Advanced Computer Graphics (Spring 2012)

CS 283, Guest Lecture: Global Illumination

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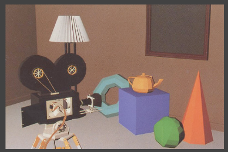
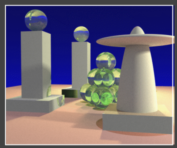




Some images courtesy Henrik Jensen
Some slide ideas courtesy Pat Hanrahan

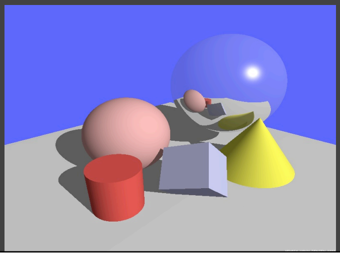
Illumination Models

So far considered mainly local illumination

- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

Global Illumination: multiple bounces (indirect light)

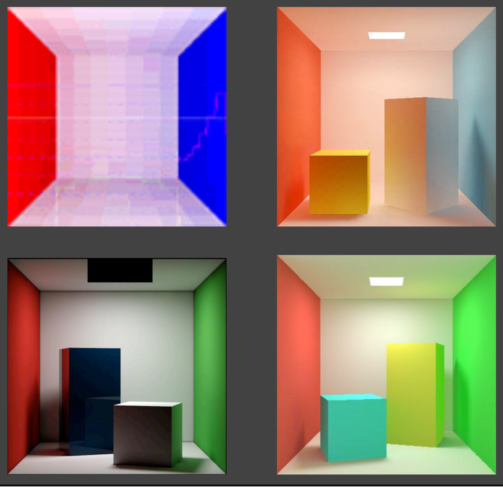
- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)



Some images courtesy Henrik Wann Jensen

Diffuse Interreflection

Diffuse interreflection, color bleeding [Cornell Box]



Radiosity

Caustics

Caustics: Focusing through specular surface

- Major research effort in 80s, 90s till today

Overview of lecture

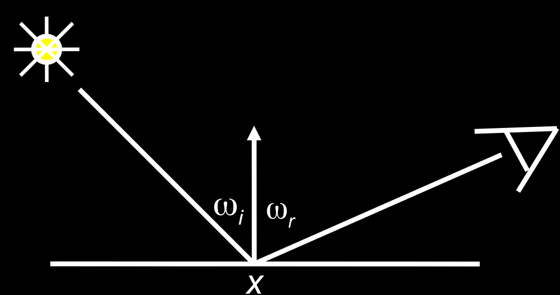
- ***Theory*** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive ***Rendering Equation*** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in textbooks

Outline

- *Reflectance Equation (review)*
- *Global Illumination*
- *Rendering Equation*
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

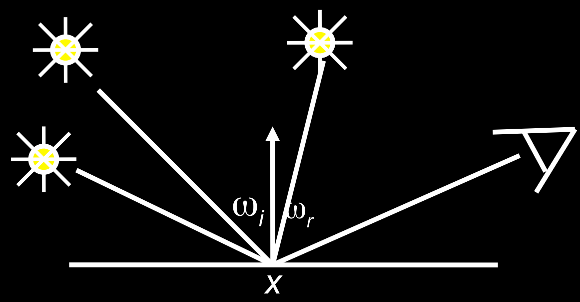
Reflectance Equation (review)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflectance Equation (review)

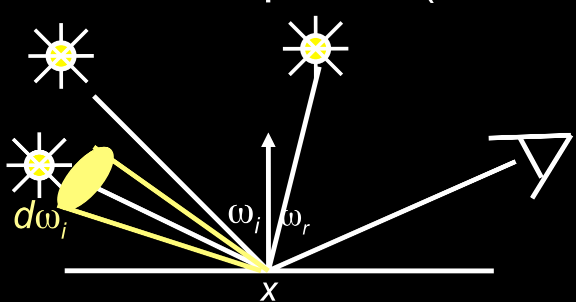


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflectance Equation (review)



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

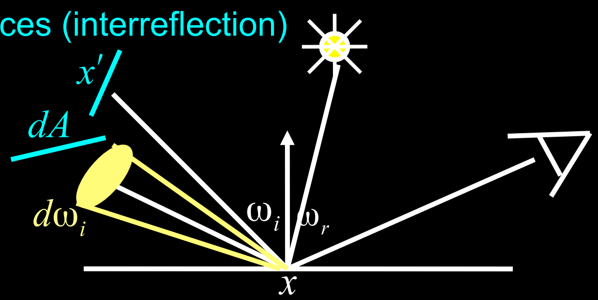
The Challenge

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing the reflected radiance from surfaces

Global Illumination

Surfaces (interreflection)



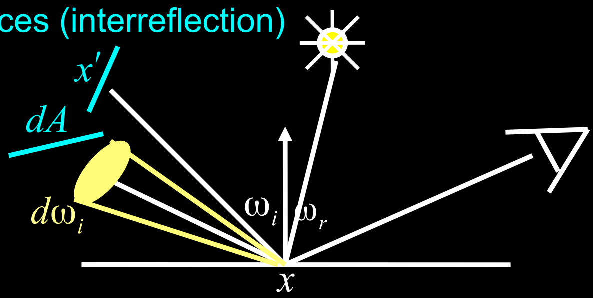
$$\omega_i \sim x' - x$$

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light (from surface)	BRDF	Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 86)

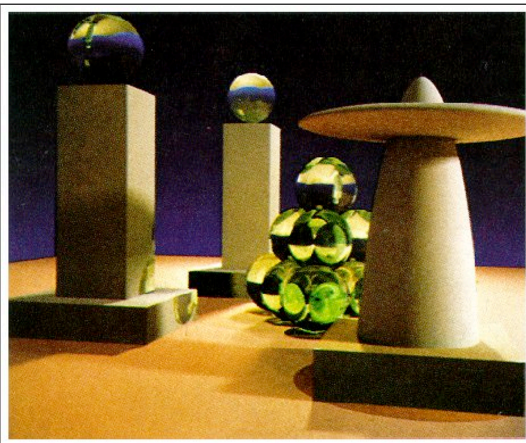


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- *As a general Integral Equation and Operator*
- *Approximations (Ray Tracing, Radiosity)*
- Surface Parameterization (Standard Form)

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\boxed{K(u,v)dv}$$

Kernel of equation

Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations

$$h(u) = (M \circ f)(u)$$

M is a linear operator.
f and h are functions of u

- Basic linearity relations hold

a and b are scalars
f and g are functions

$$M \circ (af + bg) = a(M \circ f) + b(M \circ g)$$

- Examples include integration and differentiation

$$(K \circ f)(u) = \int k(u,v) f(v)\, dv$$

$$(D \circ f)(u) = \frac{\partial f}{\partial u}(u)$$

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v)dv}$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can also be discretized to simple matrix equation
[or system of simultaneous linear equations]
(L, E are vectors, K is the light transport matrix)

Solving the Rendering Equation

$$L = E + KL$$
$$IL - KL = E$$
$$(I - K)L = E$$
$$L = (I - K)^{-1} E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \ldots)E$$
$$L = E + KE + K^2 E + K^3 E + \ldots$$

Term n corresponds to n bounces of light

Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element

- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect
[Caustics etc]

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

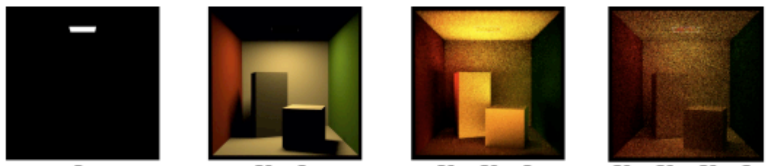
Direct Illumination
on surfaces

OpenGL
Shading

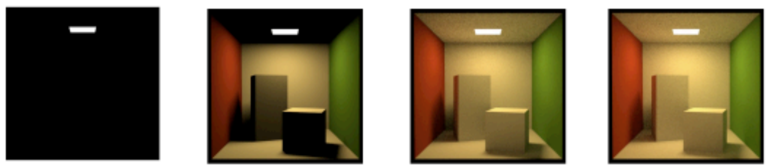
Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect
[Caustics etc]

Successive Approximation



L_e | $K \circ L_e$ | $K \circ K \circ L_e$ | $K \circ K \circ K \circ L_e$



L_e | $L_e + K \circ L_e$ | $L_e + \cdots K^2 \circ L_e$ | $L_e + \cdots K^3 \circ L_e$

Outline

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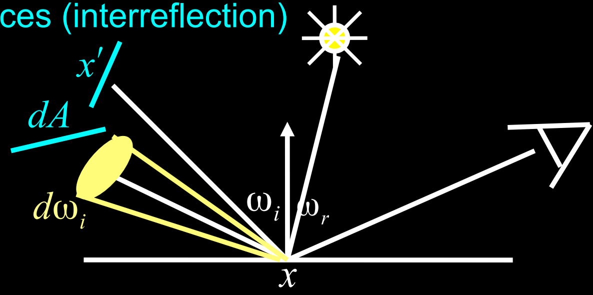
- *Surface Parameterization (Standard Form)*

Rendering Equation

Surfaces (interreflection)



$\omega_i \sim x' - x$

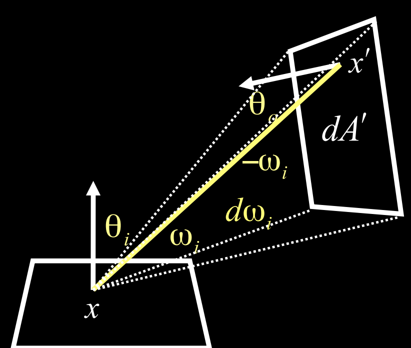
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r)\boxed{\cos\theta_i\, d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)



$$d\omega_i = \frac{dA' \cos\theta_o}{|x - x'|^2}$$

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA}$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Rendering Equation: Standard Form

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA}$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

Same as equation 2.52 Cohen Wallace. It swaps primed And unprimed, omits angular args of BRDF, - sign.
Same as equation above 19.3 in Shirley, except he has no emission, slightly diff. notation

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Radiosity Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

Drop angular dependence (diffuse Lambertian surfaces)

$$L_r(x) = L_e(x) + f(x)\int_S L_r(x') G(x,x') V(x,x')\, dA'$$

Change variables to radiosity (B) and albedo (ρ)

$$B(x) = E(x) + \rho(x)\int_S B(x') \frac{G(x,x')V(x,x')}{\pi}\, dA'$$

Expresses conservation of light energy at all points in space

Same as equation 2.54 in Cohen Wallace handout (read sec 2.6.3)
Ignore factors of π which can be absorbed.

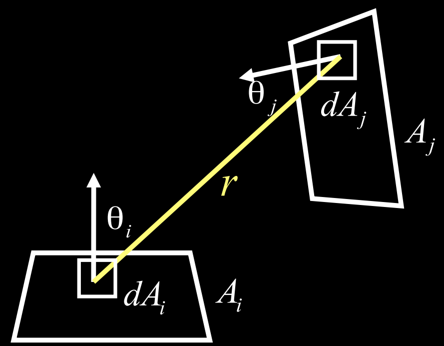
Discretization and Form Factors

$$B(x) = E(x) + \rho(x)\int_S B(x') \frac{G(x,x')V(x,x')}{\pi}\, dA'$$

$$B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i} \frac{A_j}{A_i}$$

F is the ***form factor.*** It is dimensionless and is the fraction of energy leaving the entirety of patch j (*multiply by area of* j to get total energy) that arrives anywhere in the entirety of patch i (*divide by area of i* to get energy per unit area or radiosity).

Form Factors



$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x - x'|^2}$$

Matrix Equation

$$B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i} \frac{A_j}{A_i}$$

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$B_i = E_i + \rho_i \sum_j B_j F_{i \rightarrow j}$$

$$B_i - \rho_i \sum_j B_j F_{i \rightarrow j} = E_i$$

$$\sum_j M_{ij} B_j = E_i \qquad MB = E \qquad M_{ij} = I_{ij} - \rho_i F_{i \rightarrow j}$$

Summary

- ***Theory*** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive ***Rendering Equation*** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

- Next: Practical solution using Monte Carlo methods