Topological Quantum Computing

C191 Final Presentation

by

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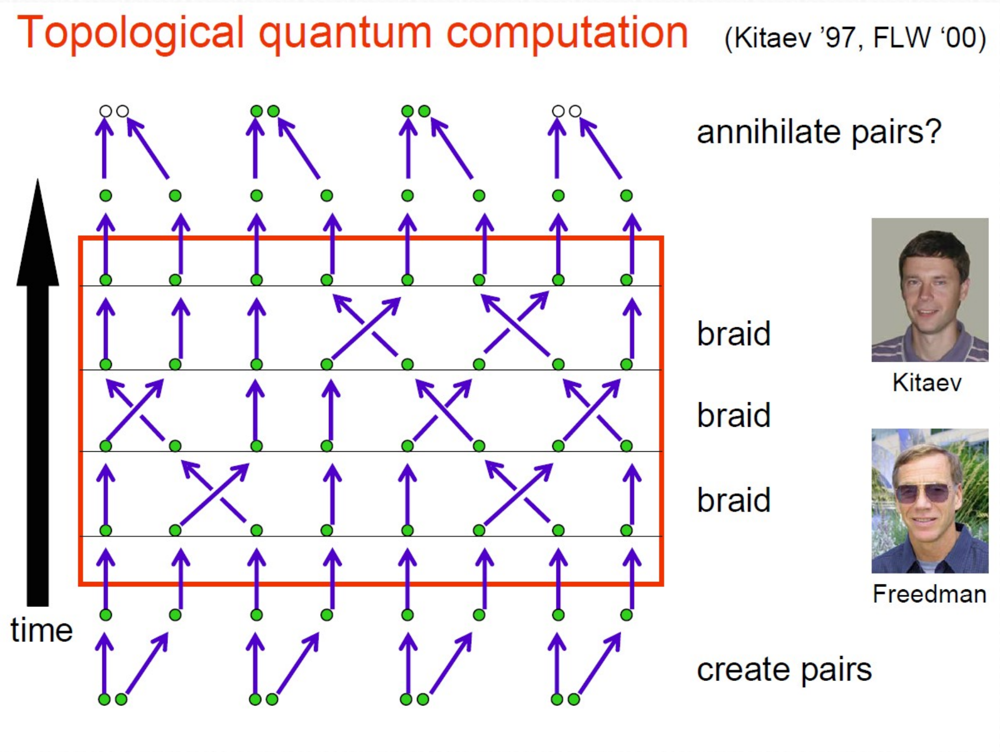
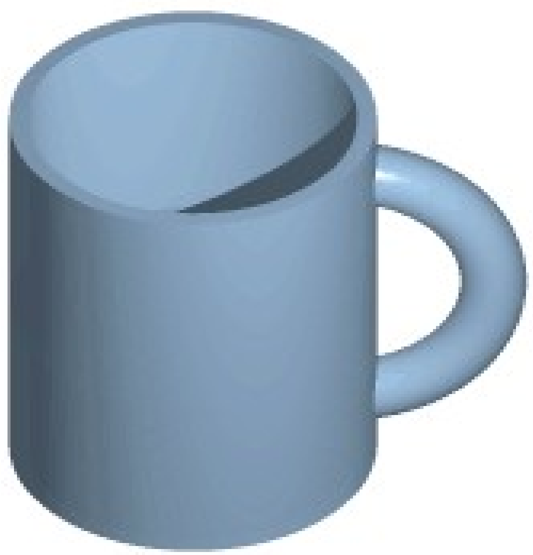
What is it?

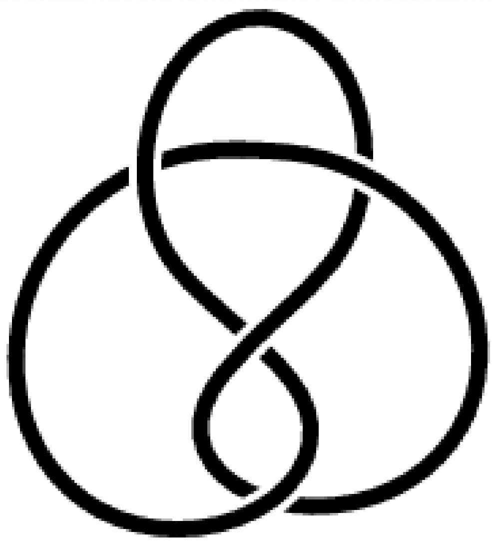
- New paradigm of fault-tolerant quantum computing
- For efficient quantum computing: error rate 10^{-4} (still not achieved) $\rightarrow$ error correction algorithms
- Possible estimated error rate as low as 10^{-30}!!!
- No need for error correction.



What studies Topology?

Properties that are not changed by smooth deformations

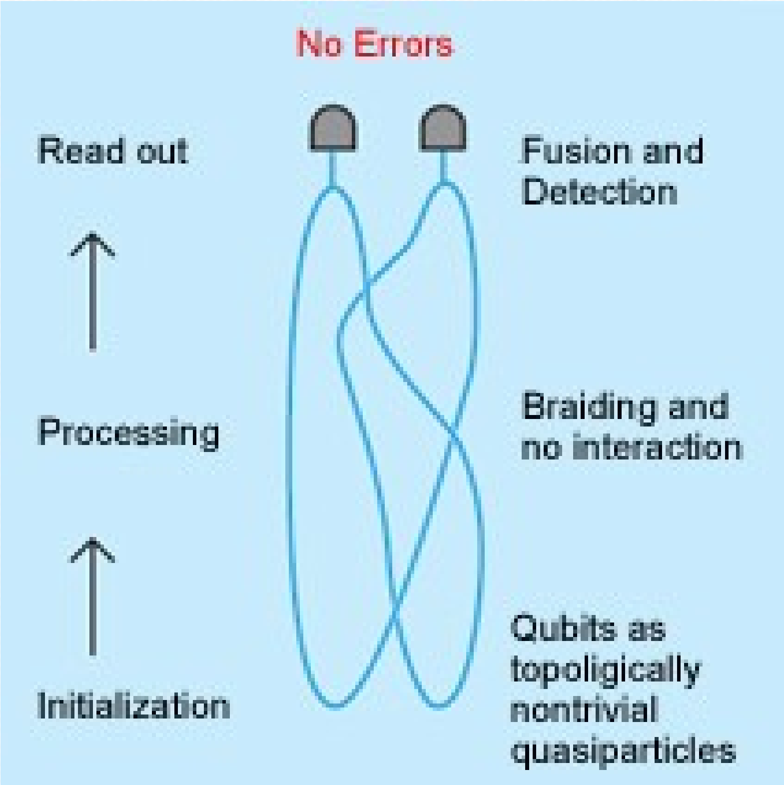




Topological properties are robust against small perturbations

The basic idea of TQC

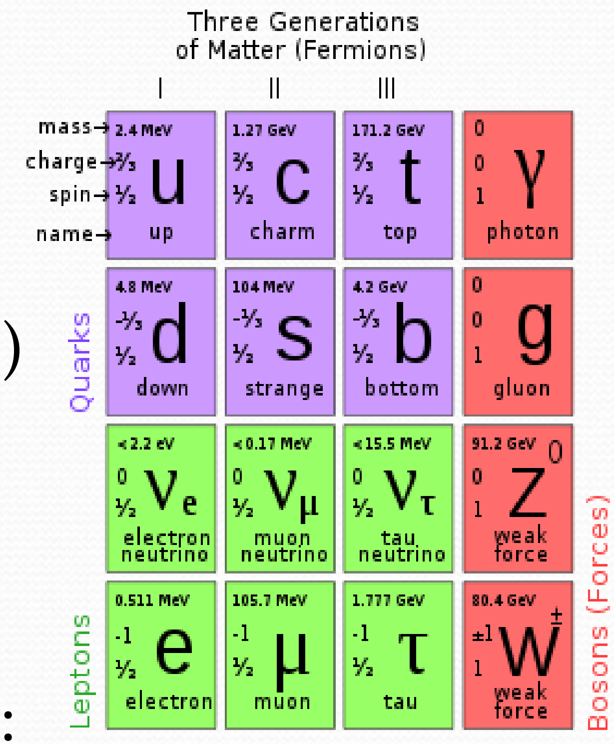
- Create particles from vacuum (initialization)
- Thread their world line (unitary operations)
  This step is not based on interactions between the two particles
- Measure the result (measure)



Physics Particles

- Fermions - Particles of Matter:
  - Elementary: Up and Down Quarks,
    Electrons, Muons
  - Composite: Protons, Neutrons (3 quarks)
    - Obey Fermi-Dirac Statistics
    - Have half-integer spins (½ for electron)
    - No two can be in exactly same state

    (Pauli Exclusion Principle)



- Bosons (Particles of Force for Elementary):
  - Elementary: Photons, Gluons
  - Composite: Mesons (quark+antiquark), nucleus like He^4
    - Obey Bose-Einstein Statistics
    - Have integer spins
    - Two can be in same state

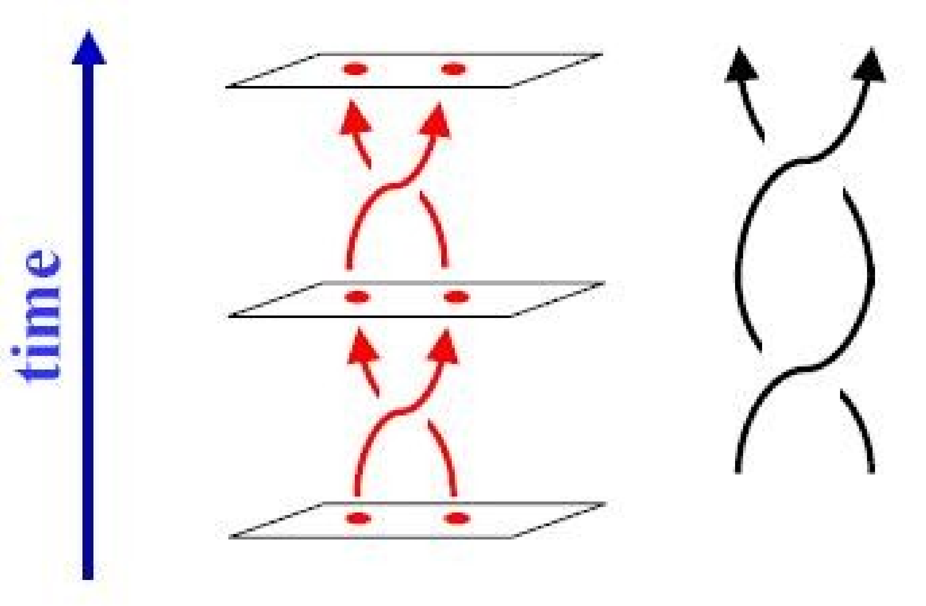
Bosons, fermions, anyons

- If we exchange two fermions: $|\psi\rangle$ becomes $-|\psi\rangle$

  Single particle properties: unchanged, but it interferes differently with other particles.
- 3D: only Bosons and fermions
- 2D: we could have also anyons, they can acquire any complex phase $a+ib = \exp(i\pi\theta)$

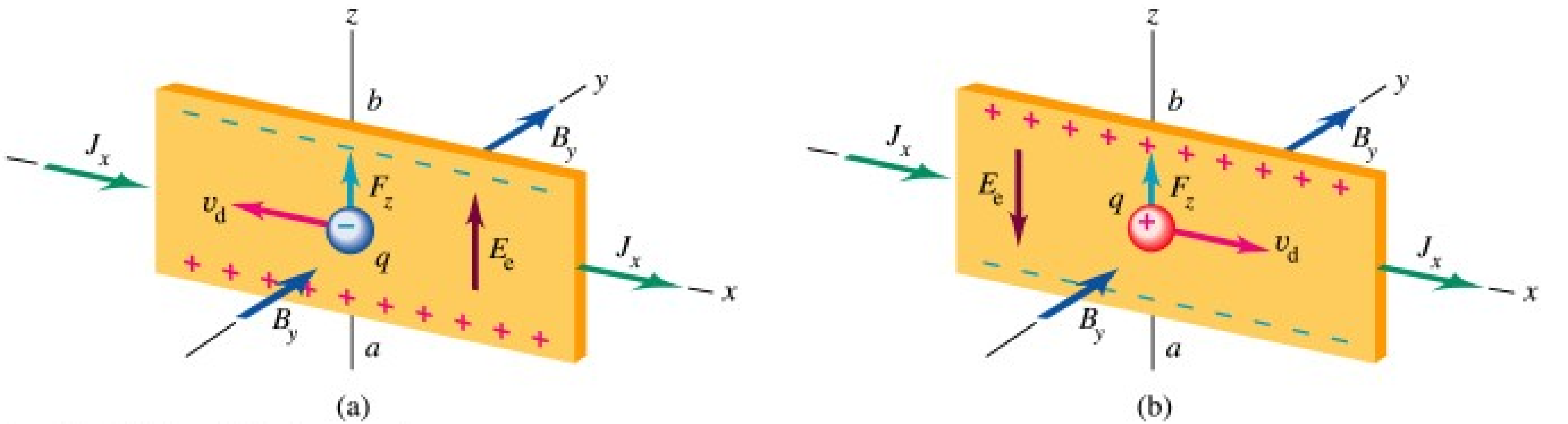
  ANY+ons
- θ depends on the kind of particle and it's fixed
- We may have instead of just a phase, an unitary U

Braiding in 2+1 dimensions



Clockwise swapping ≠ counterclockwise swapping
Classified by winding number (a topological invariant)

Classical Hall Effect



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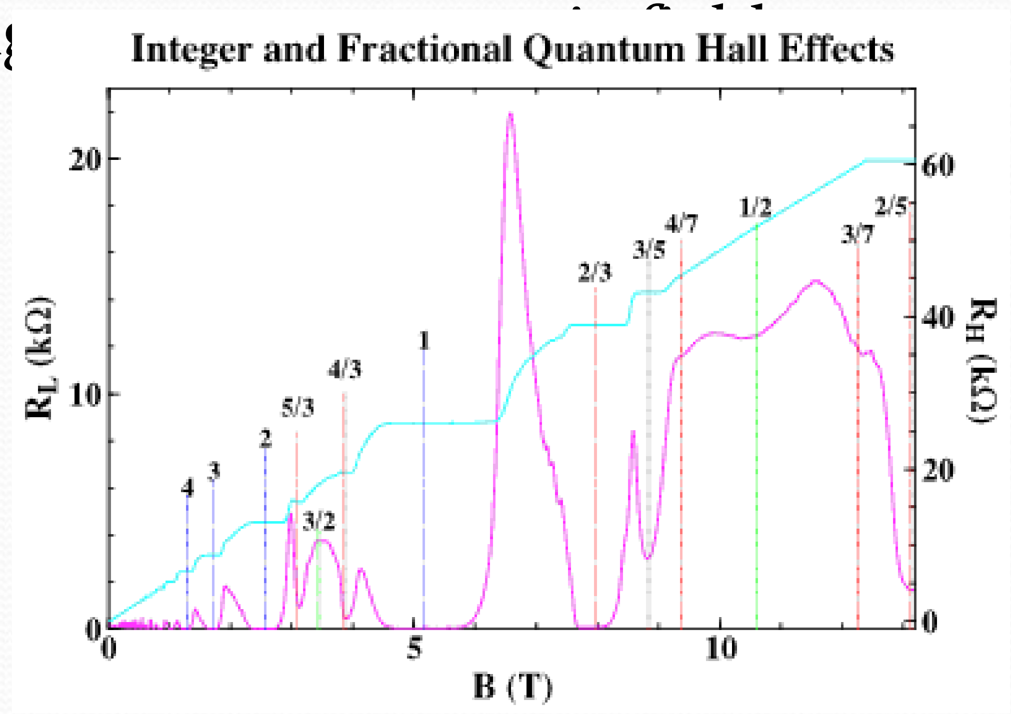
Fractional Quantum Hall Effect

- Electron gas at the interface in a GaAs heterojunction,
- T = 10 mK, strong

J=σE

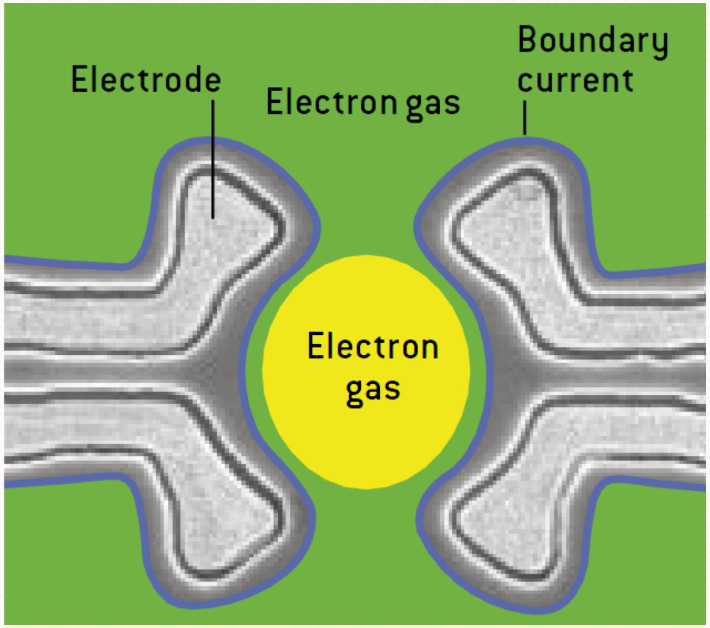
$$\sigma = \nu \frac{e^2}{h}$$



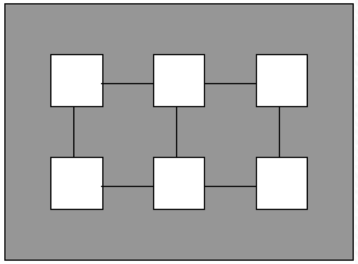
Experimental confirmations

- FQH collective excitations are quasiparticles such that the ratio f between electrons and magnetic flux quanta is a fractional number
- If we circle a f=1/3 particle around a f=2/5 particle we can find a relative statistic of $\theta = -1/15$
- So, they behave like anyons!

  (Goldman,2006)



Lattice of Abelian Anyons



- Physical system for complete quantum computation are abelians anyons on a lattice (they could be quantum Hall effect excitations)
- Lattice site: $|+\rangle_j$ is occupied by anyon,

  $|-\rangle_j$ is not occupied
- b_j $b_j^\dagger$ are creation and annihilation operators.

  $A_j = b_j^\dagger b_j$ applied on $|-\rangle_j$ gives 0, while $|+\rangle_j$ is the eigenvector with eigenvalue 1
- $B_{jk} = b_j^\dagger b_k + b_j^\dagger b_k$ used to swap the states in j and k.
- Anyon circled around another anyon gets factor $e^{i\Theta}$

Qubits and 1-qubit Gates

- Qubit is a combination of an anyon-occupied site and a unoccupied one on sites j and j': $|0\rangle_j = |-+\rangle_{jj'}$ $|1\rangle_j = |+-\rangle_{jj'}$
- This "number operator" generates rotation on z-axis $A_j = |1\rangle_j\langle 1| = (\sigma_z^j + 1)/2$
- Swapping allows to do $\sigma_x = B_{jj'} = |0\rangle\langle 1| + |1\rangle\langle 0|$
- By applying the hamiltonian $B_{jj'}$ for a certain time we can generate the rotation $\exp(i\theta\sigma_x/2)$ around the x axis. Applying A_j will generate the rotations around the z axis. The A_j and B_{jk} do not involve interactions with other anyons
- Consequently with rotations about X and Z axis we have a full set of 1-qubit operations

Universal Set of Gates

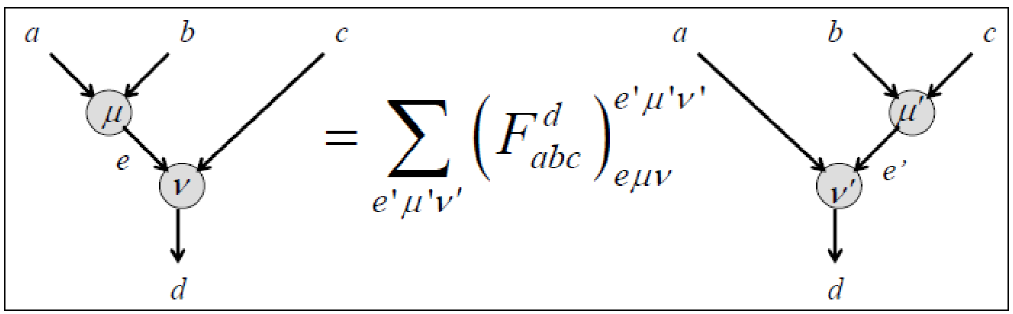
- Two qubit control gate for $|x\rangle_j\ |y\rangle_k$ is done by repeated swaps - we circle the content of the first site of j qubit around the first site of the k qubit.
- The phase of -1 for state of two quibits is obtained if and only if the second sites of j and k both contain anyons corresponding to $|1\rangle = |-+\rangle$ giving CPHASE gate:

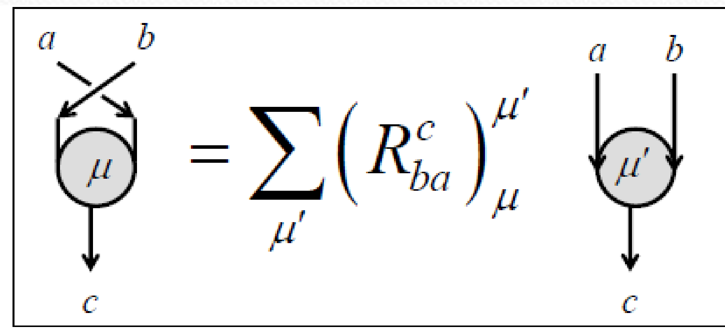
$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |10\rangle$$
$$|11\rangle \rightarrow e^{i\phi}|11\rangle$$

- The size of the orbits used when circling is unimportant: the phase is a topological effect, not due to interactions.
- We have a universal set of quantum gates!
- 1 qubit operations: local $\longrightarrow$ unprotected
- 2 qubit operations: topologically protected

Non Abelian Anyons

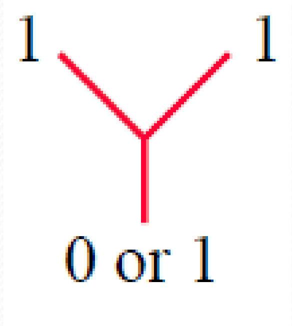
- For Abelian anyons 2 qubit operations are topologically protected from errors but single qubit operations are local and unprotected
- To have fully protected operations we need Non-Abelian anyons. They operate on fusion spaces (interaction of 3 or more anyons) and acquire a unitary instead of a phase



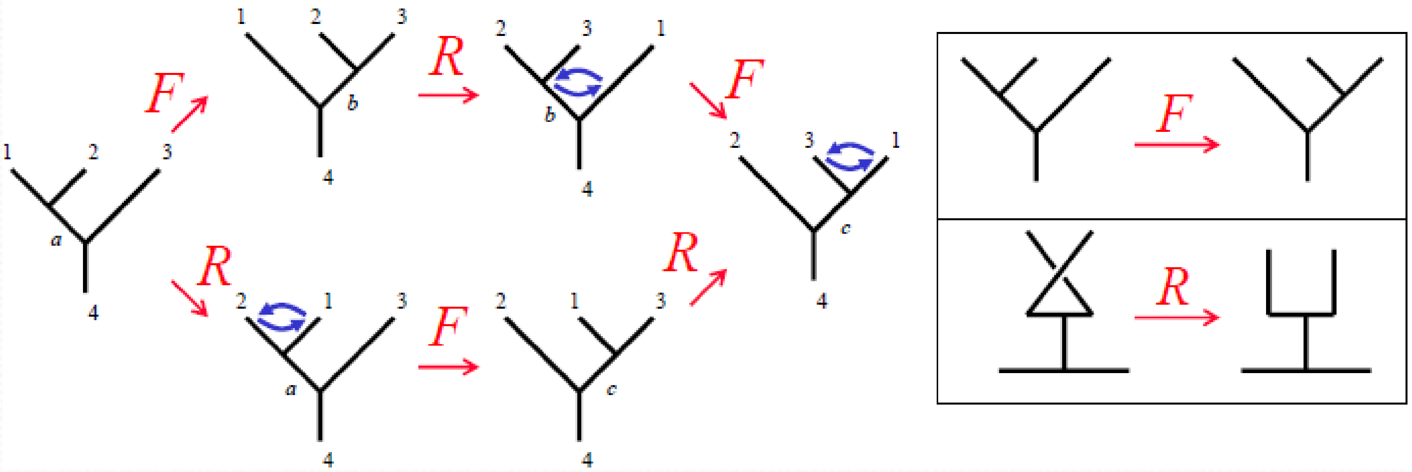


- The “braid group” is generated by moving up/down every thread
- With opportune generators, we can build a dense subset of SU(4) and SU(2): topologically protected single and two qubit operators!

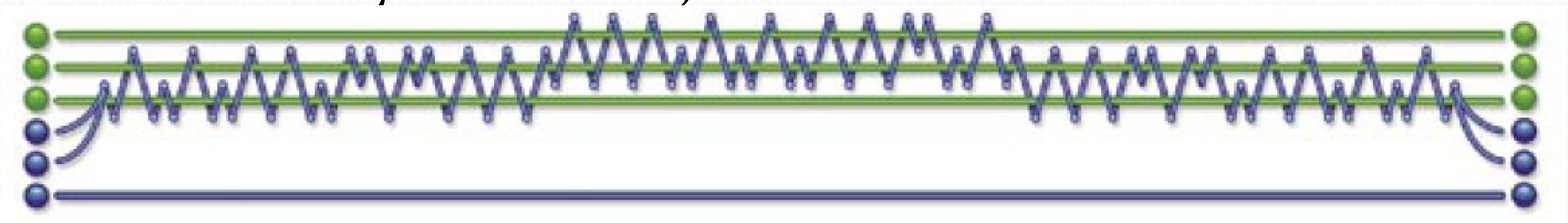
Fibonacci Model



- A simplest of Non-Abelian anyons models is Yang-Lee (Fibonacci) model.
- Two anyons can fuse in either of two ways i.e. 1 x 1 = 0 + 1.



- The resulting Hilbert space has dimensions that are Fibonacci numbers. Qubits encoded in one anyon: $\log_2 \phi = \log_2\left[\left(1+\sqrt{5}\right)/2\right] = \log_2(1.618) = .694$
- Fibonacci anyons CNOT, accurate to 10^{-3} :

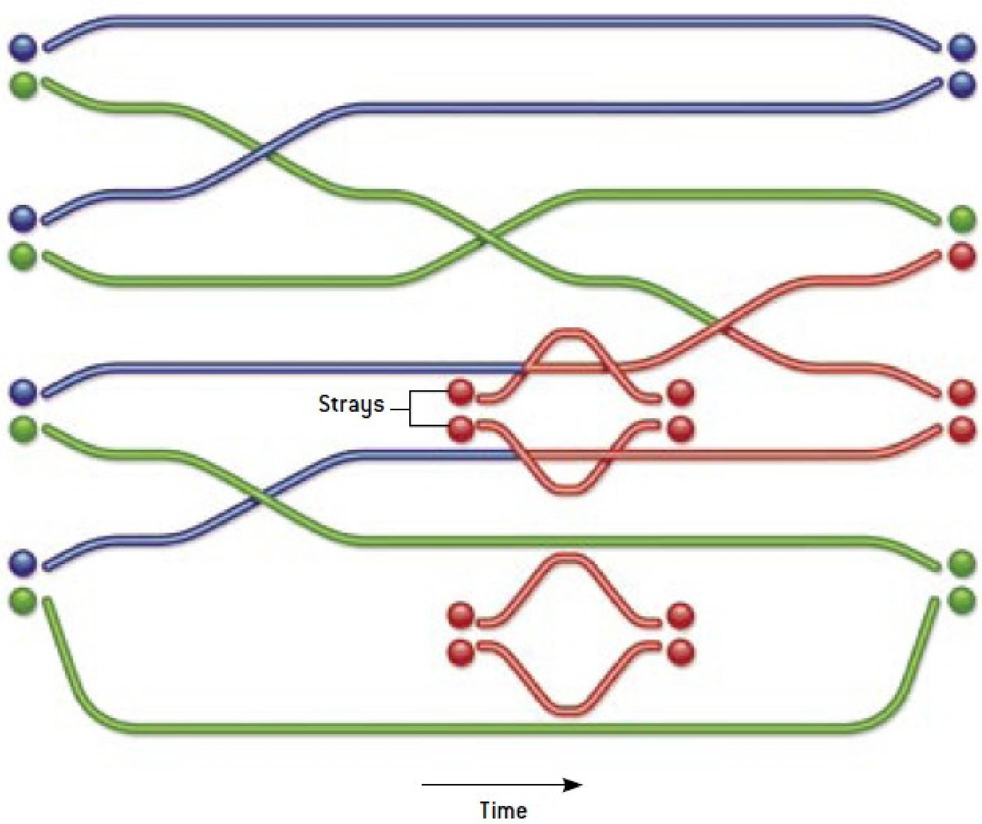


Future Perspectives

- Experimentalists think that the FQH excitations with f=5/2 (easier to investigate) and 12/5 (harder to investigate) are non abelians
- The 5/2 particles will not (probably) generate a dense subspace of SU(2). The 12/5 particle should work.
- At the moment: still trying to measure non-abelian anyons
- In 2000 they proved that topological quantum computers and ordinary quantum computer are equivalent (can simulate each other)

Possible Error Sources

- Errors occur if thermal fluctuations generate pair of anyons.
- Errors are exponentially small for low temperature
- Probability of errors decreases exponentially with distance
- The rate of errors can be minimized to almost 0 with low temperature and keeping anyons sufficiently far apart



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