Kinematic Synthesis

October 6, 2015

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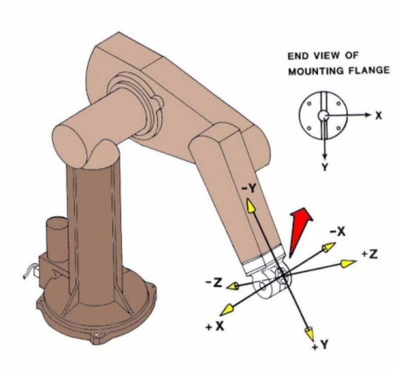
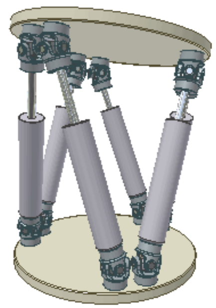
Classifying Mechanisms

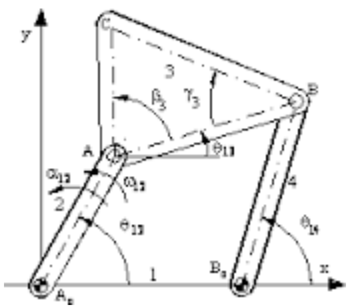
Several dichotomies

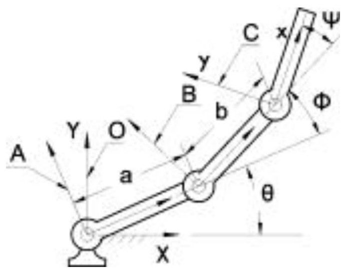
Serial and Parallel



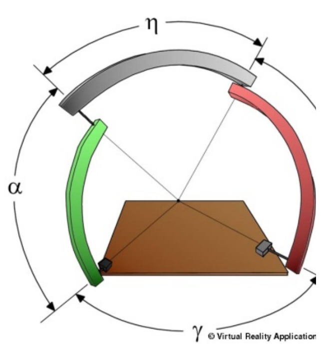


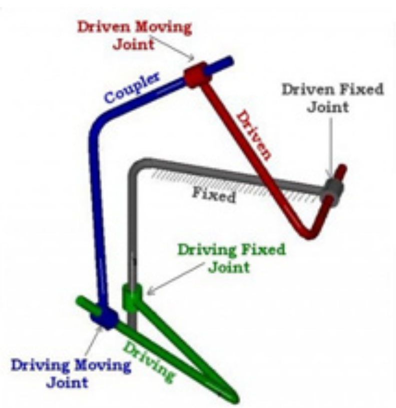
Few DOFS and Many DOFS





Planar/Spherical and Spatial

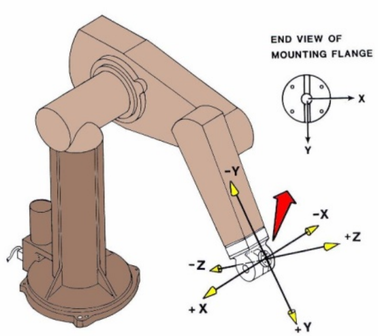




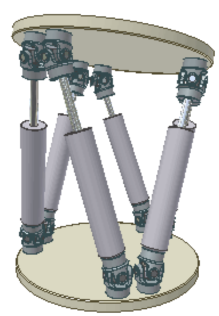
Rigid and Compliant

Mechanism Trade-offs

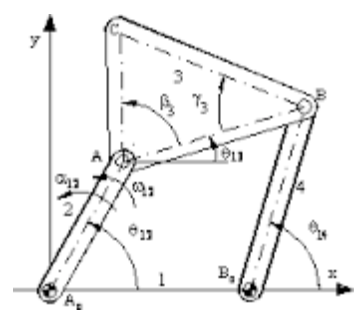
	Workspace	Rigidity	Designing Kinematics	No. of Actuators	Flexibility of Motion	Complexity of Motion
Serial	Large	Low	Simple	Depends	Depends	Depends
Parallel	Small	High	Complex	Depends	Depends	Depends
Few DOF	Small	Depends	Complex	Few	Little	Less
Many DOF	Large	Depends	Simple	Many	A lot	More



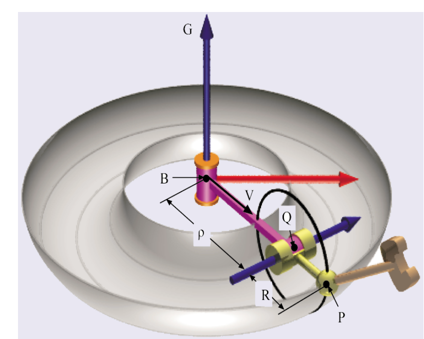
Serial, Many DOF



Parallel, Many DOF



Parallel, Few DOF



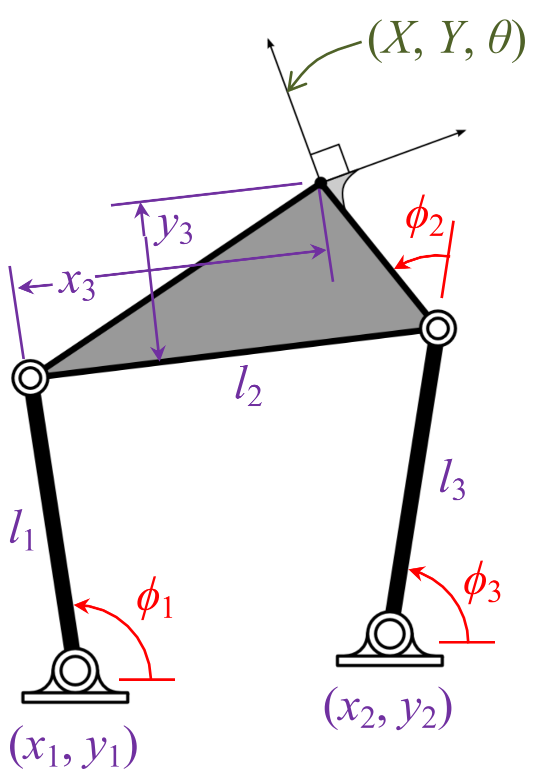
Serial, Few DOF

Problems in Kinematics

Dimensions

Joint Parameters

End Effector Coordinates



Forward Kinematics
Known: Dimensions, Joint Parameters
Solve for: End Effector Coordinates

Inverse Kinematics
Known: Dimensions, End Effector Coordinates
Solve for: Joint Parameters

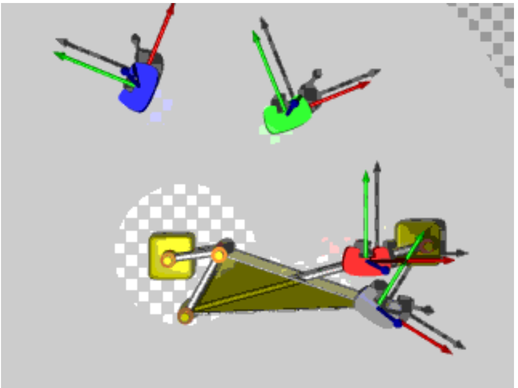
Synthesis
Known: End Effector Coordinates
Solve for: Dimensions, Joint Parameters

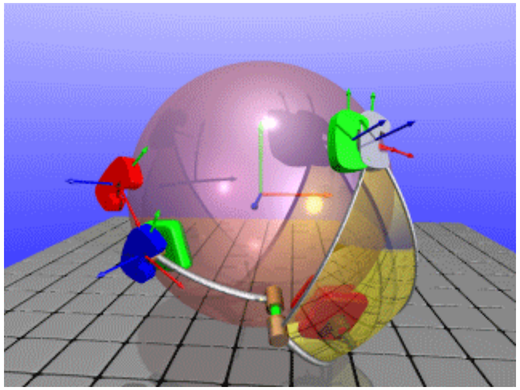
Challenges in Kinematics

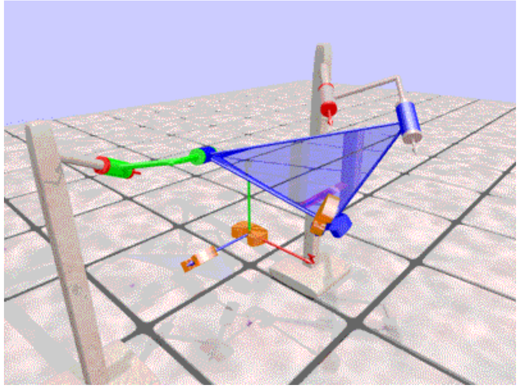
- Using sweeping generalizations, how difficult is it to solve
  - forward kinematics
  - inverse kinematics
  - synthesis

  over different types of mechanisms?
- Ranked on a scale of 1 to 4 with 4 being the most difficult:

	Forward Kinematics		Inverse Kinematics		Synthesis	
	Serial	Parallel	Serial	Parallel	Serial	Parallel
Planar	1	2	2	1	3	3.5
Spherical	1	2	2	1	3	3.5
Spatial	1.5	2.5	2.5	1.5	3.5	4


Planar


Spherical

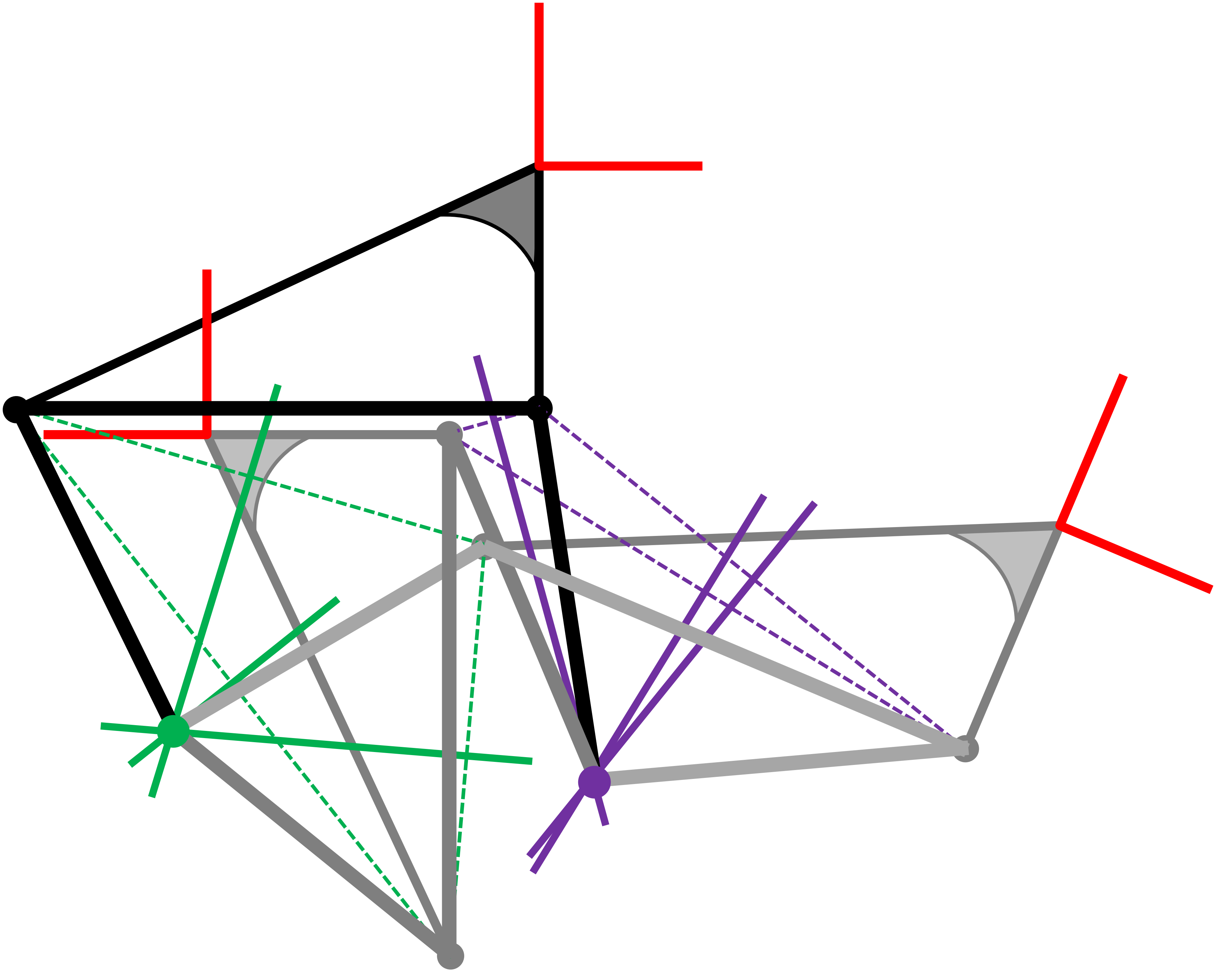

Spatial

Synthesis Approaches

- Synthesis equations are hard to solve because almost nothing is known about the mechanism beforehand

Some Methods for Synthesis

- Graphical constructions – 1 soln per construction
- Use atlases (libraries) (see http://www.saltire.com/LinkageAtlas/)
- Evolutionary algorithms – multiple solutions
- Optimization – 1 soln, good starting approximation required
- Sampling potential pivot locations
- Resultant elimination methods – all solutions, limited to simpler systems
- Groebner Bases – all solutions, limited to simpler systems
- Interval analysis – all solutions within a box of useful geometric parameters
- Homotopy – all solutions, can handle degrees in the millions and possibly greater with very recent developments

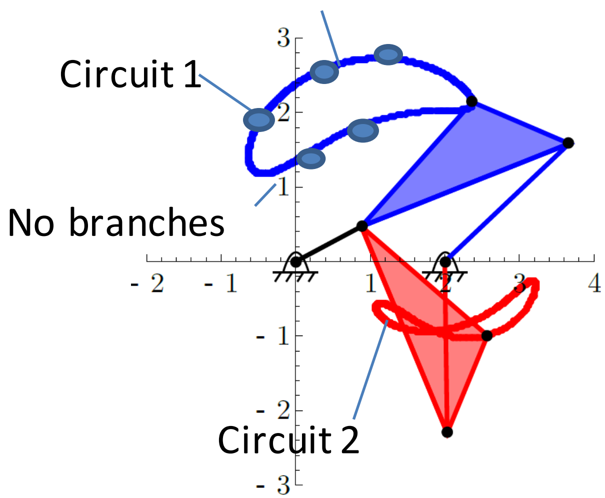


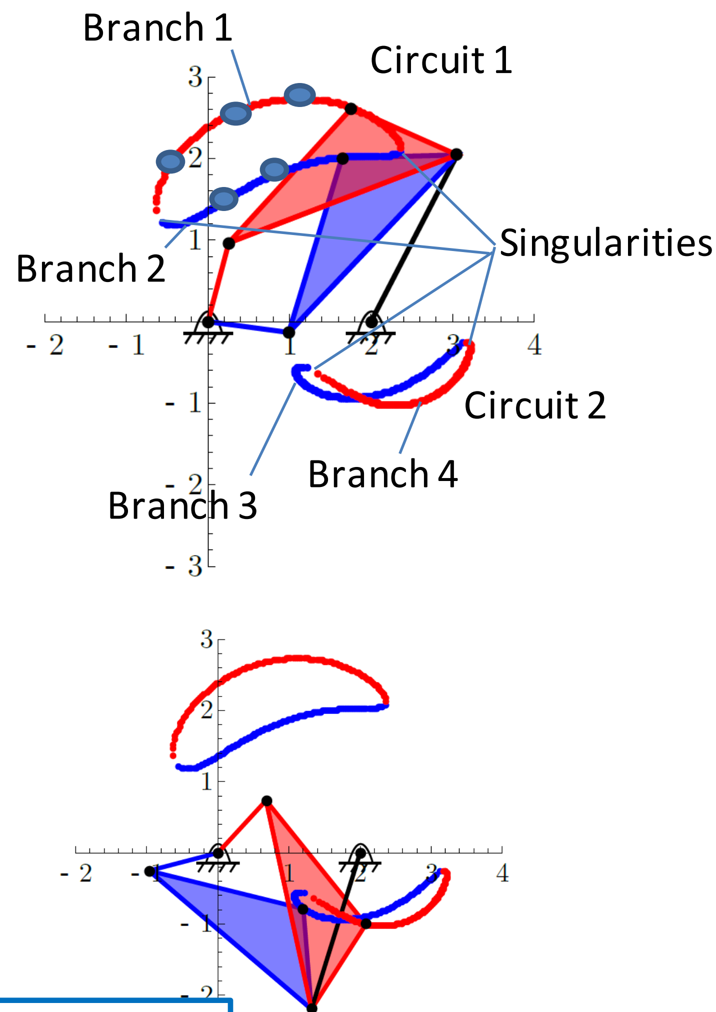
Configuration Space of a Linkage

<u>Terminology:</u>

Circuits- not dependent on input link specification

Branches- dependent on input link specification





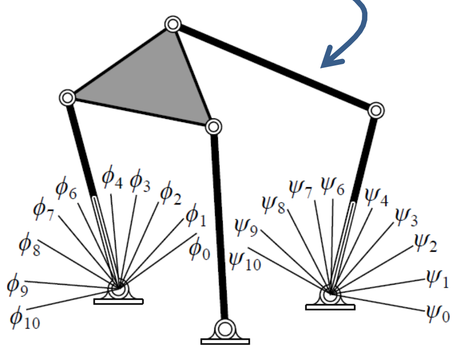
(c)

Circuit and branches can lead to linkage defects

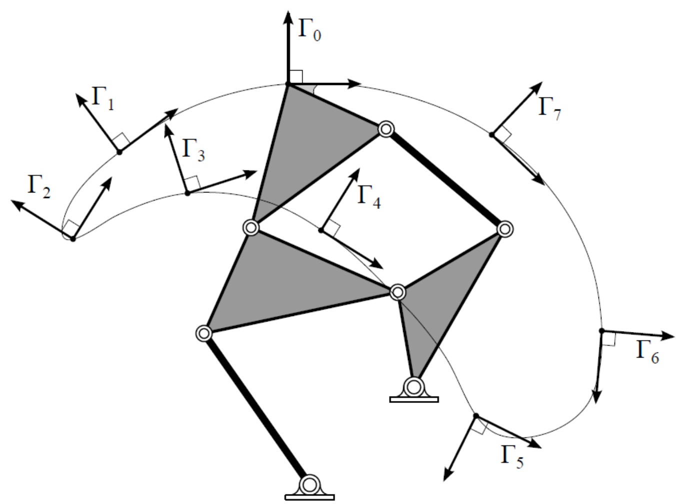
Types of Synthesis Problems

a) Function generation: set of input angles and output angles;
b) Motion generation: set of positions and orientations of a workpiece;
c) Path generation: set of points along a trajectory in the workpiece.

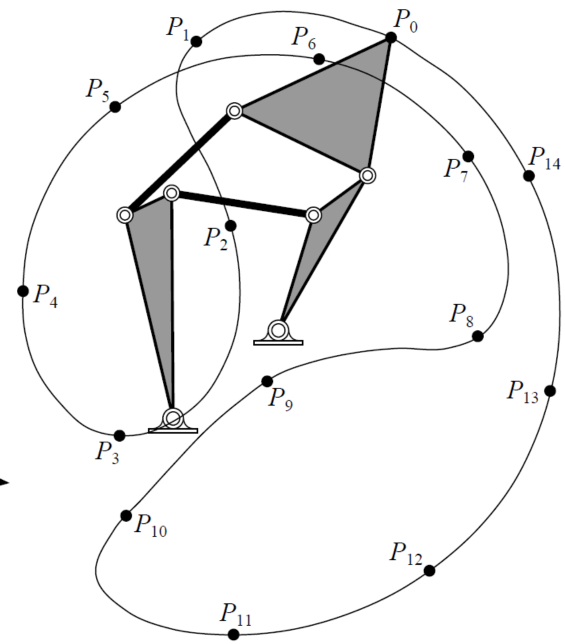
Gives control of mechanical advantage



Function Generation



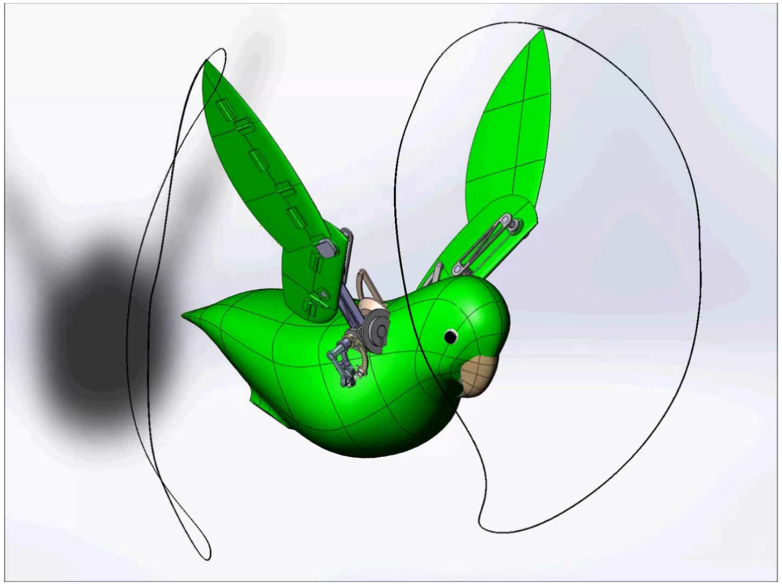
Motion Generation

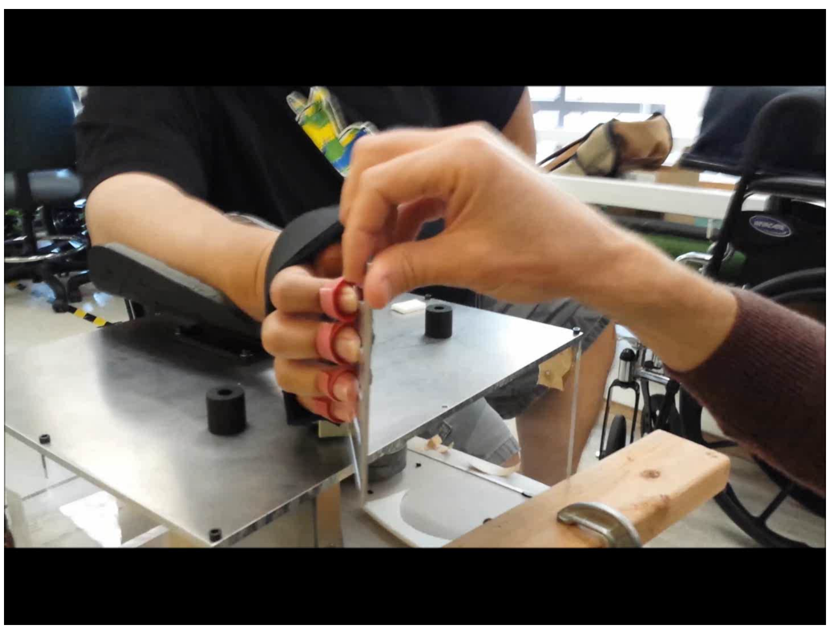


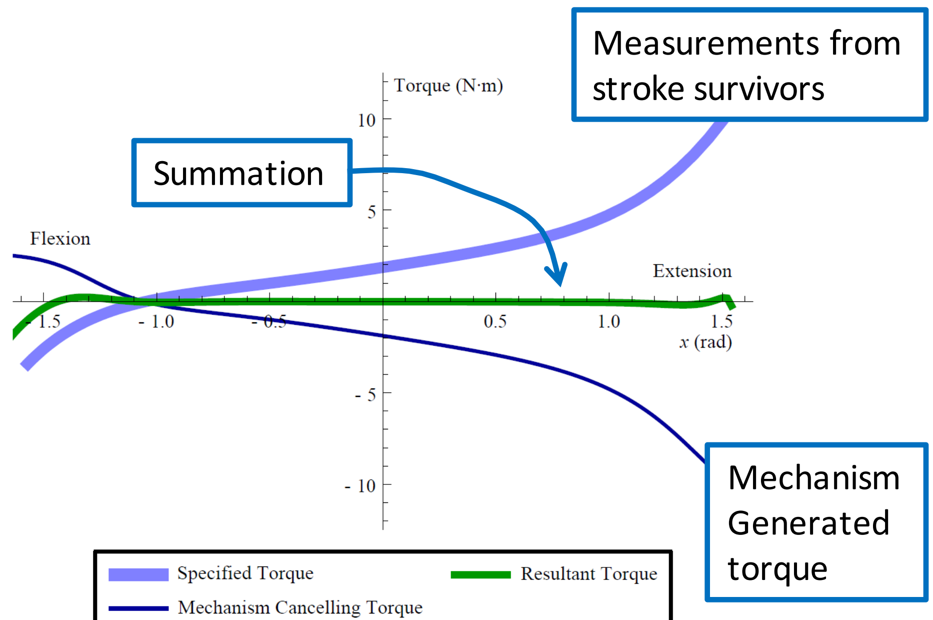
Path Generation

Above are examples of function, motion, and path generation for planar six-bar linkages.
Analogous problems exist for spherical and spatial linkages of all bars.

Examples of Function Generation



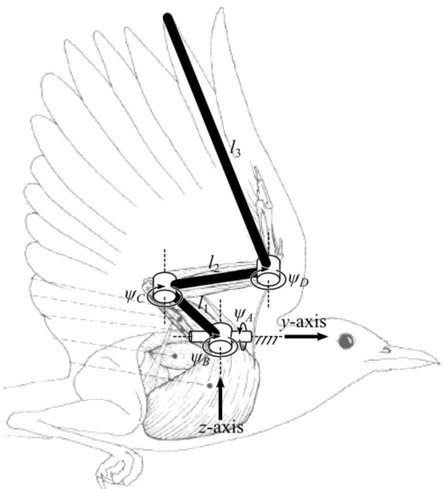




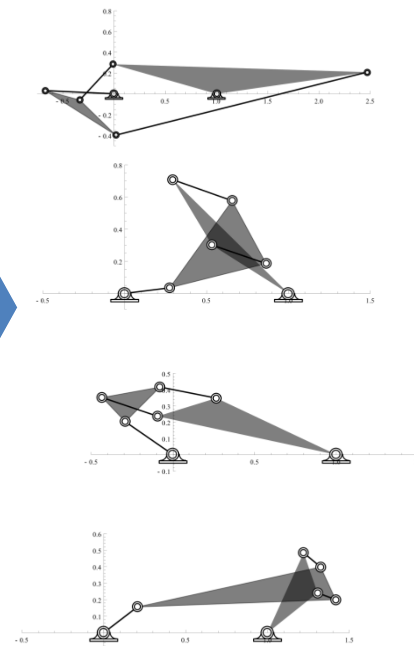
The Bird Example Technique

- Spatial chains are constrained by six-bar function generators

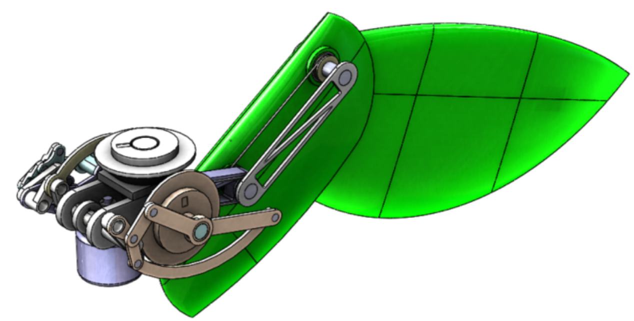
Spatial chain
4 DOF



Function generators to
control joint angles

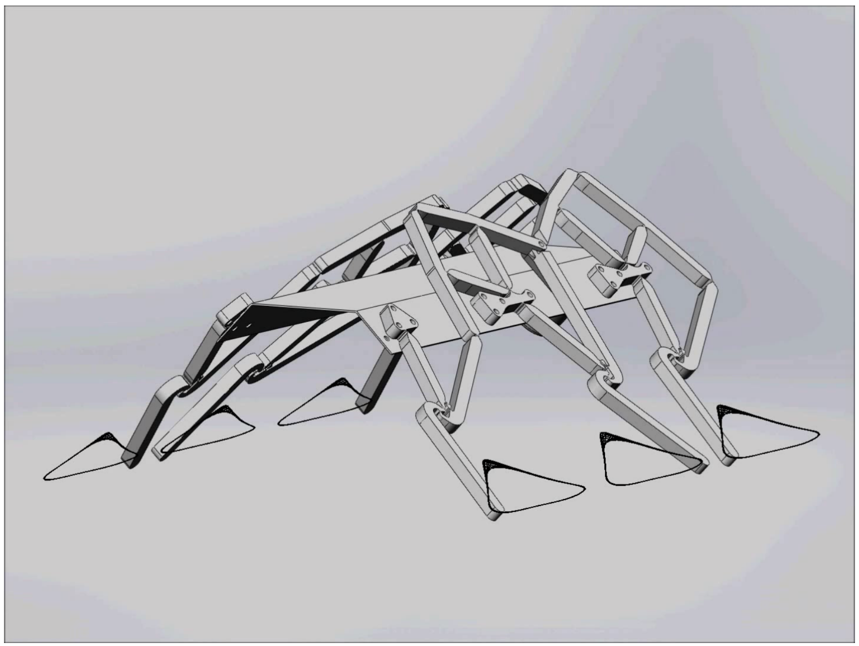


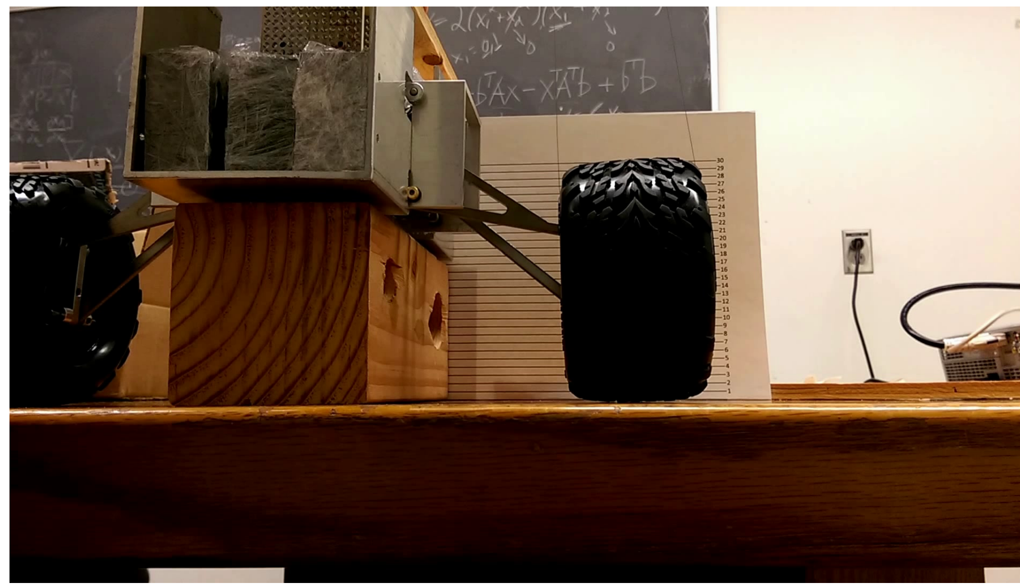
A single DOF
constrained
spatial chain



Goal: achieve accurate biomimetic motion

Examples of Motion Generation and Path Generation

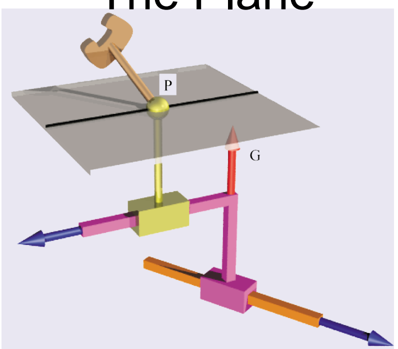




Kinematics and Polynomials

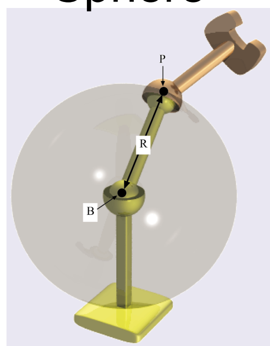
- Kinematics are intimately linked with polynomials because they are composed of revolute and prismatic joints which describe circles and lines in space, which are algebraic curves
- These lines and circles combine to describe more complex algebraic surfaces

The Plane



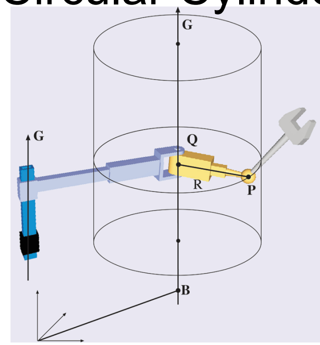
PPS

Sphere



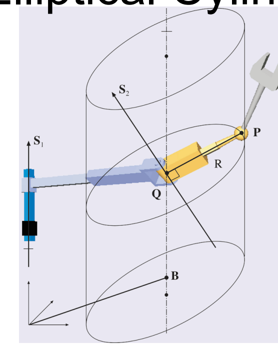
TS

Circular Cylinder



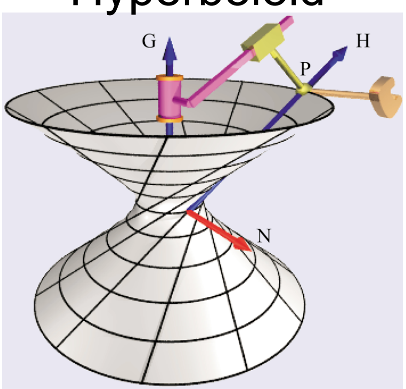
CS

Elliptical Cylinder



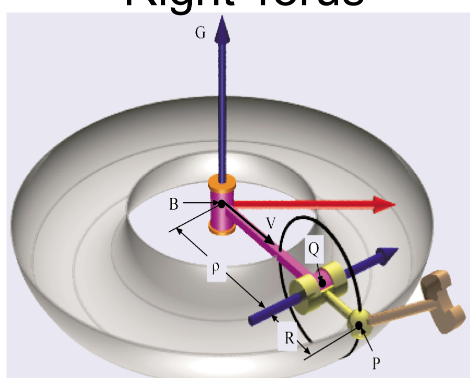
PRS

Hyperboloid



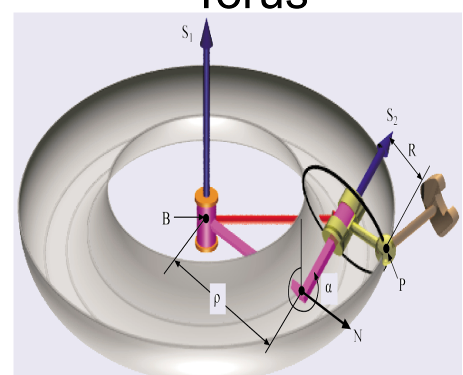
RPS

Right Torus



RRS

Torus

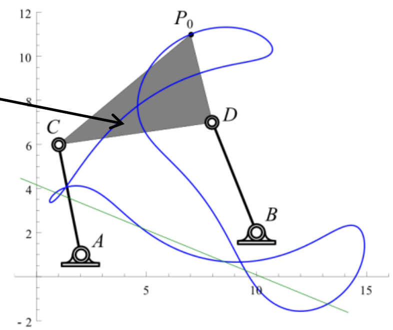


RRS

Polynomials and Complexity

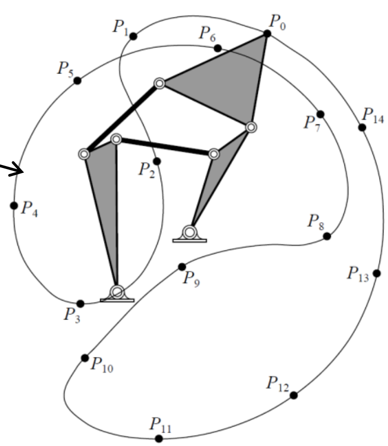
- Linkages can always be expressed as **polynomials**
- When new links are added, the complexity of synthesis rapidly increases

Degree 6 polynomial curve



Four-bar

Degree 18 polynomial curve



Six-bar

Degree 6 polynomial system

Degree 264,241,152 polynomial system

Synthesis Problems	Max Number of Positions		
	Function	Motion	Path
Four-bar	**5**	**5**	**9**
Stephenson I	5	5	15
Stephenson II	11	5	15
Stephenson III	11	5	15

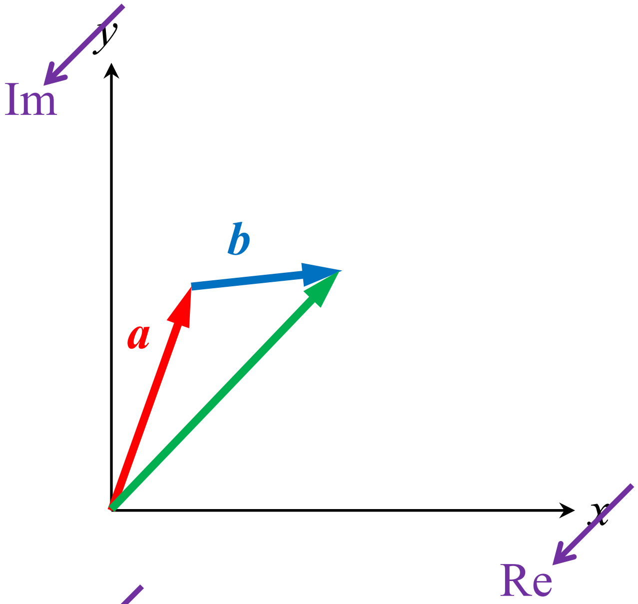
The degree of polynomial synthesis equations increases rapidly when links are added

Ways to Model Kinematics

- Planar
  - Rotation matrices, homogeneous transforms, vectors
  - Planar quaternions
  - Complex numbers
- Spherical
  - Rotation matrices
  - Quaternions
- Spatial
  - Rotation matrices, homogeneous transforms, vectors
  - Dual quaternions

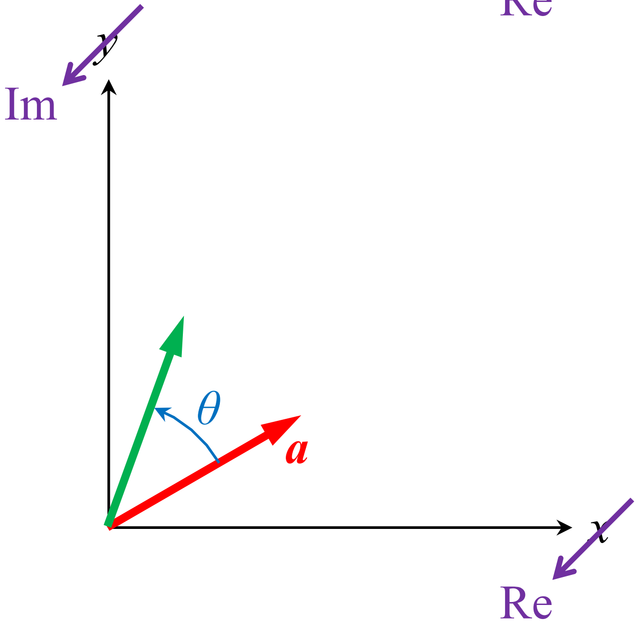
- All methods create equivalent systems, although they might look different. Different conveniences are made available by how kinematics are modelled

Planar Kinematics With Complex Numbers



$$\begin{Bmatrix} a_x \\ a_y \end{Bmatrix} + \begin{Bmatrix} b_x \\ b_y \end{Bmatrix} = \begin{Bmatrix} a_x + b_x \\ a_y + b_y \end{Bmatrix}$$

$$(a_x + ia_y) + (b_x + ib_y) = (a_x+b_x) + i(a_y+b_y)$$



$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} a_x \\ a_y \end{Bmatrix} = \begin{Bmatrix} a_x\cos\theta - a_y\sin\theta \\ a_x\sin\theta + a_y\cos\theta \end{Bmatrix}$$

$$\begin{aligned} e^{i\theta}(a_x + ia_y) &= (\cos\theta + i\sin\theta)(a_x + ia_y) \\ &= (a_x\cos\theta - a_y\sin\theta) + i(a_x\sin\theta + a_y\cos\theta) \end{aligned}$$