Homographies and Mosaics

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with a lot of slides stolen from Steve Seitz and Rick Szeliski

CS194: Intro to Computer Vision and Comp. Photo Alexei Efros, UC Berkeley, Fall 2020
Why Mosaic?

Are you getting the whole picture?
  
  • Compact Camera FOV = 50 x 35°
Why Mosaic?

Are you getting the whole picture?

- Compact Camera FOV = 50 x 35°
- Human FOV = 200 x 135°
Why Mosaic?

Are you getting the whole picture?

- Compact Camera FOV = 50 x 35°
- Human FOV = 200 x 135°
- Panoramic Mosaic = 360 x 180°
Mosaics: stitching images together

virtual wide-angle camera
Naïve Stitching

Translations are not enough to align the images
A pencil of rays contains all views

Can generate any synthetic camera view as long as it has **the same center of projection**!
Image reprojection

The mosaic has a natural interpretation in 3D
- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*
Image reprojection

Basic question

• How to relate two images from the same camera center?
  – how to map a pixel from PP1 to PP2

Answer

• Cast a ray through each pixel in PP1
• Draw the pixel where that ray intersects PP2

But don’t we need to know the geometry of the two planes in respect to the eye?

Observation:
Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another
Back to Image Warping

Which t-form is the right one for warping PP1 into PP2?

e.g. translation, Euclidean, affine, projective

Translation

Affine

Perspective

2 unknowns

6 unknowns

8 unknowns
Homography

A: Projective – mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren’t
- but must preserve straight lines
- same as: unproject, rotate, reproject

called Homography

\[
\begin{bmatrix}
wx' \\
wv' \\
w
\end{bmatrix} = \begin{bmatrix}
* & * & * \\
* & * & * \\
* & * & * \\
\end{bmatrix} \begin{bmatrix}
x \\
y \\
l
\end{bmatrix}
\]

To apply a homography \( H \)

- Compute \( p' = Hp \) (regular matrix multiply)
- Convert \( p' \) from homogeneous to image coordinates
Image warping with homographies
To unwarp (rectify) an image

- Find the homography $H$ given a set of $p$ and $p'$ pairs
- How many correspondences are needed?
- Tricky to write $H$ analytically, but we can solve for it!
  - Find such $H$ that “best” transforms points $p$ into $p'$
  - Use least-squares!
Least Squares Example

Say we have a set of data points \((p1,p1'), (p2,p2'), (p3,p3'),\) etc. (e.g. person’s height vs. weight)

We want a nice compact formula (a line) to predict \(p'\) from \(p\):

\[ px_1 + x_2 = p' \]

We want to find \(x_1\) and \(x_2\)

How many \((p,p')\) pairs do we need?

\[
\begin{align*}
p_1x_1 + x_2 &= p_1' \\
p_2x_1 + x_2 &= p_2'
\end{align*}
\]

\[
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix}
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
p_1' \\
p_2'
\end{bmatrix}
\]

\[ Ax = b \]
Least Squares Example

Say we have a set of data points \((p_1,p_1'), (p_2,p_2'), (p_3,p_3'), \text{ etc.} \) (e.g. person’s height vs. weight)

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\[
\begin{align*}
  p_1 x_1 + x_2 &= p_1' \\
  p_2 x_1 + x_2 &= p_2'
\end{align*}
\]

What if the data is noisy?

\[
\begin{align*}
  \begin{bmatrix}
    p_1 & 1 \\
    p_2 & 1 \\
    p_3 & 1 \\
    \vdots & \vdots
  \end{bmatrix}
  \begin{bmatrix}
    x_1 \\
    x_2
  \end{bmatrix}
  &=
  \begin{bmatrix}
    p_1' \\
    p_2' \\
    p_3'
  \end{bmatrix}
\end{align*}
\]

\[
\min ||Ax - b||^2
\]

overconstrained
**Least-Squares**

- **Solve:**
  \[ A \mathbf{x} = \mathbf{b} \]
  \[
  (N,d)(d,1) = (N,1)
  \]

- **Normal equations**
  \[ A^T A \mathbf{x} = A^T \mathbf{b} \]
  \[
  (d,N)(N,d)(d,1) = (d,N)(N,1)
  \]

- **Solution:**
  \[ \mathbf{x} = (A^T A)^{-1} A^T \mathbf{b} \]
  
  rank(A) ≤ min(d,N)  
  assume rank(A) = d  
  implies rank(A^T A) = d  
  A^T A is invertible
Solving for homographies

\[ \mathbf{p'} = H \mathbf{p} \]

\[
\begin{bmatrix}
wx' \\
wy' \\
w
\end{bmatrix} =
\begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

Can set scale factor \( i = 1 \). So, there are 8 unknowns.

Set up a system of linear equations:

\[ A \mathbf{h} = \mathbf{b} \]

where vector of unknowns \( \mathbf{h} = [a, b, c, d, e, f, g, h]^T \)

Need at least 8 eqs, but the more the better…

Solve for \( \mathbf{h} \). If overconstrained, solve using least-squares:

\[ \min \| A \mathbf{h} - \mathbf{b} \|^2 \]

Can be done in Matlab using “\" command

- see “help \Imdivide”
Fun with homographies

Original image

St. Petersburg photo by A. Tikhonov

Virtual camera rotations
Analysing patterns and shapes

What is the shape of the b/w floor pattern?

The floor (enlarged)

Homography

Automatically rectified floor

Slide from Criminisi
Analysing patterns and shapes

From Martin Kemp *The Science of Art*
(manual reconstruction)

2 patterns have been discovered!
Analysing patterns and shapes

What is the (complicated) shape of the floor pattern?

Automatically rectified floor

St. Lucy Altarpiece, D. Veneziano
Slide from Criminisi
Analysing patterns and shapes

From Martin Kemp, *The Science of Art* (manual reconstruction)

Slide from Criminisi
Panoramas

1. Pick one image (red)
2. Warp the other images towards it (usually, one by one)
3. blend
changing camera center

Does it still work?

synthetic PP

PP1

PP2
Planar scene (or far away)

PP3 is a projection plane of both centers of projection, so we are OK!
This is how big aerial photographs are made
Planar mosaic

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Image Compositing for Tele-Reality

1. Introduction
2. Previous work
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10. Discussion & Conclusions
Julian Beever: Manual Homographies

http://users.skynet.be/J.Beever/pave.htm
Holbein, *The Ambassadors*
Homographies and Panoramic Mosaics
• Capture photographs (and possibly video)
  • Might want to use tripod
• Compute homographies (define correspondences)
  • will need to figure out how to setup system of eqs.
• (un)warp an image (undo perspective distortion)
• Produce panoramic mosaics (with blending)
• Do some of the Bells and Whistles
Bells and Whistles

Blending and Compositing

• use homographies to combine images or video and images together in an interesting (fun) way. E.g.
  – put fake graffiti on buildings or chalk drawings on the ground
  – replace a road sign with your own poster
  – project a movie onto a building wall
  – etc.
Bells and Whistles

Virtual Camera rotation

- Similar to face morphing, produce a video of virtual camera rotation from a single image
- Can also do it for translation, if looking at a planar object

Other interesting ideas?

- Talk to me
From previous year’s classes

Ben Hollis, 2004

Ben Hollis, 2004

Matt Pucevich, 2004

Eunjeong Ryu (E.J), 2004
Bells and Whistles

Capture creative/cool/bizzare panoramas

• Example from UW (by Brett Allen):

• Ever wondered what is happening inside your fridge while you are not looking?

Capture a 360 panorama (quite tricky…)

[Image of a refrigerator shelf with various items]
Example homography final project
Setting alpha: simple averaging

Alpha = .5 in overlap region
Setting alpha: center seam

\[ \text{Alpha} = \text{logical}(\text{dtrans1} > \text{dtrans2}) \]
Setting alpha: blurred seam

Distance transform

Alpha = blurred
Simplification: Two-band Blending

Brown & Lowe, 2003

- Only use two bands: high freq. and low freq.
- Blends low freq. smoothly
- Blend high freq. with no smoothing: use binary alpha
2-band “Laplacian Stack” Blending

Low frequency ($\lambda > 2$ pixels)

High frequency ($\lambda < 2$ pixels)
Linear Blending
2-band Blending