Subdivision Surfaces

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Why Subdivide?

Arbitrary topology - no trimming or seams between patches

Inifinite level of detail - figures can be arbitrarily close or far from camare without polygonalization showing

Numerical quality - C^1 continuity can be guaranteed all over the surface

Ease of editing - local control and mult-resolution editing make design intuitive

History of Subdivision Surfaces

1974 Chaikin - quadratic B-spline

1978 Catmull and Clark - Bicubic surfaces

1978 Doo and Sabin - Biquadratic surfaces

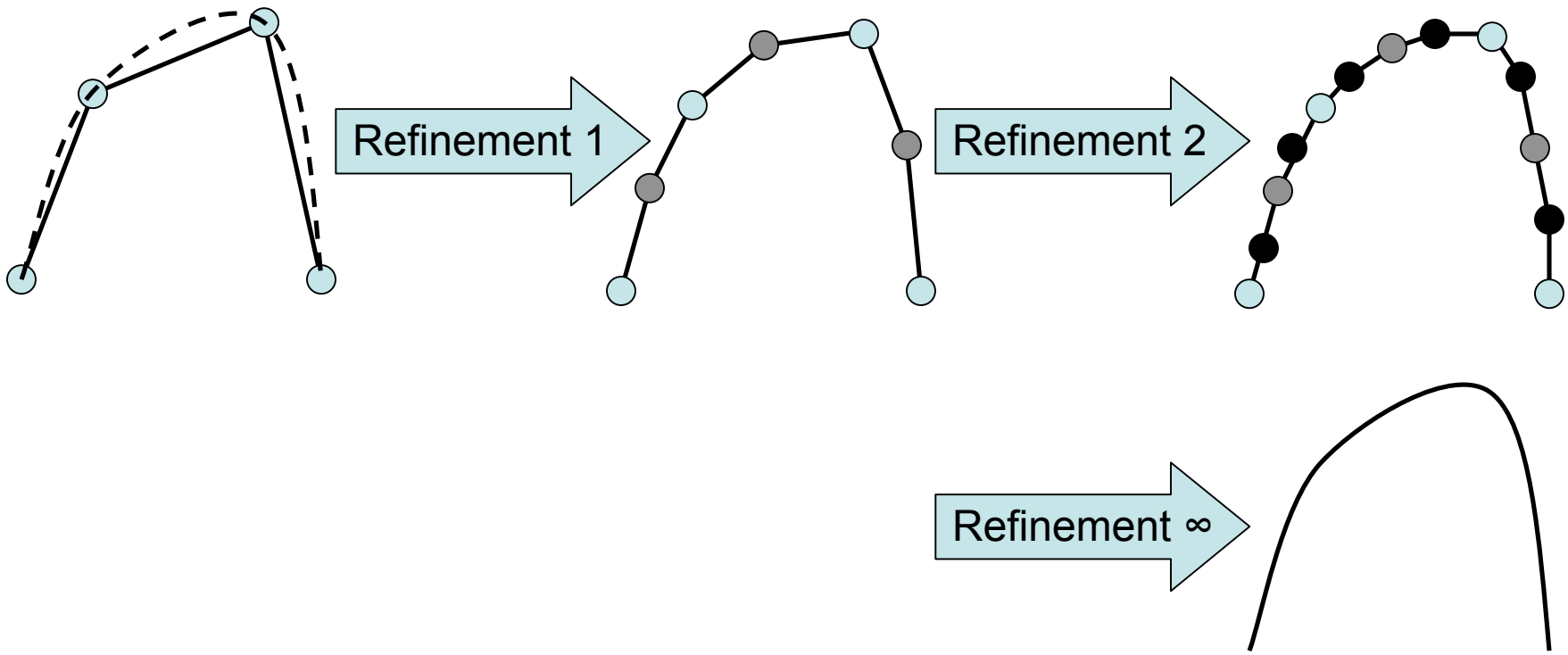
1990 Dyn, Levin, and Gregory - Butterfly

1996 Zorin and Schröer - Modified Butterfly

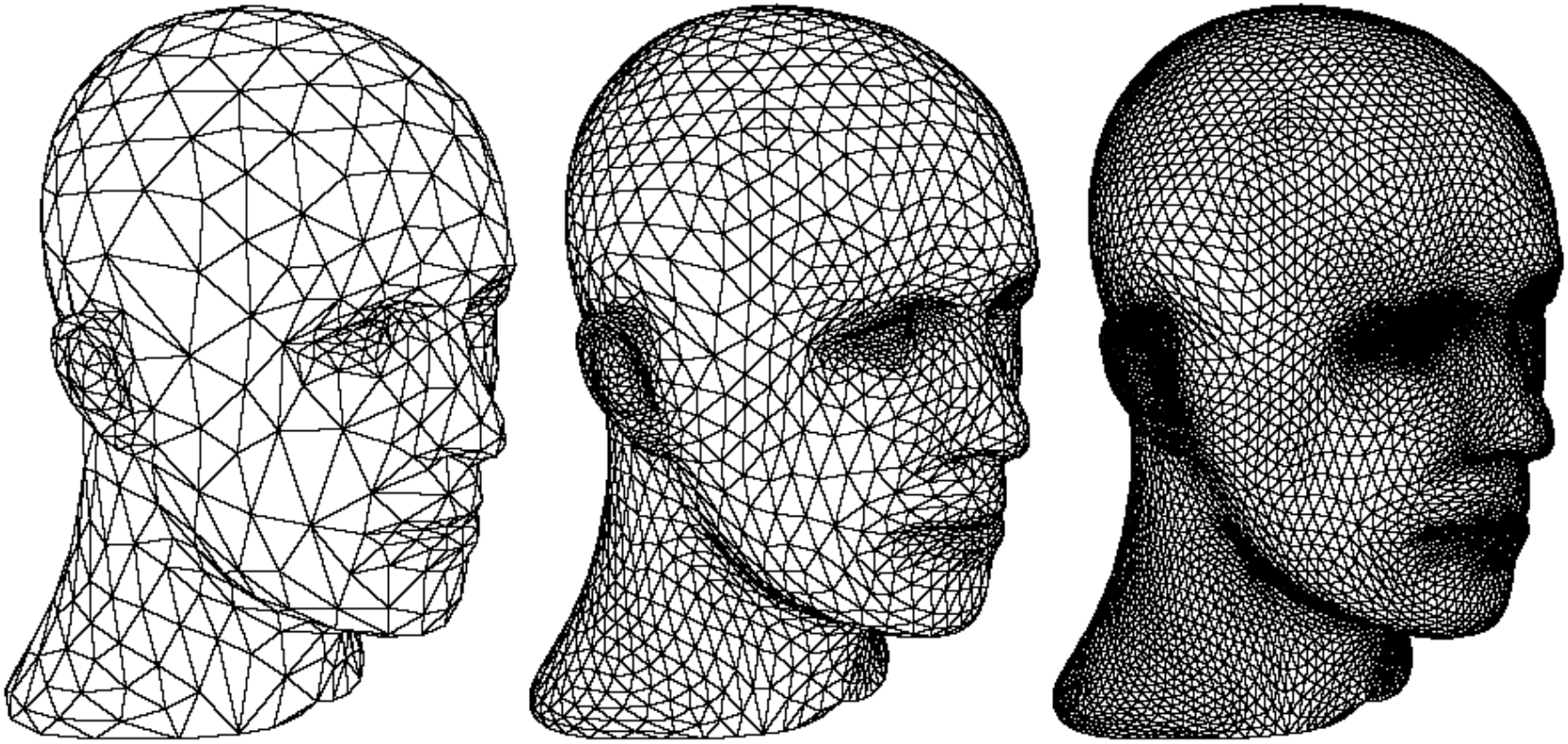
1998 DeRose and Kass - Animation

1998 Sederberg et alia - Non-uniform

The Basic Idea: Iterative Surface

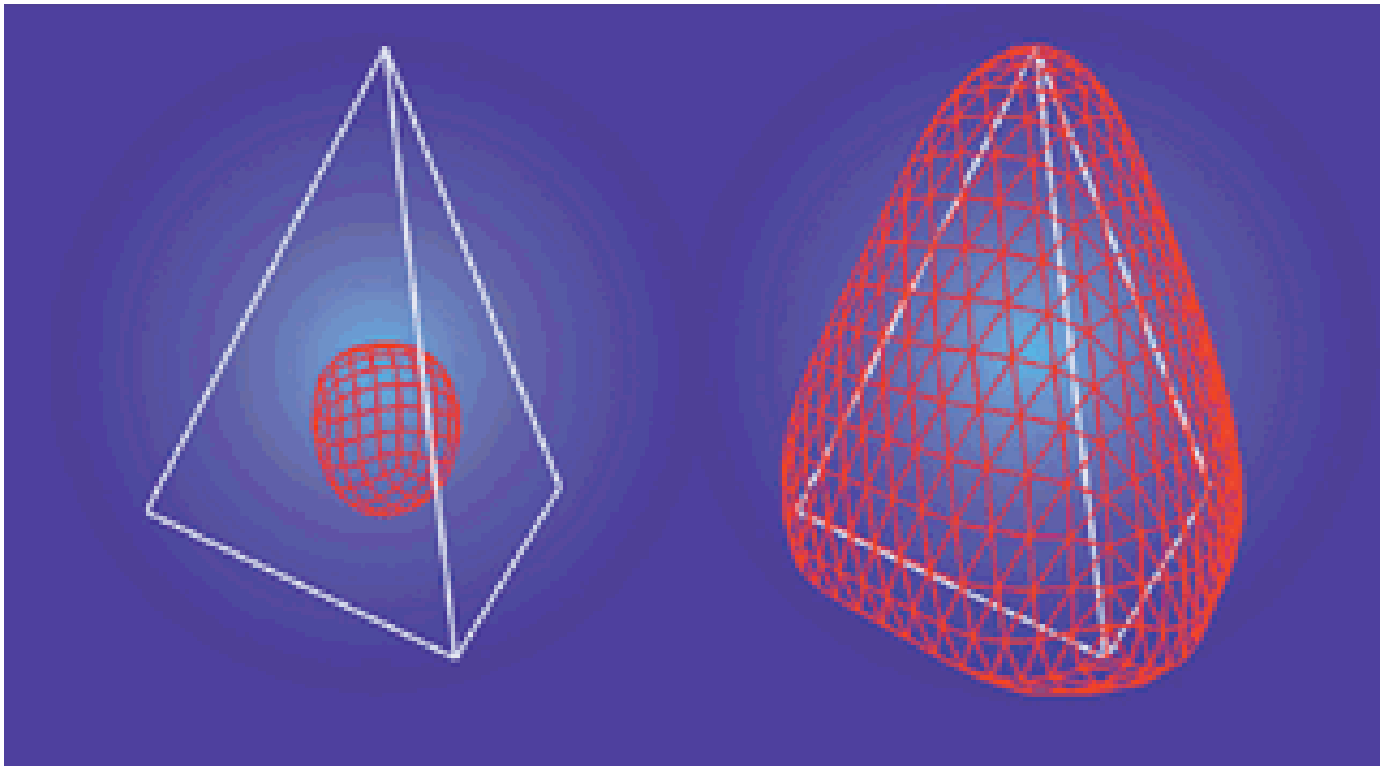


Also works in 3D

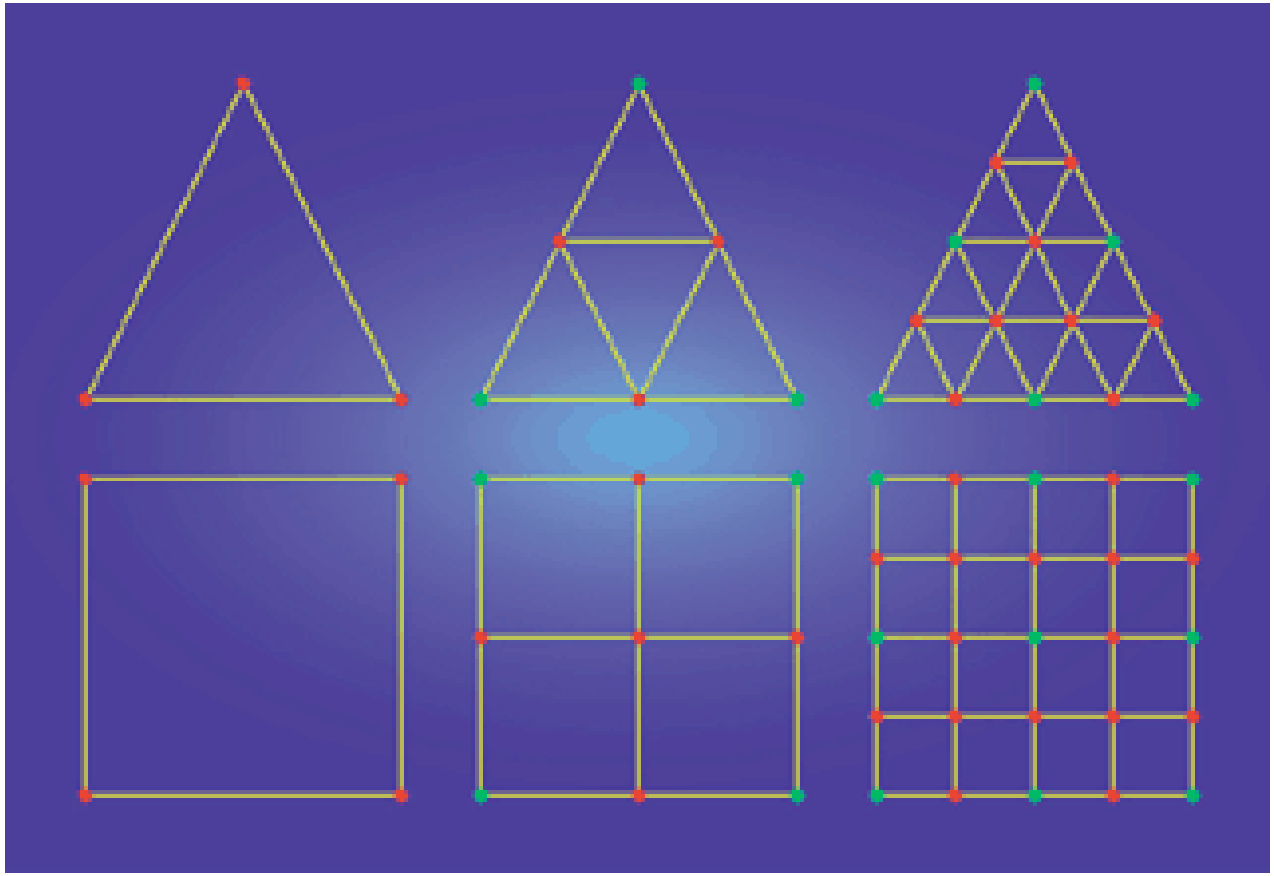


From SIGGRAPH ‘98 short course

Interpolate or Approximate?



Triangular or Quadrilateral?

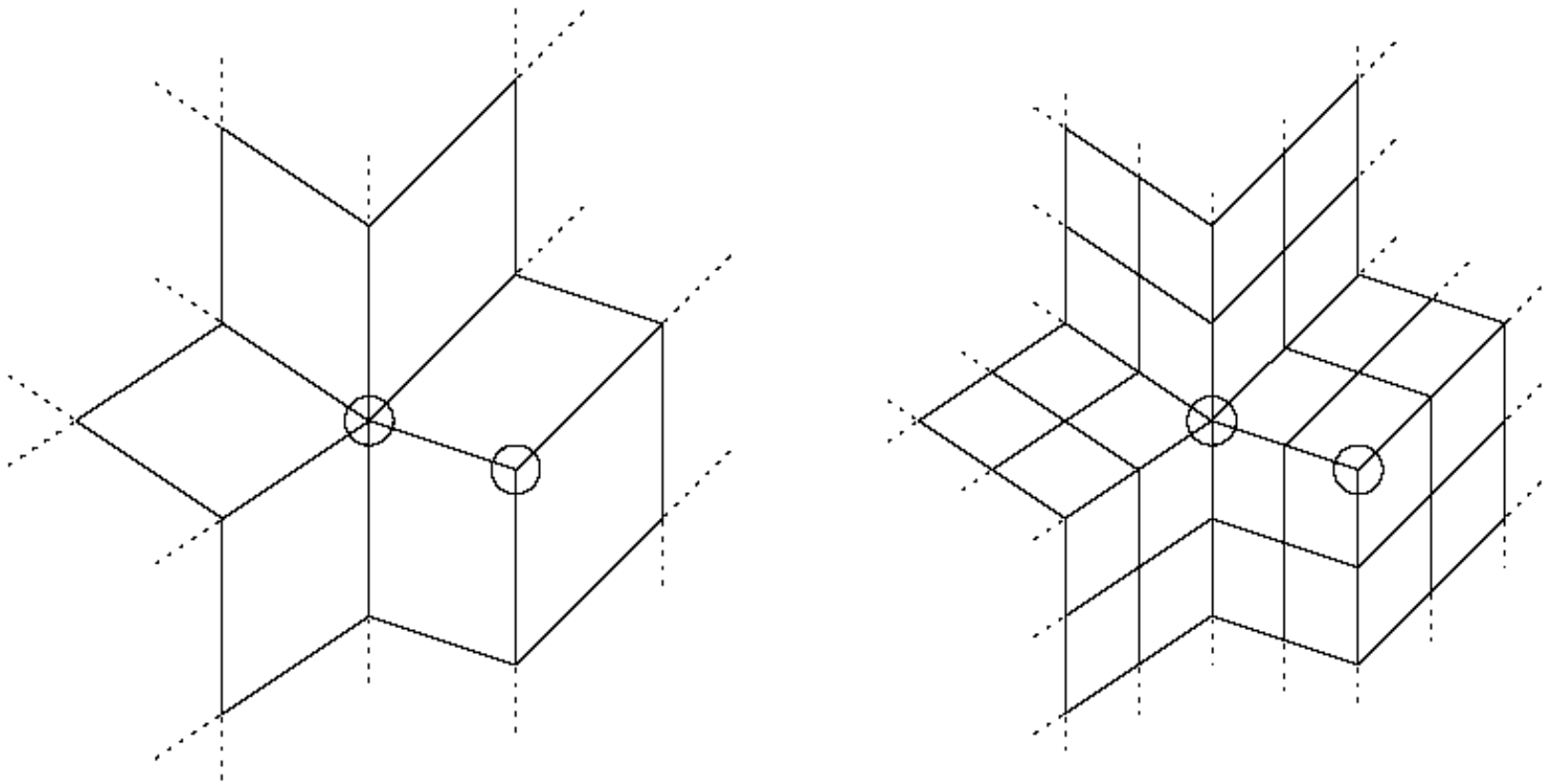


More Properties

Stationary - same algorithm applied at each step
Non-Stationary - algorithm may change between steps

Uniform - all areas of the control net treated the same
Non-Stationary - algorithm may differ across net

Extraordinary Vertices



From SIGGRAPH ‘98 short course

Notation

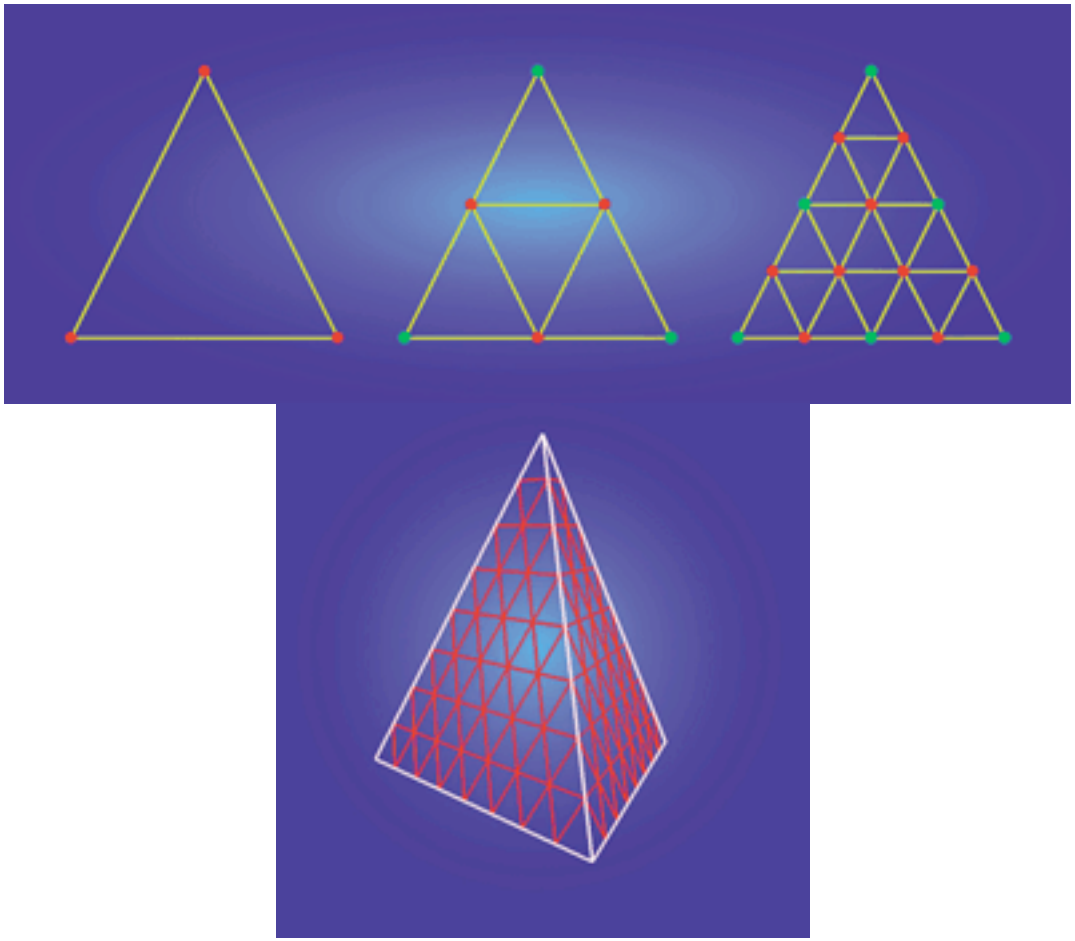
v is a vertex

v^i is the vertex after the ith subdivision step

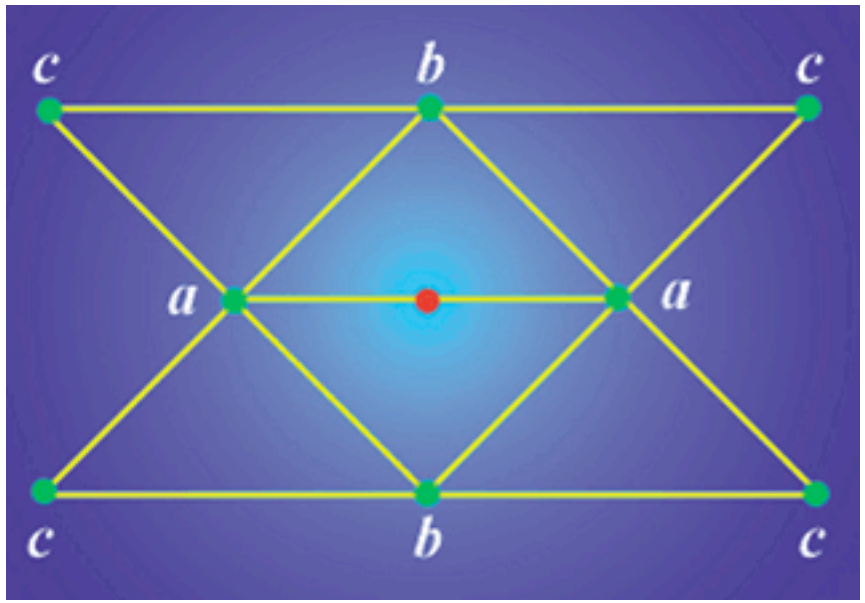
N is the valence of the vertex

e_j is a neighbor vertex to v where $j = [0..N-1]$

Example: Polyhedral



Example: Butterfly

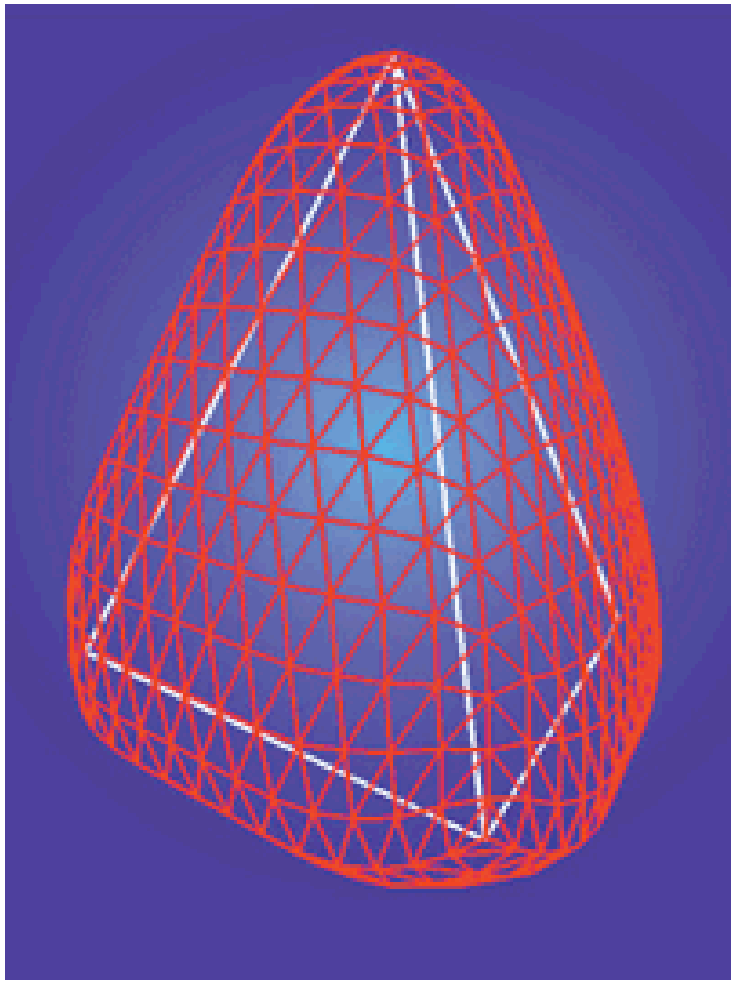


$$a : \frac{1}{2},\ b : \frac{1}{8} + 2w,\ c : -\frac{1}{16} - w$$

Butterfly Properties

- Proposed 1990 by Dyn, Levin, and Gregory
- Interpolates control net
- Allows “tension” control through *w*
- What about extraordinary vertices? No C^1
- Modified in ‘93 and ‘96 to accommodate extraordinary vertices and ensure global C^1 continuity

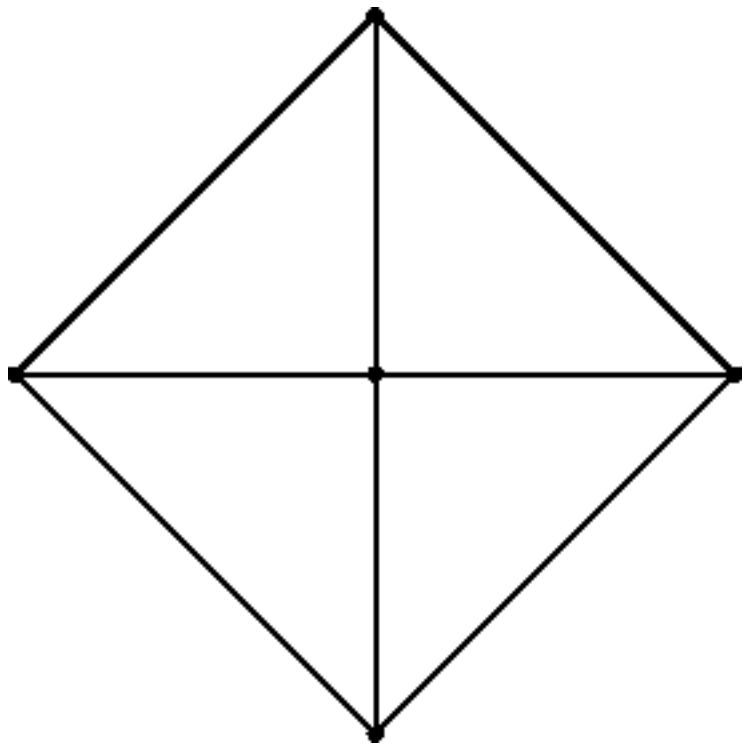
Butterfly Results



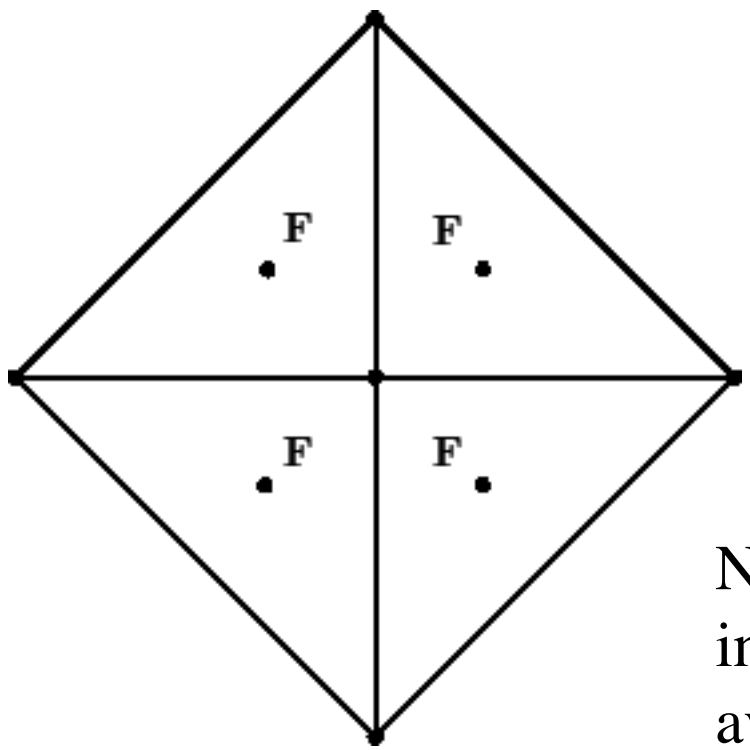
Example: Catmull-Clark

- Approximates control net
- Reduces meshes of arbitrary topology to quadrilateral meshes.
- Bicubic B-spline subdivision can be used to further refine mesh for non-extraordinary vertices.

Catmull-Clark

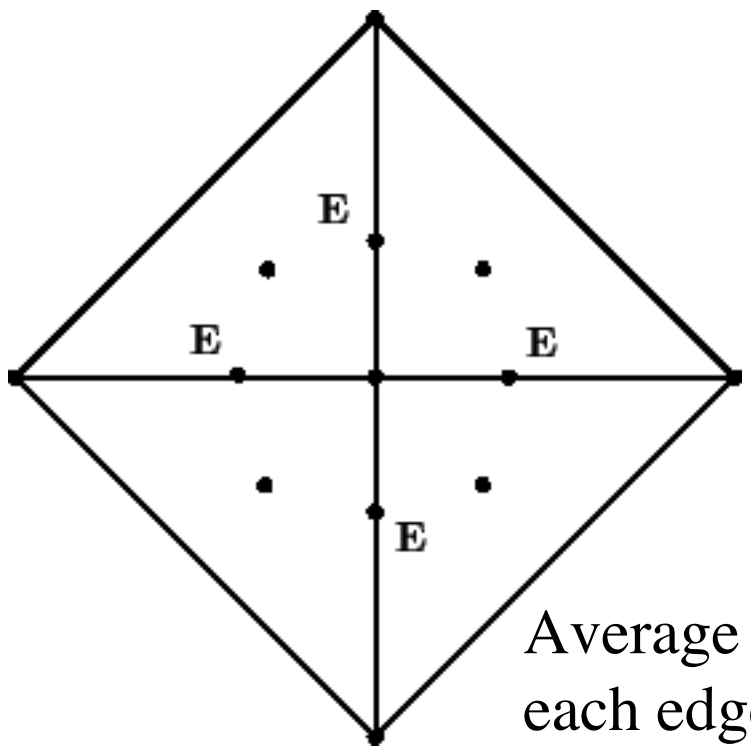


Catmull-Clark: Face Points



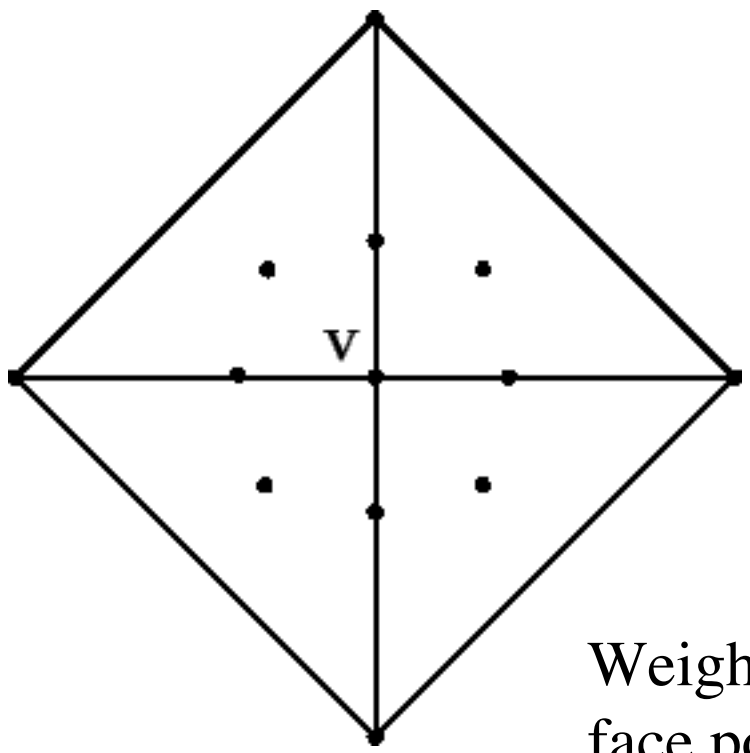
New face points inserted as average of vertices for each face

Catmull-Clark: Edge Points



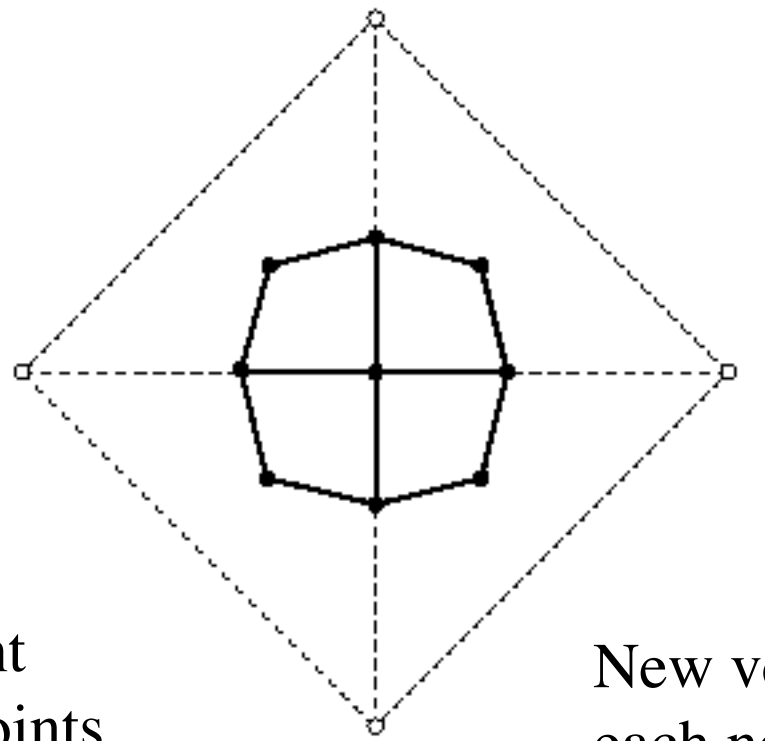
Average of midpoints of each edge and the two new face points on adjacent faces

Catmull-Clark: New Vertex



Weighted average of face points, original edge endpoints, and original vertices

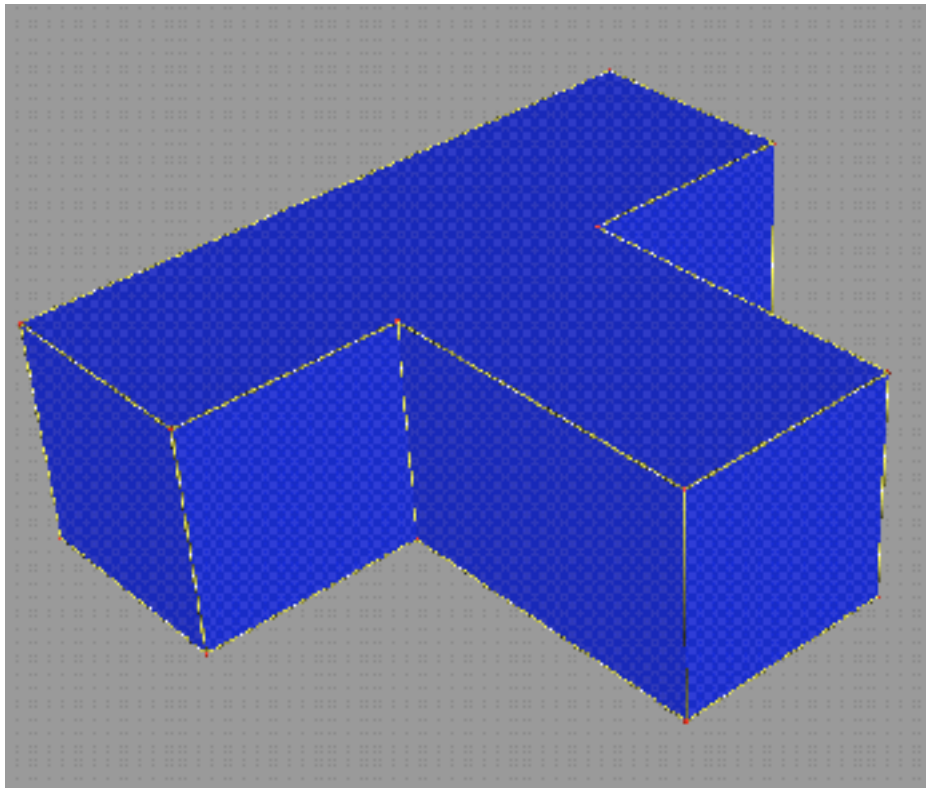
Catmull-Clark: Reconnect



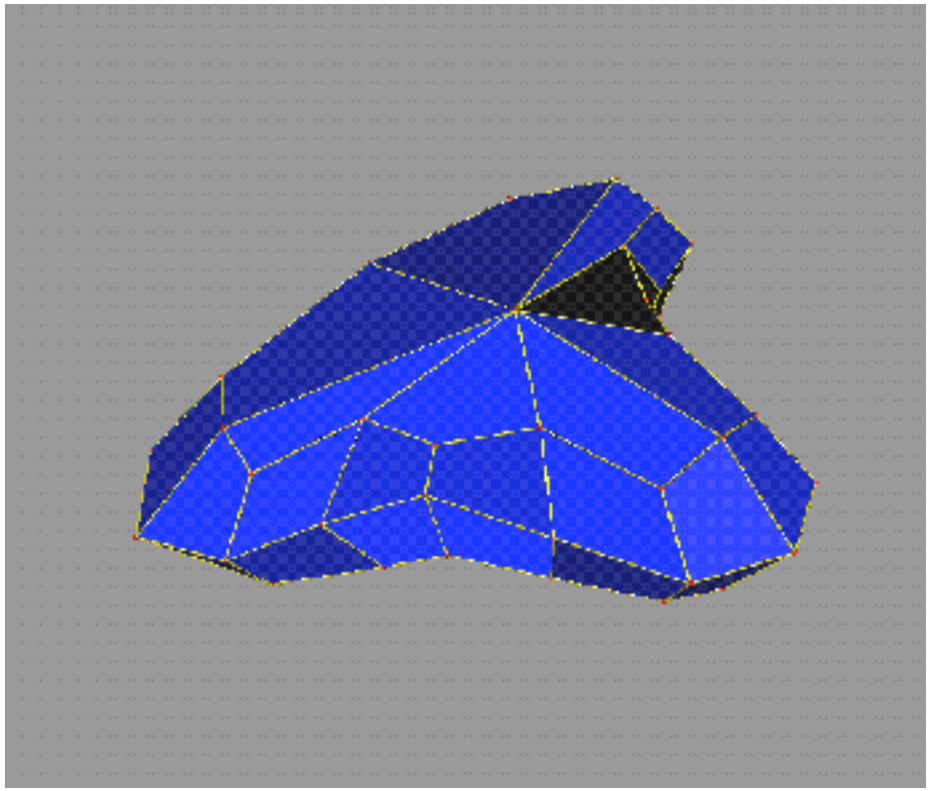
Each new face point connects to edge points from edges that gave rise to them

New vertex connects to each new edge point for edges incident to it

Catmull-Clark: Results



Catmull-Clark: Results



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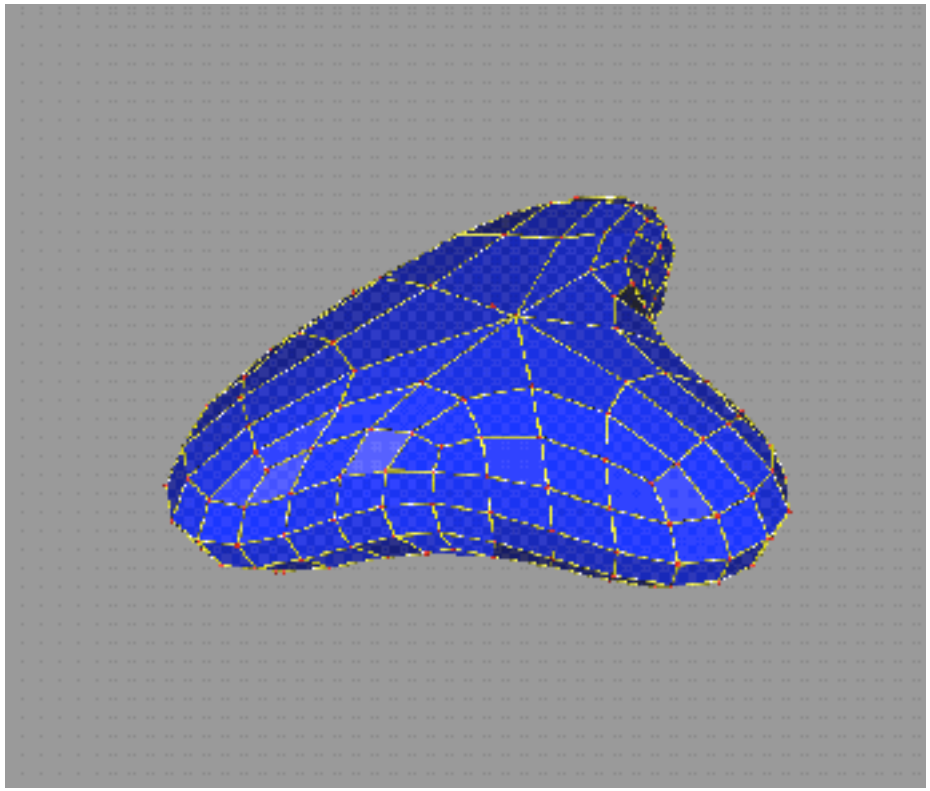
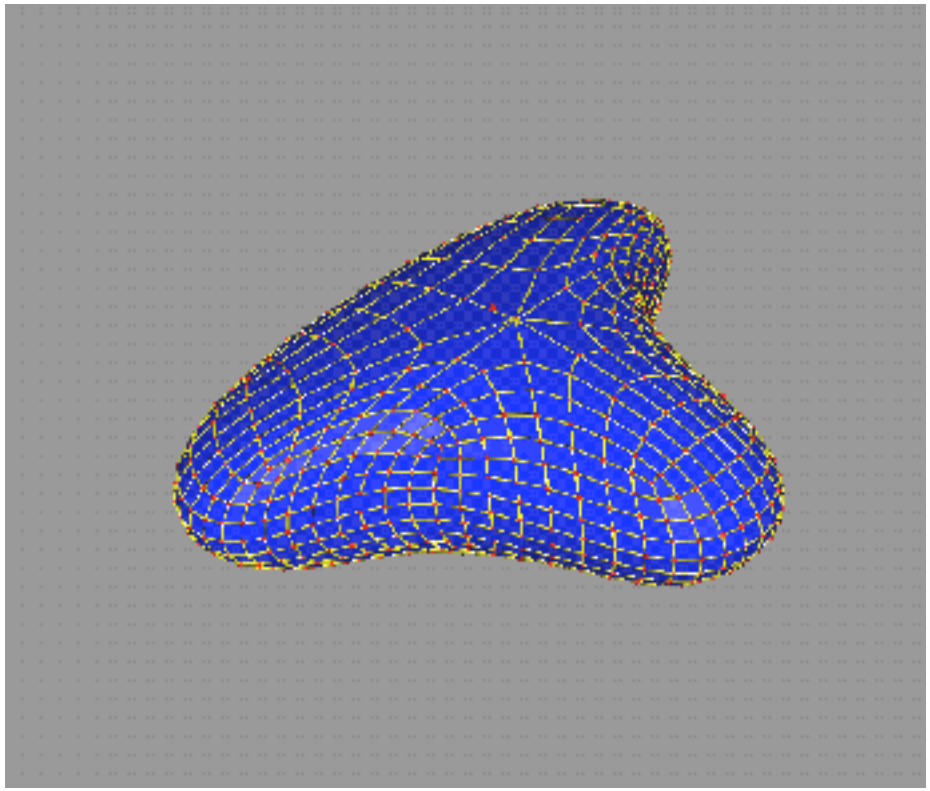


Catmull-Clark: Results

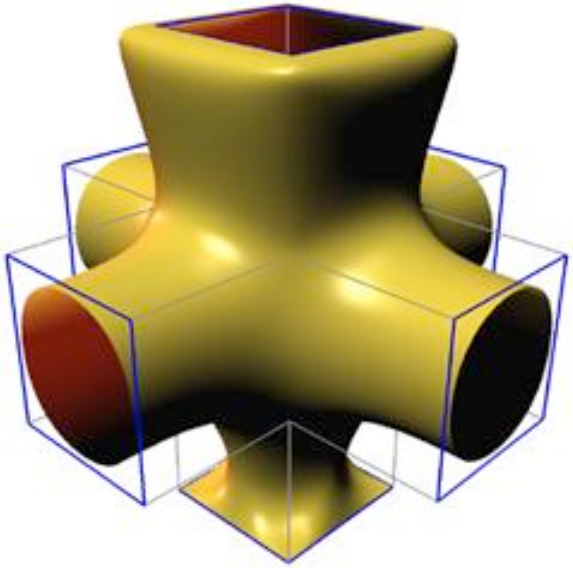


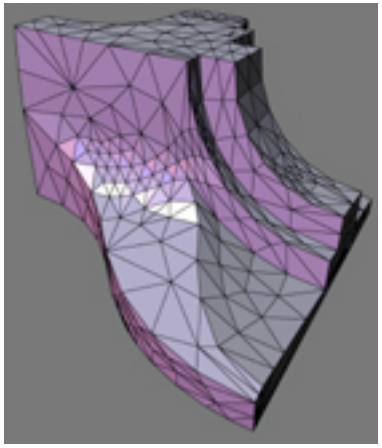
Catmull-Clark: Notes

- Extraordinary vertices can be created in first step.
- After the first step, we have all quadrilaterals, and no more extraordinary vertices will be added.
- Everywhere there is a 4x4 rectangular grid of control vertices, simply use bicubic B-spline surfaces.

Further Reading

What about places where surfaces *aren't* smooth?





SIGGRAPH '98 and '00 short course on subdivision surfaces - must read!