Secure Quantum Cryptosystems

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Outline

- Protocols in quantum cryptography
- Error correction and privacy amplification
- Security proofs

Protocols

- Prepare-and-measure protocols:

1. BB84
2. six-state
3. two-state

- EPR protocols

Error correction and privacy amplification

Quantum Bit Error Rate (QBER)

$$\text{QBER} = \frac{\text{Number of (sifted) incorrect counts}}{\text{Number of overall (sifted) counts}}$$

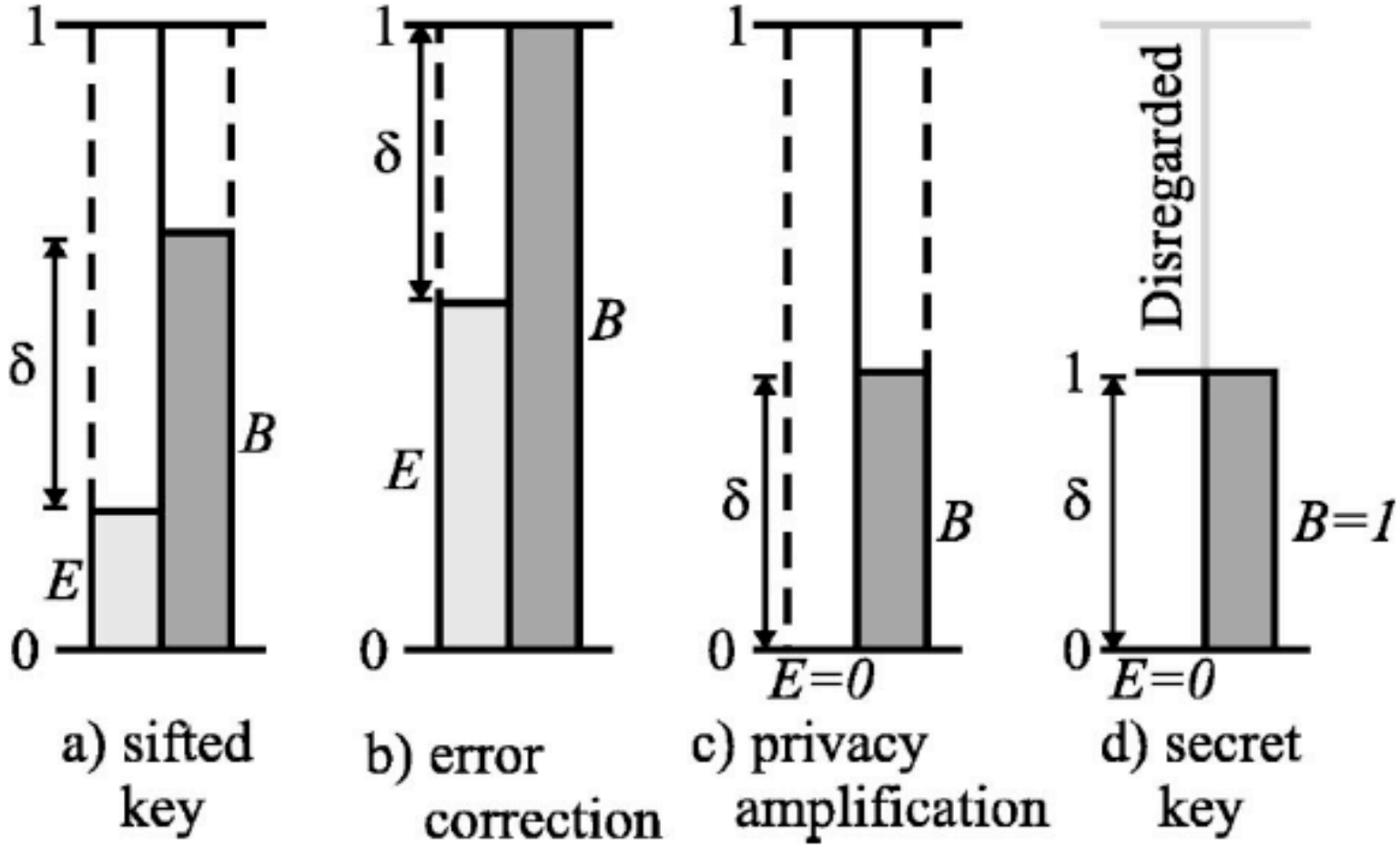
Ideas from Class cryptography

The outcome of their measurements provide Alice, Bob, and Eve random variables a, b, and e, respectively, with a joint probability distribution $P(a, b, e)$.

Theorem:

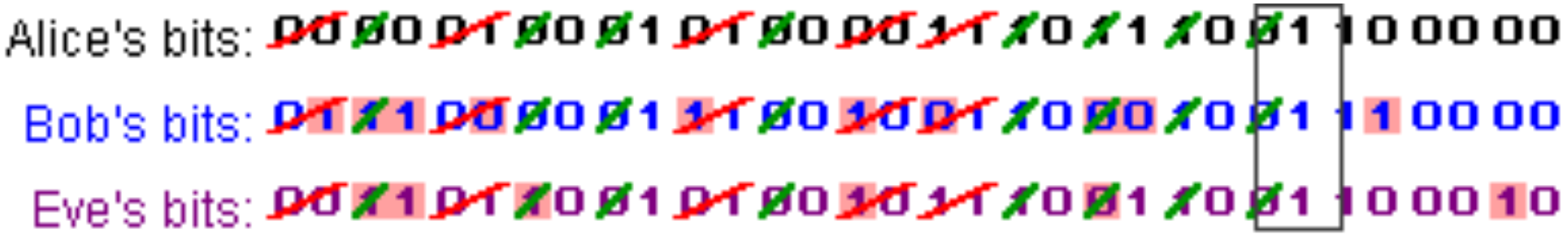
For a given P(a,b,e), Alice and Bob can establish a secret key (using only error correction and classical privacy amplification) if and only if $I(a, b) \geq I(a, e)$ *or* $I(a, b) \geq I(b, e)$, *where* $I(a, b)$ *is the mutual information between a and b.*

Error correction and privacy amplification



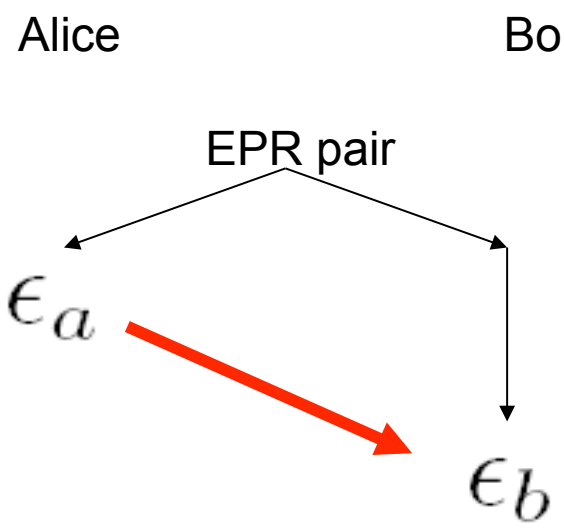
Advantage Distillation

In fact, Alice and Bob can still establish a secret key by using advantage distillation even if the conditions in the theorem are not satisfied.

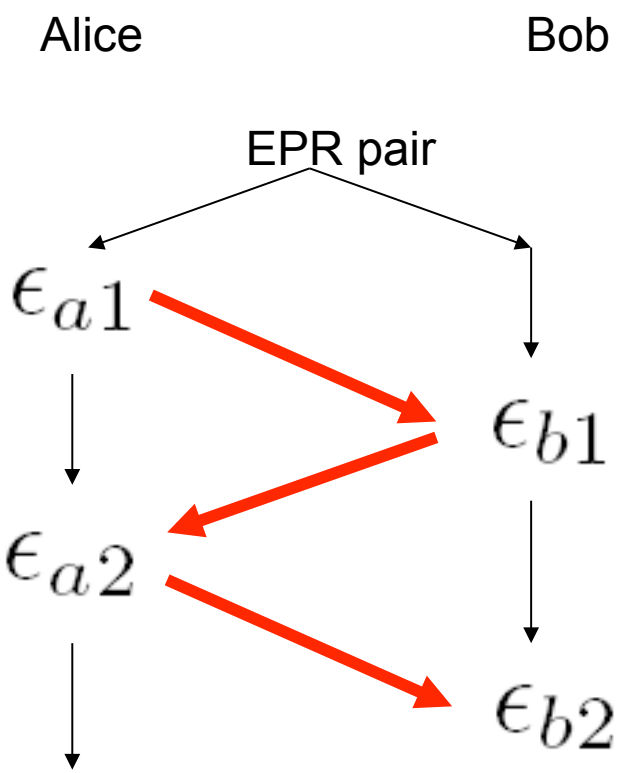


Quantum Privacy Amplification: Entanglement Purification Protocols (EPP)

One-way EPP



Two-way EPP



Security Proofs

Types of Attacks

- Individual attacks
- Joint attacks

Practical proofs vs. ultimate proofs

A conditional proof of the BB84

-Assuming individual attacks

$$I(a,b) = 1 + D\log_2 D + (1-D)\log_2(1-D)$$

$$I(a,e) + I(a,b) \leq 1.$$

$$I(a,b) \geq I(a,e) \Longrightarrow \text{QBER: } D \leq 11\%$$

Unconditional proofs

Assumptions:

- Alice and Bob have perfect photon generators and detectors
- Eve cannot access Alice and Bob's encoding and decoding devices.
- The random number generators used by Alice and Bob must be trusted and truly random
- The classical communication channel must be authenticated using an unconditionally secure authentication scheme.

A one-way EPP: the Modified Lo-Chau

1: Alice creates 2n EPR pairs in the state (|00>+|11>)/sqrt(2).
2: Alice selects a random 2n bit string b, and performs a Hadamard transformation on the second half of each EPR pair for which b is 1.
3: Alice sends the second half of each EPR pair to Bob.
4: Bob receives the qubits and publicly announces this fact.
5: Alice randomly selects n of the 2n encoded EPR pairs to serve as check bits to test for Eve's interference.
6: Alice announces the bit string b, and which n EPR pairs are to be check bits.
7: Bob performs Hadamards on the qubits where b is 1.
8: Alice and Bob each measure their halves of the n check EPR pairs in the Z basis and share the results. If more than t of these measurements disagree, they abort the protocol.
9: Alice and Bob measure their remaining n qubits according to the same check matrix for a pre-determined [n,m] CSS quantum code correcting up to t errors. Alice sends the error syndromes to Bob, and they transform their states so as to obtain m nearly perfect EPR pairs.
10: Alice and Bob measure the EPR pairs in the Z basis to obtain a shared secret key.

Shannon's Bound:

- CSS codes exist with asymptotic rate :

$$k/n = 1 - 2H(t/n)$$

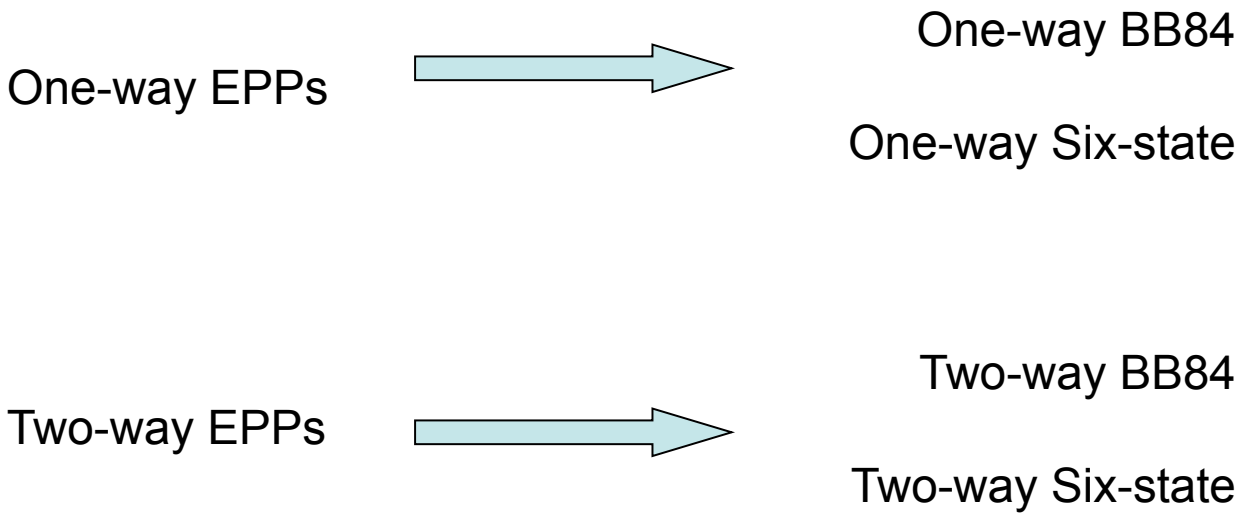
where $H(p) = -p \log p - (1-p) \log (1-p)$.

$1 - 2H(t/n) > 0$  $t/n <= 11\%$

The BB84 Protocol Reduced from the Modified Lo-Chau

1: Alice creates $(4 + \delta)n$ random bits.
2: Alice chooses a random $(4 + \delta)n$-bit string b. For each bit, she creates a state in the Z basis (if the corresponding bit of b is 0) or the X basis (if the bit of b is 1).
3: Alice sends the resulting qubits to Bob.
4: Bob receives the $(4+\delta)n$ qubits, measuring each in Z or X basis at random.
5: Alice announces b.
6: Bob discards any results where he measured a different basis than Alice prepared. With high probability, there are at least 2n bits left (if not, abort the protocol). Alice decides randomly on a set of 2n bits to use for the protocol, and chooses at random n of these to be check bits.
7: Alice and Bob announce the values of their check bits. If more than t of these values disagree, they abort the protocol.
8: Alice announces u + v, where v is the string consisting of the remaining non-check bits, and u is a random codeword in C1.
9: Bob subtracts u + v from his code qubits, (v + error), and corrects the result, (u + error), to a codeword in C1.
10: Alice and Bob use the coset of u + C2 as the key.

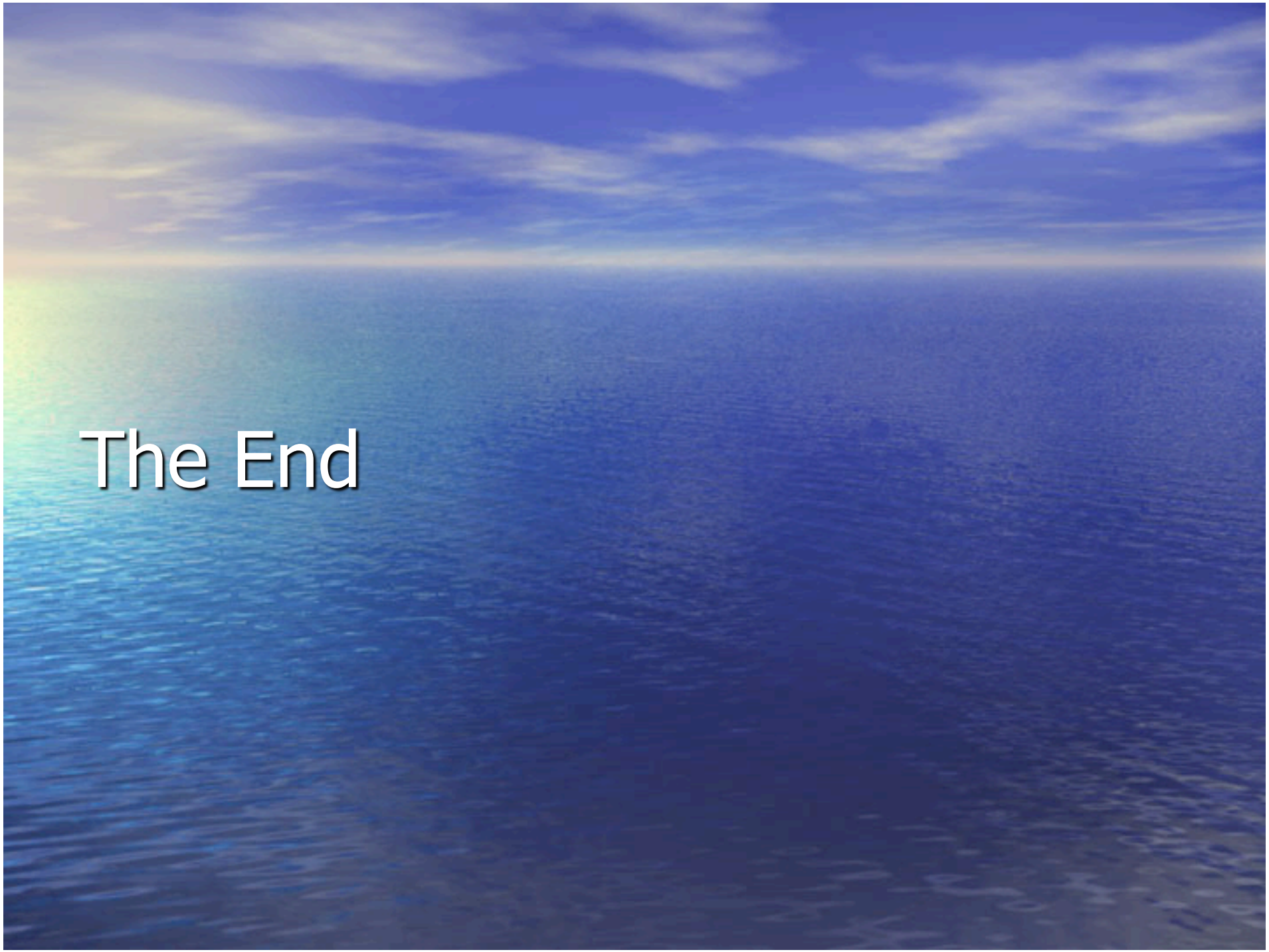
EPP to prepare-and-measure reductions



Bounds on QBER

The BB84 Protocol		
	one-way	two-way
Upper bound	14.6%	1/4
Lower bound	11.0%	18.9%

The six-state Protocol		
	one-way	two-way
Upper bound	1/6	1/3
Lower bound	12.7%	27.6%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