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CS 282 **Prof. R. Fateman**
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Assignment 4: Simplification, Integration

Due: Wed, 27 March, 2002

1. Experiment with the integration programs in Maple, Mathematica, and/or Macsyma. See if you can come up with examples of indefinite integrals that exist in closed form in terms of elementary functions but which they do not find. One way of doing this is to start with expressions, and differentiate (and rearrange). One heuristic that sometimes causes problems is integrating a sum of terms by integrating term-by-term. What if the individual terms are not integrable, but the sum is?

Can you outline some region(s) of "expression space" that characterize your discoveries?

2. Definite improper integrals (limits including ∞, or integrals with singularities at the endpoints or in the middle), cause problems sometimes. Consider the integrand $1/(x-a)^2$ and various endpoints. What should the answer be? Experiment with the integration programs in Maple, Mathematica, and/or Macsyma. See if you can come up with examples of definite (symbolic) integrals with parameters that they do not get correct.

3. All univariate real polynomials can be factored "numerically" into (at worst) quadratic factors over the reals, and into (at worst) linear factors over the complex numbers. This trivializes rational function integration. Attack and/or defend this position.

4. This problem concerns iteration in a power series domain. The short Macsyma program below uses "Picard's method" (described in any elementary differential equations book) to solve a first-order ordinary differential equation by integration, but in a power-series domain. Write a (short) program that will solve simple second-order ODEs using the same technique. (Solving simple ODEs would include, for example, finding $x(t)$ where

$$\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt}, t)$$

for $f()$ is a polynomial in three variables.) `Picard`, under some restrictions, solves the equation

$$\frac{dx}{dt} = f(x,t)$$

where $x(0) = a0$ as a series to degree n in t. For example, you could try `picard(x,x,t,1,5);`

```
picard(f,x,t,a0,n):=
block([s:a0,deg:0],
 while deg < n do
 (deg:deg+1, s:ratdisrep(s),
  s:integrate(taylor(subst(x=s,f),t,0,deg),t)+a0),
 return(taylor(s,t,0,n)));
```

To better understand `picard`, you can use debugging information from `setcheck:[s,deg];` and `trace(integrate,taylor);` etc. There is an analogy to p-adic convergence in this business, but you need not discuss it.

Show that your program can solve $y'' + y = 0$, $y(0) = 0$, $y'(0) = 1$.

Feel free to use Mathematica or Maple or Mupad instead of Macsyma. If you need help in deciphering the program, see me.

5. Consider the algorithm discussed by Moses for testing for zero equivalence of an expression in a class which differs from Brown's REX expressions in that it involves no i, only a single variable x but allows the function $\log(|x|)$. The log as well as the exponential can be nested to any depth. This algorithm (also due to D. Richardson) works by reducing the decision process to one requiring the solution to "the constant problem" of determining whether an expression consisting entirely of constants is zero or not.

(a) What limitations are relevant on this algorithm. Give examples where these limitations come into play. (Hint: Is this expression a constant: $\arctan(x) + \arctan(1/x)$?)

(b) Describe alternative algorithms or heuristics for solving this constant problem.

(c) What properties of the derivative are used by Richardson?

(d) Consider the extension to Richardson's results suggested by Moses so that the class can include functions specifically defined by a differential equation of the form $y' = a(x)y + b(x)$. Describe the algorithmic details for this, and if you can, write a program for it.

For example, the equation $y' = (2/\sqrt{\pi})e^{-x^2}$ has as its solution a function $y = \mathrm{erf}(x)$ that is an integral not expressible in terms of elementary functions. Can you show that $\mathrm{erf}(x) + \mathrm{erf}(-x)$ is zero?