

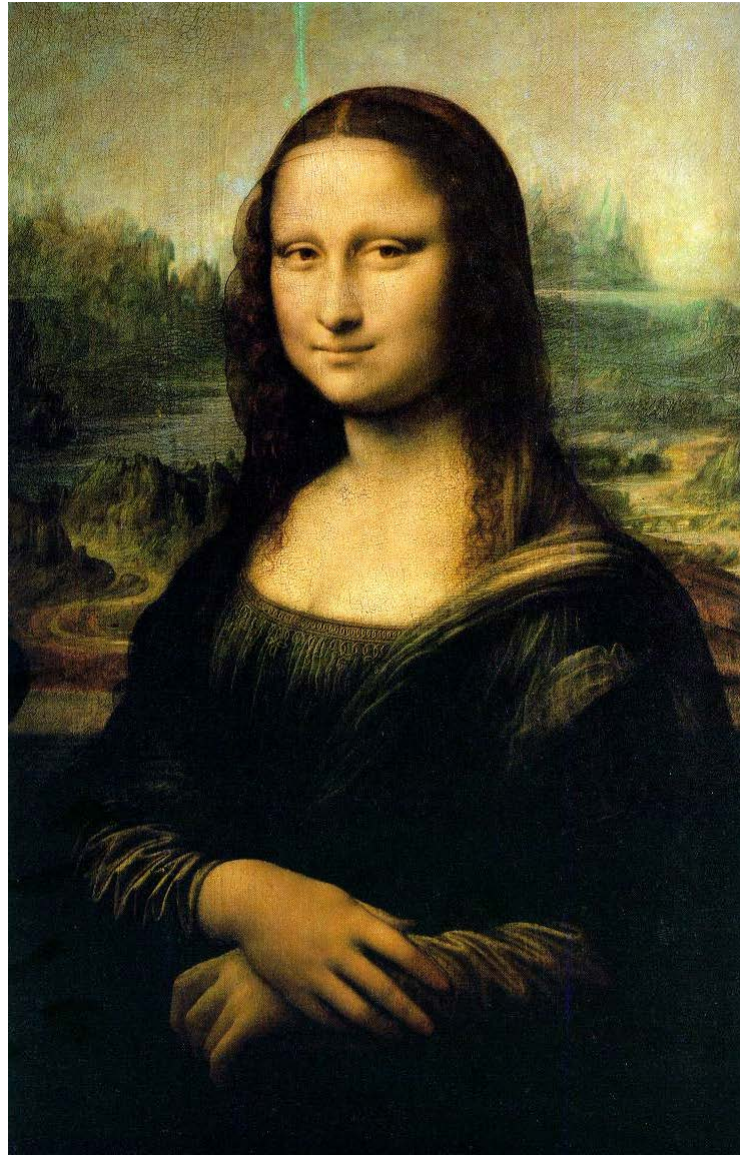
# Spatial Frequencies and Human Perception



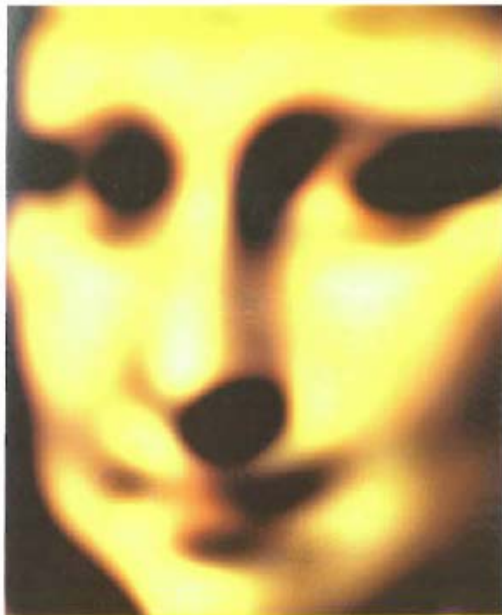
CS194: Intro to Computer Vision and Comp. Photo  
Alexei Efros, UC Berkeley, Fall 2020

# Da Vinci and Peripheral Vision

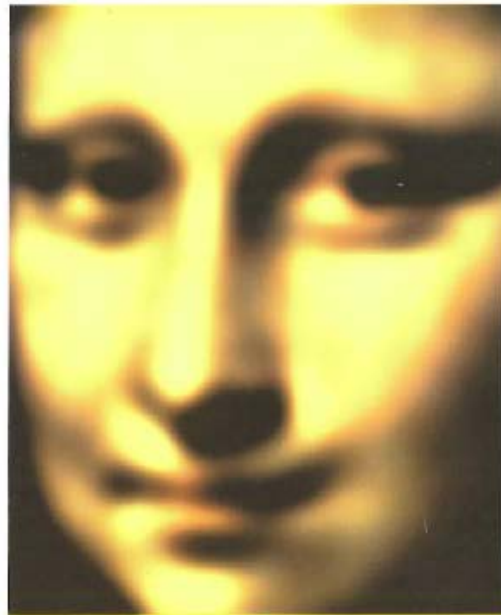
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[https://en.wikipedia.org/wiki/Speculations\\_about\\_Mona\\_Lisa#Smile](https://en.wikipedia.org/wiki/Speculations_about_Mona_Lisa#Smile)



coarse components  
(peripheral vision)



medium components  
(near peripheral vision)

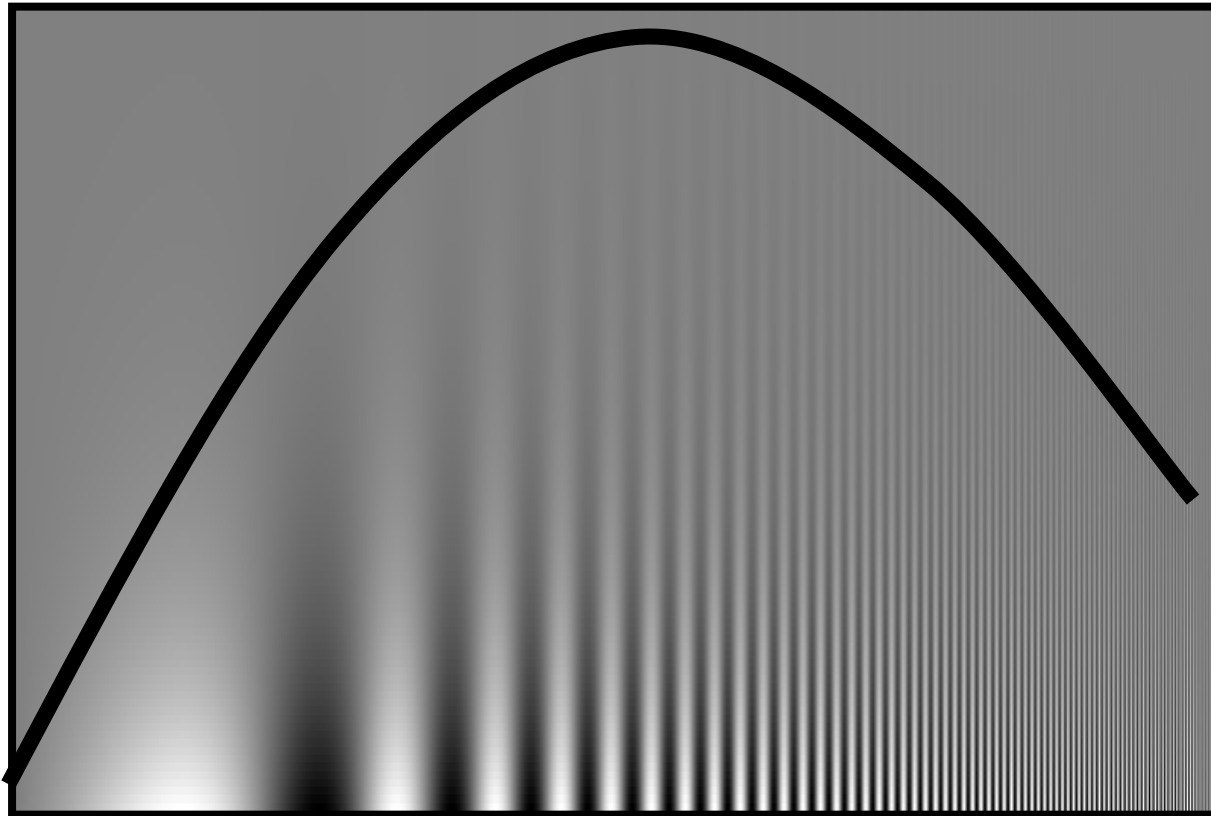


fine details  
(central vision)

Leonardo playing with peripheral vision

# Frequency Domain and Perception

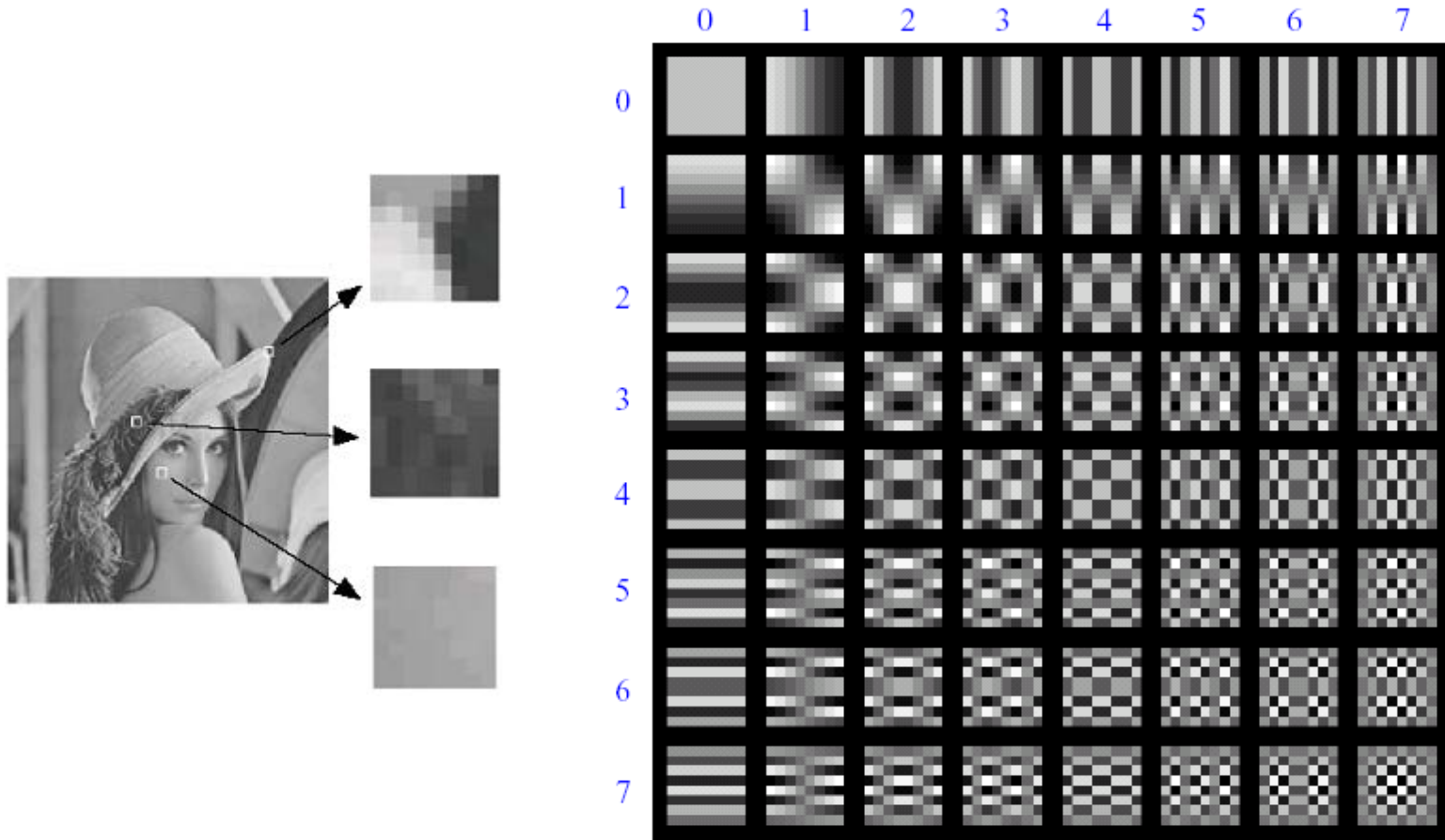
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Campbell-Robson contrast sensitivity curve

# Lossy Image Compression (JPEG)

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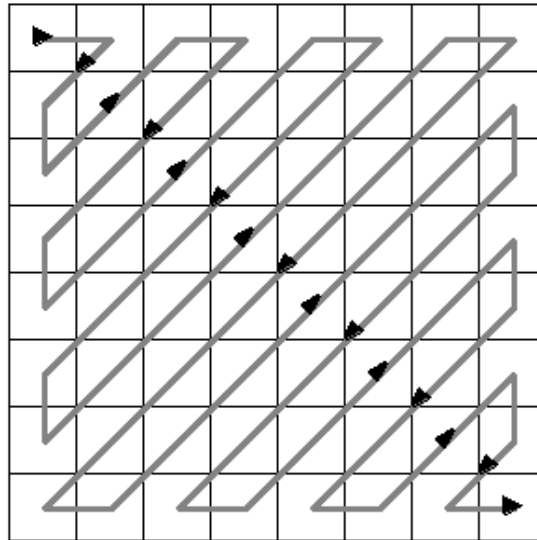
Block-based Discrete Cosine Transform (DCT)

# Using DCT in JPEG

---

The first coefficient  $B(0,0)$  is the DC component, the average intensity

The top-left coeffs represent low frequencies, the bottom right – high frequencies



# Image compression using DCT

## Quantize

- More coarsely for high frequencies (which also tend to have smaller values)
- Many quantized high frequency values will be zero

## Encode

- Can decode with inverse dct

## Filter responses

$$G = \begin{matrix} & & & \begin{matrix} u \\ \longrightarrow \end{matrix} \\ \begin{matrix} \left[ \begin{array}{cccccccc} -415.38 & -30.19 & -61.20 & 27.24 & 56.13 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.88 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{array} \right] \end{matrix} & \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \downarrow v \end{matrix} \end{matrix}$$



## Quantized values

$$B = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

## Quantization table

$$Q = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$



# JPEG Compression Summary

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Subsample color by factor of 2

- People have bad resolution for color

Split into blocks (8x8, typically), subtract 128

For each block

- a. Compute DCT coefficients
- b. Coarsely quantize
  - Many high frequency components will become zero
- c. Encode (e.g., with Huffman coding)



# Block size in JPEG

---

## Block size

- small block
  - faster
  - correlation exists between neighboring pixels
- large block
  - better compression in smooth regions
- It's 8x8 in standard JPEG

# JPEG compression comparison

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89k



12k

# Image Warping

---



<http://www.jeffrey-martin.com>

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# Image Transformations

---

image filtering: change **range** of image

$$g(x) = T(f(x))$$

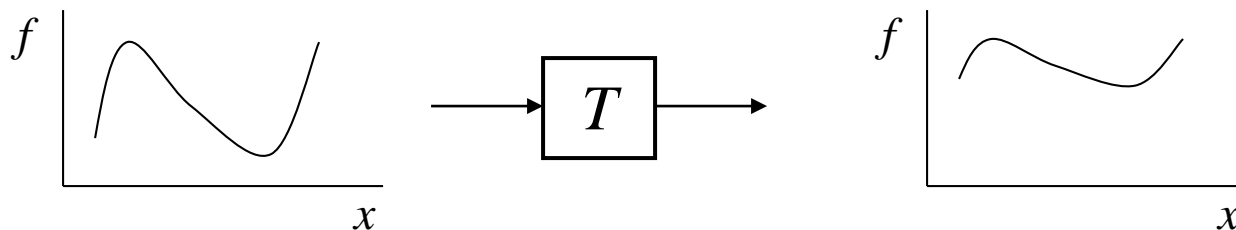
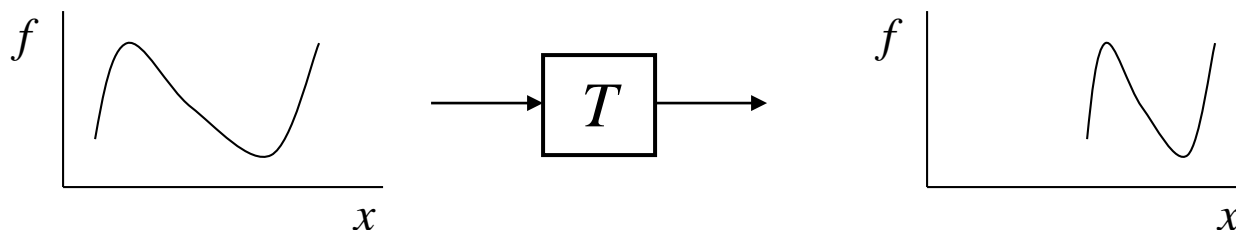


image warping: change **domain** of image

$$g(x) = f(T(x))$$



# Image Transformations

---

image filtering: change **range** of image

$$g(x) = T(f(x))$$

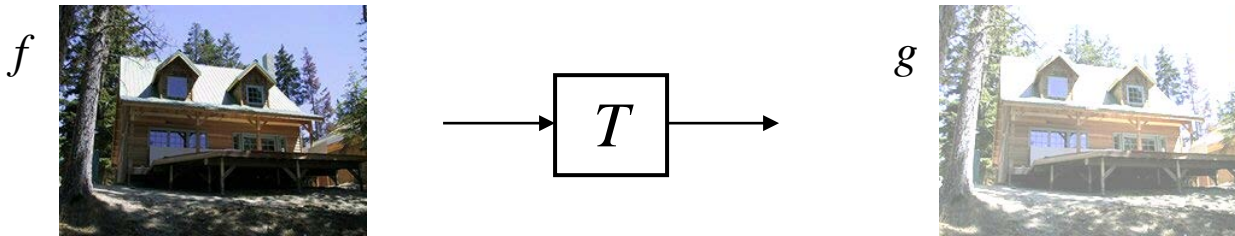
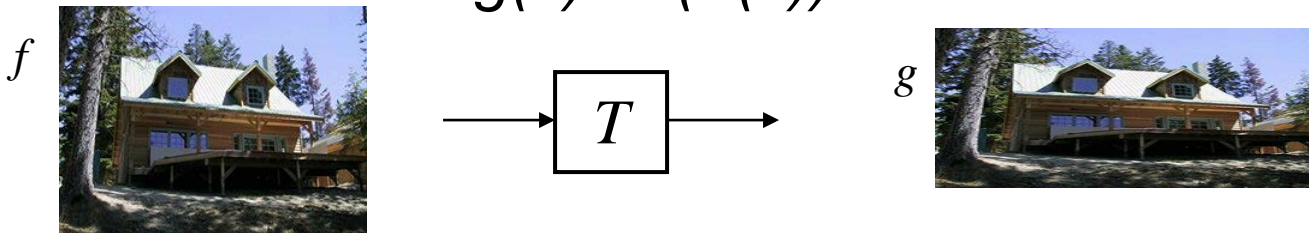


image warping: change **domain** of image

$$g(x) = f(T(x))$$



# Parametric (global) warping

---

Examples of parametric warps:



translation



rotation



aspect



affine



perspective



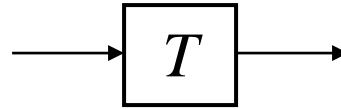
cylindrical

# Parametric (global) warping

---



$$\mathbf{p} = (x, y)$$



$$\mathbf{p}' = (x', y')$$

Transformation  $T$  is a coordinate-changing machine:

$$\mathbf{p}' = T(\mathbf{p})$$

What does it mean that  $T$  is global?

- Is the same for any point  $\mathbf{p}$
- can be described by just a few numbers (parameters)

Let's represent a linear  $T$  as a matrix:

$$\mathbf{p}' = \mathbf{M}\mathbf{p}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

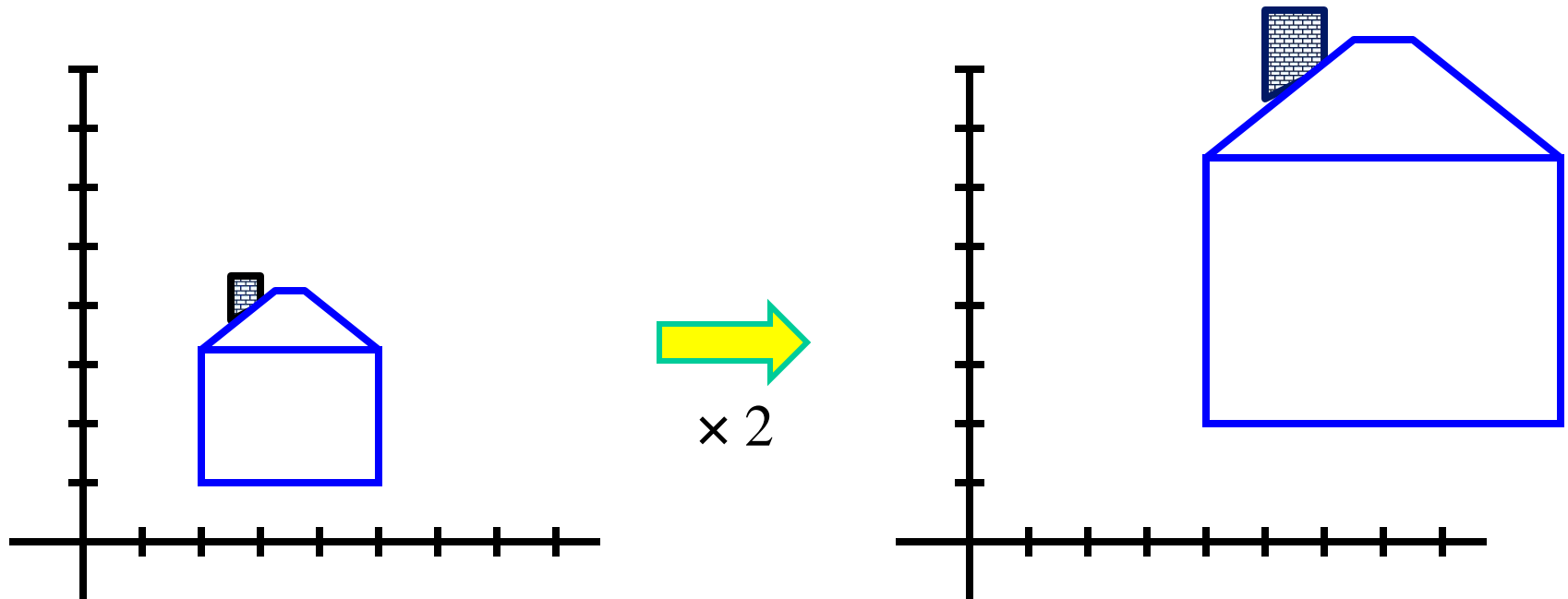


# Scaling

---

*Scaling* a coordinate means multiplying each of its components by a scalar

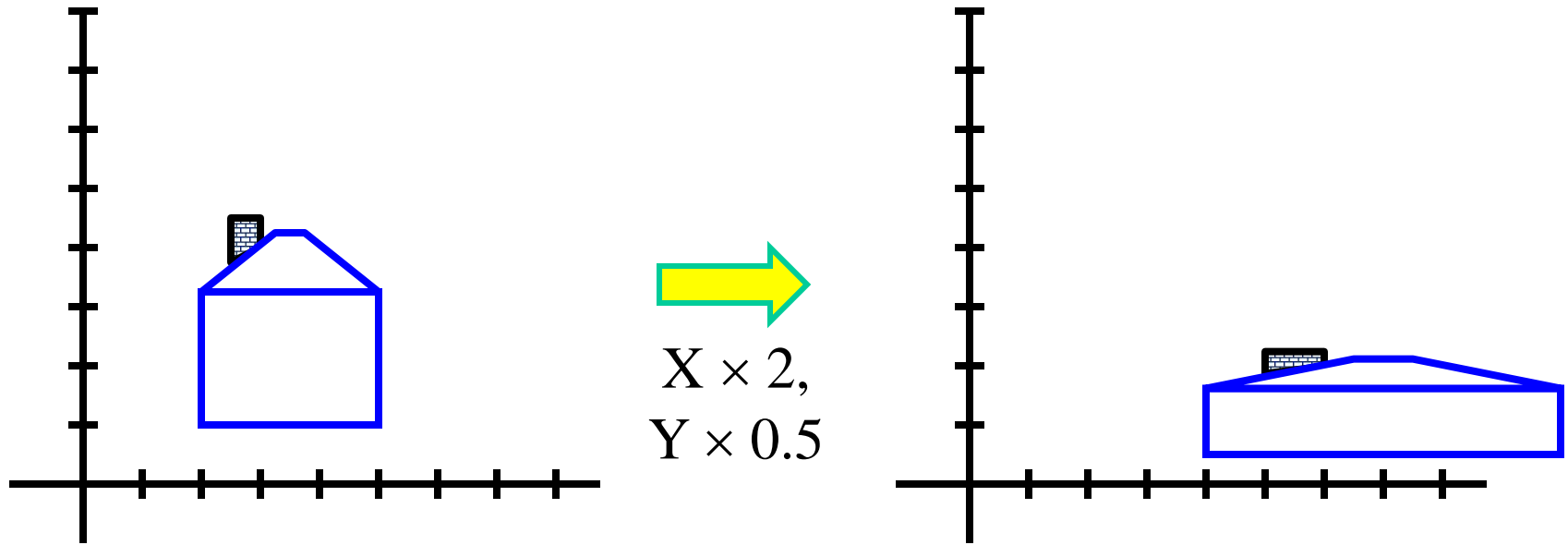
*Uniform scaling* means this scalar is the same for all components:



# Scaling

---

*Non-uniform scaling*: different scalars per component:



# Scaling

---

Scaling operation:

$$x' = ax$$

$$y' = by$$

Or, in matrix form:

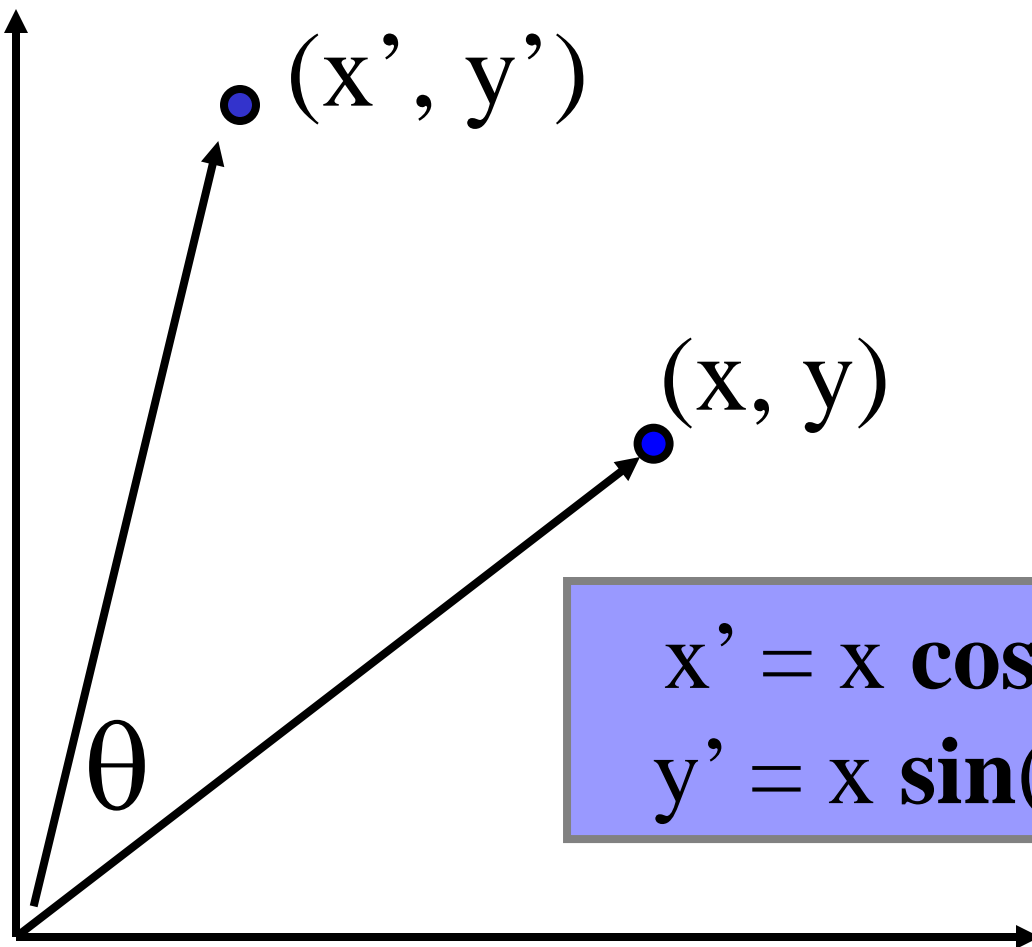
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\text{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

*scaling matrix S*

What's inverse of S?

# 2-D Rotation

---



$$\begin{aligned}x' &= x \cos(\theta) - y \sin(\theta) \\y' &= x \sin(\theta) + y \cos(\theta)\end{aligned}$$

# 2-D Rotation

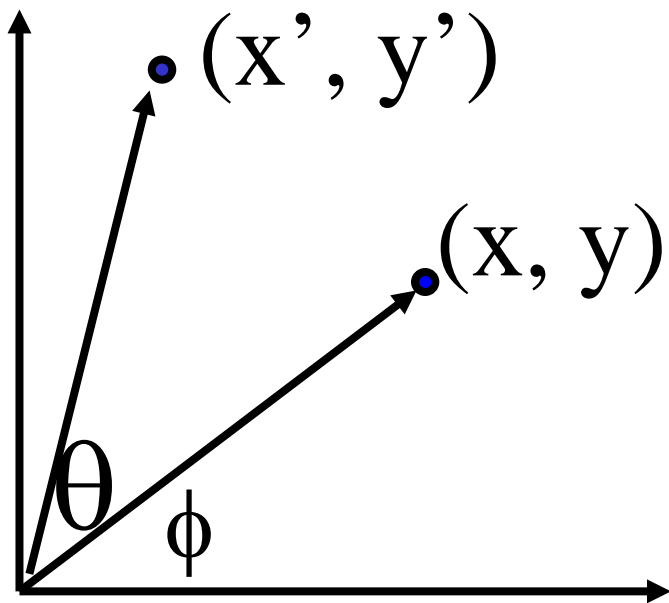
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$$x = r \cos (\phi)$$

$$y = r \sin (\phi)$$

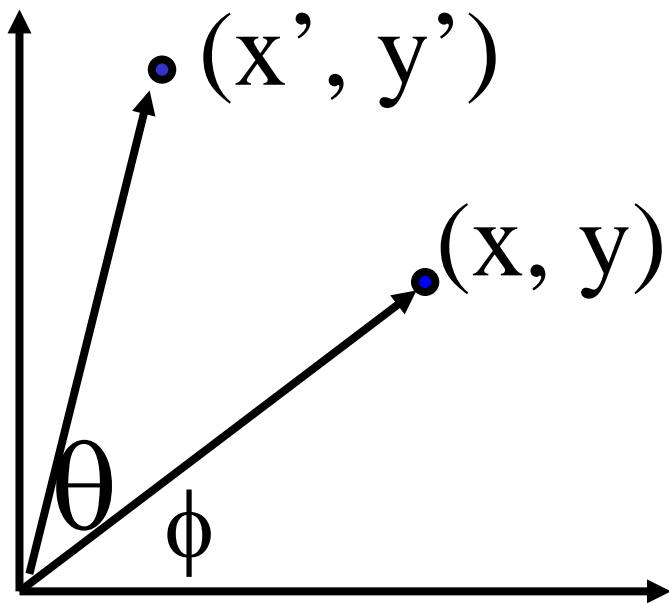
$$x' = r \cos (\phi + \theta)$$

$$y' = r \sin (\phi + \theta)$$



# 2-D Rotation

---



$$x = r \cos(\phi)$$

$$y = r \sin(\phi)$$

$$x' = r \cos(\phi + \theta)$$

$$y' = r \sin(\phi + \theta)$$

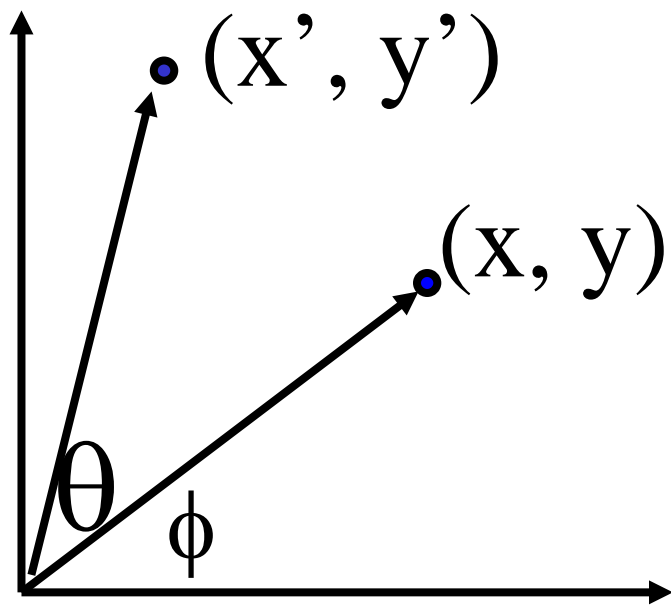
Trig Identity...

$$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$$

$$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$$

# 2-D Rotation

---



$$x = r \cos(\phi)$$

$$y = r \sin(\phi)$$

$$x' = r \cos(\phi + \theta)$$

$$y' = r \sin(\phi + \theta)$$

Trig Identity...

$$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$$

$$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$$

Substitute...

$$x' = x \cos(\theta) - y \sin(\theta)$$

$$y' = x \sin(\theta) + y \cos(\theta)$$



# 2-D Rotation

---

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though  $\sin(\theta)$  and  $\cos(\theta)$  are nonlinear functions of  $\theta$ ,

- ***$x'$  is a linear combination of  $x$  and  $y$***
- ***$y'$  is a linear combination of  $x$  and  $y$***

What is the inverse transformation?

- Rotation by  $-\theta$
- For rotation matrices  **$\mathbf{R}^{-1} = \mathbf{R}^T$**

# 2x2 Matrices

---

What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$\begin{aligned}x' &= x \\ y' &= y\end{aligned}\quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\begin{aligned}\mathbf{x}' &= s_x * \mathbf{x} \\ \mathbf{y}' &= s_y * \mathbf{y}\end{aligned}\quad \begin{bmatrix} \mathbf{x}' \\ \mathbf{y}' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$$

# 2x2 Matrices

---

What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$\begin{aligned}x' &= \cos \Theta * x - \sin \Theta * y \\y' &= \sin \Theta * x + \cos \Theta * y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$x' = x + sh_x * y$$

$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# 2x2 Matrices

---

What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$\begin{aligned}x' &= -x \\ y' &= y\end{aligned}\quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$\begin{aligned}x' &= -x \\ y' &= -y\end{aligned}\quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# 2x2 Matrices

---

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$\begin{aligned}x' &= x + t_x \\ y' &= y + t_y\end{aligned} \quad \text{NO!}$$

Only linear 2D transformations  
can be represented with a 2x2 matrix

# All 2D Linear Transformations

---

Linear transformations are combinations of ...

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

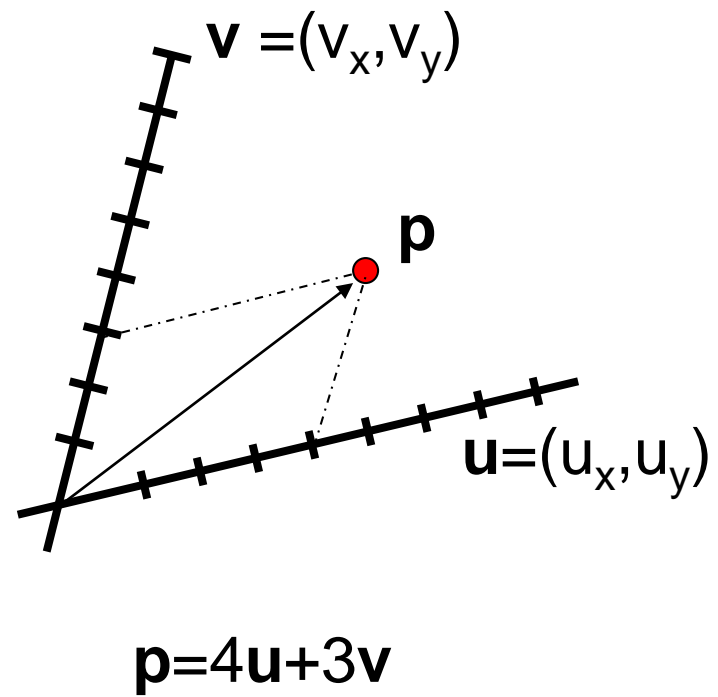
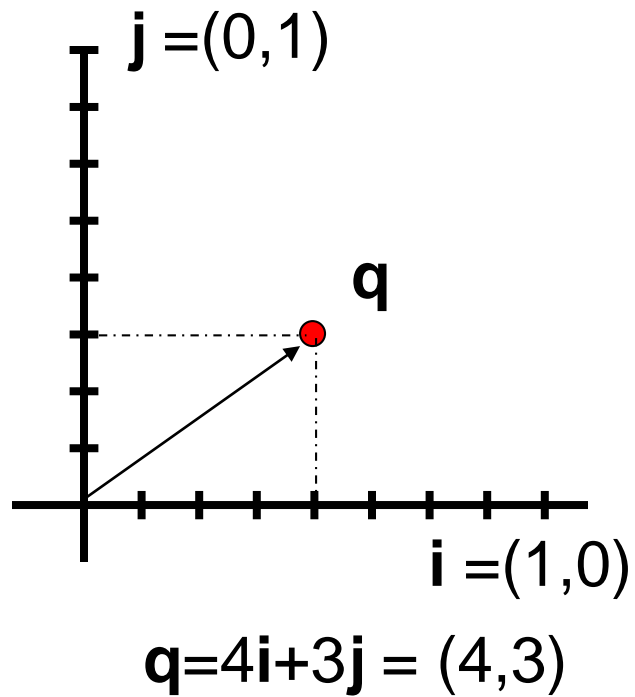
Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

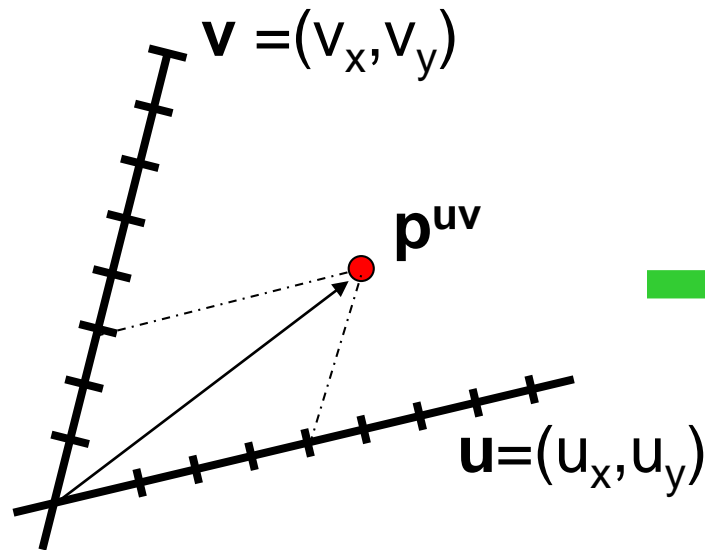
# Consider a different Basis

---





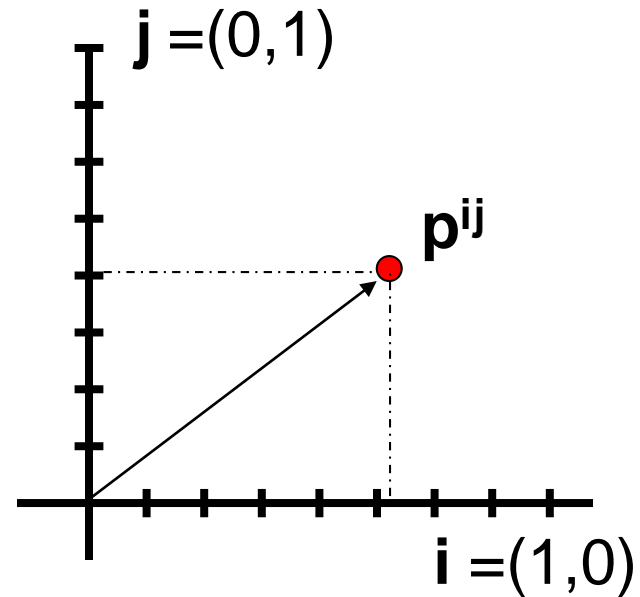
# Linear Transformations as Change of Basis



$$\mathbf{p}^{uv} = (4, 3)$$

$$p_x = 4u_x + 3v_x$$

$$p_y = 4u_y + 3v_y$$

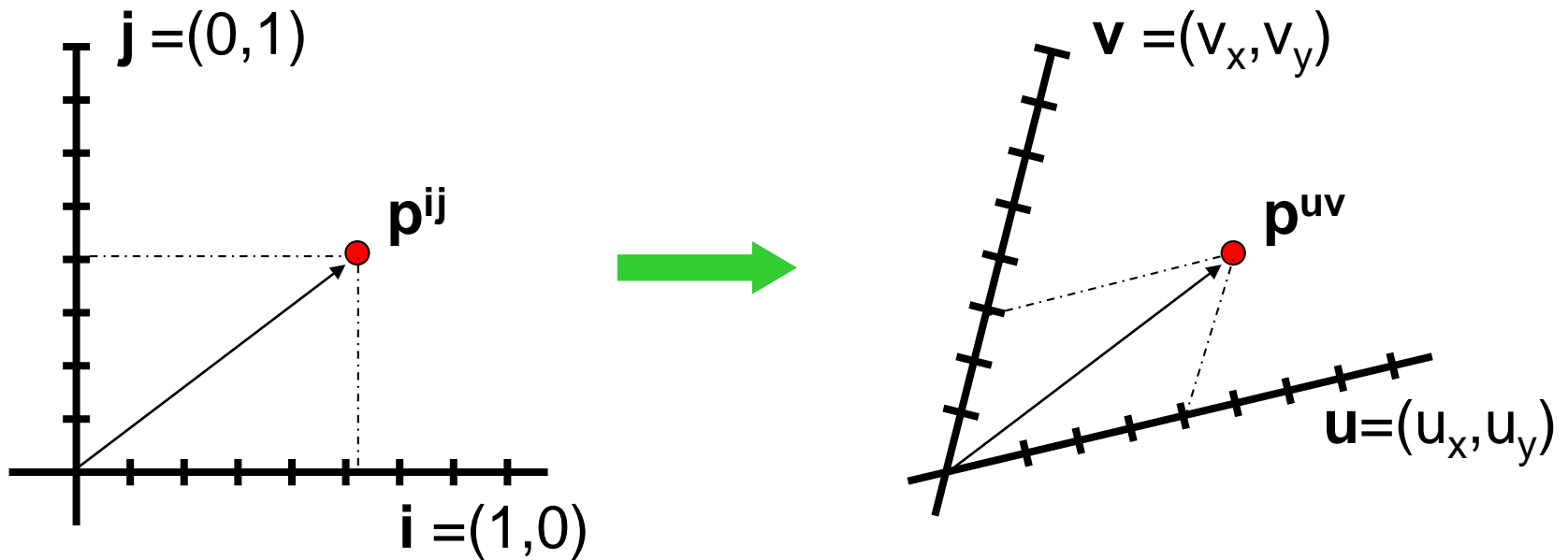


$$\mathbf{p}^{ij} = 4\mathbf{u} + 3\mathbf{v}$$

$$\mathbf{p}^{ij} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix} \mathbf{p}^{uv}$$

Any linear transformation is a basis!!!

# What's the inverse transform?



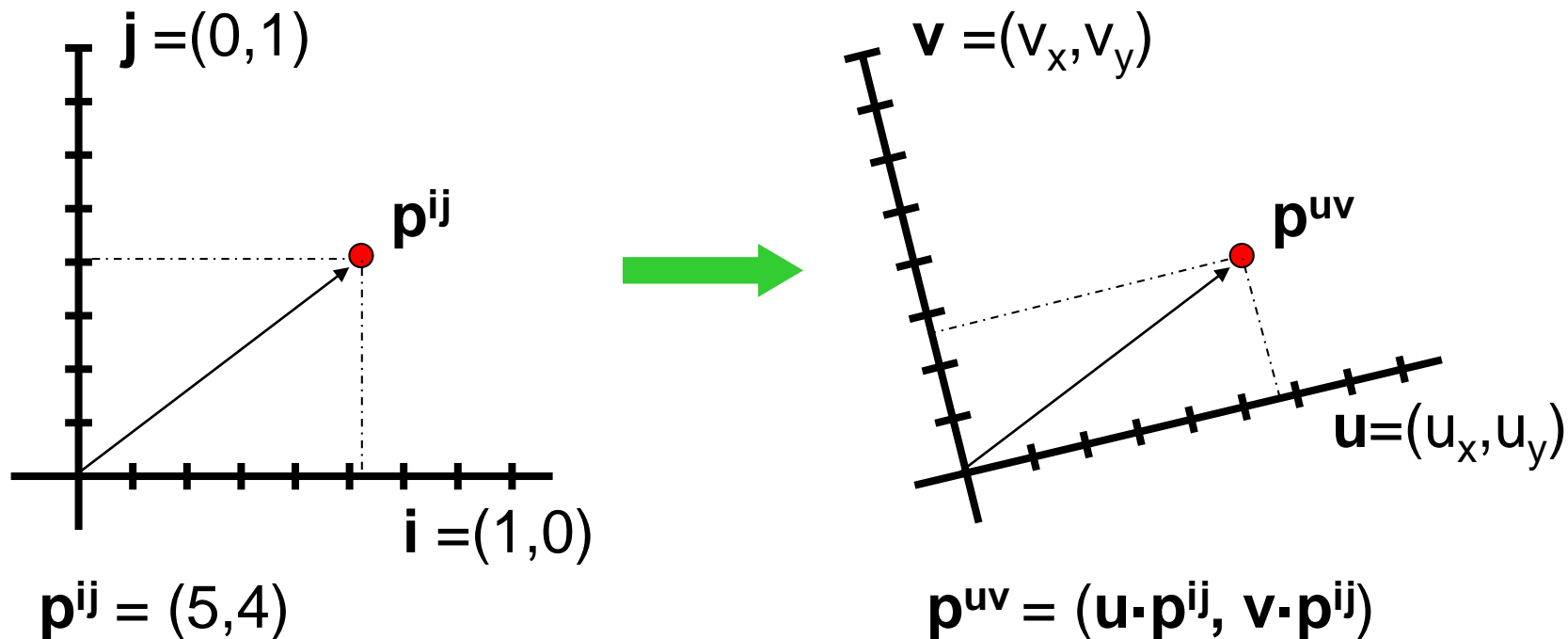
$$\mathbf{p}^{ij} = (5, 4) = p_x \mathbf{u} + p_y \mathbf{v}$$

$$\mathbf{p}^{uv} = (p_x, p_y) = ?$$

$$\mathbf{p}^{uv} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix}^{-1} \mathbf{p}^{ij}$$

- How can we change from any basis to any basis?
- What if the basis are orthogonal?

# Projection onto orthogonal basis



$$\mathbf{p}^{uv} = \begin{bmatrix} u_x & v_x \\ v_y & v_y \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} \mathbf{p}^{ij}$$

# Homogeneous Coordinates

---

**Q: How can we represent translation as a 3x3 matrix?**

$$x' = x + t_x$$

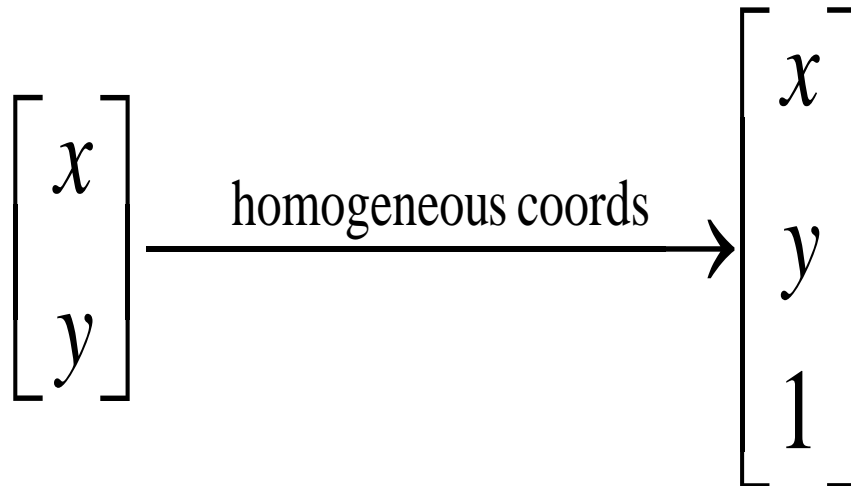
$$y' = y + t_y$$

# Homogeneous Coordinates

---

## *Homogeneous coordinates*

- represent coordinates in 2 dimensions with a 3-vector

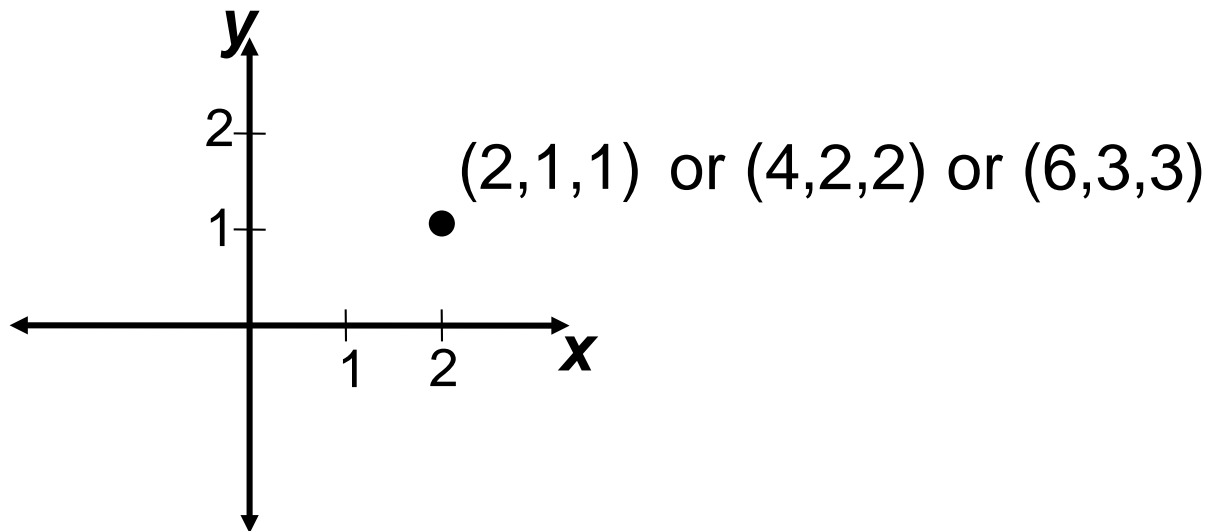


# Homogeneous Coordinates

---

Add a 3rd coordinate to every 2D point

- $(x, y, w)$  represents a point at location  $(x/w, y/w)$
- $(x, y, 0)$  represents a point at infinity
- $(0, 0, 0)$  is not allowed



Convenient  
coordinate system to  
represent many  
useful  
transformations

# Homogeneous Coordinates

---

**Q: How can we represent translation as a 3x3 matrix?**

$$x' = x + t_x$$

$$y' = y + t_y$$

**A: Using the rightmost column:**

$$\mathbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

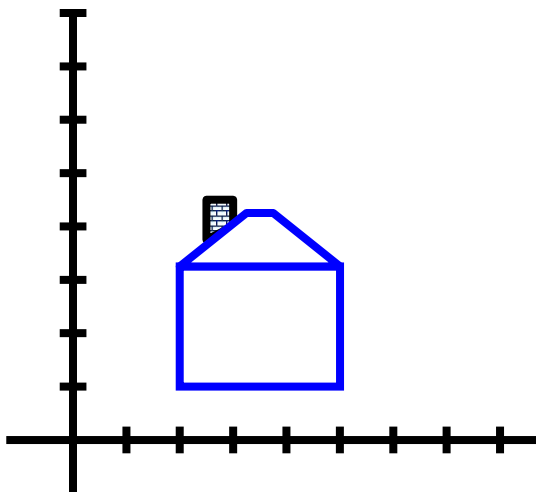
# Translation

---

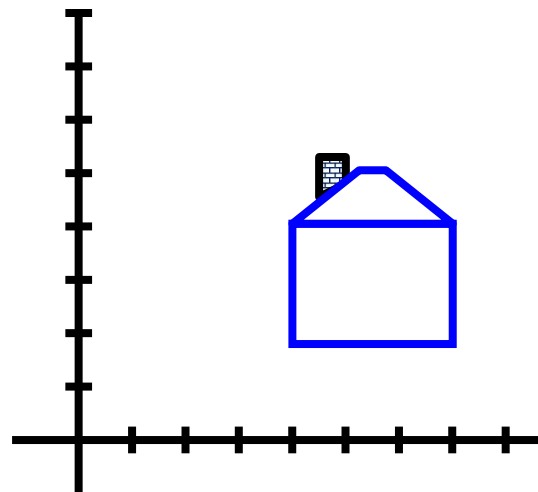
Example of translation

Homogeneous Coordinates

$$\begin{array}{c} \blacktriangledown \qquad \qquad \qquad \blacktriangledown \qquad \qquad \qquad \blacktriangledown \\ \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} \end{array}$$



$$\begin{array}{l} t_x = 2 \\ t_y = 1 \end{array}$$





# Basic 2D Transformations

---

Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

# Matrix Composition

---

Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left( \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$\mathbf{p}' = \mathbf{T}(t_x, t_y) \mathbf{R}(\Theta) \mathbf{S}(s_x, s_y) \mathbf{p}$

Does the order of multiplication matter?

# Affine Transformations

---

Affine transformations are combinations of ...

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Models change of basis

Will the last coordinate  $w$  always be 1?

# Projective Transformations

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Projective transformations ...

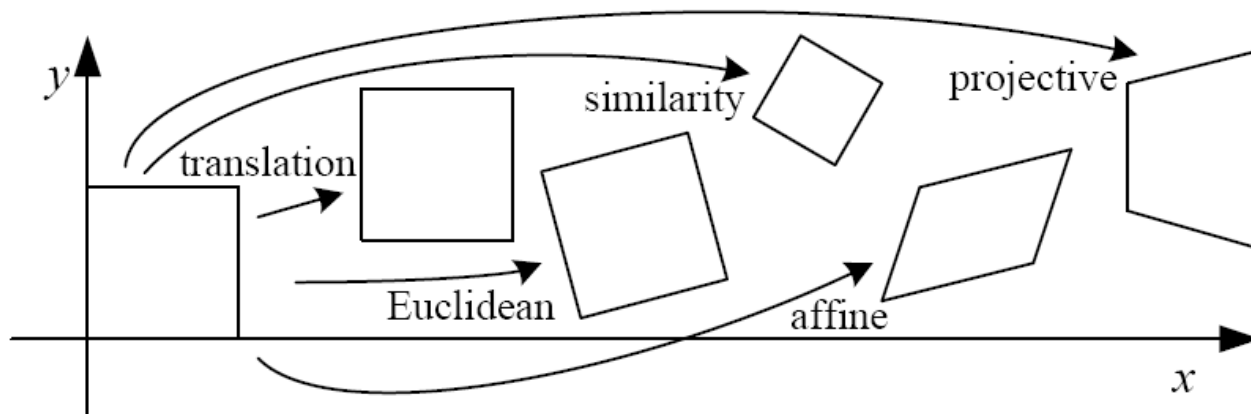
- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis

# 2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I &   & t \end{bmatrix}_{2 \times 3}$			
rigid (Euclidean)	$\begin{bmatrix} R &   & t \end{bmatrix}_{2 \times 3}$			
similarity	$\begin{bmatrix} sR &   & t \end{bmatrix}_{2 \times 3}$			
affine	$\begin{bmatrix} A \end{bmatrix}_{2 \times 3}$			
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3 \times 3}$			



These transformations are a nested set of groups

- Closed under composition and inverse is a member