Advanced Computer Graphics (Fall 2009)

CS 294, Rendering Lecture 9:

Frequency Analysis and Signal Processing for Rendering

Ravi Ramamoorthi

http://inst.eecs.berkeley.edu/~cs294-13/fa09

Motivation

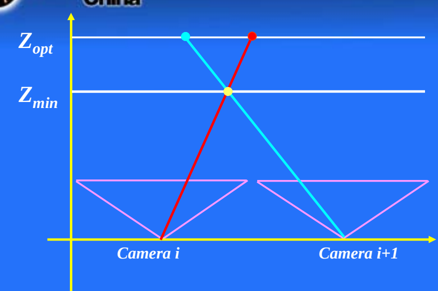
- Signal-processing provides new understanding
- Methods based on (Spherical) Fourier analysis
- Allows understanding of sampling rates (in IBR)
- Frequency-domain algorithms like convolution

- This lecture high-level, mostly conceptual ideas.
  - Follow original papers for details, applications

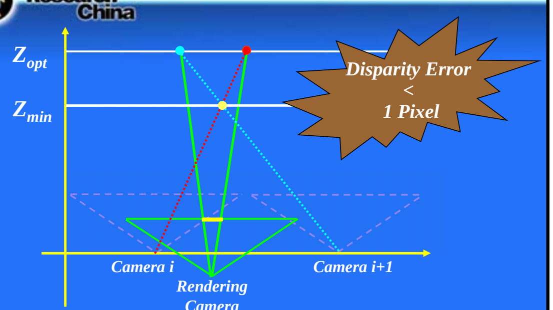
Plenoptic Sampling

- Plenoptic Sampling. *Chai, Tong, Chan, Shum 00*
- Signal-processing on light field
- Minimal sampling rate for antialiased rendering
- Relates to depth range, Fourier analysis
- Fourer spectra derived for 2D light fields for simplicity. Same ideas extend to 4D

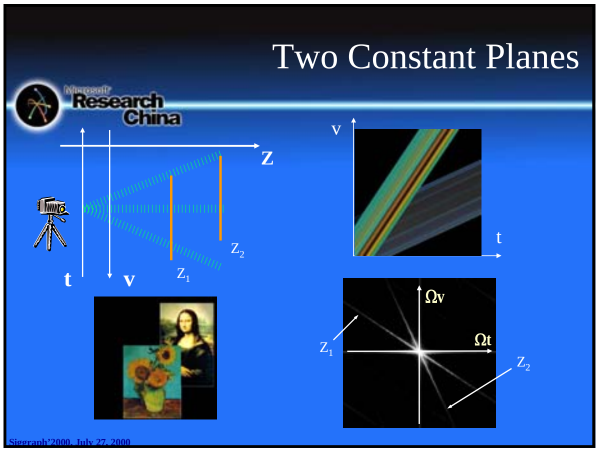
A Geometrical Intuition

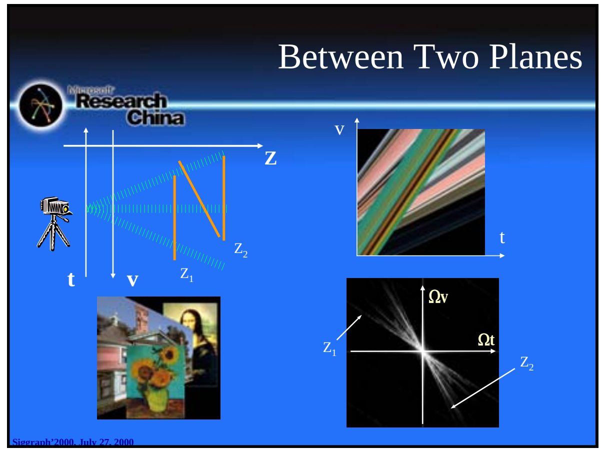


A Geometrical Intuition



A Constant Plane


Two Constant Planes
Research China
Z
v
t
v
Z1
Z2
t
Ωv
Ωt
Z1
Z2
Siggraph'2000, July 27, 2000


Between Two Planes
Research China
Z
v
t
v
Z1
Z2
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Ωv
Ωt
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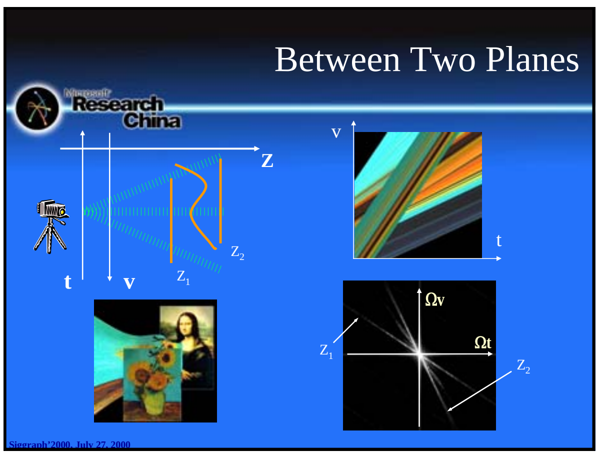
Siggraph'2000, July 27, 2000


Between Two Planes
Research China
Z
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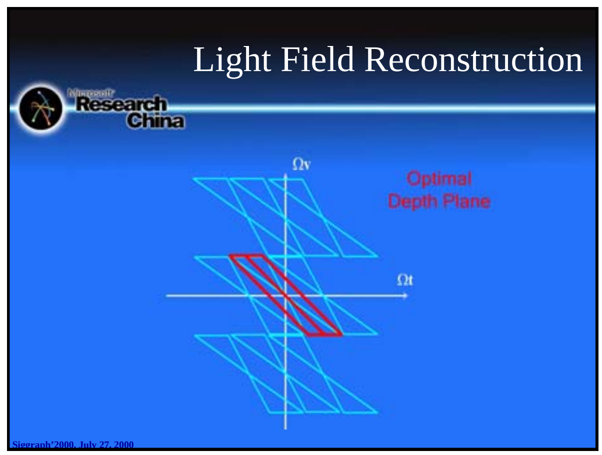
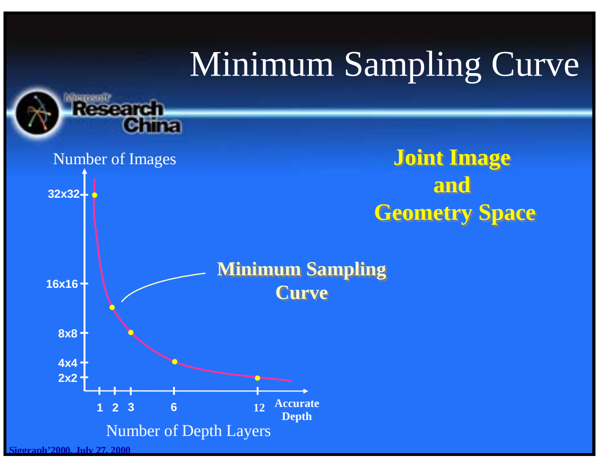
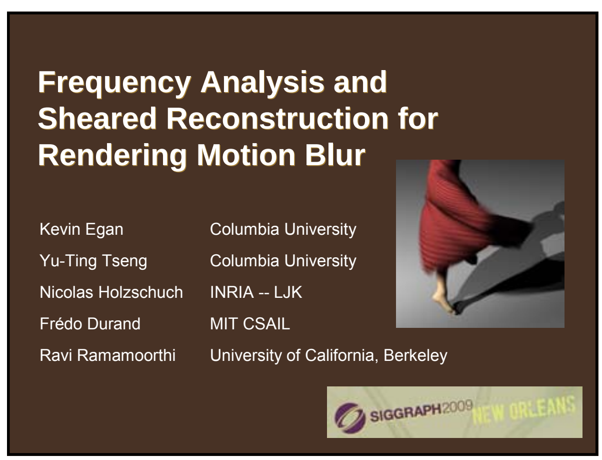
Siggraph'2000, July 27, 2000

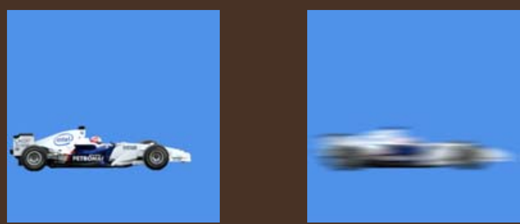

Light Field Reconstruction
Research China
Ωv
Optimal
Depth Plane
Ωt
Siggraph'2000, July 27, 2000


Minimum Sampling Curve
Research China
Number of Images
Joint Image
and
Geometry Space
32x32
16x16
8x8
4x4
2x2
Minimum Sampling
Curve
1 2 3 6 12
Accurate Depth
Number of Depth Layers
Siggraph'2000, July 27, 2000


Frequency Analysis and
Sheared Reconstruction for
Rendering Motion Blur
Kevin Egan Columbia University
Yu-Ting Tseng Columbia University
Nicolas Holzschuch INRIA -- LJK
Frédo Durand MIT CSAIL
Ravi Ramamoorthi University of California, Berkeley
SIGGRAPH2009 NEW ORLEANS

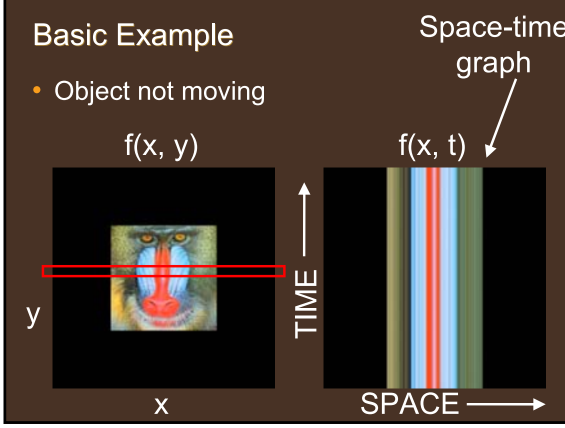
Observation

- Motion blur is expensive
- Motion blur *removes* spatial complexity



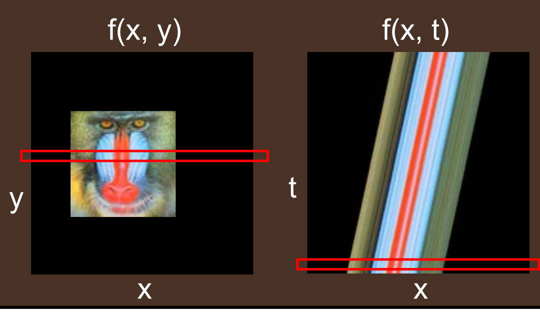
Basic Example

- Object not moving



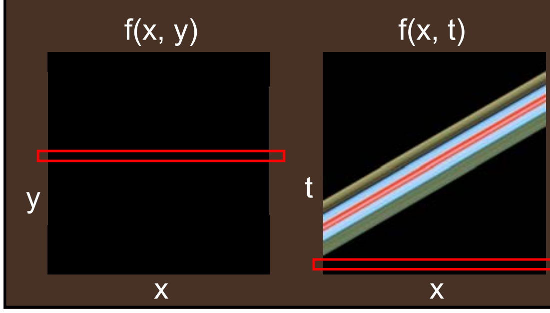
Basic Example

- Low velocity, **t** ∈ **[0.0, 1.0]**



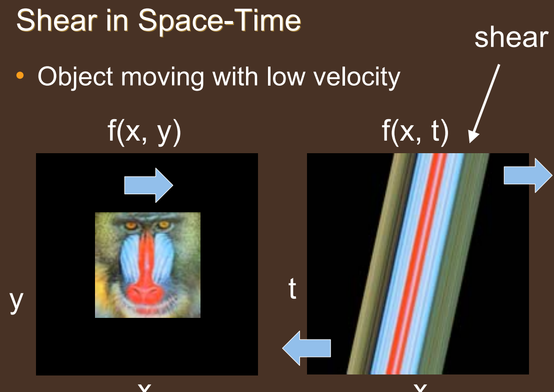
Basic Example

- High velocity, **t** ∈ **[0.0, 1.0]**



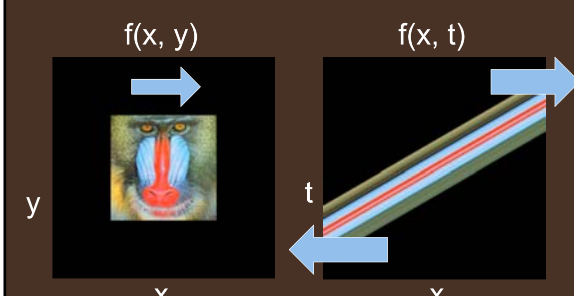
Shear in Space-Time

- Object moving with low velocity



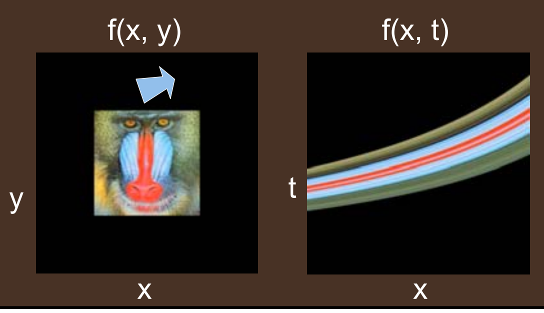
Shear in Space-Time

- Object moving with high velocity



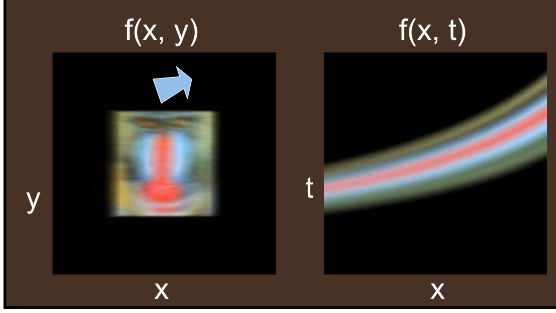
Shear in Space-Time

- Object moving away from camera



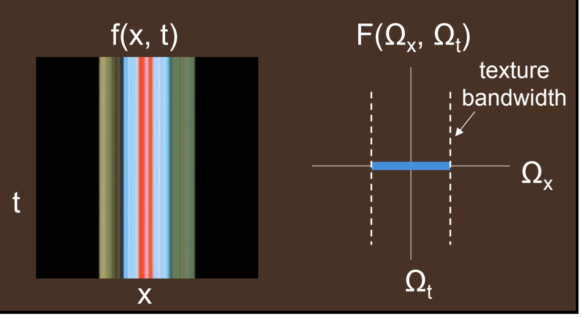
Basic Example

- Applying shutter blurs across time



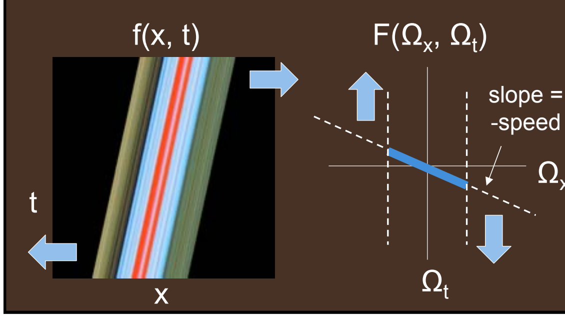
Basic Example – Fourier Domain

- Fourier spectrum, zero velocity



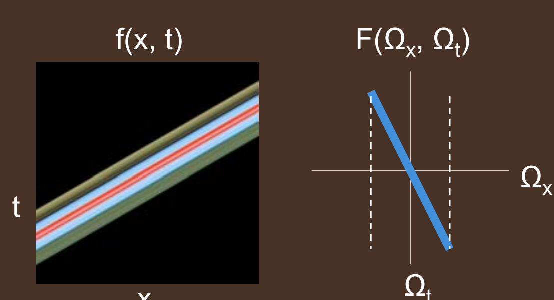
Basic Example – Fourier Domain

- Low velocity, small shear in both domains



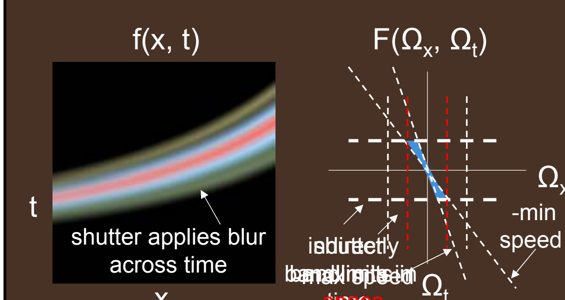
Basic Example – Fourier Domain

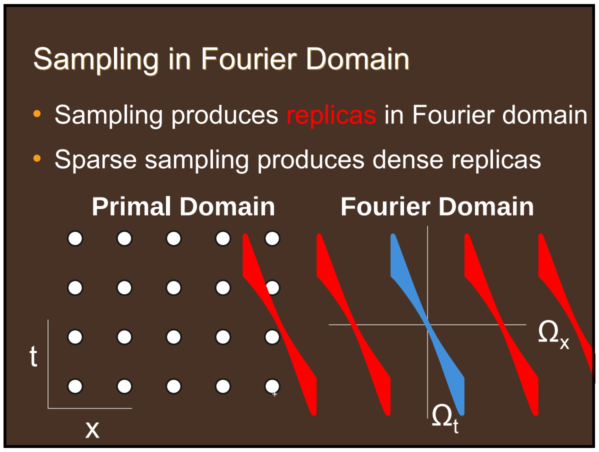
- Large shear

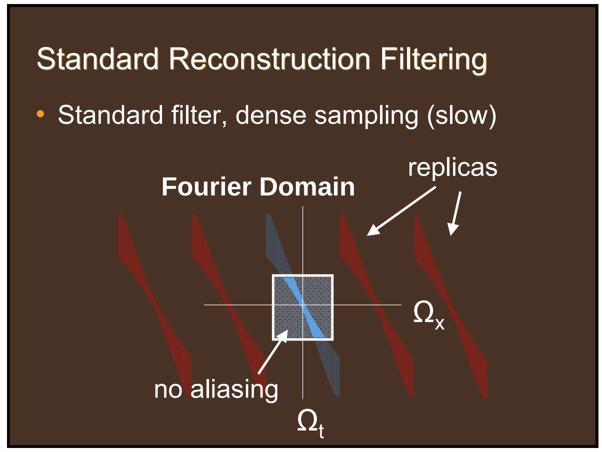


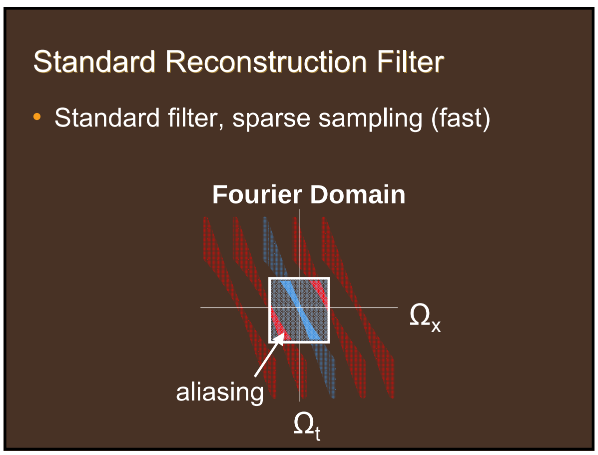
Basic Example – Fourier Domain

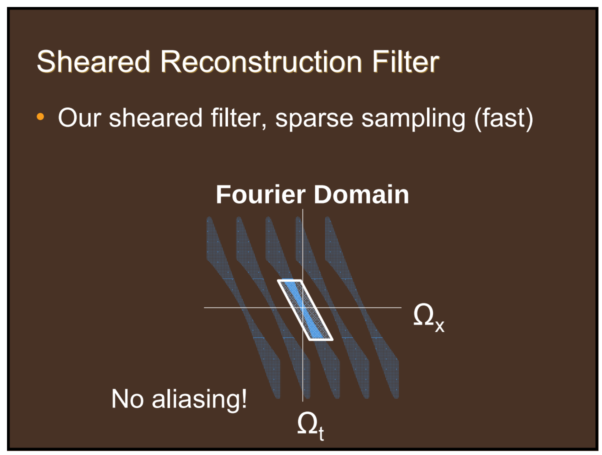
- Non-linear motion, wedge shaped spectra

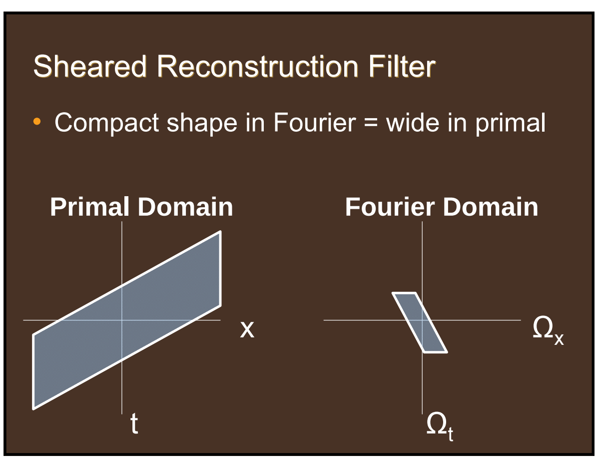


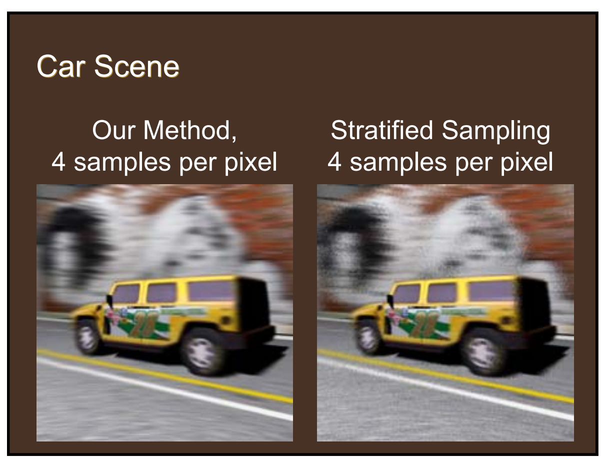

Sampling in Fourier Domain
Sampling produces replicas in Fourier domain
Sparse sampling produces dense replicas
Primal Domain
Fourier Domain
t
x
Ωx
Ωt


Standard Reconstruction Filtering
Standard filter, dense sampling (slow)
Fourier Domain
replicas
Ωx
no aliasing
Ωt


Standard Reconstruction Filter
Standard filter, sparse sampling (fast)
Fourier Domain
Ωx
aliasing
Ωt


Sheared Reconstruction Filter
Our sheared filter, sparse sampling (fast)
Fourier Domain
Ωx
No aliasing!
Ωt

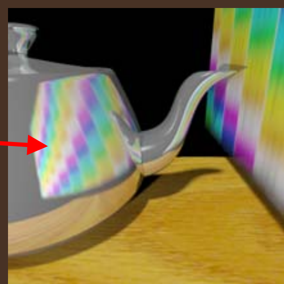

Sheared Reconstruction Filter
Compact shape in Fourier = wide in primal
Primal Domain
Fourier Domain
x
t
Ωx
Ωt


Car Scene
Our Method,
4 samples per pixel
Stratified Sampling
4 samples per pixel

Teapot Scene

Our Method
8 samples / pix

motion blurred reflection



Reflection as Convolution

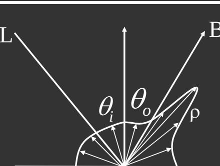
- My PhD thesis (A signal-processing framework for forward and inverse rendering Stanford 2002)
- Rewrite reflection equation on curved surfaces as a convolution with frequency-space product form
- Theoretical underpinning for much work on relighting (next lecture), limits of inverse problems
- Low-dimensional lighting models for Lambertian

Assumptions

- Known geometry
- Convex curved surfaces: no shadows, interreflection
- Distant illumination
- Homogeneous isotropic materials

Later precomputed methods: relax many assumptions

Reflection



$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\ d\theta_i$$

Reflected Light Field **Lighting** **BRDF**

Reflection as Convolution (2D)

$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\ d\theta_i$$

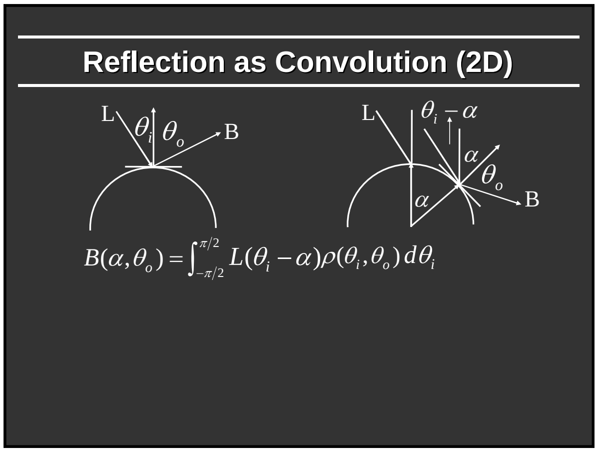
Reflected Light Field **Lighting** **BRDF**

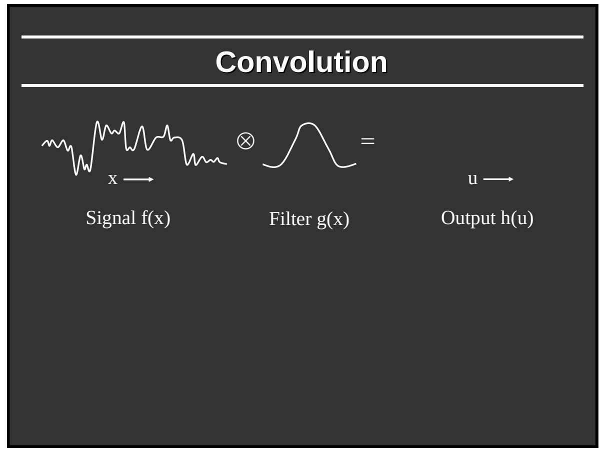
Reflection as Convolution (2D)

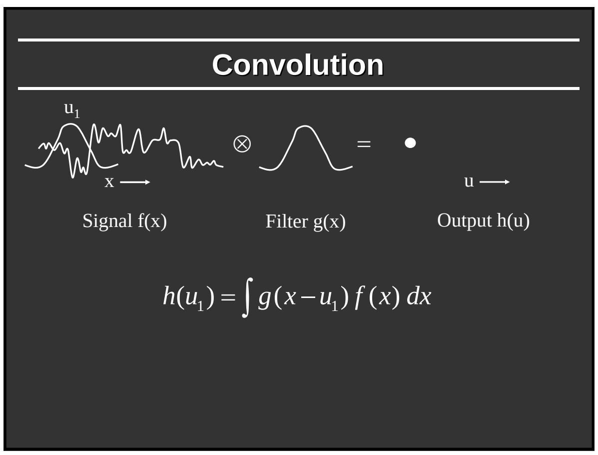
$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\ d\theta_i$$

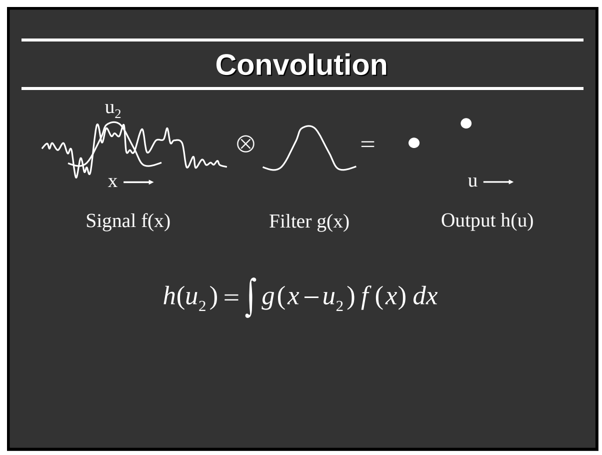
Reflected Light Field **Lighting** **BRDF**

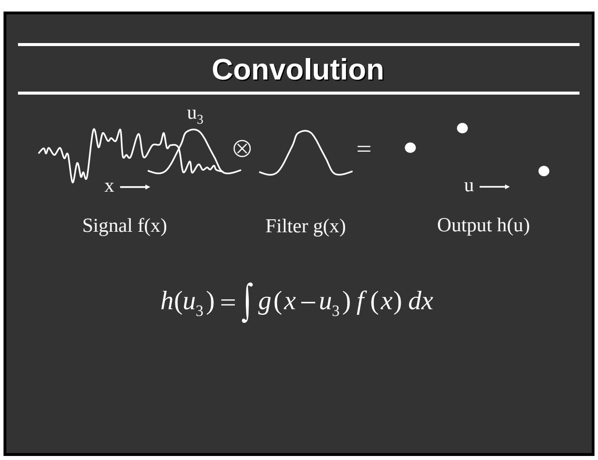
$$B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \rho(\theta_i, \theta_o)\, d\theta_i$$

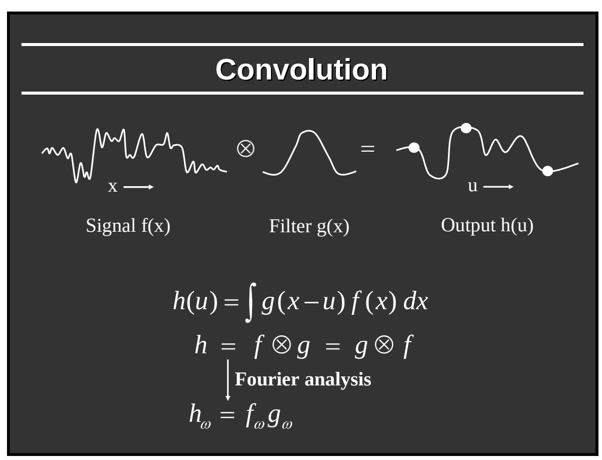

Reflection as Convolution (2D)
L
B
θ_i
θ_o
$\theta_i - \alpha$
α
$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha)\rho(\theta_i,\theta_o)\,d\theta_i$


Convolution
⊗
=
x
u
Signal f(x)
Filter g(x)
Output h(u)

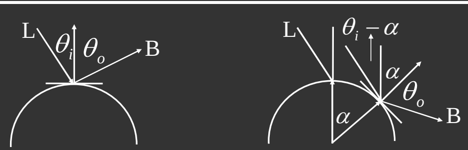

Convolution
u_1
⊗
=
x
u
Signal f(x)
Filter g(x)
Output h(u)
$h(u_1) = \int g(x - u_1)\, f(x)\, dx$


Convolution
u_2
⊗
=
x
u
Signal f(x)
Filter g(x)
Output h(u)
$h(u_2) = \int g(x - u_2)\, f(x)\, dx$


Convolution
u_3
⊗
=
x
u
Signal f(x)
Filter g(x)
Output h(u)
$h(u_3) = \int g(x - u_3)\, f(x)\, dx$


Convolution
⊗
=
x
u
Signal f(x)
Filter g(x)
Output h(u)
$h(u) = \int g(x - u)\, f(x)\, dx$
$h = f \otimes g = g \otimes f$
Fourier analysis
$h_\omega = f_\omega g_\omega$

Reflection as Convolution (2D)



$$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha)\rho(\theta_i,\theta_o)\,d\theta_i$$

$$B = L \otimes \rho$$

Spatial: integral

Fourier analysis

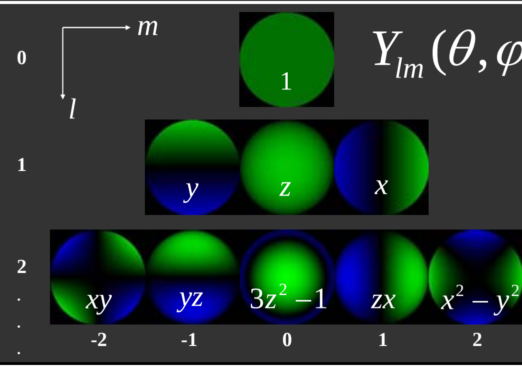
$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

Frequency: product

R. Ramamoorthi and P. Hanrahan "Analysis of Planar Light Fields from Homogeneous Convex Curved Surfaces under Distant Illumination" SPIE Photonics West 2001: Human Vision and Electronic Imaging VI pp 195-208

Spherical Harmonics

$$Y_{lm}(\theta,\varphi)$$



Spherical Harmonic Analysis

2D:

$$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \quad \rho(\theta_i,\theta_o) \quad d\theta_i$$

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

3D:

$$B(\alpha,\beta,\theta_o,\varphi_o) = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} L(R_{\alpha,\beta}[\theta_i,\varphi_i])\rho(\theta_i,\varphi_i,\theta_o,\varphi_o)\,d\theta_i d\varphi_i$$

$$B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq}$$

Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Filter is Delta function : Output = Signal

Mirror BRDF : Image = Lighting

[Miller and Hoffman 84]

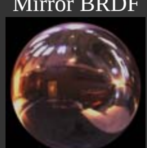


Image courtesy Paul Debevec

Insights: Signal Processing

Signal processing framework for reflection

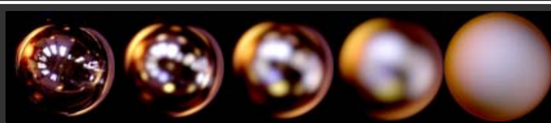
- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Signal is Delta function : Output = Filter

Point Light Source : Images = BRDF

[Marschner et al. 00]

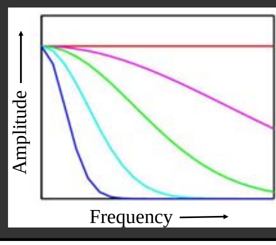
Phong, Microfacet Models



Mirror

Roughness →

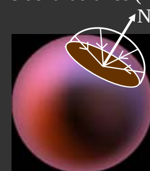
Illumination estimation
ill-posed for rough surfaces



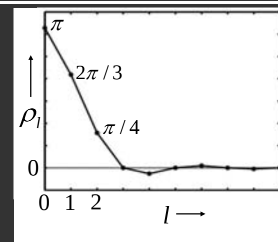
Analytic formulae in R. Ramamoorthi and P. Hanrahan
"A Signal-Processing Framework for Inverse Rendering"
SIGGRAPH 2001 pp 117-128

Lambertian

Incident radiance (mirror sphere)



Irradiance (Lambertian)



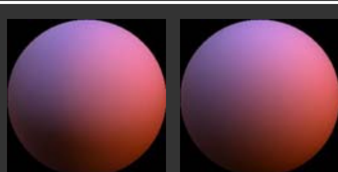
$$A_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)}\left[\frac{l!}{2^l\left(\frac{l}{2}!\right)^2}\right] \quad l \text{ even}$$

R. Ramamoorthi and P. Hanrahan "On the Relationship between Radiance and Irradiance: Determining the Illumination from Images of a Convex Lambertian Object" Journal of the Optical Society of America A 18(10) Oct 2001 pp 2448-2459

R. Basri and D. Jacobs "Lambertian Reflectance and Linear Subspaces" ICCV 2001 pp 383-390

9 Parameter Approximation

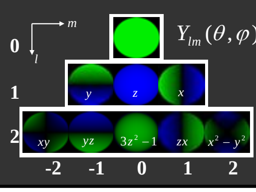
Exact image



Order 2
9 terms

RMS Error = 1%

For any illumination, average error < 3% [Basri Jacobs 01]



Ramamoorthi and Hanrahan 01b

Convolution for general materials

$$B(\vec{N},\vec{V}) = \int_{\Omega} L\left(R(\vec{N})\,\vec{l}\right)\rho\left(\vec{l},\vec{V}\right)dl$$

$$B = L \otimes \rho$$

Spherical harmonic analysis ↓

$$B_{ij} = L_i \rho_{ij}$$

Spatial: integral ↓ Frequency: product

Ramamoorthi and Hanrahan 01

Real-Time Rendering

Motivation: Interactive rendering with natural illumination and realistic, measured materials

Ramamoorthi and Hanrahan 02

Normal Map Filtering and Rendering

Han, Sun, Ramamoorthi, Grinspun 07

More Frequency Analysis

- Many other papers
  - Shadows: Soler and Sillion 98, Ramamoorthi et al. 04
  - Gradients: Ward, Heckbert, Arvo, Igehy, Holzschuch, Chen, Ramamoorthi, …
  - Wavelets: Gortler et al. 93, …, Ng et al. 03
- Full frequency analysis of light transport
  - Durand et al. 05
  - Space and Angle, generalizes previous work
- Many recent papers in computational imaging [Levin et al. 08, 09]