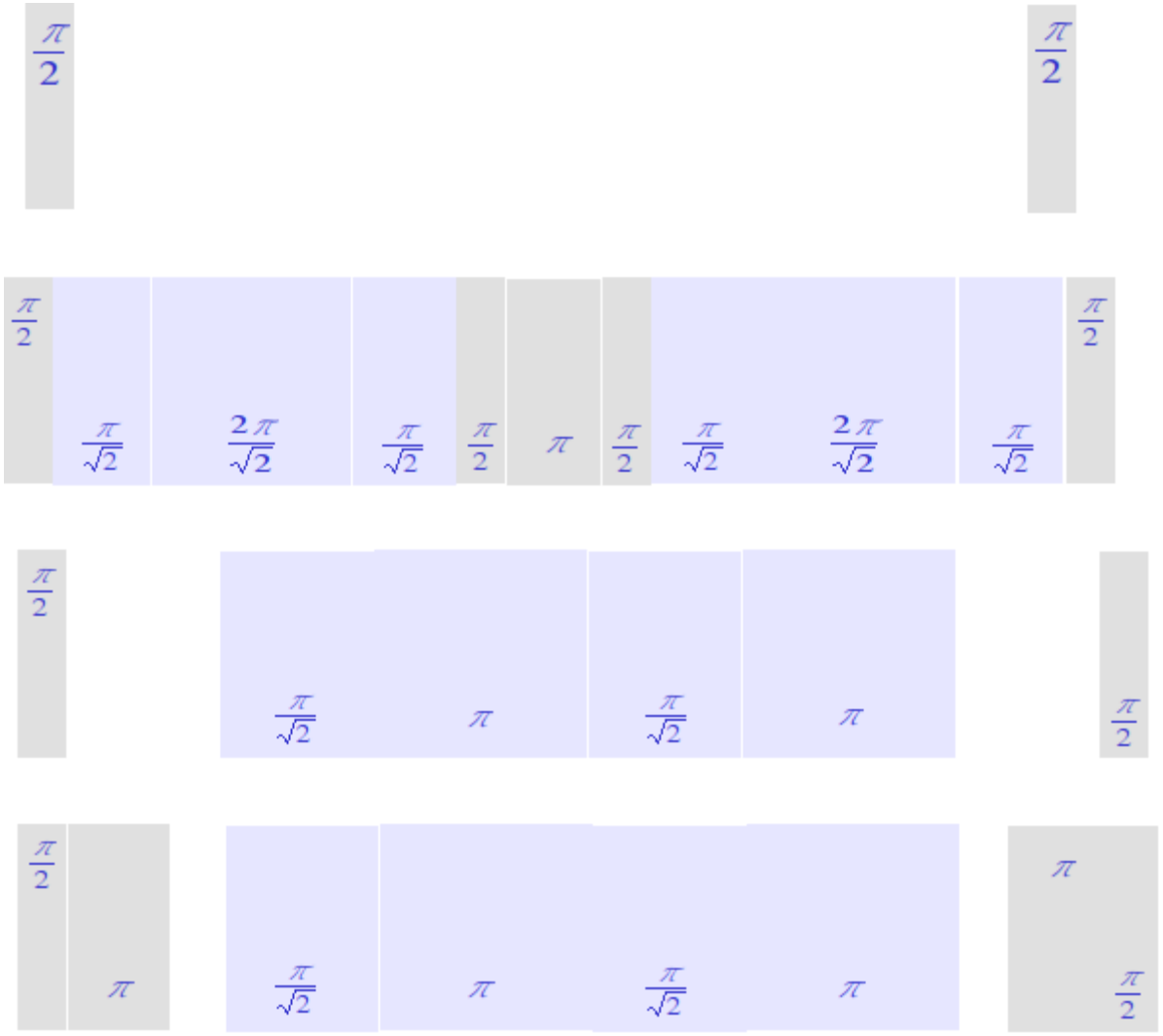

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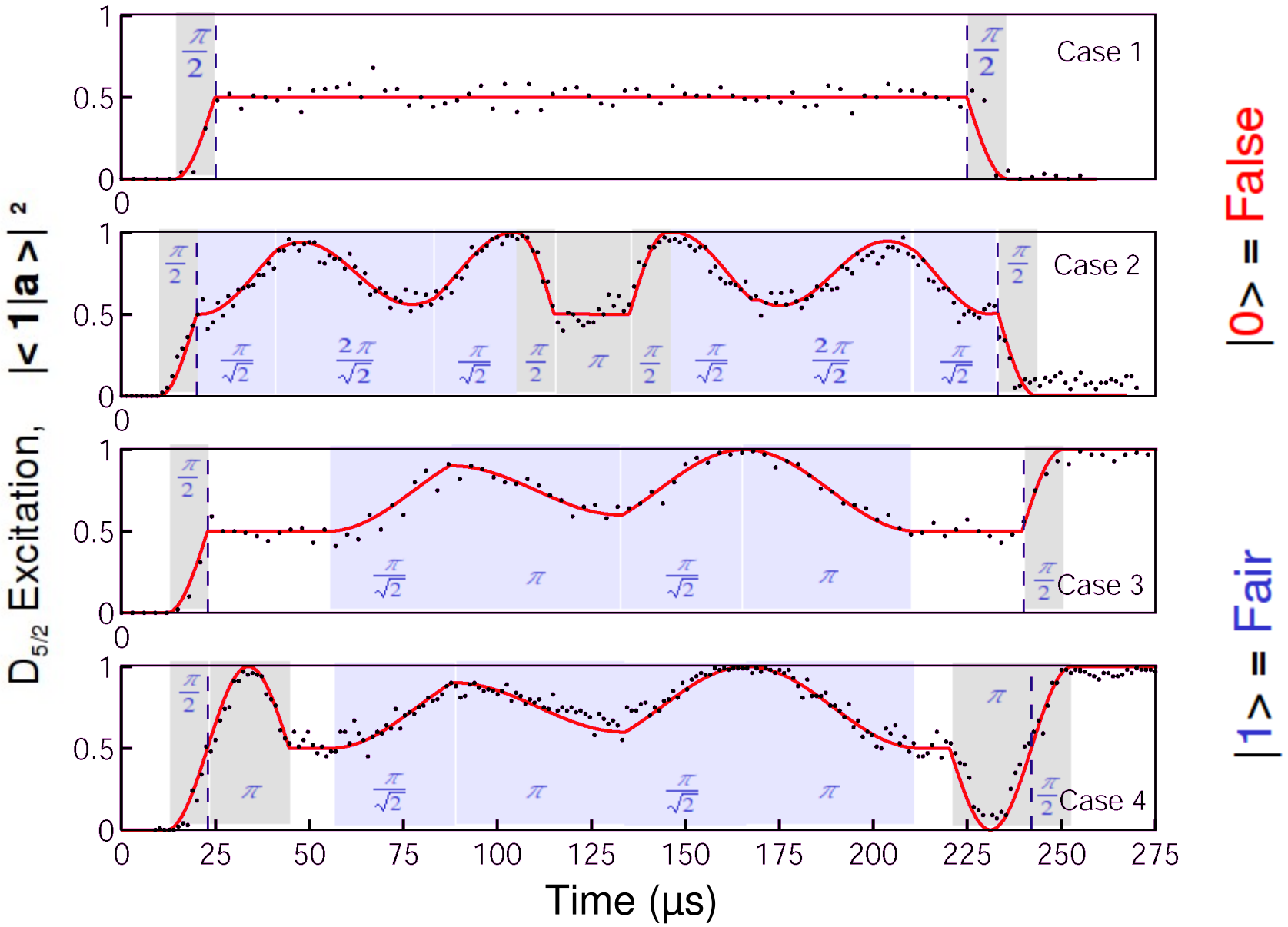

Deutsch Jozsa: Result

	Constant		Balanced	
	Case 1	Case 2	Case 3	Case 4
expected / $\langle 1\|a\rangle\|^2$	0	0	1	1
measured / $\langle 1\|a\rangle\|^2$	0.019(6)	0.087(6)	0.975(4)	0.975(2)
expected / $\langle 1\|w\rangle\|^2$	1	1	1	1
measured / $\langle 1\|w\rangle\|^2$	--	0.90(1)	0.931(9)	0.986(4)

S. Gulde et al., Nature 412, 48 (2003)

Conclusions

- Basics of ion trap quantum computing
- Measuring a density matrix
- Quantum gates
- Deutsch Algorithm

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Berkeley, Nov 25th 2008