Homographies and Panoramas

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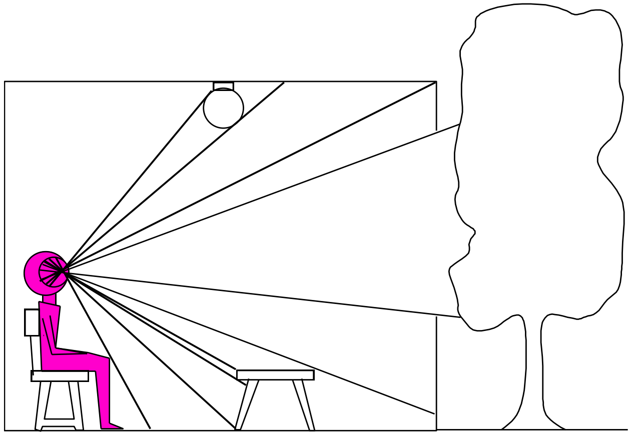
CS194: Intro to Computer Vision and Comp. Photo
Alexei Efros, UC Berkeley, Fall 2022

with a lot of slides stolen from
Steve Seitz and Rick Szeliski

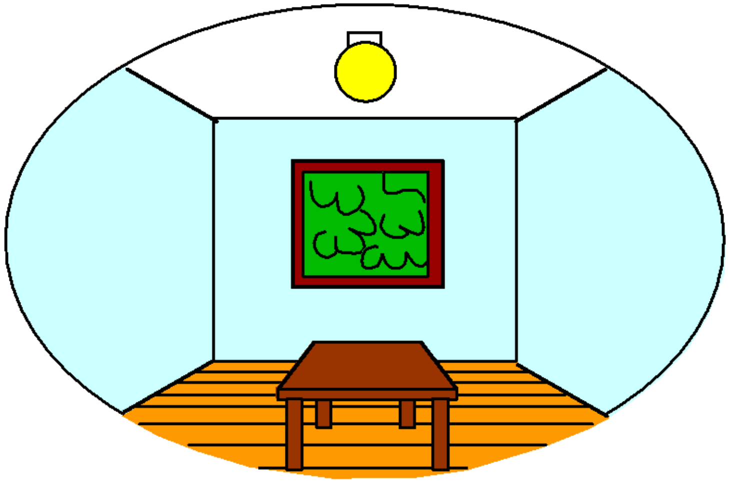
What do we see?

3D world



Point of observation

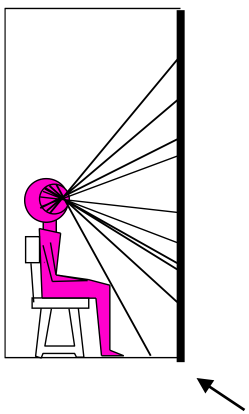
2D image



Figures © Stephen E. Palmer, 2002

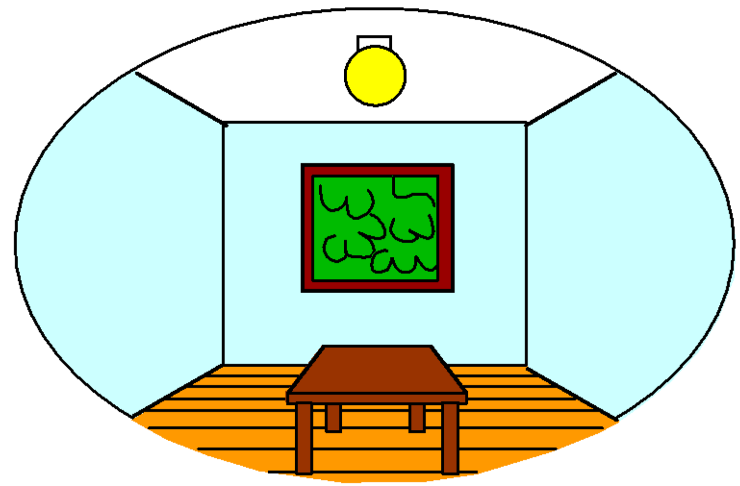
What do we see?

3D world



Painted
backdrop

2D image



On Simulating the Visual Experience

Just feed the eyes the right data

- No one will know the difference!

Philosophy:

- Ancient question: "Does the world really exist?"

Science fiction:

- Many, many, many books on the subject, e.g. *slowglass* from "Light of Other Days"
- Latest take: *The Matrix*

Physics:

- *Slowglass* might be possible?

Computer Science:

- Virtual Reality

To simulate we need to know:

What does a person see?

The Plenoptic Function

Figure by Leonard McMillan

Q: What is the set of all things that we can ever see?

A: The Plenoptic Function (Adelson & Bergen)

Let’s start with a stationary person and try to parameterize everything that he can see…

Grayscale snapshot

$$P(\theta,\phi)$$

is intensity of light

- Seen from a single view point
- At a single time
- Averaged over the wavelengths of the visible spectrum

(can also do *P(x,y),* but spherical coordinate are nicer)

Color snapshot

$$P(\theta,\phi,\lambda)$$

is intensity of light

- Seen from a single view point
- At a single time
- As a function of wavelength

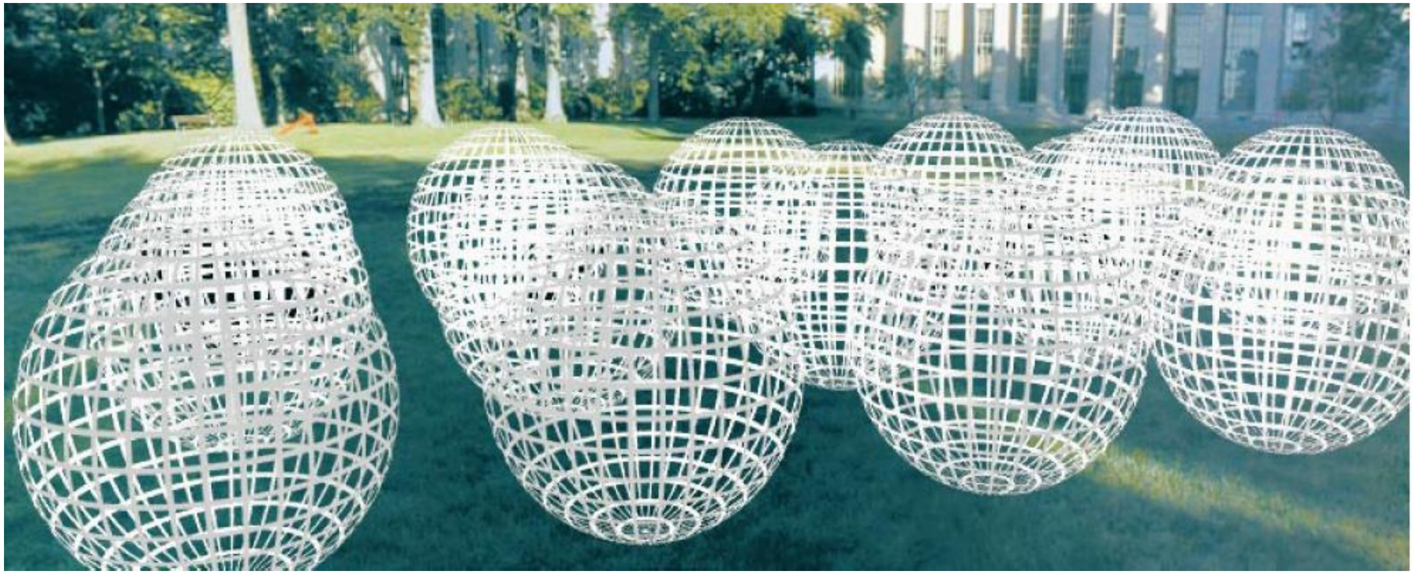
A movie

$$P(\theta,\phi,\lambda,t)$$

is intensity of light

- Seen from a single view point
- Over time
- As a function of wavelength

Holographic movie

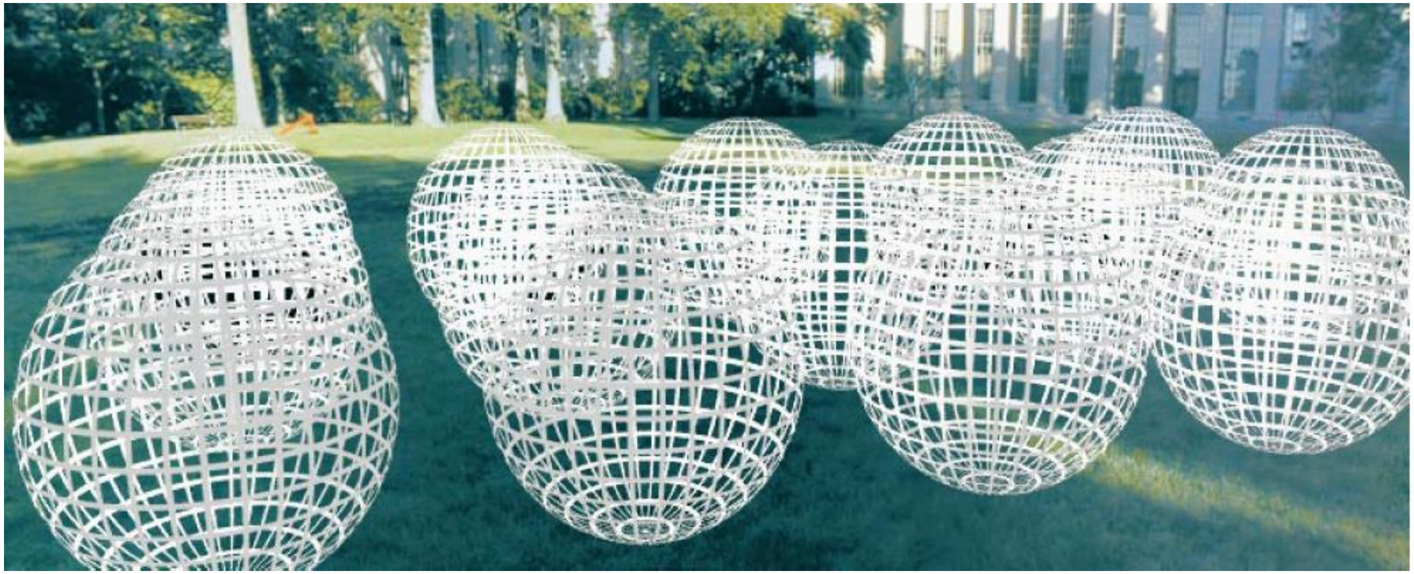


$$P(\theta,\phi,\lambda,t,V_X,V_Y,V_Z)$$

is intensity of light

- Seen from ANY viewpoint
- Over time
- As a function of wavelength

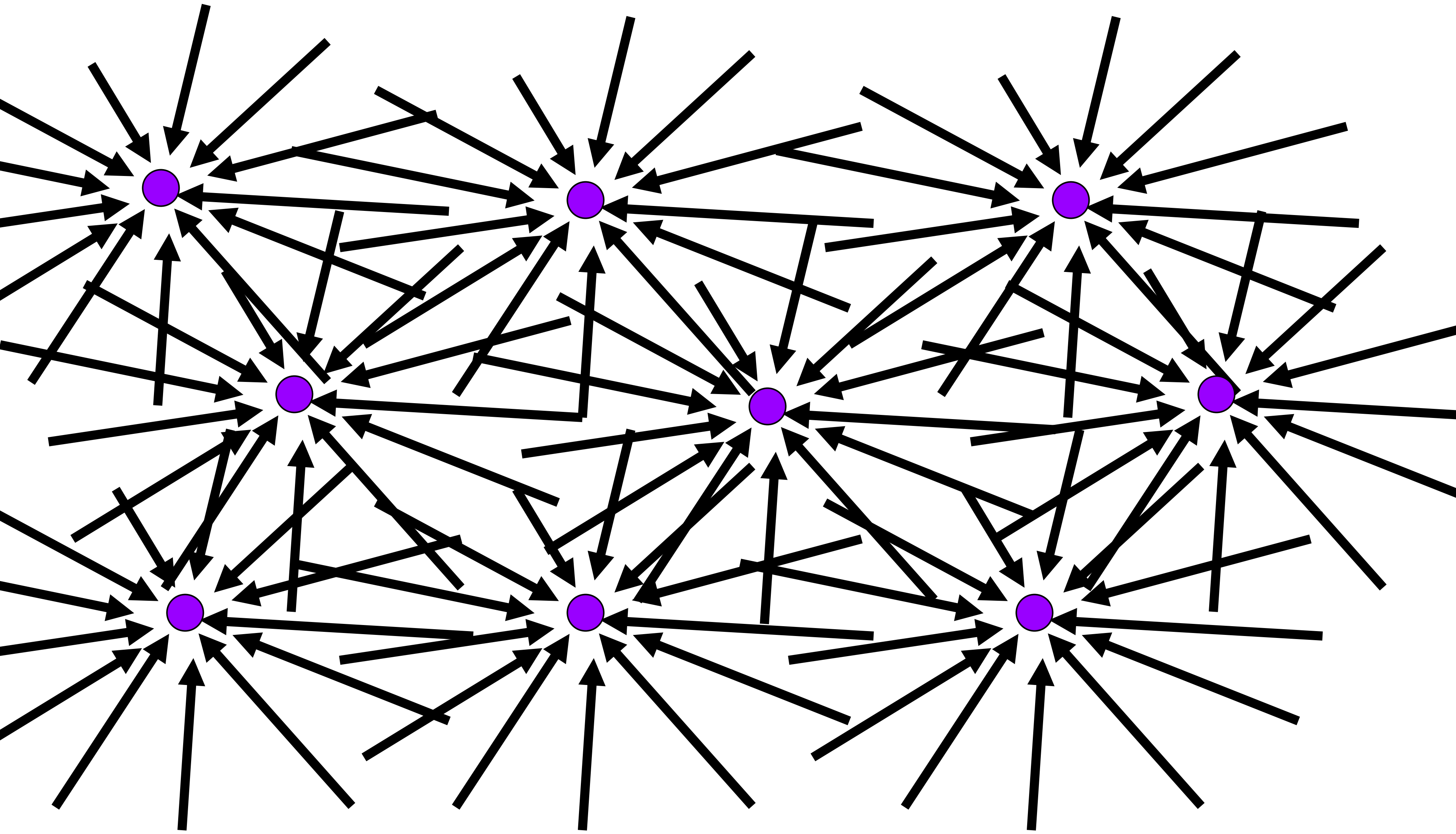
The Plenoptic Function



$$P(\theta,\phi,\lambda,t,V_X,V_Y,V_Z)$$

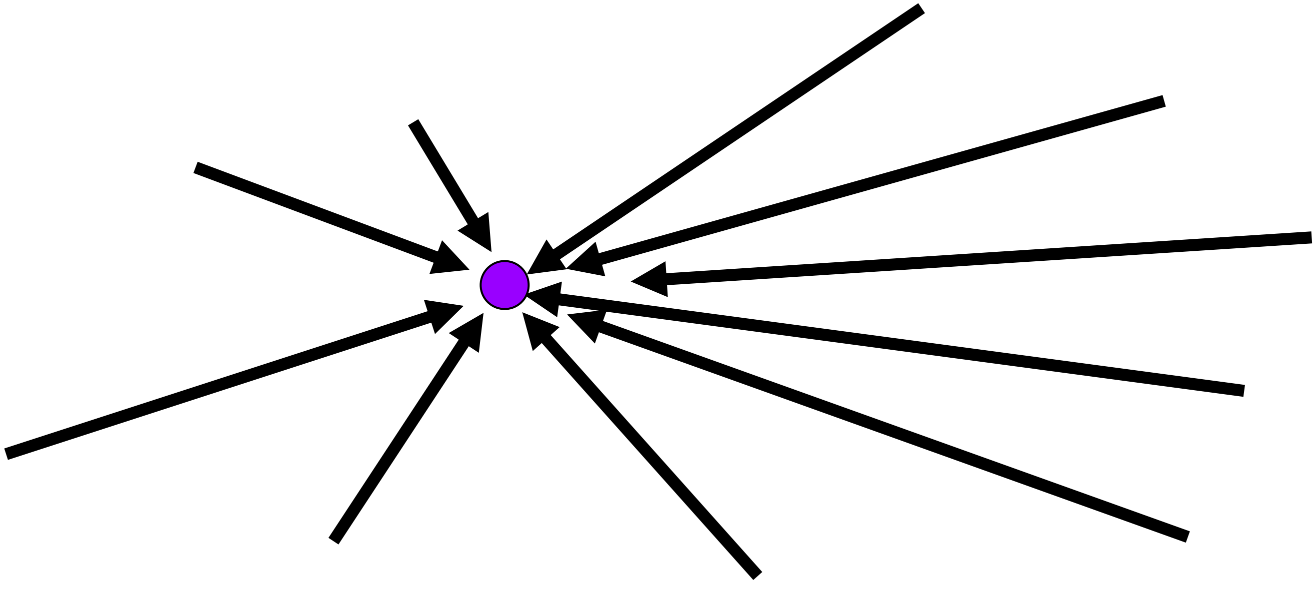
- Can reconstruct every possible view, at every moment, from every position, at every wavelength
- Contains every photograph, every movie, everything that anyone has ever seen! it completely captures our visual reality!   Not bad for a function…

Sampling Plenoptic Function (top view)



Just lookup -- Quicktime VR

What is an image?



Slide by Rick Szeliski and Michael Cohen

Spherical Panorama

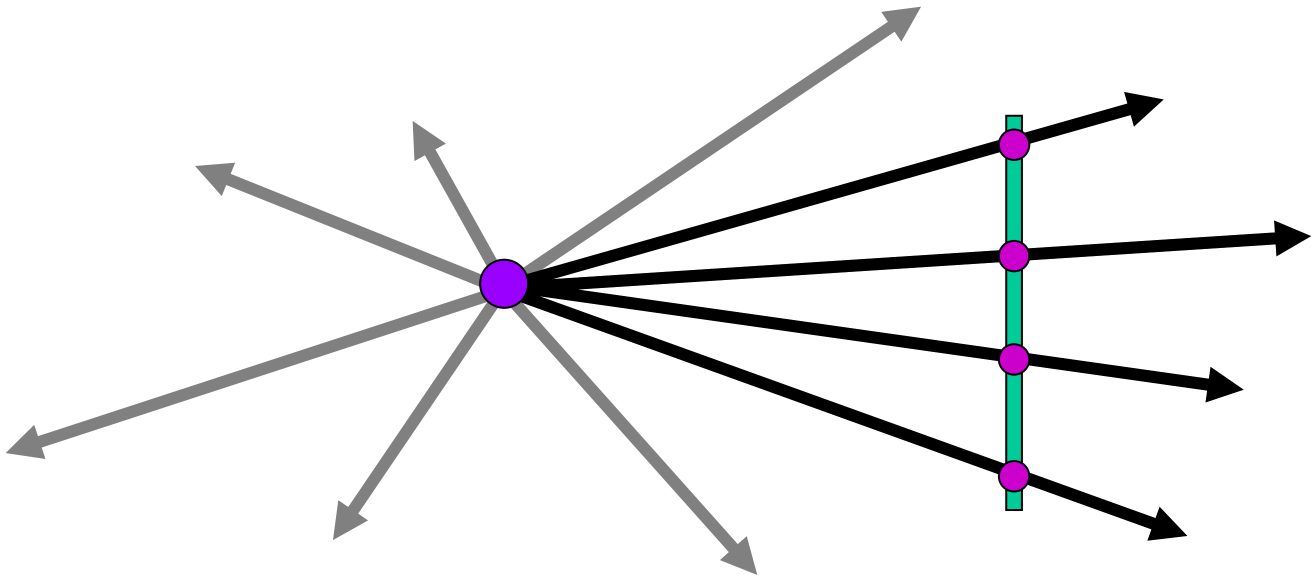
See also: 2003 New Years Eve
http://www.panoramas.dk/New-Year/times-square.html

All light rays through a point form a ponorama

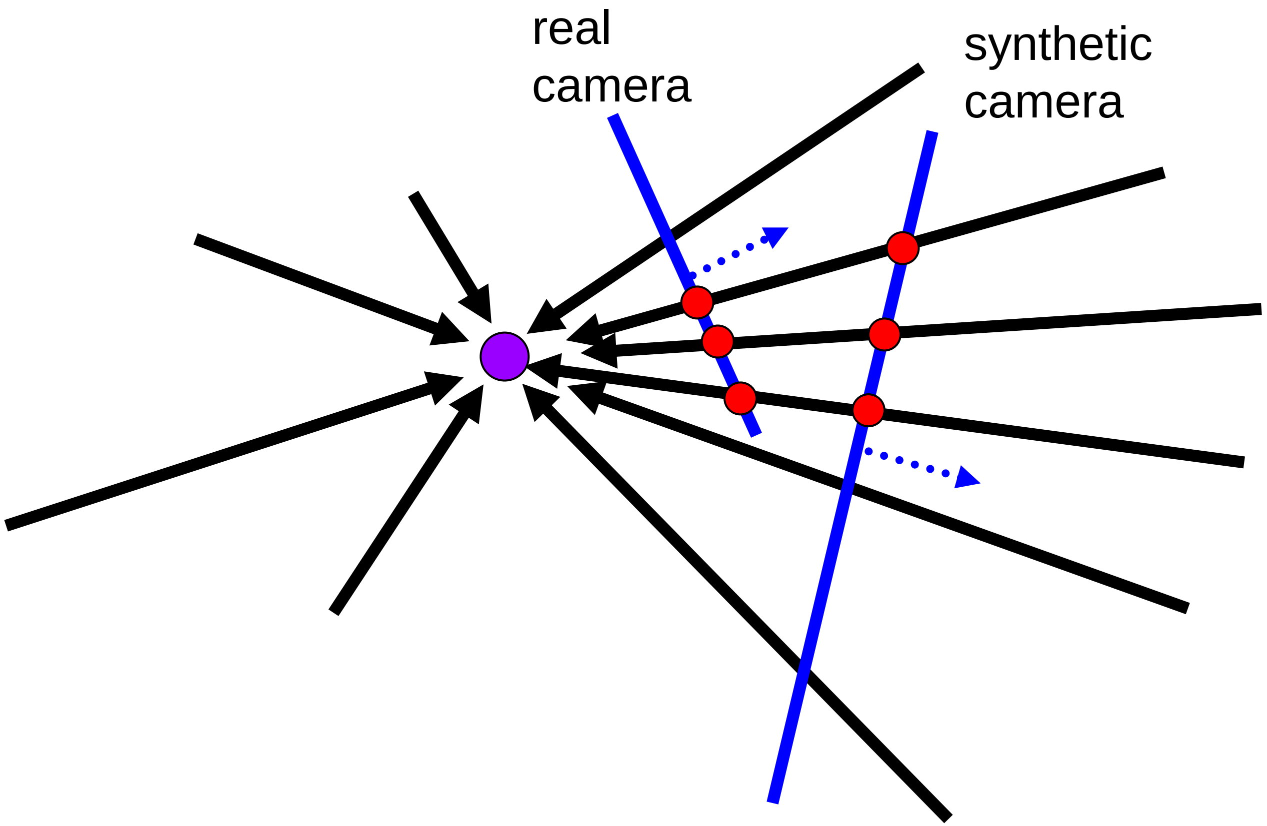
Totally captured in a 2D array -- $P(\theta,\phi)$

Where is the geometry???

What is an Image?



A pencil of rays contains all views

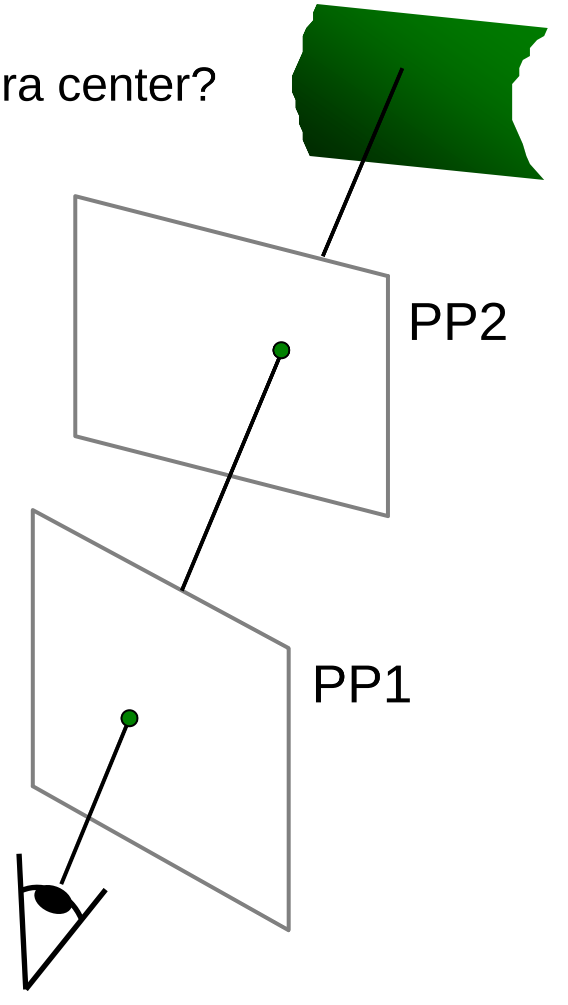


Can generate any synthetic camera view
as long as it has **the same center of projection**!

Image reprojection

Basic question

- How to relate two images from the same camera center?
  - how to map a pixel from PP1 to PP2



Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

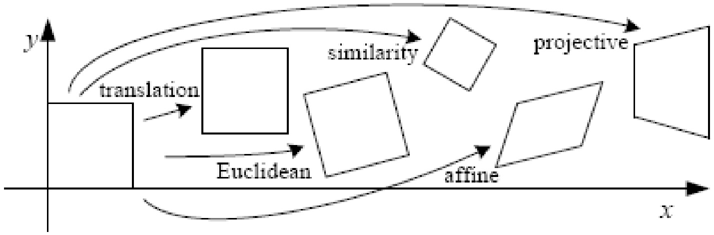
But don’t we need to know the geometry of the two planes in respect to the eye?

Observation:
Rather than thinking of this as a 3D reprojection, think of it as a 2D **image warp** from one image to another

Back to Image Warping

Which t-form is the right one for warping PP1 into PP2?

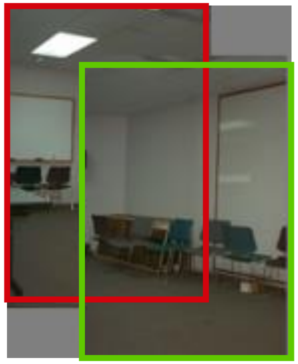
e.g. translation, Euclidean, affine, projective

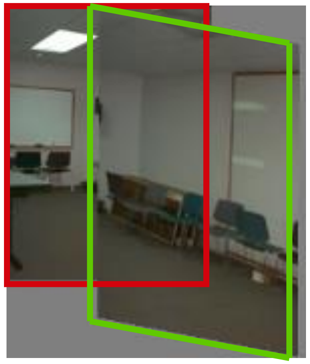


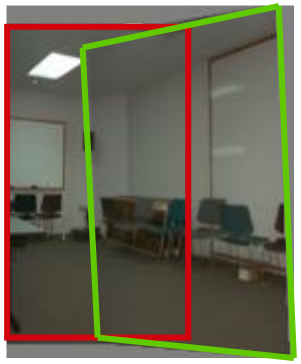
Translation

Affine

Perspective







2 unknowns

6 unknowns

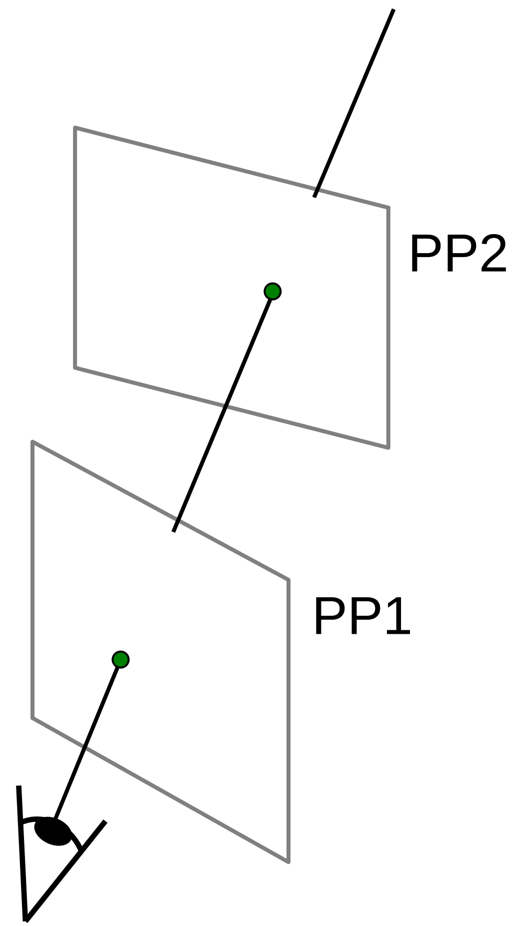
8 unknowns

Homography

A: Projective – mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren't
- but must preserve straight lines
- same as: unproject, rotate, reproject

called Homography

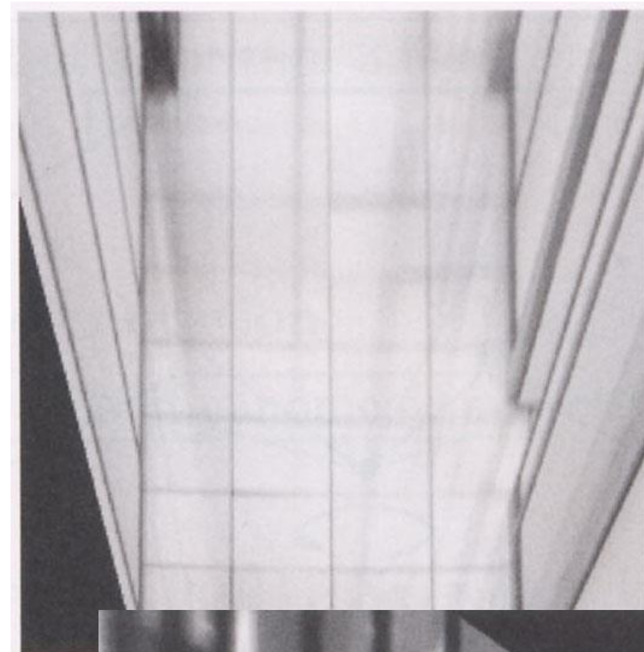


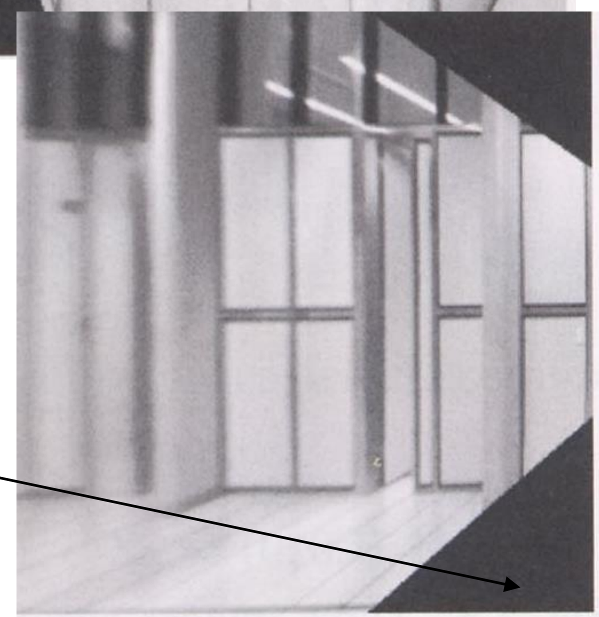
$$\underset{\mathbf{p'}}{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}} = \underset{\mathbf{H}}{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}} \underset{\mathbf{p}}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

To apply a homography **H**

- Compute **p'** = **Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

Image warping with homographies





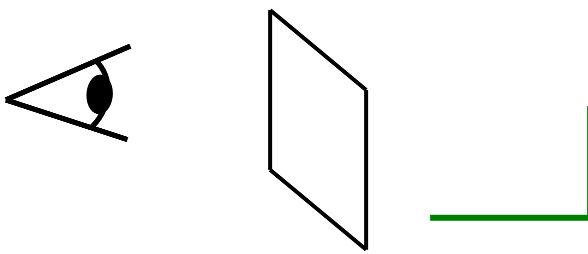
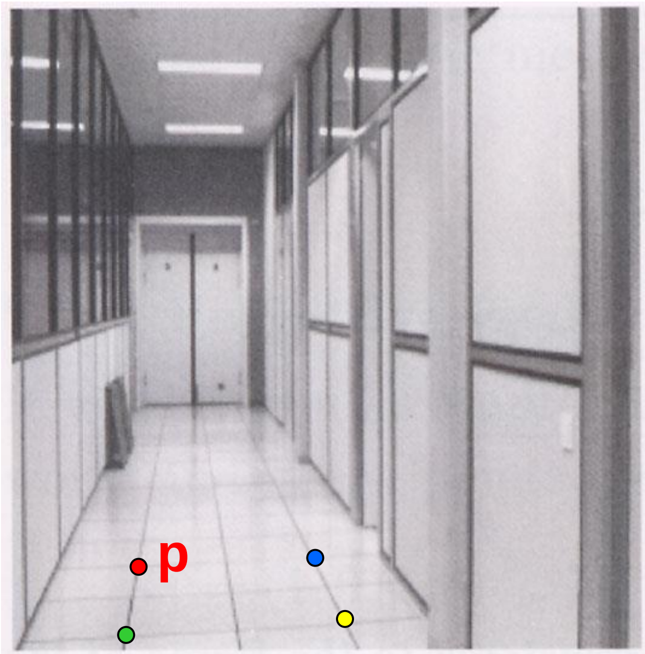
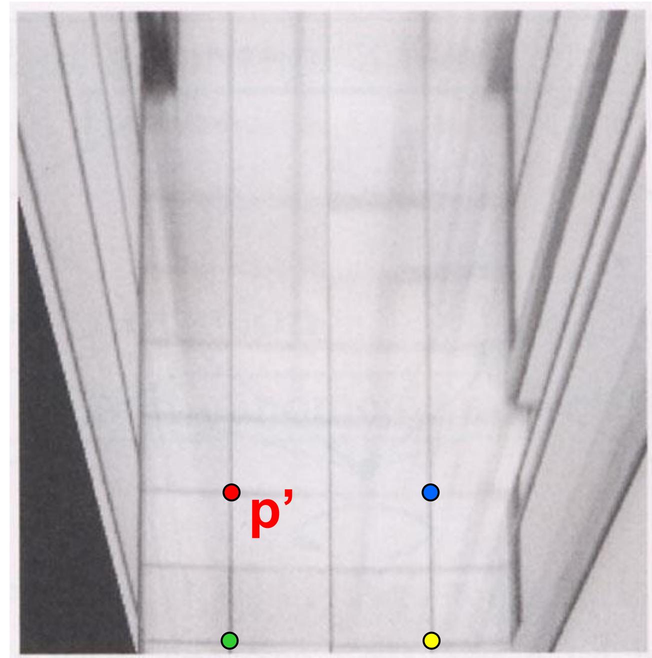


image plane in front

black area
where no pixel
maps to

Image rectification





To unwarp (rectify) an image

- Find the homography **H** given a set of **p** and **p’** pairs
- How many correspondences are needed?
- Tricky to write H analytically, but we can <u>solve</u> for it!
  - Find such H that “best” transforms points p into p’
  - Use least-squares!

Least Squares Example

Say we have a set of data points (p1,p1'), (p2,p2'), (p3,p3'), etc. (e.g. person's height vs. weight)

We want a nice compact formula (a line) to predict p' from p: $px_1 + x_2 = p'$

We want to find x_1 and x_2

How many (p,p') pairs do we need?

$$p_1 x_1 + x_2 = p_1^{'}$$
$$p_2 x_1 + x_2 = p_2^{'}$$

$$\begin{bmatrix} p_1 & 1 \\ p_2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1^{'} \\ p_2^{'} \end{bmatrix}$$

$$\mathrm{Ax} = b$$

Least Squares Example

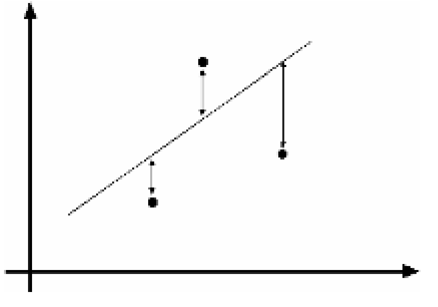
- Say we have a set of data points (p1,p1'), (p2,p2'), (p3,p3'), etc. (e.g. person's height vs. weight)
- We want a nice compact formula (a line) to predict p' from p: $px_1 + x_2 = p'$
- We want to find x_1 and x_2
- How many (p,p') pairs do we need?

$$p_1 x_1 + x_2 = p_1' \qquad \begin{bmatrix} p_1 & 1 \\ p_2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1' \\ p_2' \end{bmatrix} \qquad \mathrm{Ax} = b$$

$$p_2 x_1 + x_2 = p_2'$$

- What if the data is noisy?

$$\begin{bmatrix} p_1 & 1 \\ p_2 & 1 \\ p_3 & 1 \\ \dots & \dots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1' \\ p_2' \\ p_3' \\ \dots \end{bmatrix} \qquad \min \left\| Ax - b \right\|^2$$



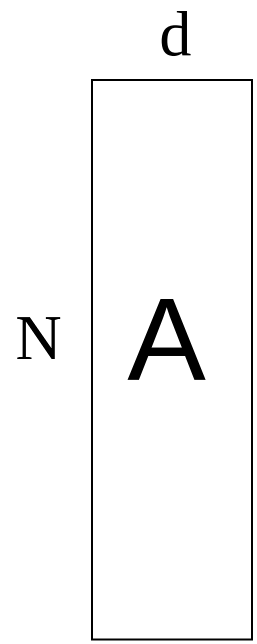
overconstrained

Least-Squares

- Solve:

$$A\,\mathbf{x} = \mathbf{b}$$

(N,d)(d,1) = (N,1)



- Normal equations

$$A^T A\,\mathbf{x} = A^T\mathbf{b}$$

(d,N)(N,d)(d,1) = (d,N)(N,1)

rank(A)≤min(d,N)
assume rank(A)=d
implies rank(A^TA)=d
A^TA is invertible

- Solution:

$$\mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}$$

Solving for homographies

p' = Hp

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Can set scale factor *i*=1. So, there are 8 unkowns.

Set up a system of linear equations:

Ah = b

where vector of unknowns h = $[a,b,c,d,e,f,g,h]^T$

Need at least 8 eqs, but the more the better…

Solve for h. If overconstrained, solve using least-squares:

$$\min \|Ah - b\|^2$$

Can be done in Matlab using "\" command

- see "help lmdivide"

Fun with homographies

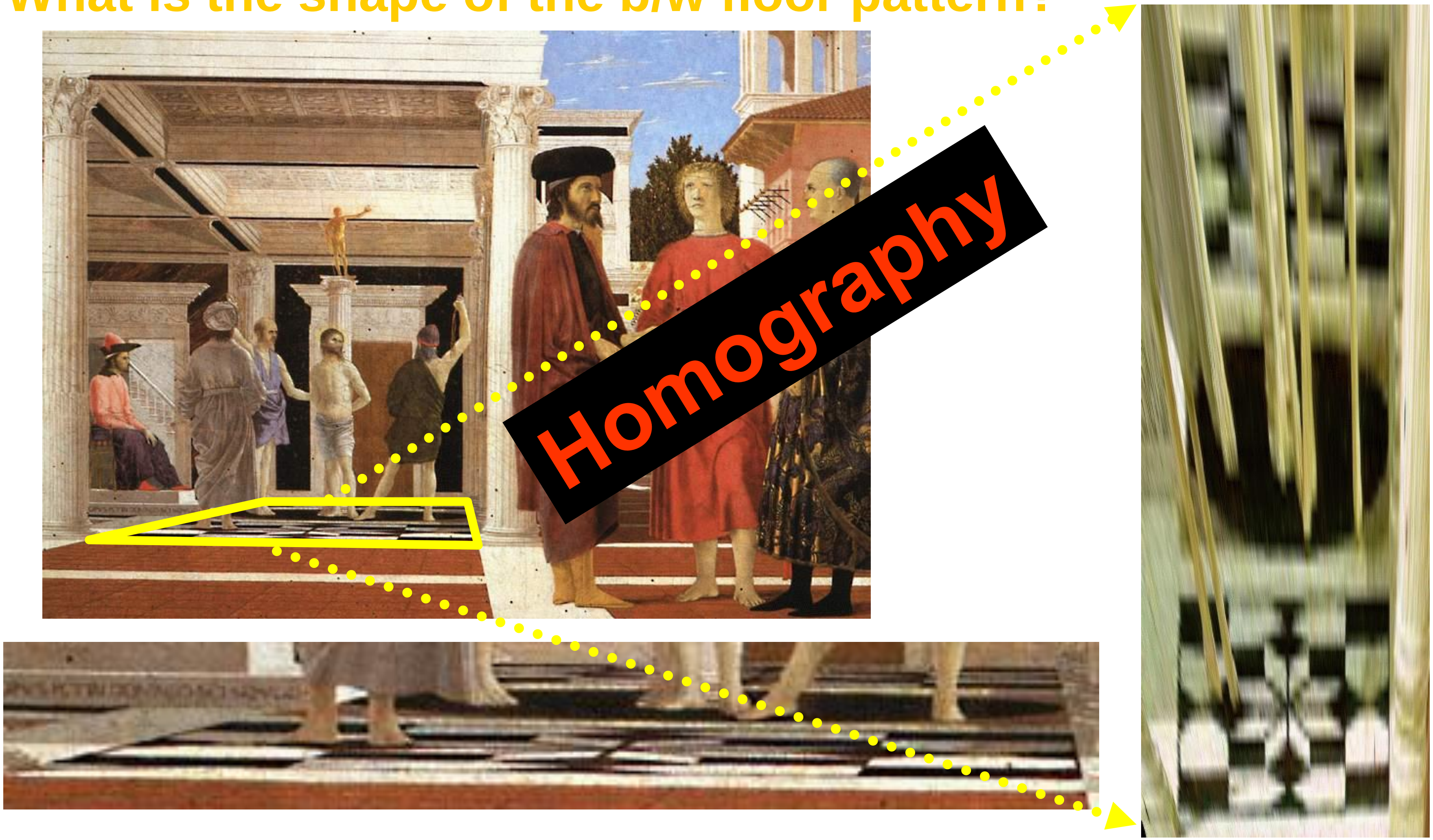
Original image

St.Petersburg
photo by A. Tikhonov

Virtual camera rotations

Analysing patterns and shapes

What is the shape of the b/w floor pattern?



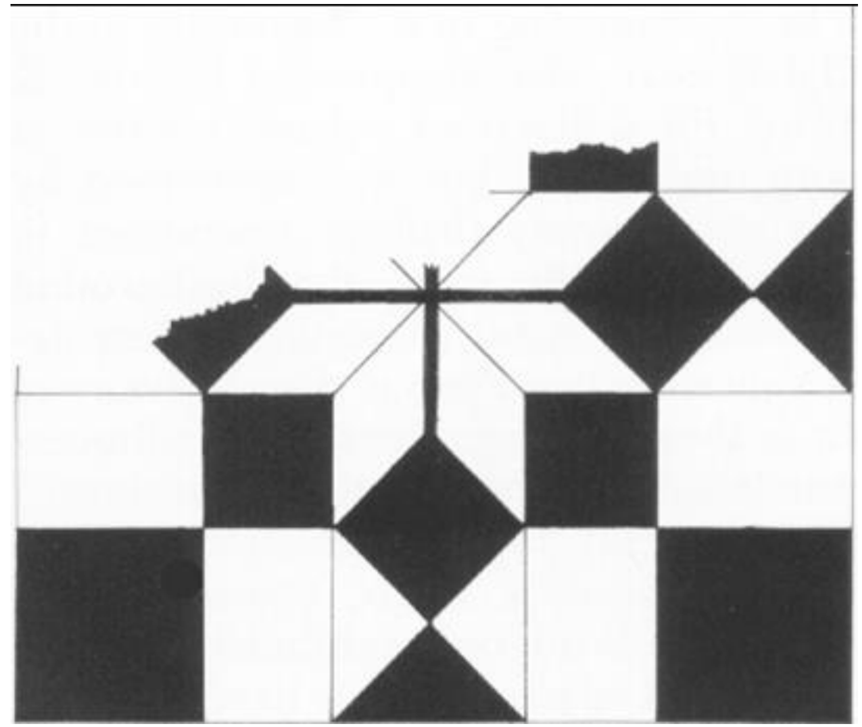
The floor (enlarged)

Automatically rectified floor

Slide from Criminisi

Analysing patterns and shapes

Automatic rectification



From Martin Kemp *The Science of Art*
(manual reconstruction)

2 patterns have been discovered !

Analysing patterns and shapes

What is the (complicated) shape of the floor pattern?

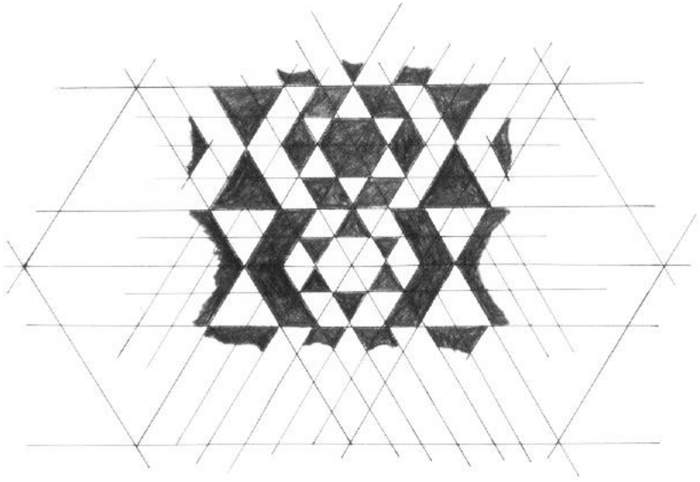
Automatically rectified floor

St. Lucy Altarpiece, **D. Veneziano**

Slide from Criminisi

Analysing patterns and shapes

Automatic rectification



From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

Mosaics: stitching images together









virtual wide-angle camera

Why Mosaic?

Are you getting the whole picture?

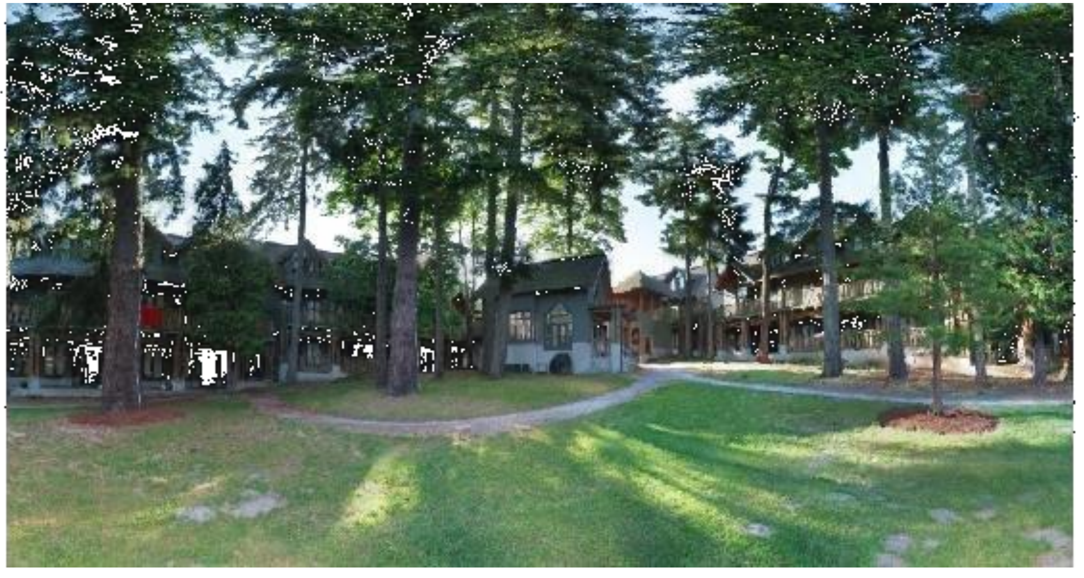
- Compact Camera FOV = 50 x 35°

Slide from Brown & Lowe

Why Mosaic?

Are you getting the whole picture?

- Compact Camera FOV = 50 x 35°
- Human FOV = 200 x 135°



Slide from Brown & Lowe

Why Mosaic?

Are you getting the whole picture?

- Compact Camera FOV = 50 x 35°
- Human FOV = 200 x 135°
- Panoramic Mosaic = 360 x 180°

Slide from Brown & Lowe

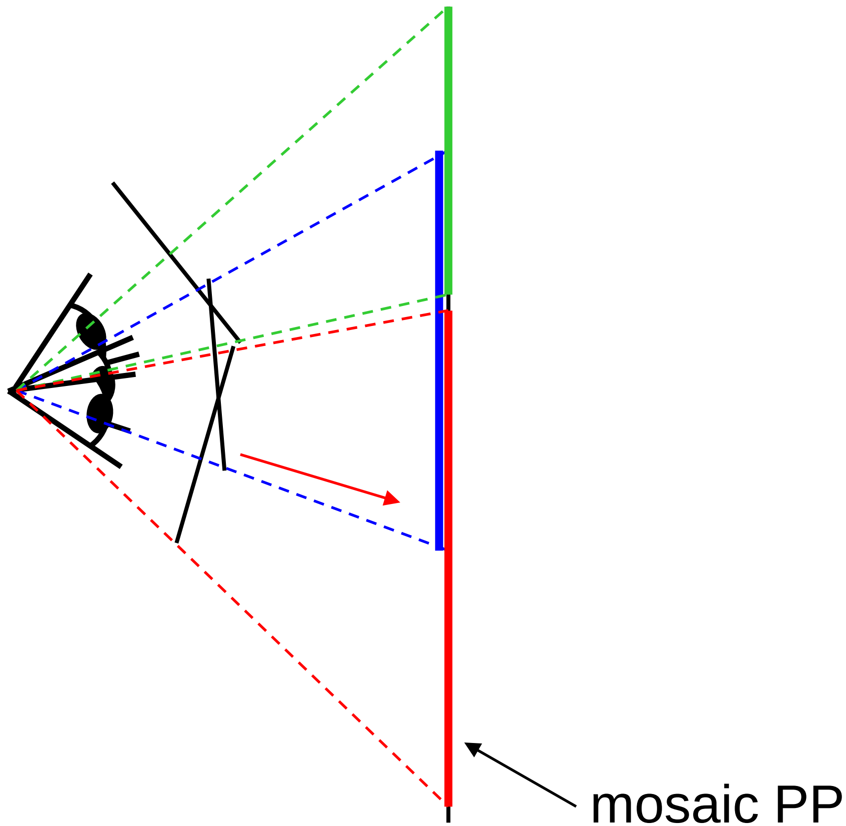
Naïve Stitching

left on top

right on top

Translations are not enough to align the images

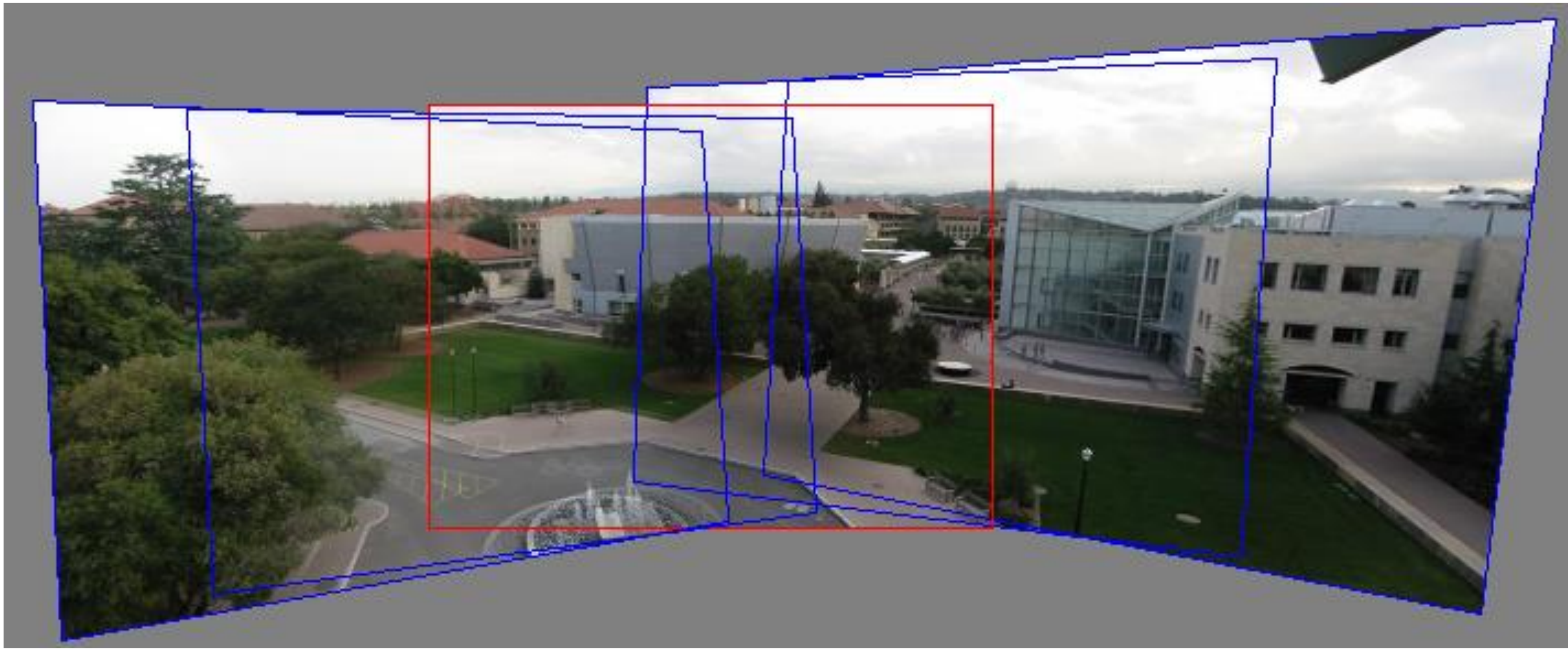
Image reprojection



The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

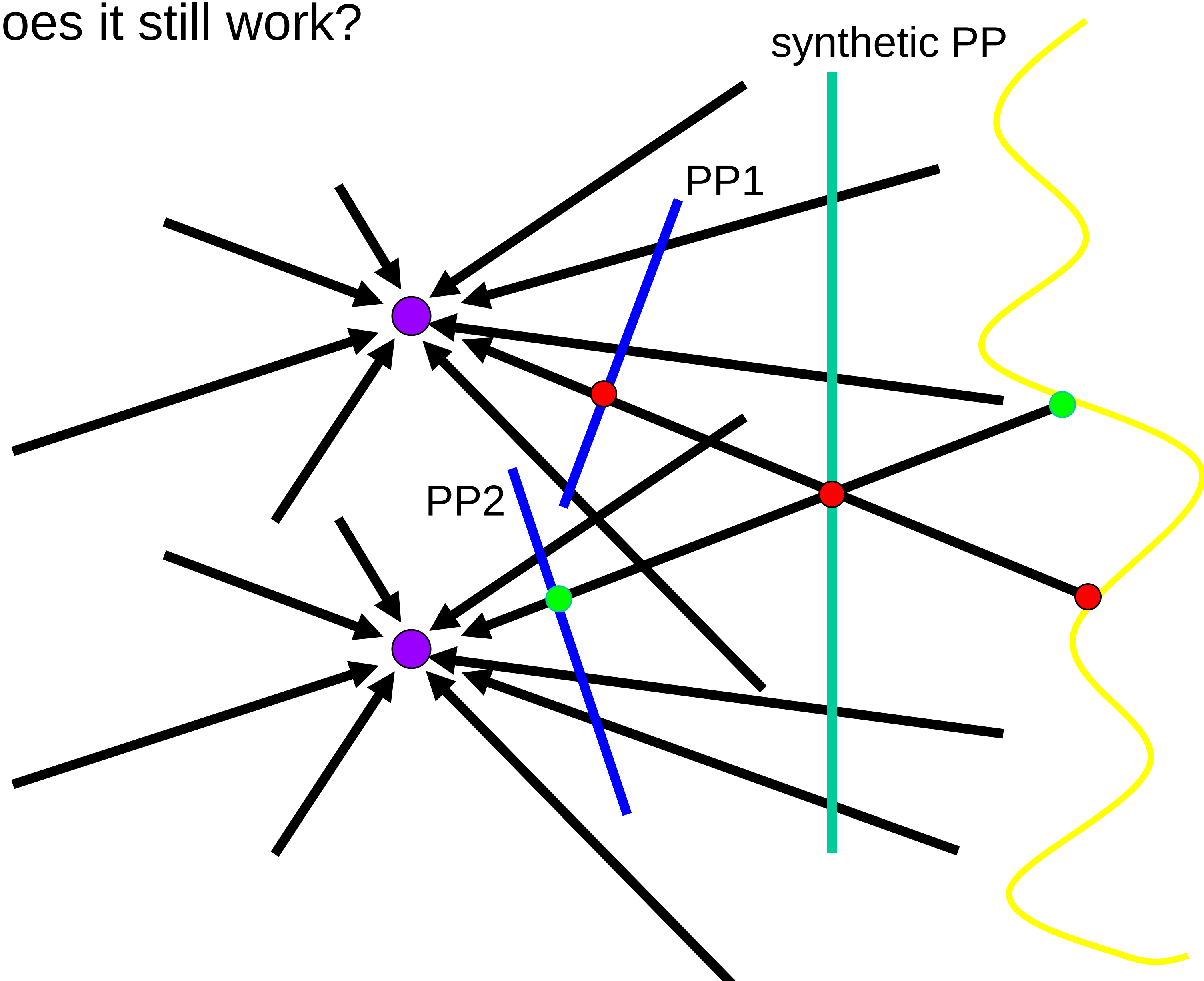
Panoramas



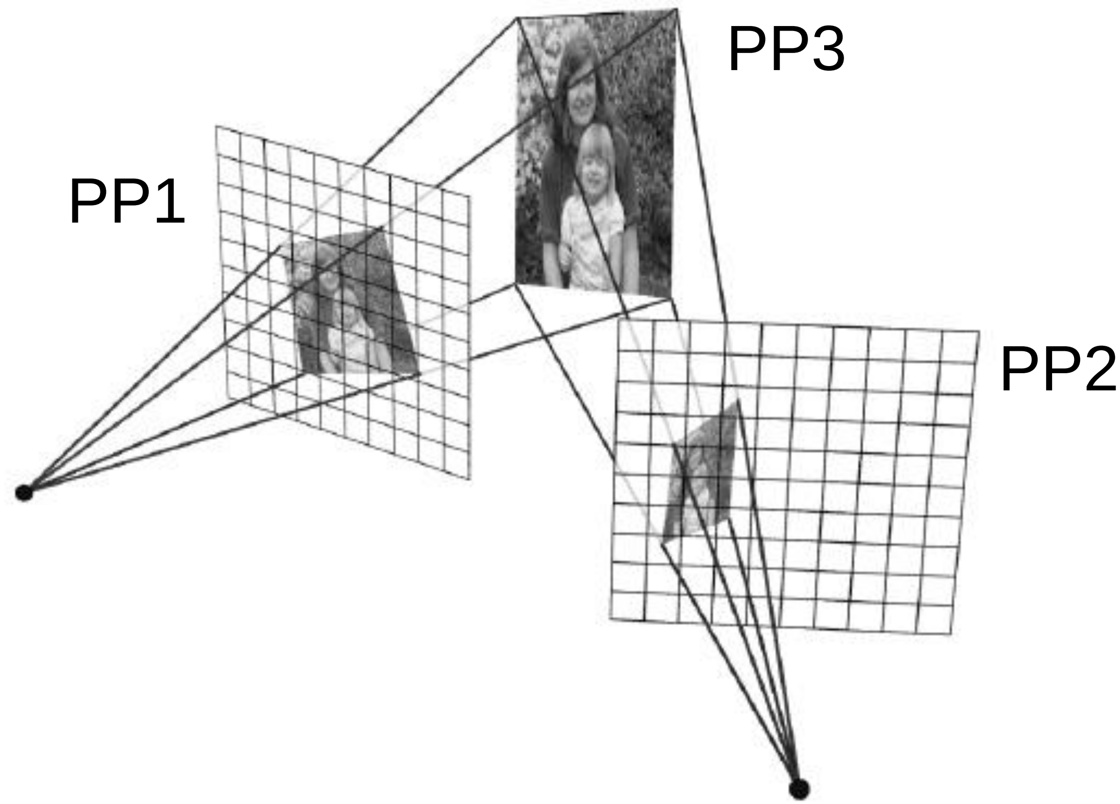
1. Pick one image (red)
2. Warp the other images towards it (usually, one by one)
3. blend

changing camera center

Does it still work?



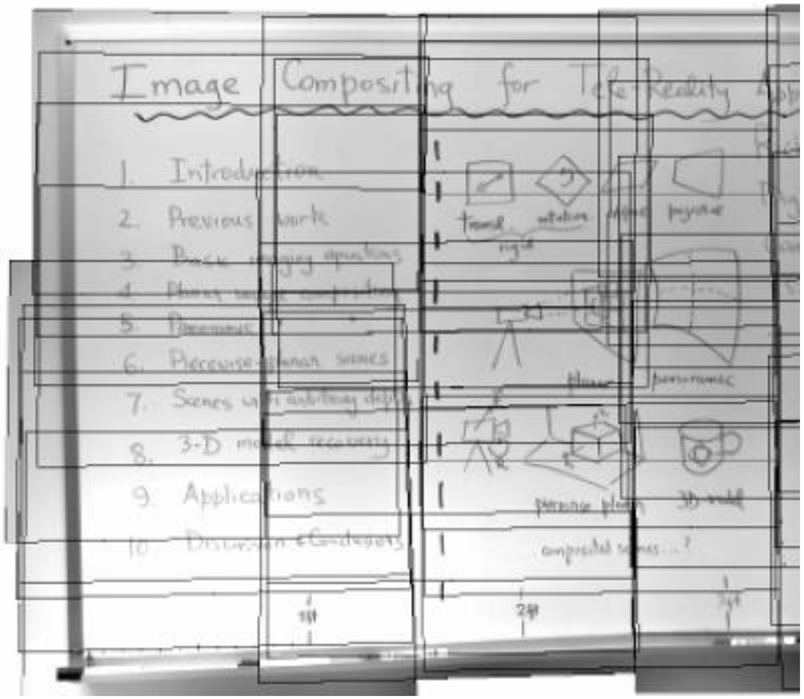
Planar scene (or far away)

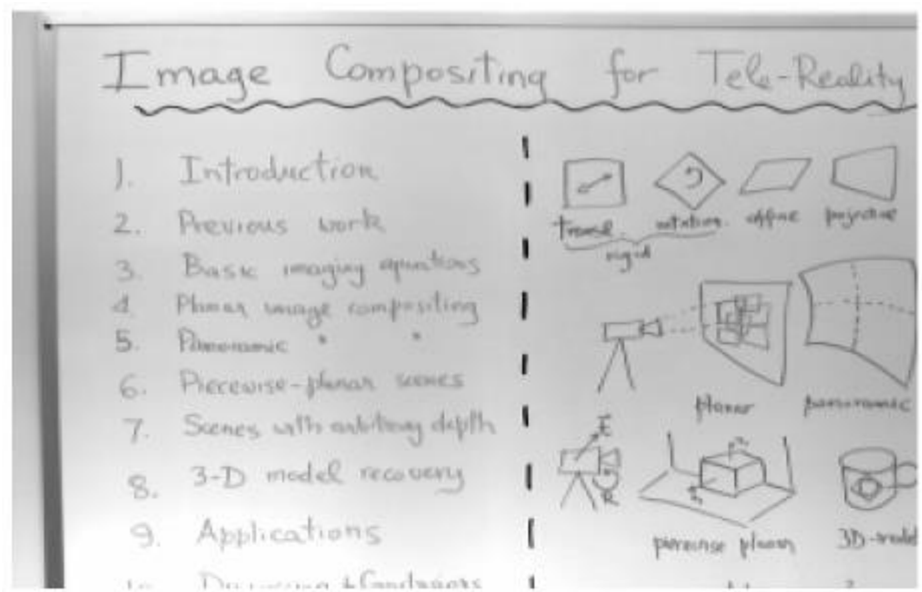


PP3 is a projection plane of both centers of projection, so we are OK!

This is how big aerial photographs are made

Planar mosaic



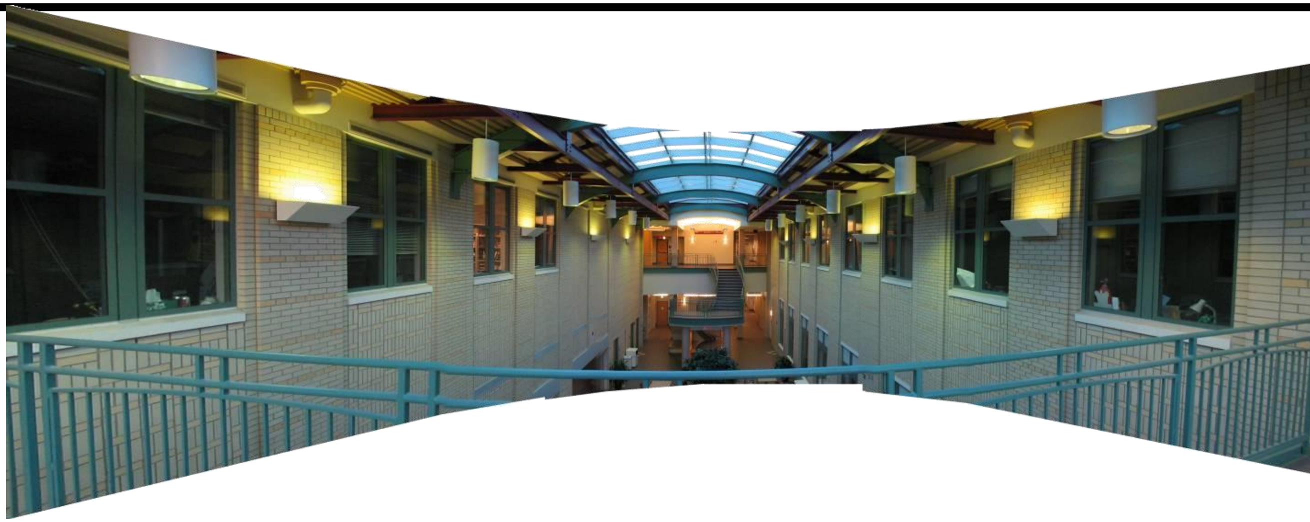


Julian Beever: Manual Homographies

http://users.skynet.be/J.Beever/pave.htm

Holbein, *The Ambassadors*

Programming Project #4 (part 1)



Homographies and Panoramic Mosaics

- Capture photographs (and possibly video)
  - Might want to use tripod
- Compute homographies (define correspondences)
  - will need to figure out how to setup system of eqs.
- (un)warp an image (undo perspective distortion)
- Produce panoramic mosaics (with blending)
- Do some of the Bells and Whistles

Bells and Whistles

Blending and Compositing

- use homographies to combine images or video and images together in an interesting (fun) way. E.g.
  - put fake graffiti on buildings or chalk drawings on the ground
  - replace a road sign with your own poster
  - project a movie onto a building wall
  - etc.

Bells and Whistles

Virtual Camera rorate

- Similar to face morphing, produce a video of virtual camera rotation from a single image
- Can also do it for translation, if looking at a planar object

Other interesting ideas?

- talk to me

From previous year's classes

Ben Hollis, 2004

Ben Hollis, 2004

Eunjeong Ryu (E.J), 2004

Matt Pucevich , 2004

Bells and Whistles

Capture creative/cool/bizzare panoramas

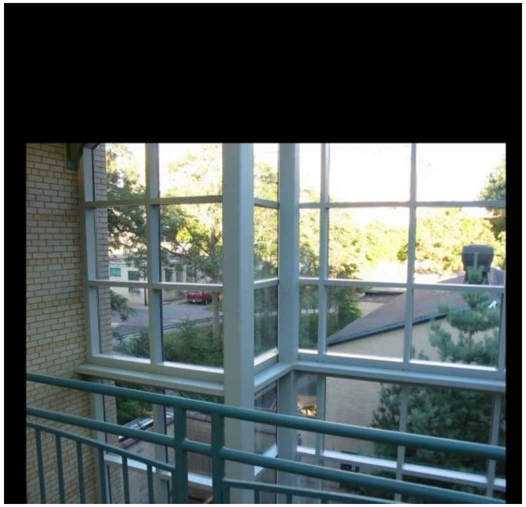
- Example from UW (by Brett Allen):

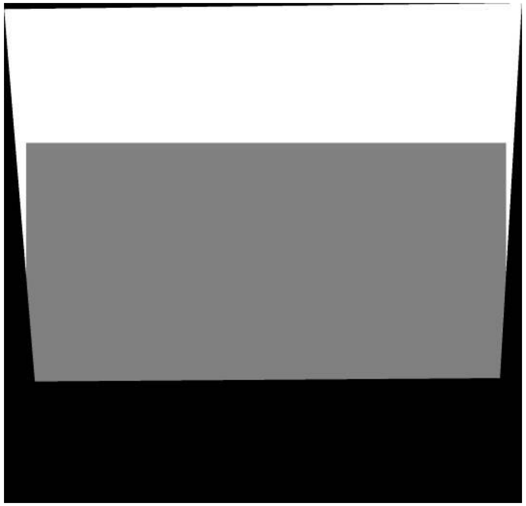
- Ever wondered what is happening inside your fridge while you are not looking?

Capture a 360 panorama (quite tricky…)

Example homography final project

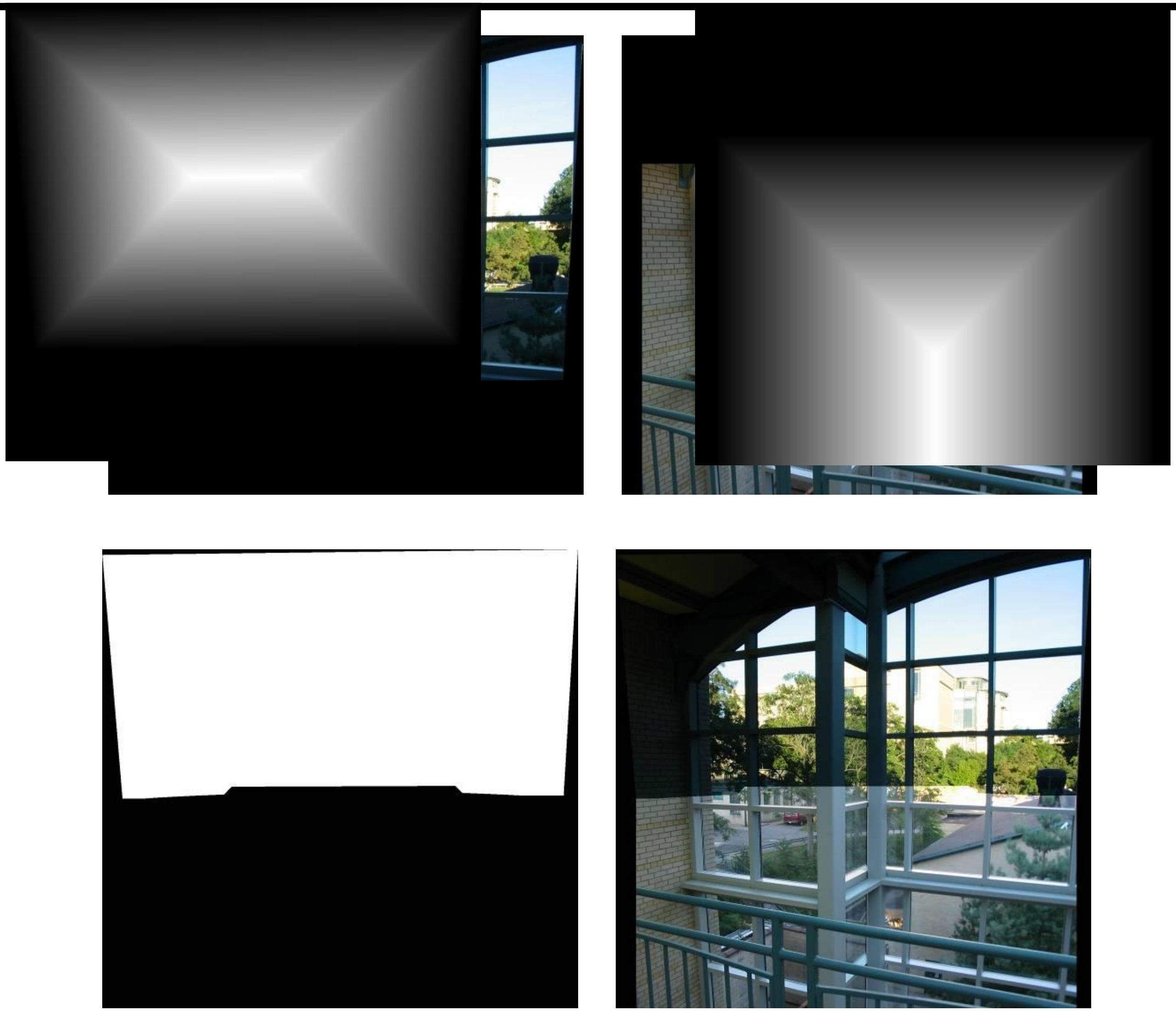
Setting alpha: simple averaging





Alpha = .5 in overlap region

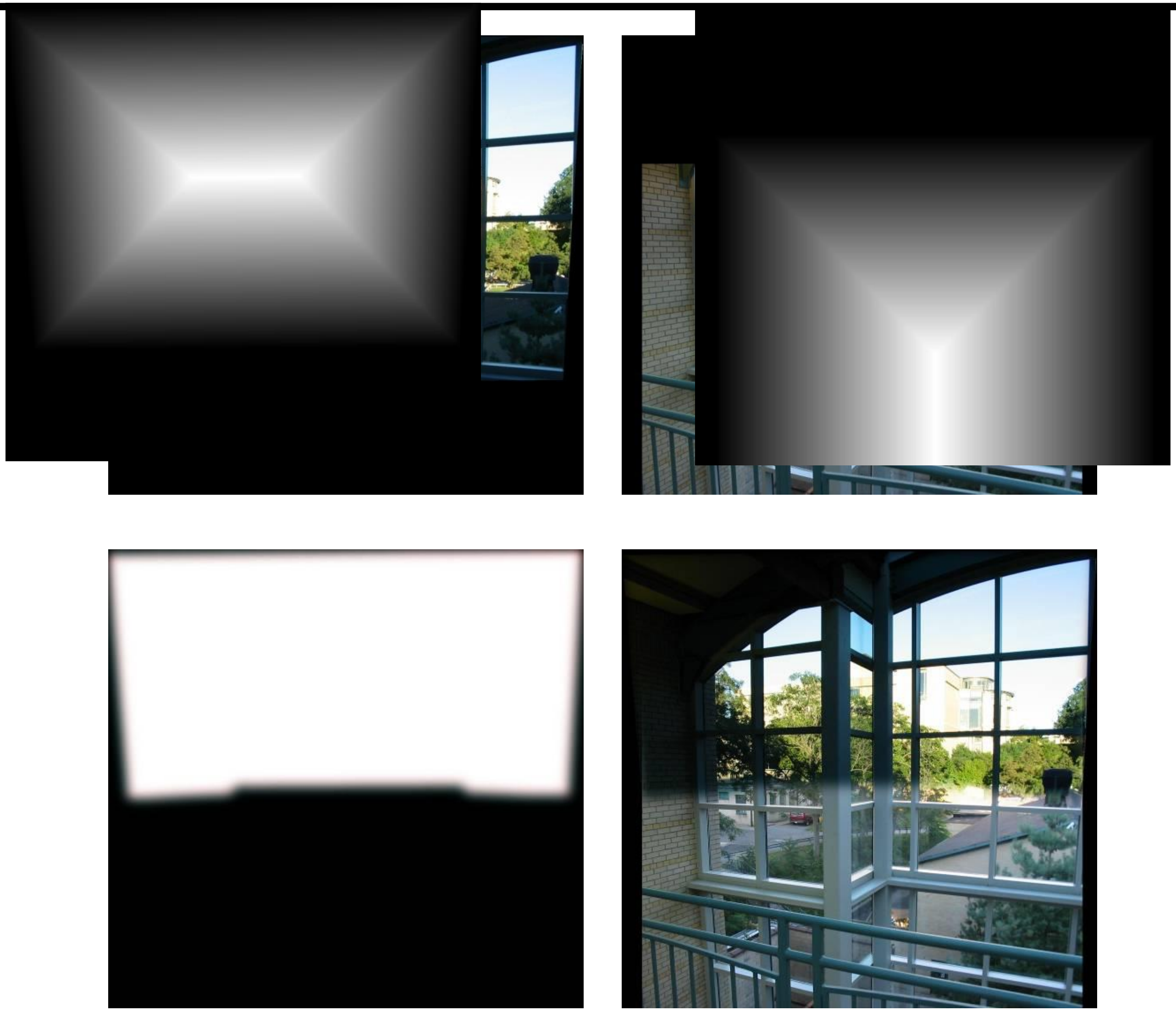
Setting alpha: center seam



Distance Transform
`bwdist`

Alpha = logical(dtrans1>dtrans2)

Setting alpha: blurred seam



Distance transform

Alpha = blurred

Simplification: Two-band Blending

Brown & Lowe, 2003

- Only use two bands: high freq. and low freq.
- Blends low freq. smoothly
- Blend high freq. with no smoothing: use binary alpha

2-band "Laplacian Stack" Blending

Low frequency (λ > 2 pixels)

High frequency (λ < 2 pixels)

Linear Blending

2-band Blending