C191 Midterm 2 November 16, 2010

NAME+SID: _____

Please remember to justify your answers and show your work

Useful Formulae:

The time evolution operator and time evolved quantum state is

$$U(t) = e^{-iHt/\hbar}, |\psi(t)\rangle = U(t)|\psi(0)\rangle \,. \tag{1}$$

The Euler formula for the matrix exponential of Pauli matrices is,

$$\exp\left(i\theta\hat{n}\cdot\vec{\sigma}\right) = I\cos\theta + i\hat{n}\cdot\vec{\sigma}\sin\theta \tag{2}$$

Reflection about mean operator:

$$D = \begin{pmatrix} -2/N+1 & -2/N & \cdots & -2/N \\ -2/N & -2/N+1 & \cdots & -2/N \\ \vdots & \vdots & \ddots & \vdots \\ -2/N & -2/N & \cdots & -2/N+1 \end{pmatrix}$$

The density matrix describing the mixture $|\psi_i\rangle$ with probability p_i is:

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i| \tag{3}$$

1. - Give a quantum circuit to create the superposition $\frac{1}{\sqrt{N}}\sum_{j=0}^{N-1}|j\rangle$. You may assume that j is represented in binary as an n bit string. You may also assume that you have a quantum circuit U_N that on input $|j\rangle|0\rangle$ outputs $|j\rangle|b\rangle$, where $b=1$ if $j<N$ and $b=0$ if $j\geq N$.

   - Let E be the set of n bit strings of even parity. i.e. $\sum_j x_j = 0 \bmod 2\}$, and O be the set of odd parity n bit strings, $\sum_j x_j = 1 \bmod 2\}$. Give a quantum circuit that outputs the superposition $\frac{1}{\sqrt{2^{n-1}}}\sum_{x\in E}|x\rangle$ with probability 1/2 and the superposition $\frac{1}{\sqrt{2^{n-1}}}\sum_{x\in O}|x\rangle$ with probability 1/2. You may assume that you have a quantum circuit U_f that on input $|x\rangle|0\rangle$ outputs $|x\rangle|b\rangle$, where $b=1$ if $x\in O$ and $b=0$ if $x\in E$.

2. Let $N = 2^n$, and let $f : \{0,1\}^n \to \{0,1\}$ be a function such that $|x : f(x) = 1| = \lceil \sqrt{N} \rceil$. Recall that Grover's algorithm consists of setting up a uniform superposition, followed by several phases, where each phase consists of a phase reflection followed by a reflection about the mean.

   - What is the state at the end of each step during the first phase of the algorithm?

   - *extra credit* How many phases should you run the algorithm if you want a constant probability (say 1/2) of finding $\{x : f(x) = 1\}$. Give your answer in big-oh notation.

3. Give one reason why liquid state NMR quantum computation is not scalable. Be as precise as possible.

4. Alice is sending Bob a continuous stream of qubits. Alice prepares the qubits in the following way: First, Alice flips a coin. She then consults one of her instruction cards (she uses the same card every for every qubit). The instruction card says that if Alice flipped heads, then she should prepare the state $|\psi_h\rangle$, while if she were to flip tails, she should prepare and send the state $|\psi_t\rangle$. Bob is trying to figure out which instruction card Alice is using. He knows that there are three possible cards,

   Card A) $|\psi_h\rangle = (1,0)$ $|\psi_t\rangle = (0,1)$

   Card B) $|\psi_h\rangle = (1,1)/\sqrt(2)$ $|\psi_t\rangle = (1,-1)/\sqrt(2)$

   Card C) $|\psi_h\rangle = (1,0)$ $|\psi_t\rangle = (1,i)/\sqrt(2)$

   Is it possible for Bob to tell which card Alice is using? Why (or why not)?

5. A spin qubit is subjected to a magnetic field in the z direction, so experiences the following Hamiltonian, $H = \gamma B \sigma_z / 2$

   a) If it is initially in the state $\psi = (1, 1)/\sqrt{2}$, what is the state as a function of time?

   b) What is the state of the qubit as a function of time if its initial state is described by the density operator,

$$\rho_0 = \frac{1}{2}(I + \frac{1}{2}\sigma_x + \frac{1}{2}\sigma_y)$$

6. Suppose a qubit is initially in the state $|\psi_0\rangle = (1,1)/\sqrt(2)$ and experiences a magnetic field in the z-direction, $H = \gamma B \sigma_z/2$ After a time, $T/2$, the qubit experiences a fast pulse which applies the following transformation: $|\psi(T/2)\rangle \rightarrow X|\psi(T/2)\rangle$. The qubit then continues to evolve under the above Hamiltonian for a further time $T/2$. What is the state of the qubit at time T ?

extra credit What are the implications for decoherence management? You can answer in just a few sentences. (No math!)