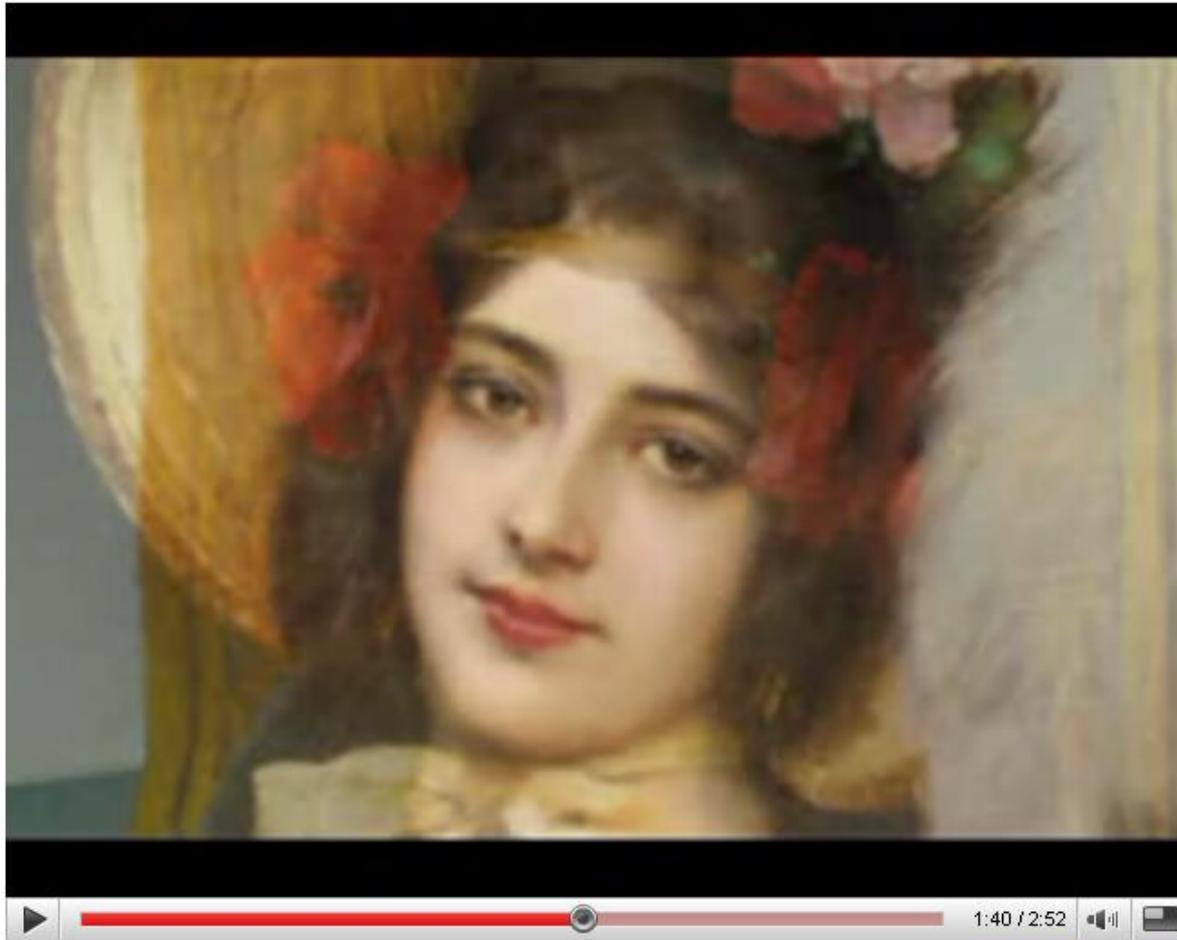


Amuse-bouche



watch in high quality

<http://youtube.com/watch?v=nUDIoN-Hxs>



<http://www.youtube.com/watch?v=L0GKp-uvjO0>

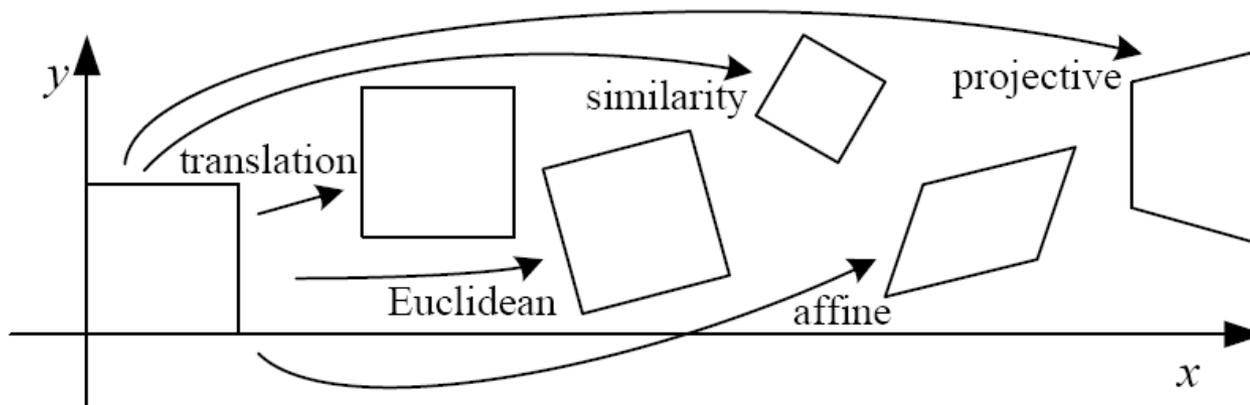
Image Warping and Morphing



© Alexey Tikhonov

CS194: Intro to Computer Vision and Comp. Photo
Alexei Efros, UC Berkeley, Fall 2021

2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$			
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$			



These transformations are a nested set of groups

- Closed under composition and inverse is a member

Image Warping in Biology

D'Arcy Thompson

<http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html>

http://en.wikipedia.org/wiki/D'Arcy_Thompson

Importance of shape and structure in evolution

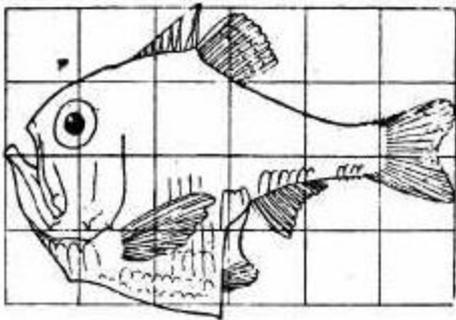
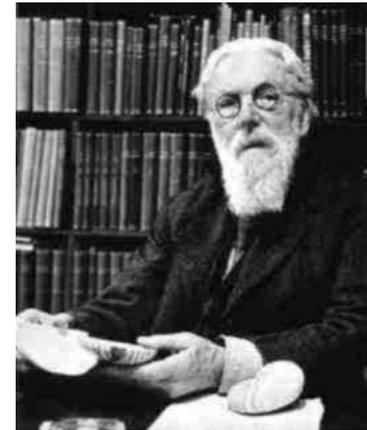
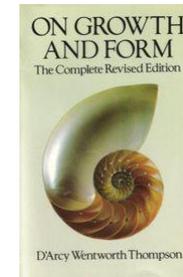


Fig. 517. *Argyropelecus Olfersi*.

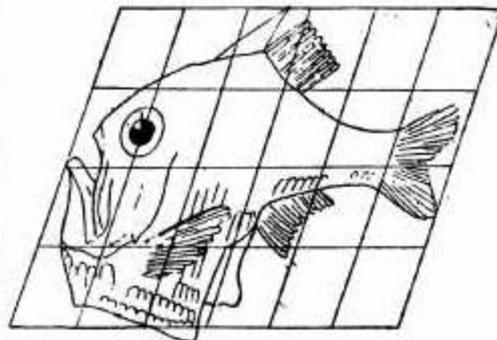
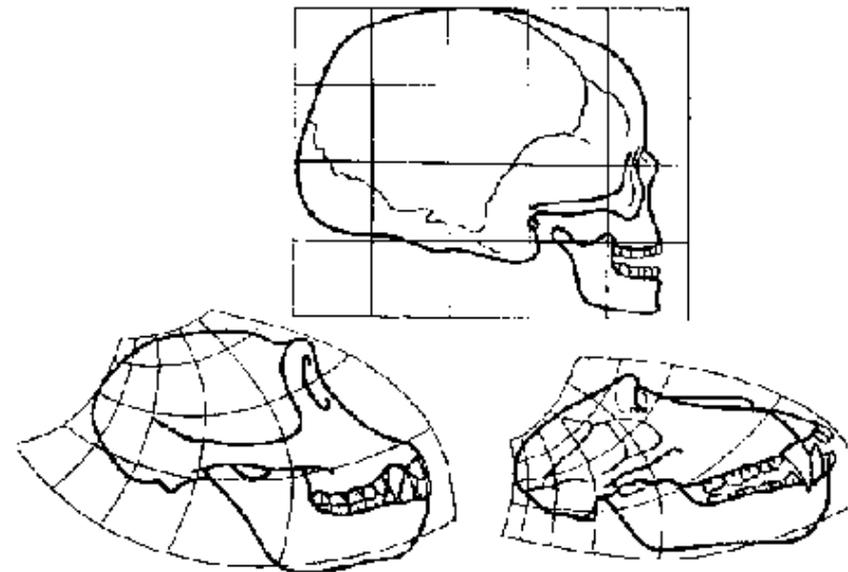
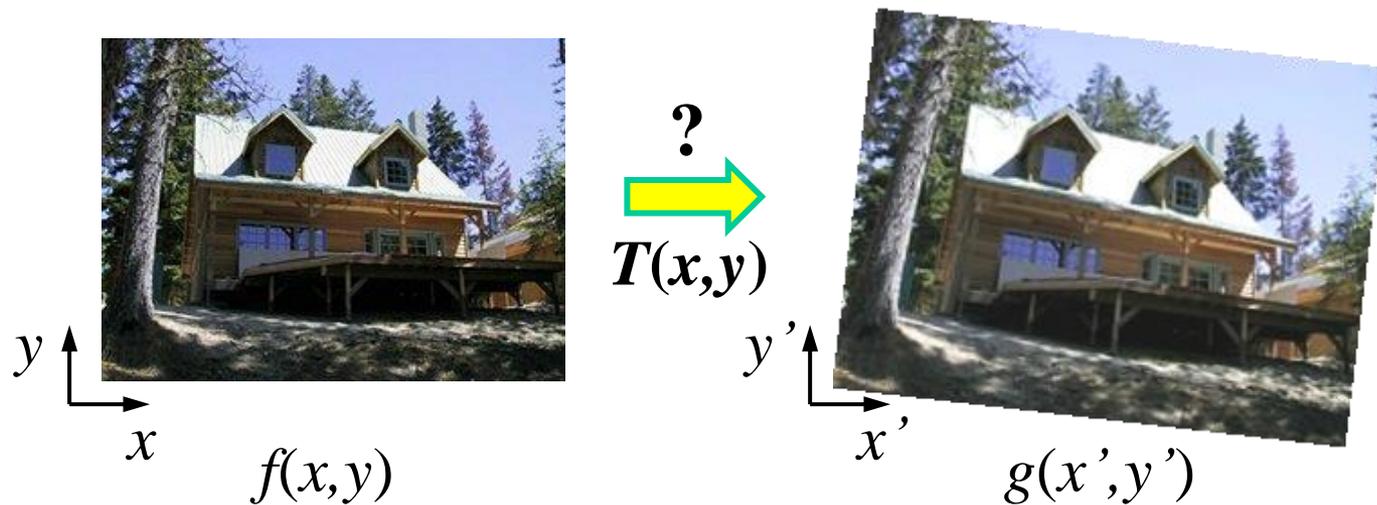


Fig. 518. *Sternoptyx diaphana*.



Skulls of a human, a chimpanzee and a baboon and transformations between them

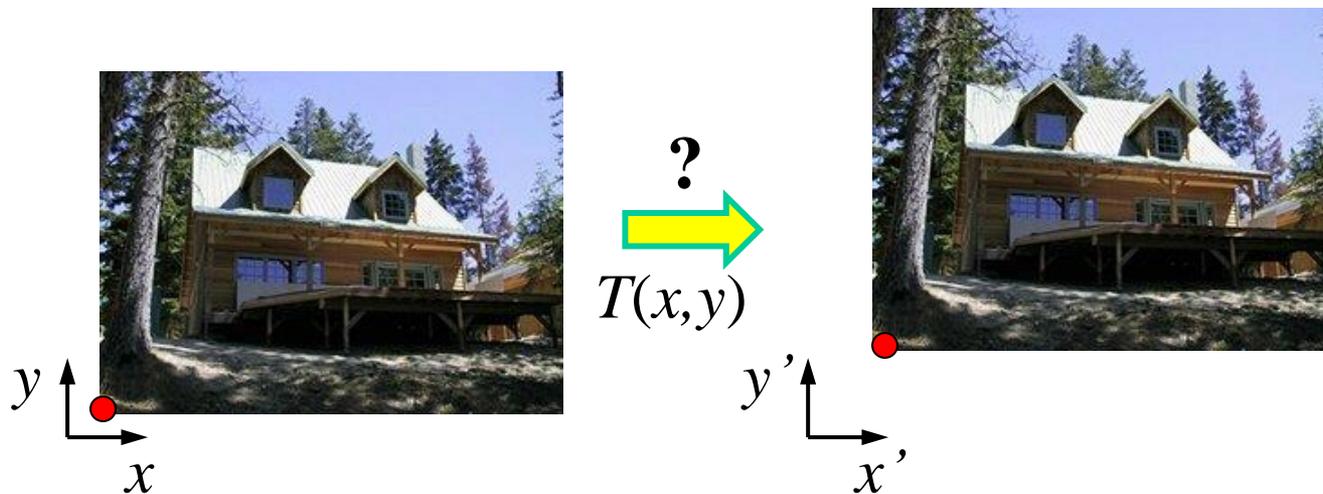
Recovering Transformations



What if we know f and g and want to recover the transform T ?

- e.g. better align images from Project 1
- willing to let user provide correspondences
 - How many do we need?

Translation: # correspondences?



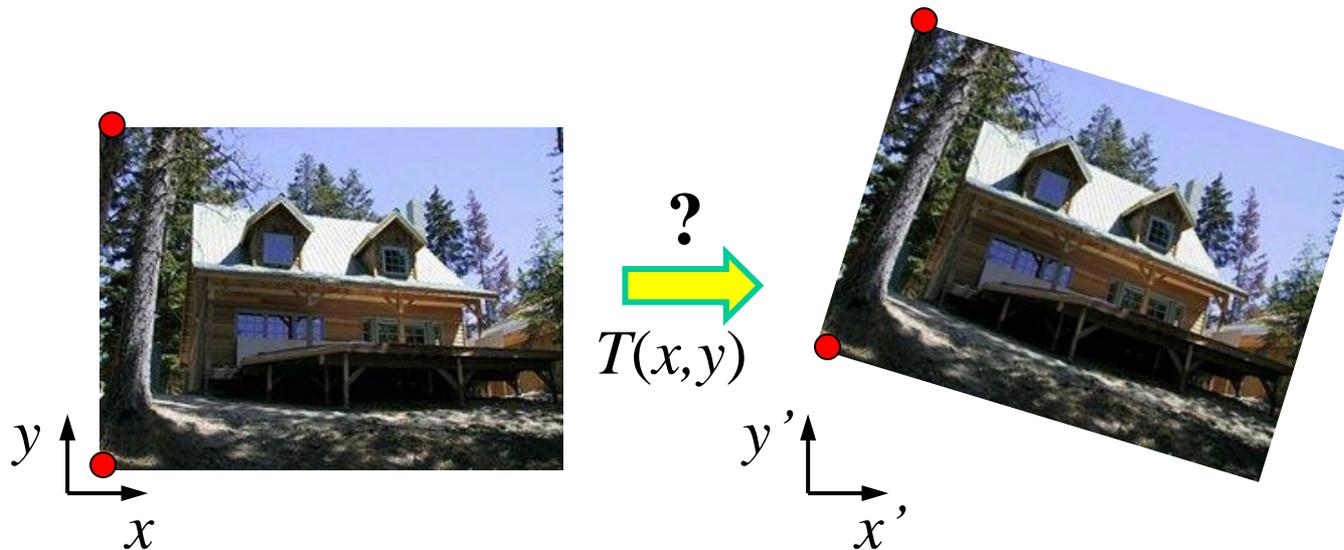
How many correspondences needed for translation?

How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

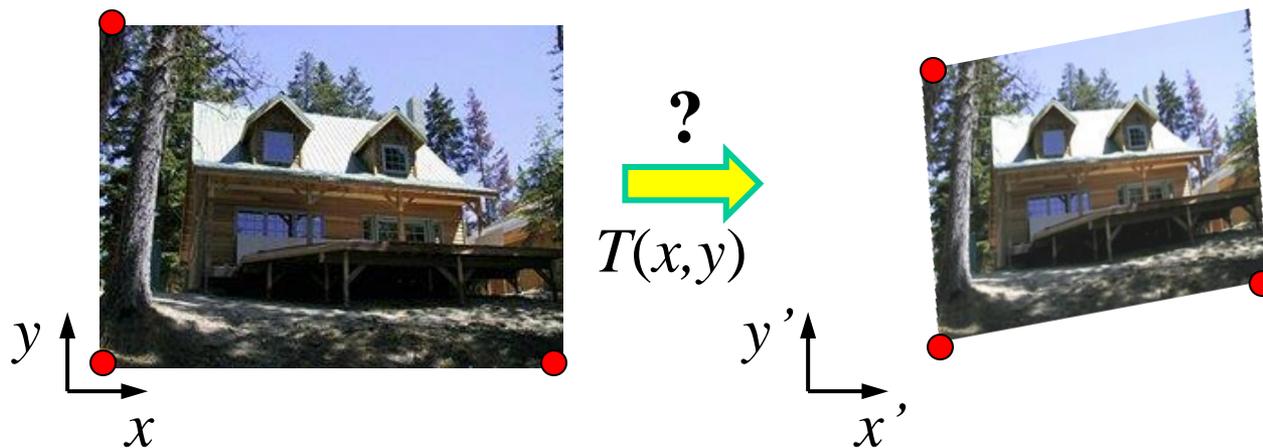
Euclidian: # correspondences?



How many correspondences needed for translation+rotation?

How many DOF?

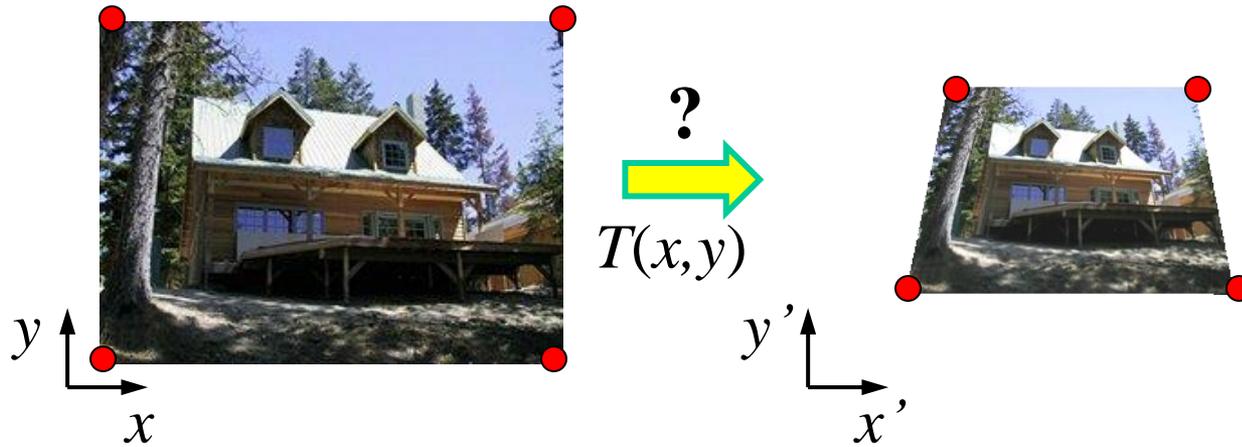
Affine: # correspondences?



How many correspondences needed for affine?

How many DOF?

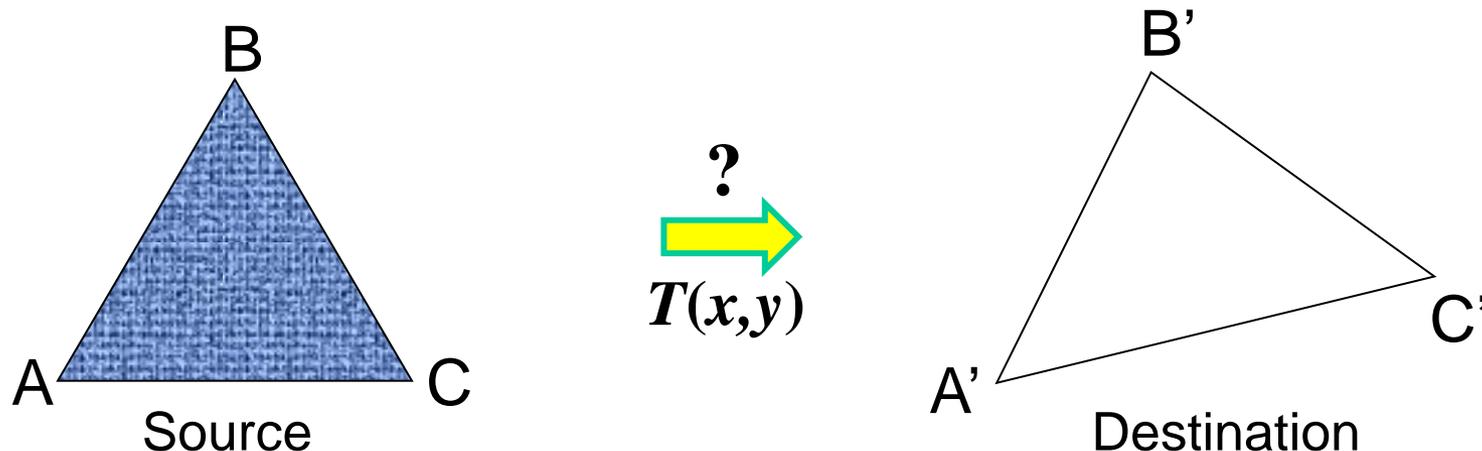
Projective: # correspondences?



How many correspondences needed for projective?

How many DOF?

Example: warping triangles



Given two triangles: ABC and $A'B'C'$ in 2D (12 numbers)
Need to find transform T to transfer all pixels from one to the other.

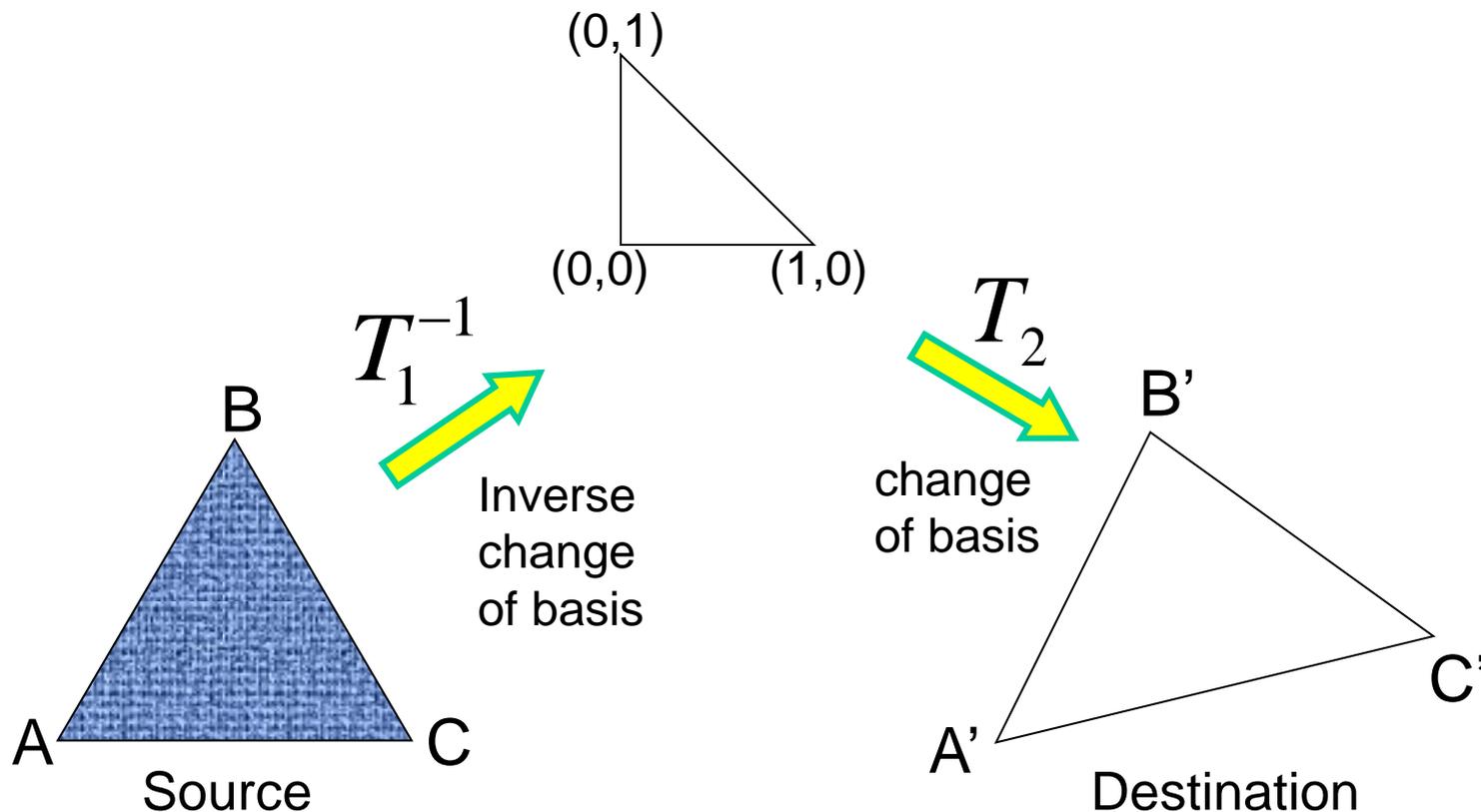
What kind of transformation is T ?

How can we compute the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two ways:
Algebraic and
geometric

warping triangles (Barycentric Coordinates)



Don't forget to move the origin too!

Very useful for Project 3... (hint, hint, nudge, nudge)

Morphing = Object Averaging



The aim is to find “an average” between two objects

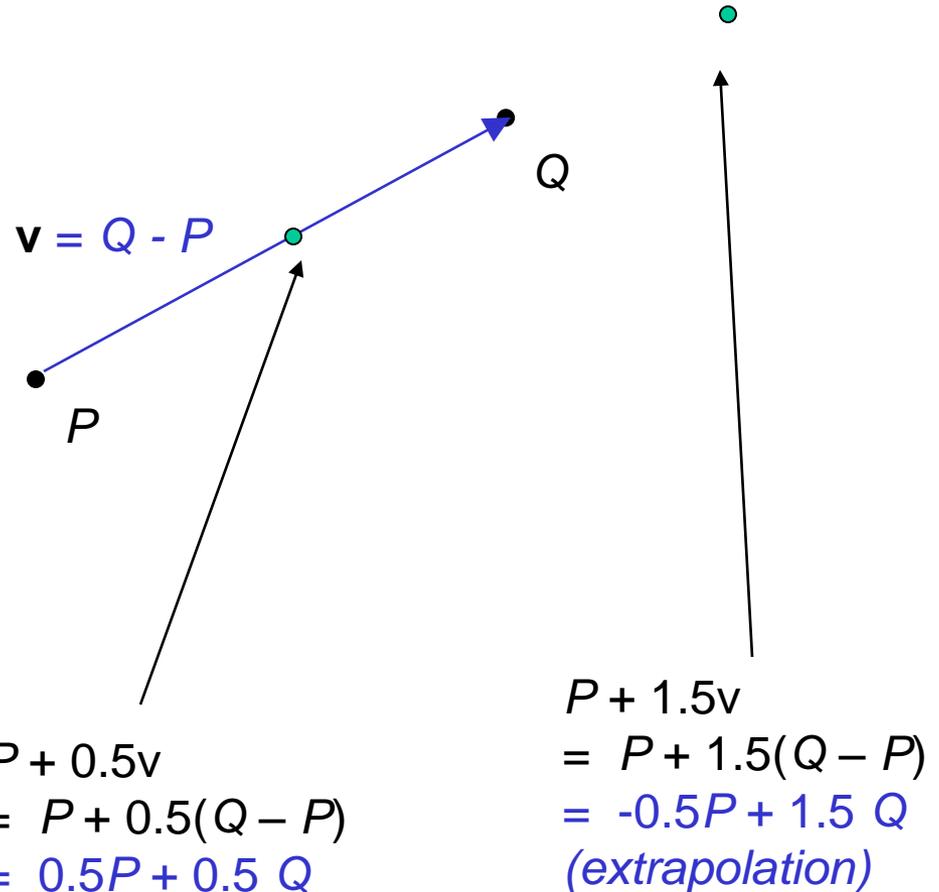
- Not an average of two images of objects...
- ...but an image of the average object!
- How can we make a smooth transition in time?
 - Do a “weighted average” over time t

How do we know what the average object looks like?

- We haven't a clue!
- But we can often fake something reasonable
 - Usually required user/artist input

Averaging Points

What's the average of P and Q?



Linear Interpolation
(Affine Combination):
New point $aP + bQ$,
defined only when $a+b = 1$
So $aP+bQ = aP+(1-a)Q$

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)... etc.

Idea #1: Cross-Dissolve



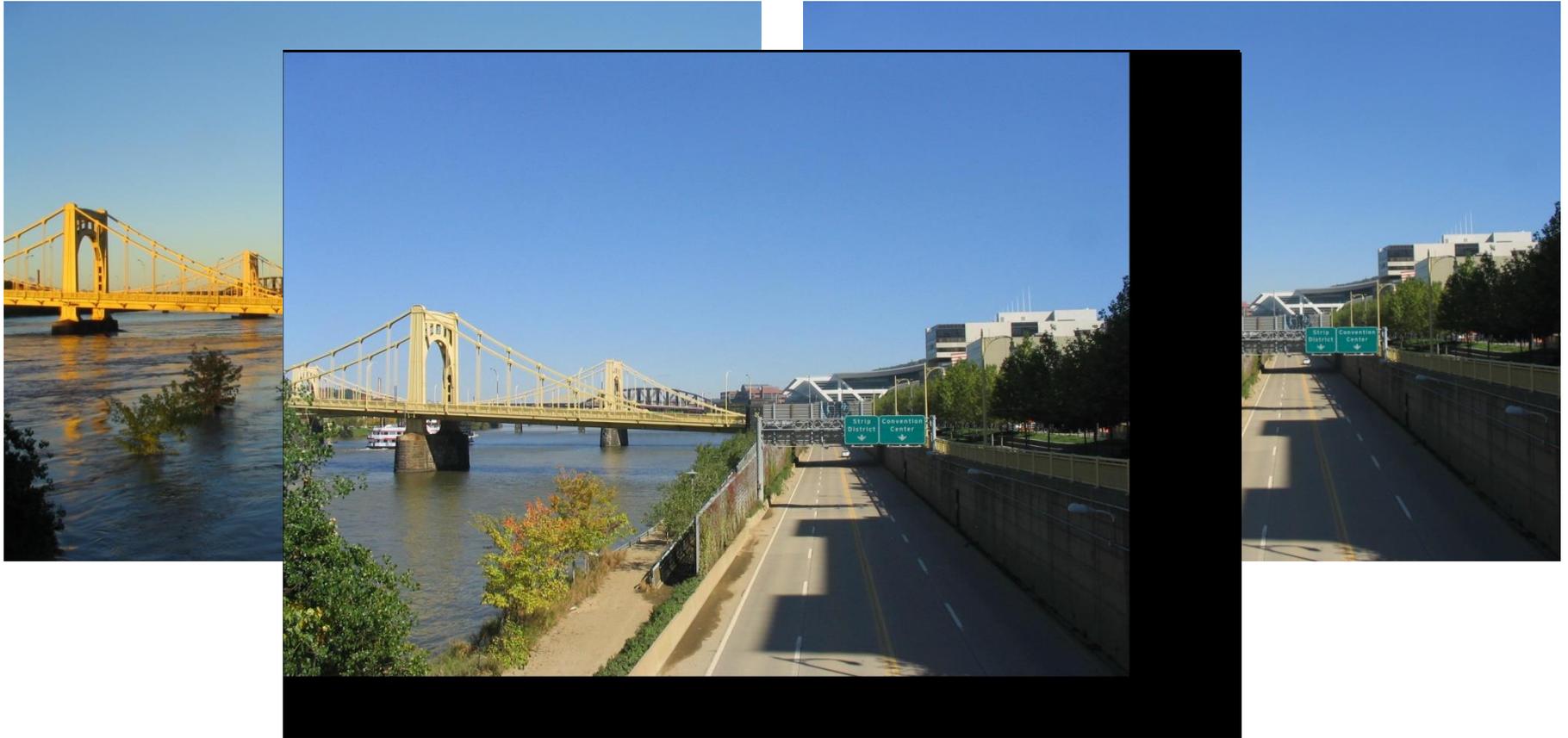
Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1-t) \cdot \text{Image}_1 + t \cdot \text{Image}_2$$

This is called **cross-dissolve** in film industry

But what if the images are not aligned?

Idea #2: Align, then cross-dissolve

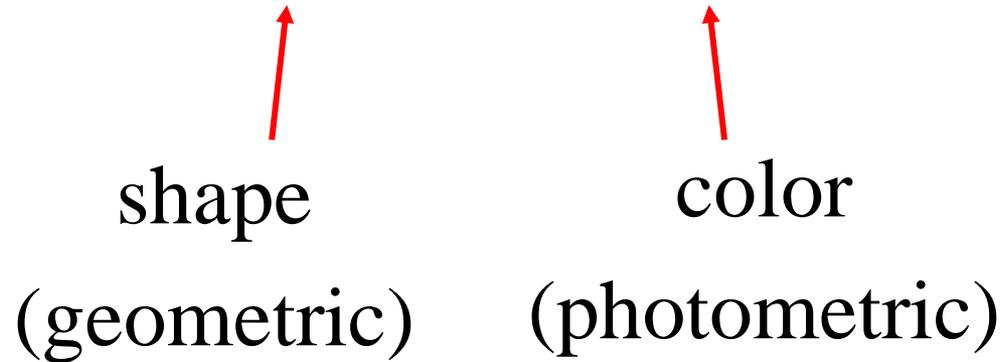


Align first, then cross-dissolve

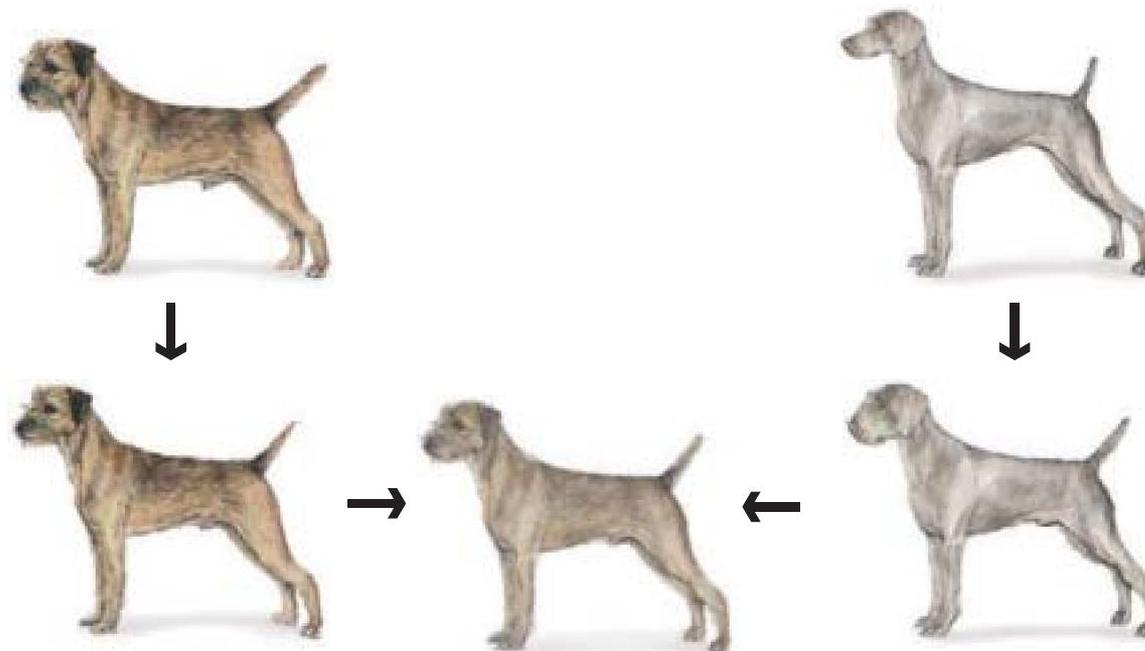
- Alignment using global warp – picture still valid

Image Morphing

Morphing = warping + cross-dissolving



Two-stage Morphing Procedure



Morphing procedure:

for every t ,

1. Find the average shape (the “mean dog” 😊)
 - warping
2. Find the average color
 - Cross-dissolve the warped images

BUT: global warp not always enough!



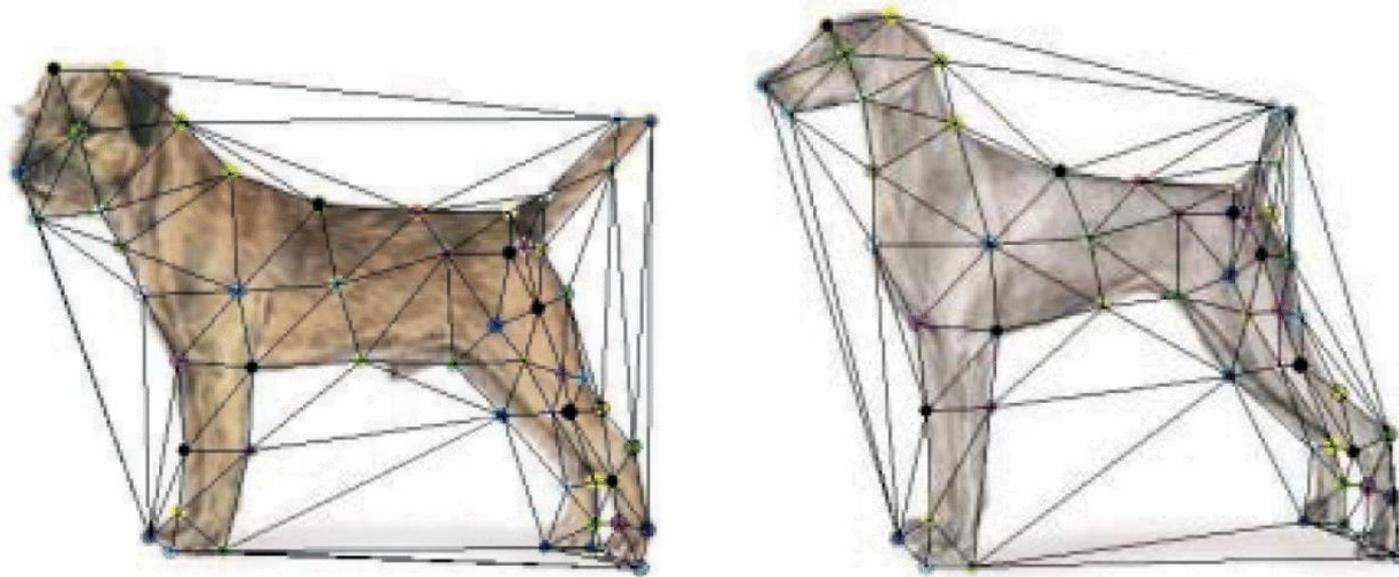
What to do?

- Cross-dissolve doesn't work
- Global alignment doesn't work
 - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

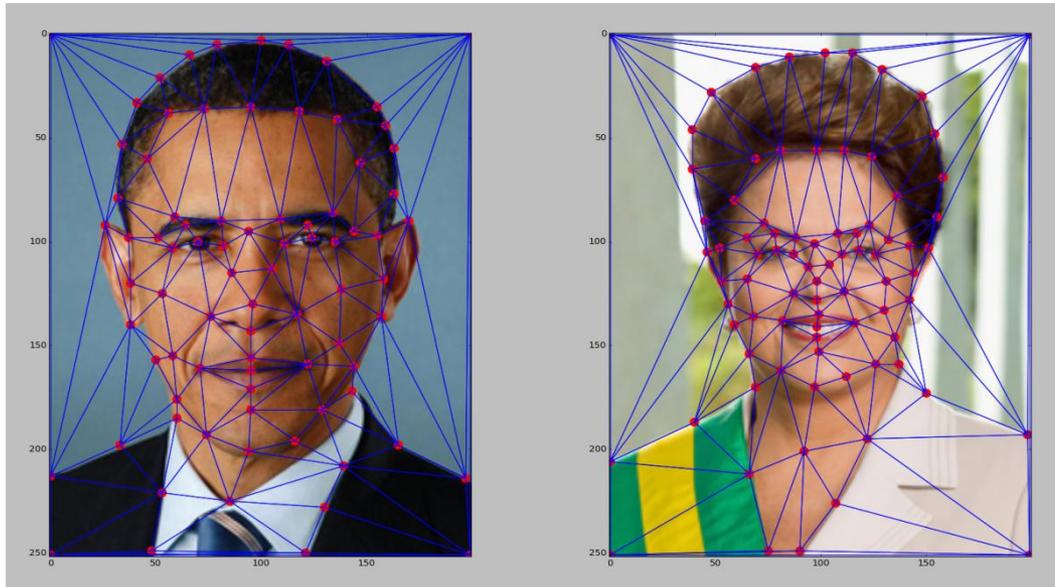
- Nose to nose, tail to tail, etc.
- But what to do with all the intermediate pixels?

Triangular Mesh



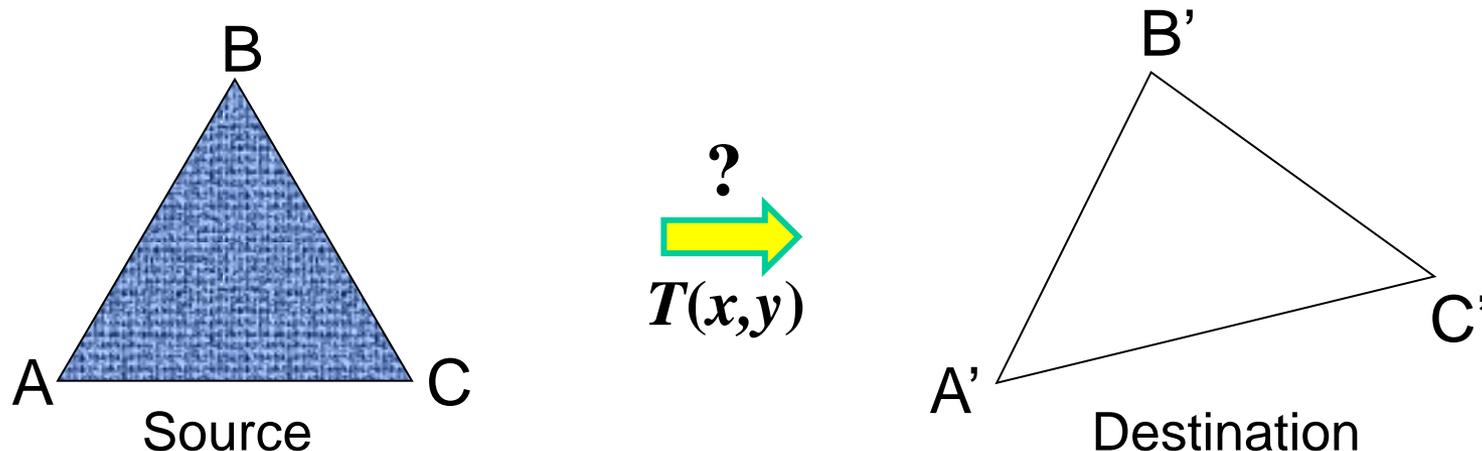
1. Input correspondences at key feature points
2. Define a triangular mesh over the points
 - Same mesh in both images!
 - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
 - How do we warp a triangle?

Full morphing result



(c) Ian Albuquerque Raymundo da Silva

Warping triangles



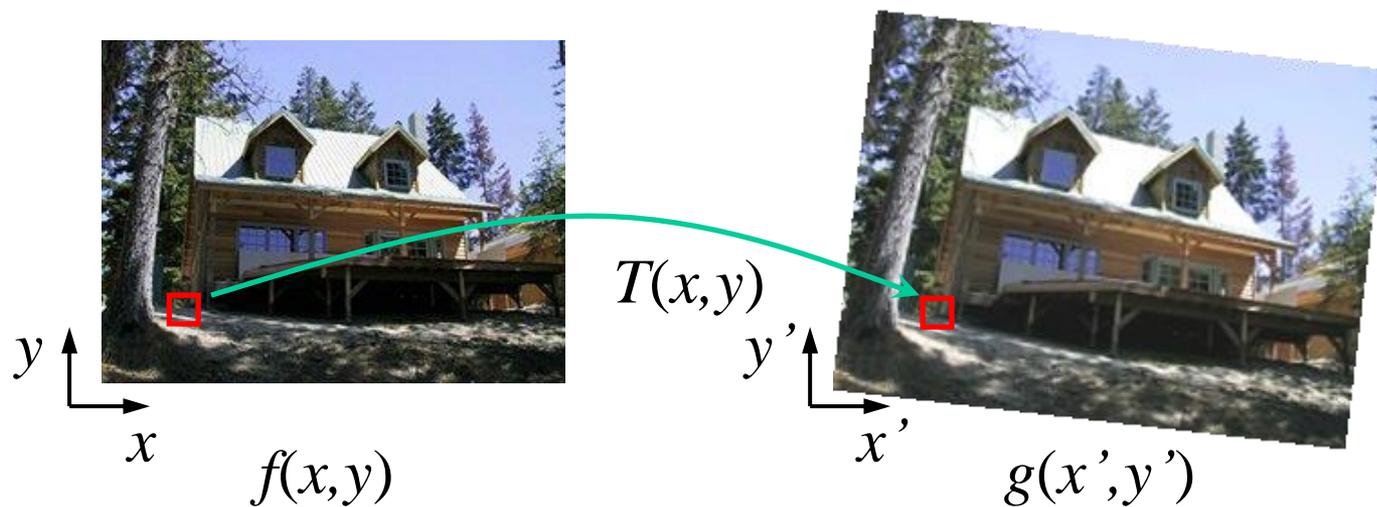
Given two triangles: ABC and $A'B'C'$ in 2D (12 numbers)
Need to find transform T to transfer all pixels from one to the other.

What kind of transformation is T ?

How can we compute the transformation matrix:

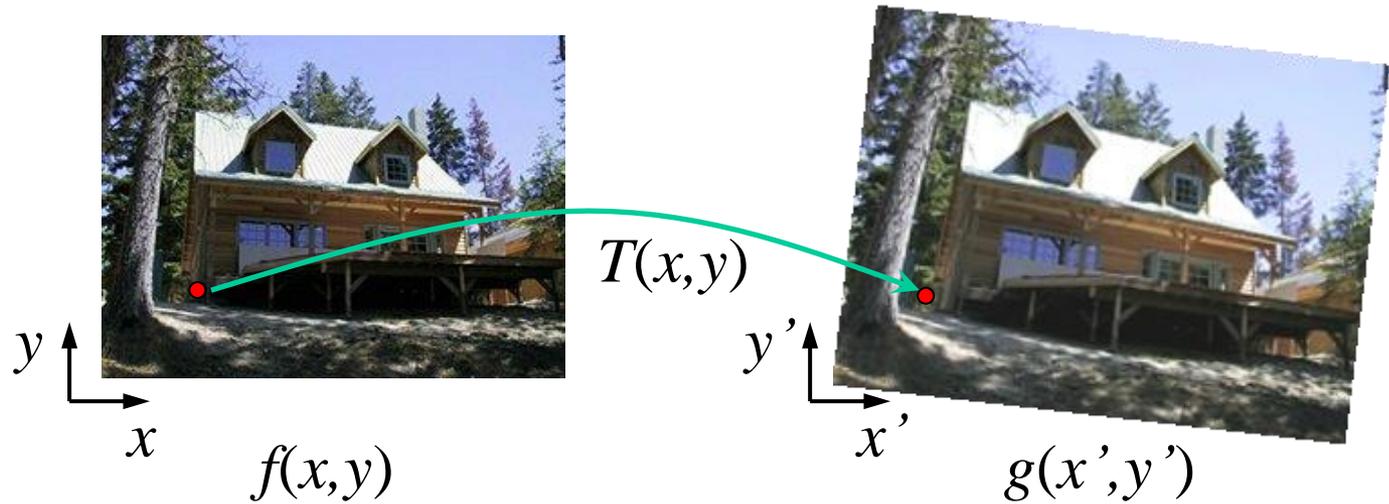
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Warping Pixels



Given a coordinate transform $(x',y') = T(x,y)$ and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

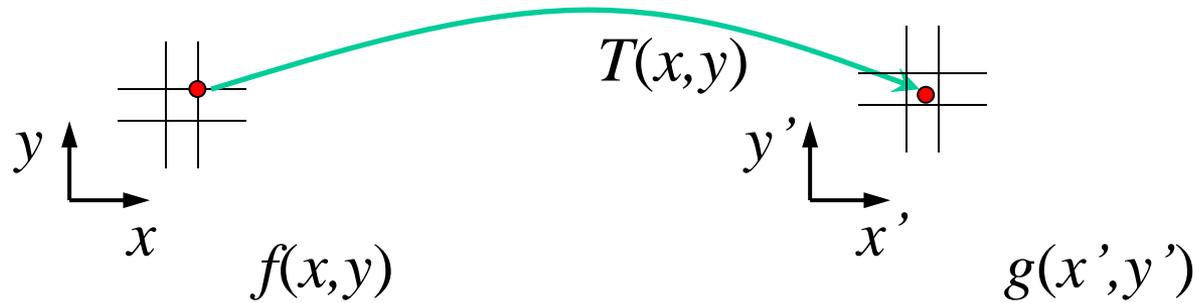
Forward warping



Send each pixel $f(x, y)$ to its corresponding location
 $(x', y') = T(x, y)$ in the second image

Q: what if pixel lands “between” two pixels?

Forward warping



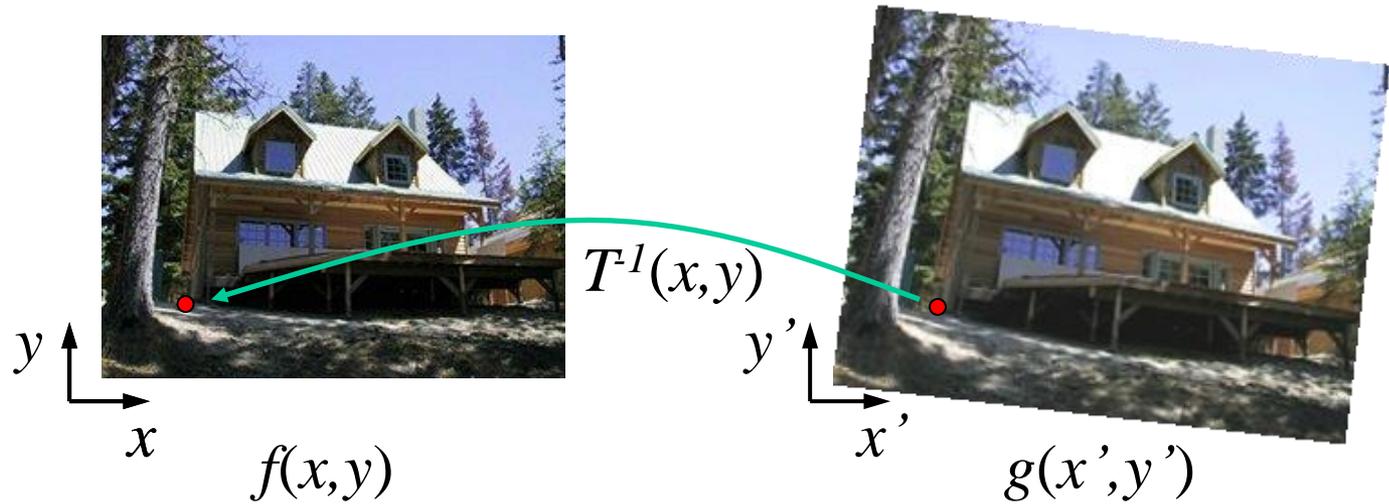
Send each pixel $f(x,y)$ to its corresponding location $(x',y') = T(x,y)$ in the second image

Q: what if pixel lands “between” two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as “splatting”
- Check out `griddata` in Matlab

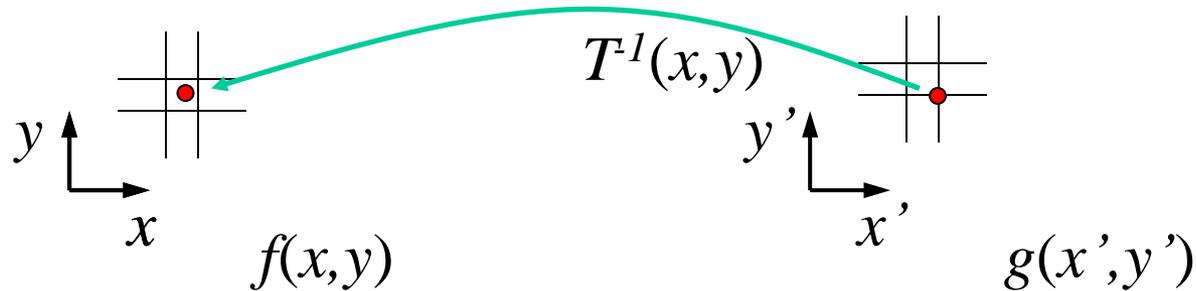
Inverse warping



Get each pixel $g(x',y')$ from its corresponding location $(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from “between” two pixels?

Inverse warping



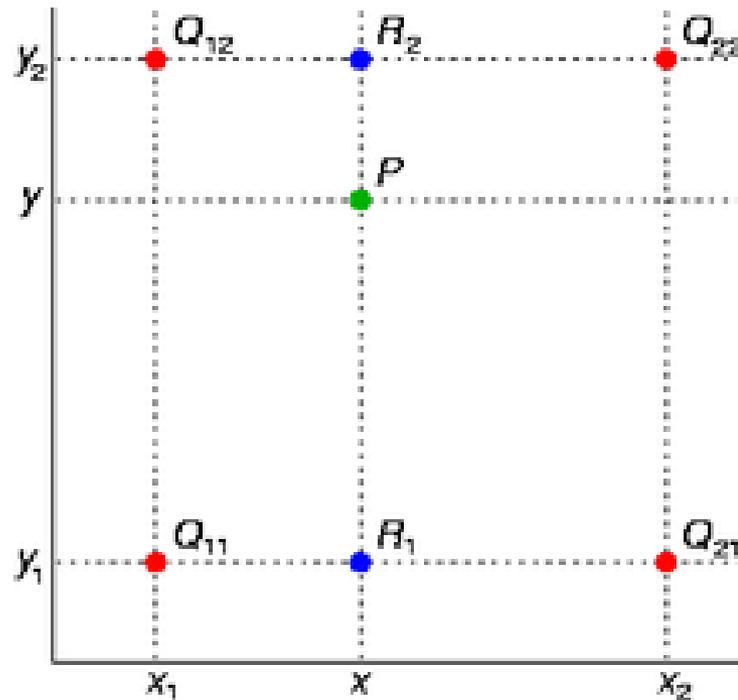
Get each pixel $g(x', y')$ from its corresponding location $(x, y) = T^{-1}(x', y')$ in the first image

Q: what if pixel comes from “between” two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab / Python

Bilinear Interpolation



http://en.wikipedia.org/wiki/Bilinear_interpolation
Help interp2

Forward vs. inverse warping

Q: which is better?

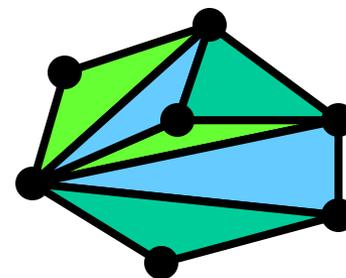
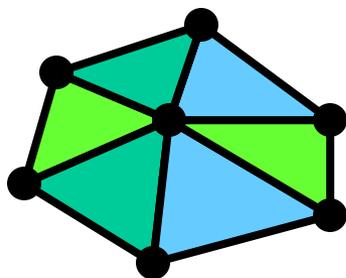
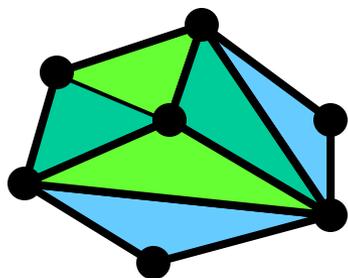
A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

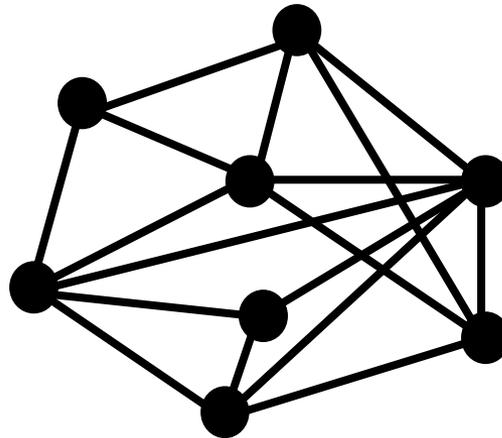
There are an exponential number of triangulations of a point set.



An $O(n^3)$ Triangulation Algorithm

Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



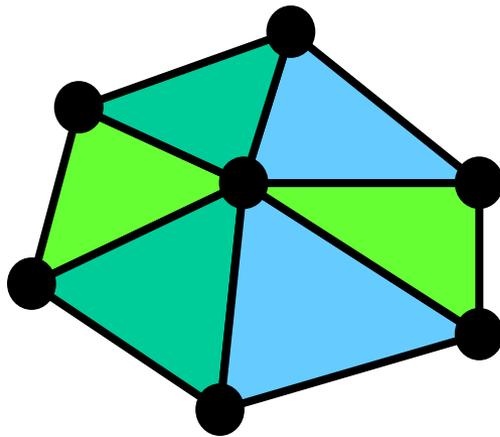
“Quality” Triangulations

Let $\alpha(T) = (\alpha_1, \alpha_2, \dots, \alpha_{3t})$ be the vector of angles in the triangulation T in increasing order.

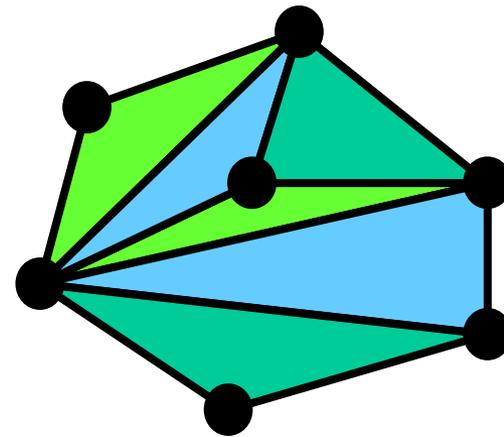
A triangulation T_1 will be “better” than T_2 if $\alpha(T_1) > \alpha(T_2)$ lexicographically.

The Delaunay triangulation is the “best”

- Maximizes smallest angles



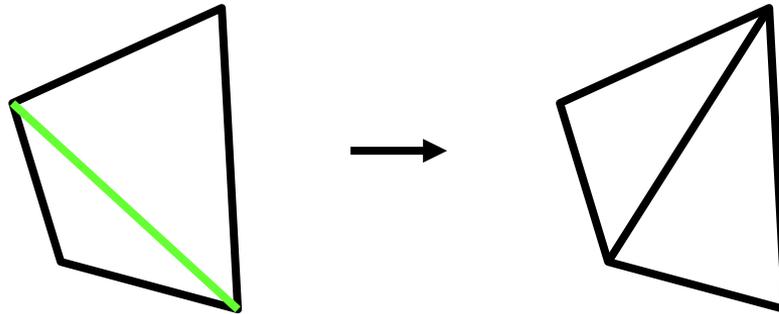
good



bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.

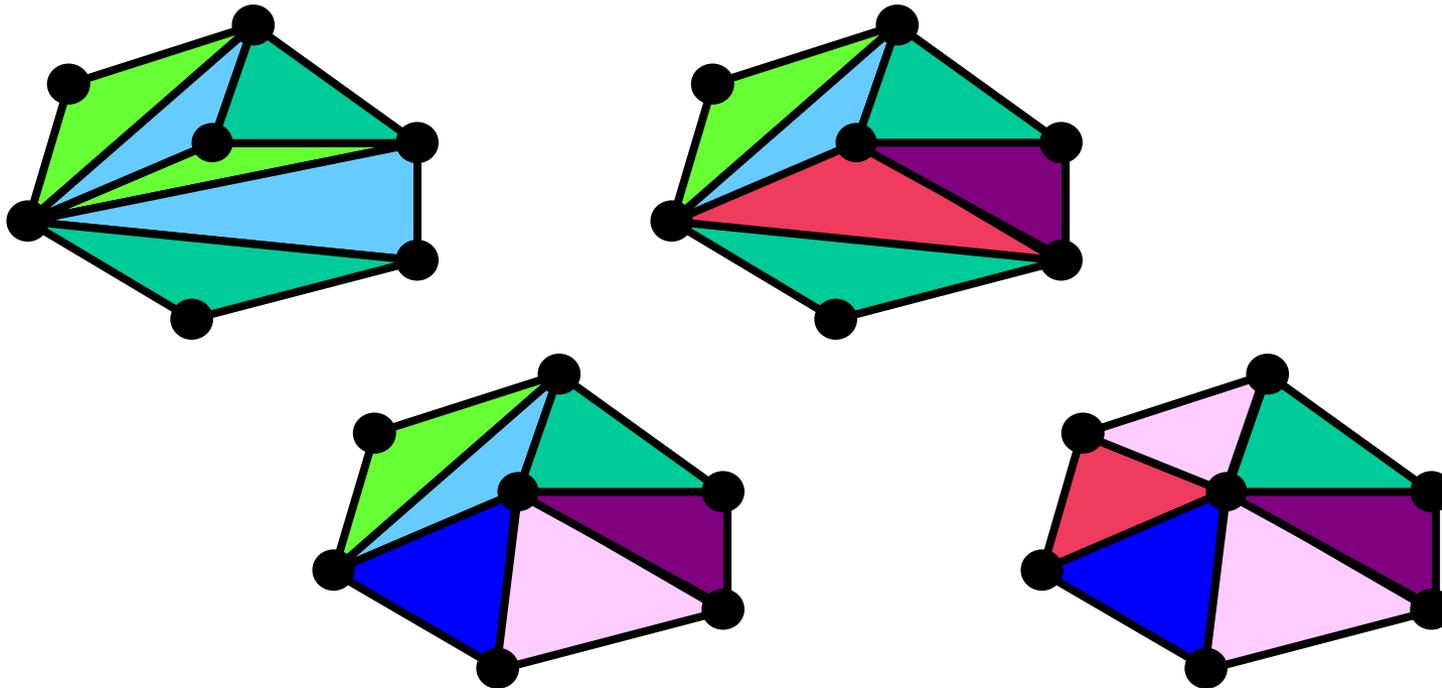


If an edge flip improves the triangulation, the first edge is called *illegal*.

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

Could take a long time to terminate.



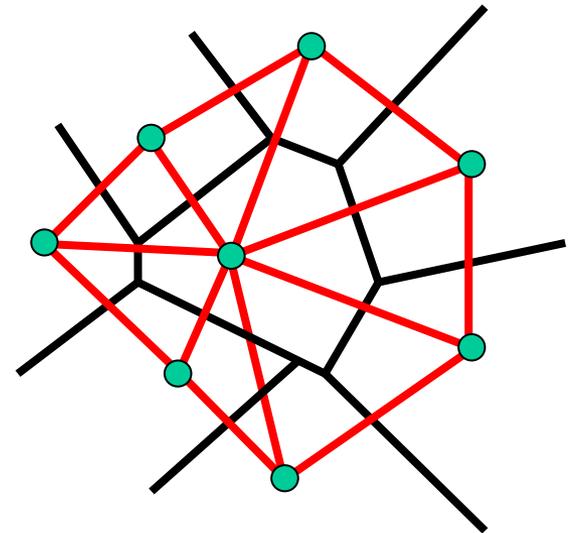
Delaunay Triangulation by Duality

General position assumption: There are no four co-circular points.

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.

Corollary: The DT may be constructed in $O(n \log n)$ time.

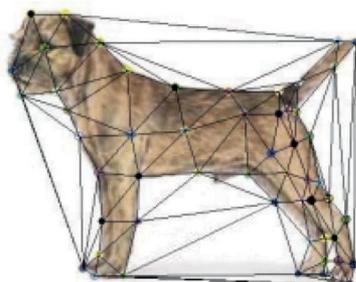
This is what Matlab's `delaunay` function uses.



1. Create Average Shape

How do we create an intermediate warp at time t ?

- Assume $t = [0,1]$
- Simple linear interpolation of each feature pair
 - $p=(x,y) \rightarrow p'(x,y)$
- $(1-t)*p+t*p'$ for corresponding features p and p'



2. Create Average Color



Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1-t) * \text{Image} + t * \text{image}'$$

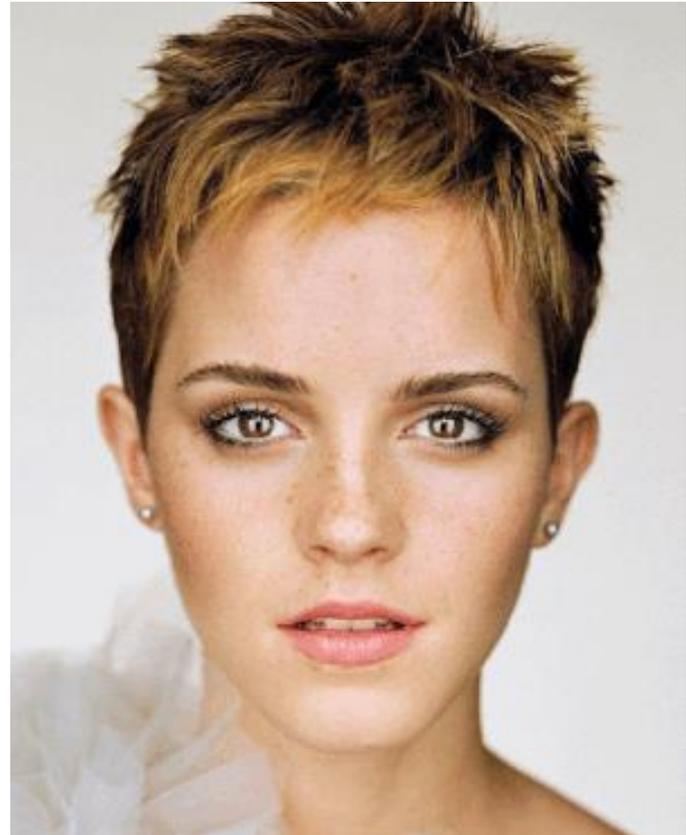
cross-dissolve!



Project #3: morphing

1. Define corresponding points
 2. Define triangulation on points
 - Use same triangulation for both images
 3. For each $t = 0:\text{step}:1$
 - a. Compute the average shape at t (weighted average of points)
 - b. For each triangle in the average shape
 - Get the affine projection to the corresponding triangles in each image
 - For each pixel in the triangle, find the corresponding points in each image and set value to weighted average (cross-dissolve each triangle)
 - c. Save the image as the next frame of the sequence
- Life-hack: can be done with just two nested loops (for t , and for each triangle). Hint: compute warps for all pixels first, then use `interp2`

Examples



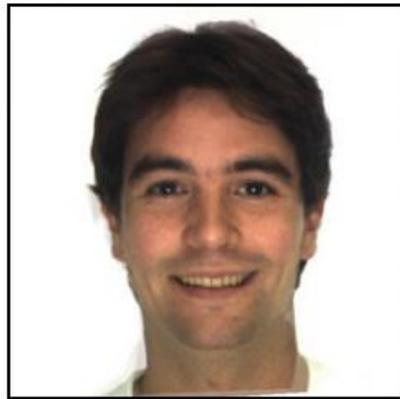
© Rachel Albert, CS194-26, Fall 2015

Morphing & matting

Extract foreground first to avoid artifacts in the background



(c) $\alpha = 0.0$



(d) $\alpha = 0.2$



(e) $\alpha = 0.4$



(f) $\alpha = 0.6$

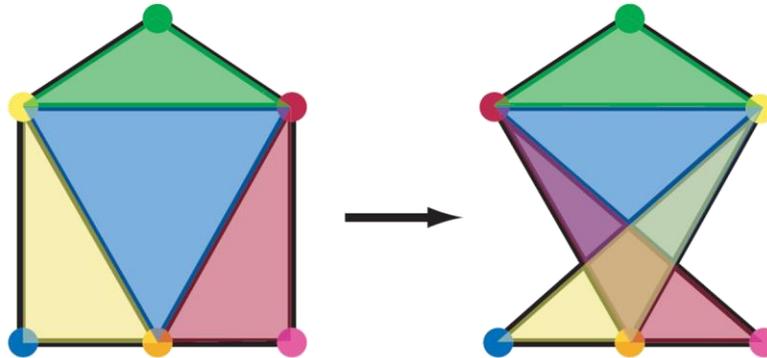


(g) $\alpha = 0.8$



(h) $\alpha = 1.0$

Other Issues



Beware of folding

- You are probably trying to do something 3D-ish

Morphing can be generalized into 3D

- If you have 3D data, that is!

Extrapolation can sometimes produce interesting effects

- Caricatures

Dynamic Scene (“Black or White”, MJ)



<http://www.youtube.com/watch?v=R4kLKv5gtxc>