Advanced Computer Graphics (Fall 2009)

CS 294, Rendering Lecture 5: Monte Carlo Path Tracing

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http://inst.eecs.berkeley.edu/~cs294-13/fa09

Acknowledgements and some slides: Szymon Rusinkiewicz and Pat Hanrahan

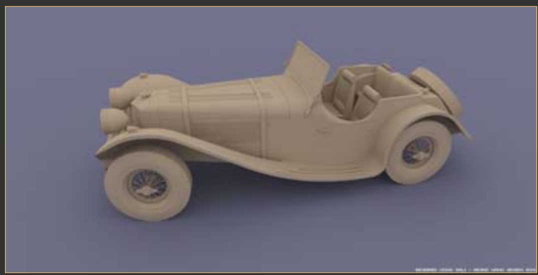
To Do

- Start working on assignment
- Find partners for this purpose

Motivation

- General solution to rendering and global illumination
- Suitable for a variety of general scenes
- Based on Monte Carlo methods
- Enumerate all paths of light transport

Monte Carlo Path Tracing



Big diffuse light source, 20 minutes

Jensen

Monte Carlo Path Tracing

1000 paths/pixel

Jensen

Monte Carlo Path Tracing

Advantages

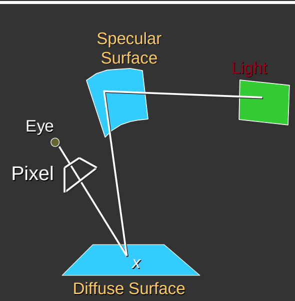
- Any type of geometry (procedural, curved, ...)
- Any type of BRDF (specular, glossy, diffuse, ...)
- Samples all types of paths (L(SD)*E)
- Accuracy controlled at pixel level
- Low memory consumption
- Unbiased - error appears as noise in final image

Disadvantages (standard Monte Carlo problems)

- Slow convergence (square root of number of samples)
- Noise in final image

Monte Carlo Path Tracing

Integrate radiance
for each pixel
by sampling paths
randomly



$$L_o(x,\vec{w}) = L_e(x,\vec{w}) + \int_{\Omega} f_r(x,\vec{w}',\vec{w}) L_i(x,\vec{w}')(\vec{w}' \bullet \vec{n}) d\vec{w}$$

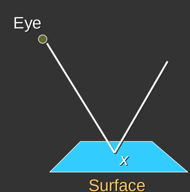
Simple Monte Carlo Path Tracer

- *Step 1: Choose a ray (u,v,θ,ϕ) [per pixel]; assign weight = 1*
- *Step 2: Trace ray to find intersection with nearest surface*
- *Step 3: Randomly choose between emitted and reflected light*
  - *Step 3a: If emitted,*
    *return weight' * Le*
  - *Step 3b: If reflected,*
    *weight'' *= reflectance*
    Generate ray in random direction
    Go to step 2

Sampling Techniques

Problem: how do we generate random points/directions during path tracing and reduce variance?

- Importance sampling (e.g. by BRDF)
- Stratified sampling



Outline

- Motivation and Basic Idea
- *Implementation of simple path tracer*
- Variance Reduction: Importance sampling
- Other variance reduction methods
- Specific 2D sampling techniques

Simplest Monte Carlo Path Tracer

For each pixel, cast n samples and average
- Choose a ray with *p*=camera, *d*=(θ,ϕ) within pixel
- Pixel color += (1/n) * TracePath(*p*, *d*)

TracePath(*p*, *d*) returns (r,g,b) [and calls itself recursively]:
- Trace ray (*p*, *d*) to find nearest intersection *p'*
- Select with probability (say) 50%:
  - Emitted:
    return 2 * (Le_{red}, Le_{green}, Le_{blue}) // 2 = 1/(50%)
  - Reflected:
    generate ray in random direction *d'*
    return 2 * $f_r(d \rightarrow d')$ * $(n \cdot d')$ * TracePath(*p'*, *d'*)

Simplest Monte Carlo Path Tracer

For each pixel, cast n samples and average over paths
- Choose a ray with *p*=camera, *d*=(θ,ϕ) within pixel
- Pixel color += (1/n) * TracePath(*p*, *d*)

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Simplest Monte Carlo Path Tracer

For each pixel, cast n samples and average
- Choose a ray with p=camera, d=(θ,ϕ) within pixel
- Pixel color += (1/n) * TracePath(p, d)

TracePath(p, d) returns (r,g,b) [and calls itself recursively]:
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- Select with probability (say) 50%:
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  - Reflected:
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    return 2 * $f_r(d \to d')$ * ($n \cdot d'$) * TracePath(p', d')

Weight = 1/probability
Remember: unbiased requires having f(x) / p(x)

Simplest Monte Carlo Path Tracer

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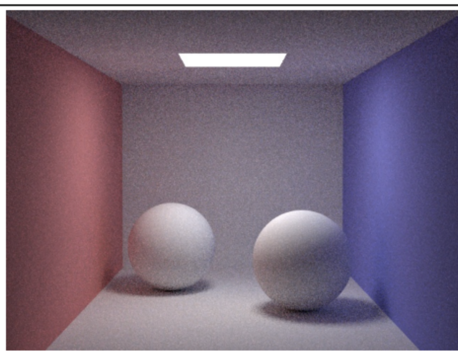
Path terminated when Emission evaluated

Path Tracing



CS348B Lecture 14 — 10 paths / pixel — Pat Hanrahan, Spring 2009

Arnold Renderer (M. Fajardo)

- Works well diffuse surfaces, hemispherical light

Advantages and Drawbacks

- Advantage: general scenes, reflectance, so on
  - By contrast, standard recursive ray tracing only mirrors
- This algorithm is *unbiased*, but horribly inefficient
  - Sample “emitted” 50% of the time, even if emitted=0
  - Reflect rays in random directions, even if mirror
  - If light source is small, rarely hit it
- Goal: improve efficiency without introducing bias
  - Variance reduction using many of the methods discussed for Monte Carlo integration last week
  - Subject of much interest in graphics in 90s till today

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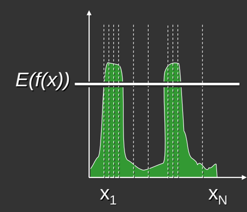
Importance Sampling

- Pick paths based on energy or expected contribution
  - More samples for high-energy paths
  - Don't pick low-energy paths
- At "macro" level, use to select between reflected vs emitted, or in casting more rays toward light sources
- At "micro" level, importance sample the BRDF to pick ray directions
- Tons of papers in 90s on tricks to reduce variance in Monte Carlo rendering

Importance Sampling

Can pick paths however we want, but contribution weighted by 1/probability

- Already seen this division of 1/prob in weights to emission, reflectance



$$\int_{\Omega} f(x)dx = \frac{1}{N}\sum_{i=1}^{N} Y_i$$

$$Y_i = \frac{f(x_i)}{p(x_i)}$$

Simplest Monte Carlo Path Tracer

For each pixel, cast n samples and average

- Choose a ray with *p*=camera, *d*=(θ,ϕ) within pixel
- Pixel color += (1/n) * TracePath(*p*, *d*)

TracePath(*p*, *d*) returns (r,g,b) [and calls itself recursively]:

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Importance sample Emit vs Reflect

TracePath(*p*, *d*) returns (r,g,b) [and calls itself recursively]:

- Trace ray (*p*, *d*) to find nearest intersection *p'*
- If Le = (0,0,0) then p_{emit}= 0 else p_{emit}= 0.9 (say)
- If random() < p_{emit} then:
  - Emitted:
    - return (1/ p_{emit}) * (Le_{red}, Le_{green}, Le_{blue})
  - Else Reflected:
    - generate ray in random direction *d'*
    - return (1/(1- p_{emit})) * $f_r(d \to d')$ * $(n \cdot d')$ * TracePath(*p'*, *d'*)

Importance sample Emit vs Reflect

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Can never be 1 unless Reflectance is 0

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More variance reduction

- Discussed "macro" importance sampling
  - Emitted vs reflected
- How about "micro" importance sampling
  - *Shoot rays towards light sources in scene*
  - Distribute rays according to BRDF

One Variation for Reflected Ray

- Pick a light source
- Trace a ray towards that light
- Trace a ray anywhere except for that light
  - Rejection sampling
- Divide by probabilities
  - 1/(solid angle of light) for ray to light source
  - (1 – the above) for non-light ray
  - Extra factor of 2 because shooting 2 rays

Russian Roulette

- Maintain current weight along path (need another parameter to TracePath)
- Terminate ray iff |weight| < const.
- Be sure to weight by 1/probability

Russian Roulette

Terminate photon with probability *p*

Adjust weight of the result by 1/(1-p)

$$E(X) = p \cdot 0 + (1-p)\frac{E(X)}{1-p} = E(X)$$

Intuition:

Reflecting from a surface with R=.5

100 incoming photons with power 2 W

1. **Reflect 100 photons with power 1 W**
2. **Reflect 50 photons with power 2 W**

Path Tracing: Include Direct Lighting

```
Step 1. Choose a camera ray r given the
   (x,y,u,v,t) sample
   weight = 1;
   L = 0
Step 2. Find ray-surface intersection
Step 3.
   L += weight * Lr(light sources)
   weight *= reflectance(r)
   Choose new ray r' ~ BRDF pdf(r)
        Go to Step 2.
```


Monte Carlo Extensions

Unbiased
- Bidirectional path tracing
- Metropolis light transport

Biased, but consistent
- Noise filtering
- Adaptive sampling
- Irradiance caching

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RenderPark

Monte Carlo Extensions

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Heinrich

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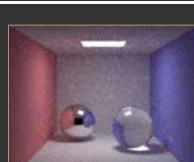
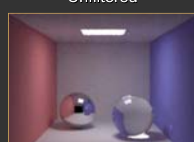


Unfiltered



Filtered

Jensen

Monte Carlo Extensions

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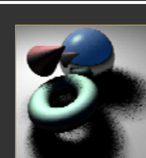
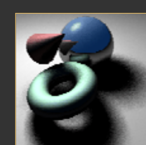


Fixed



Adaptive

Ohbuchi

Monte Carlo Extensions

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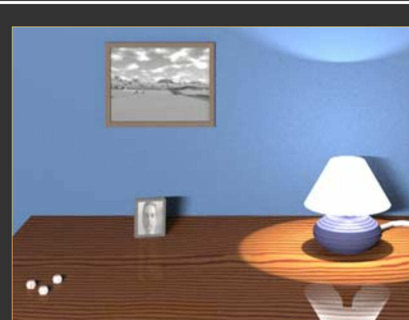
Monte Carlo Path Tracing Image



2000 samples per pixel, 30 computers, 30 hours

Jensen

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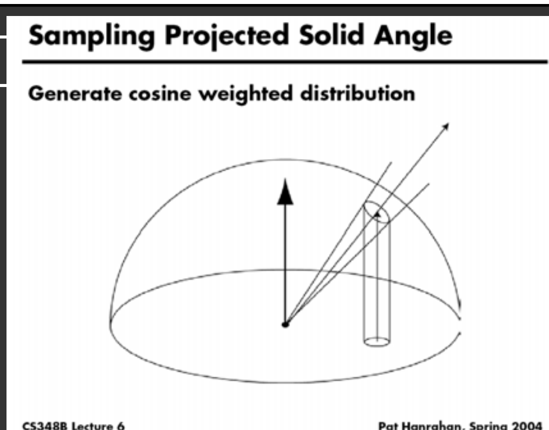
2D Sampling: Motivation

- Final step in sending reflected ray: sample 2D domain
- According to projected solid angle
- Or BRDF
- Or area on light source
- Or sampling of a triangle on geometry
- Etc.



Sampling Upper Hemisphere

- Uniform directional sampling: how to generate random ray on a hemisphere?
- Option #1: rejection sampling
  - Generate random numbers (x,y,z), with x,y,z in –1..1
  - If $x^2+y^2+z^2 > 1$, reject
  - Normalize (x,y,z)
  - If pointing into surface (ray dot n < 0), flip

Sampling Upper Hemisphere

- Option #2: inversion method
  - In polar coords, density must be proportional to sin θ (remember d(solid angle) = sin θ $d\theta\, d\phi$)
  - Integrate, invert → $\cos^{-1}$
- So, recipe is
  - Generate ϕ in 0..2π
  - Generate z in 0..1
  - Let $\theta = \cos^{-1} z$
  - $(x,y,z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

BRDF Importance Sampling

- Better than uniform sampling: importance sampling
- Because you divide by probability, ideally probability $\propto f_r * \cos\theta_i$

BRDF Importance Sampling

- For cosine-weighted Lambertian:
  - Density = cos θ sin θ
  - Integrate, invert → $\cos^{-1}$(sqrt)
- So, recipe is:
  - Generate ϕ in 0..2π
  - Generate z in 0..1
  - Let $\theta = \cos^{-1}$ (sqrt(z))

BRDF Importance Sampling

- Phong BRDF: $f_r \propto \cos^n \alpha$ where α is angle between outgoing ray and ideal mirror direction
- Constant scale = $k_s(n+2)/(2\pi)$
- Can't sample this times cos θ_i
  - Can only sample BRDF itself, then multiply by cos θ_i
  - That's OK – still better than random sampling

BRDF Importance Sampling

- Recipe for sampling specular term:
  - Generate z in 0..1
  - Let $\alpha = \cos^{-1} (z^{1/(n+1)})$
  - Generate ϕ_α in 0..2π
- This gives direction w.r.t. ideal mirror direction
- Convert to (x,y,z), then rotate such that z points along mirror dir.

Summary

- Monte Carlo methods robust and simple (at least until nitty gritty details) for global illumination
- Must handle many variance reduction methods in practice
- Importance sampling, Bidirectional path tracing, Russian roulette etc.
- Rich field with many papers, systems researched over last 10 years