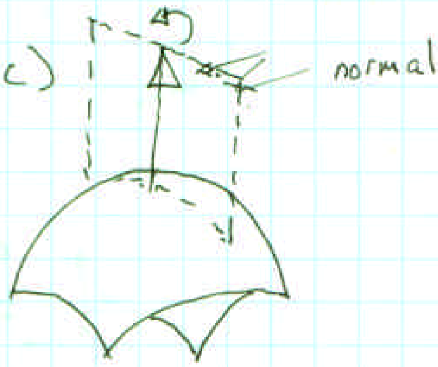
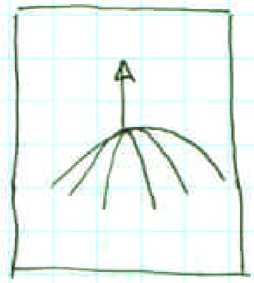
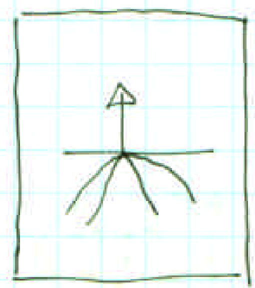
Qualitatively, there are three types of surface locally.

① (ELLIPTIC)

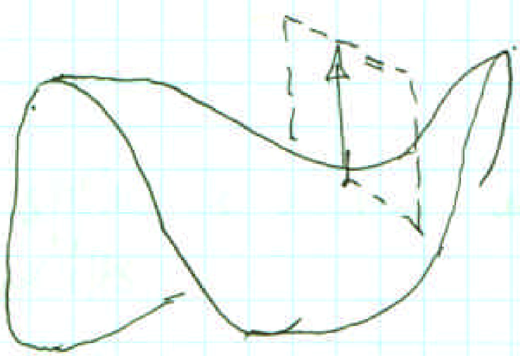


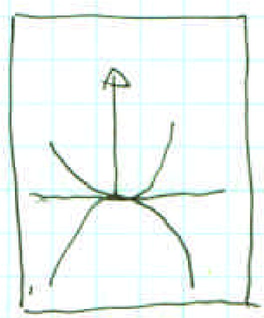


② PARABOLIC



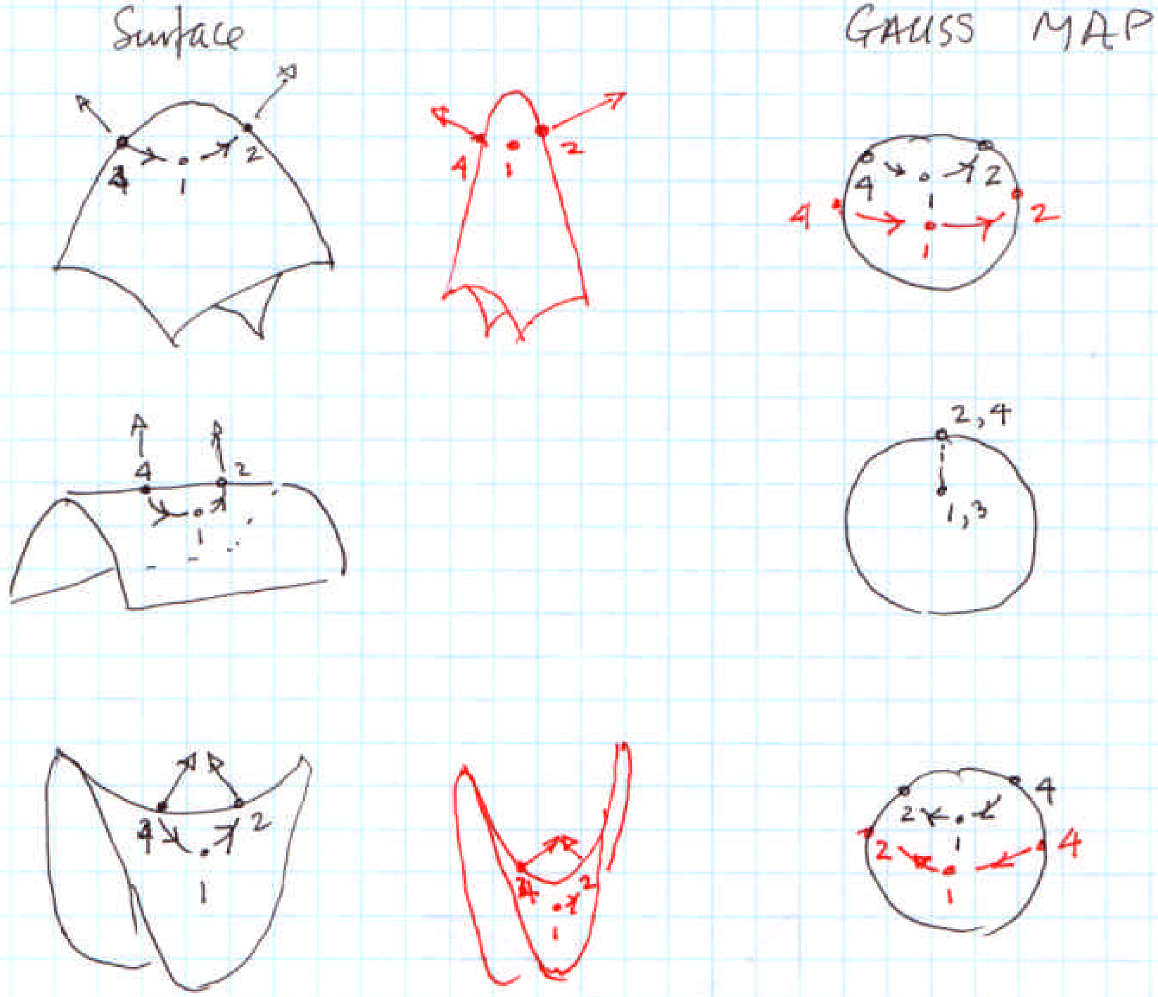
③ HYPERBOLIC





<u>Distinction</u>: drawing on right consists of sections, obtained by slicing surface with plane through pt. In ELLIPTIC, slices hang only down (or only up); in PARABOLIC, one slice is "horizontal"; in HYPERBOLIC, up and down.

② look more closely at the normal



GAUSS map applied to a small loop on the surface yields interesting results

- $\frac{\Delta A_m}{\Delta A_s}$ is bigger for highly curved surfs
- Elliptic doesn't change sign, Hyp does
- Parabolic: $\frac{\Delta A_m}{\Delta A_s} = 0$

③ GAUSSIAN CURVATURE

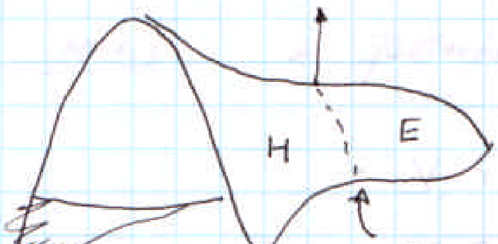
$$\boxed{K = \lim_{\Delta s \to 0} \frac{\Delta A_m}{\Delta A_s}}$$

THEOREMA EGREGIUM

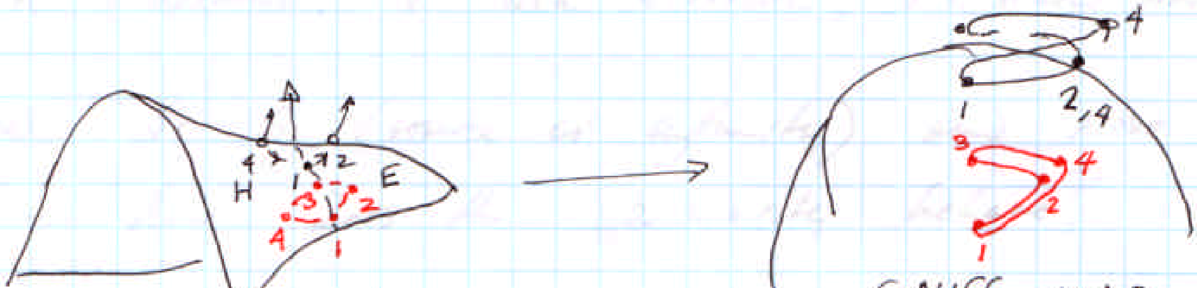
- Local isometry does not change GAUSSIAN curvature
- $\longrightarrow$ Can interpret curvature in terms of extra / missing area.

There is more to curvature – we'll return.

④ TWO APPLICATIONS.

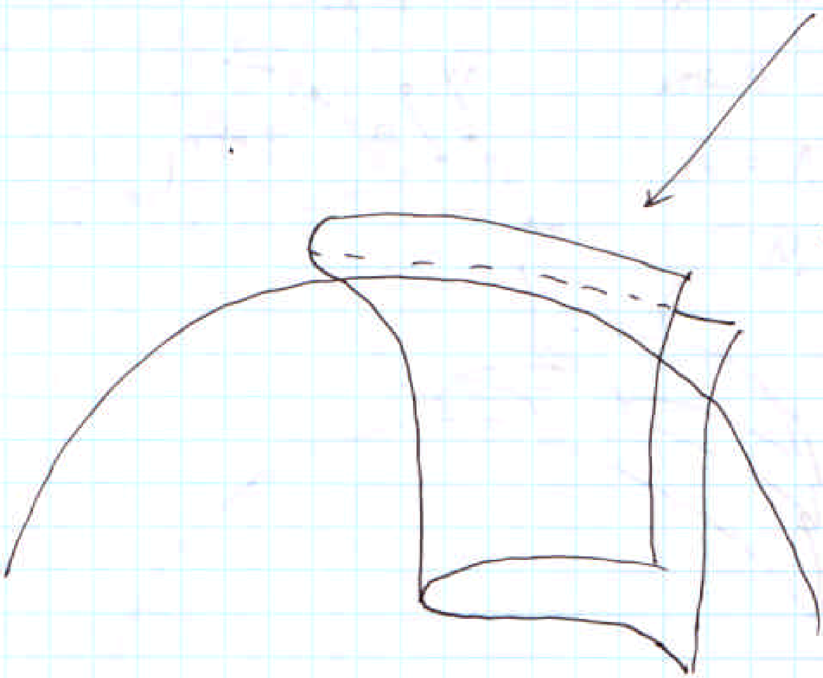


P — PARABOLIC LINE



GAUSS MAP.

join up these loops



The Gauss map <u>FOLDS</u> at parabolic lines

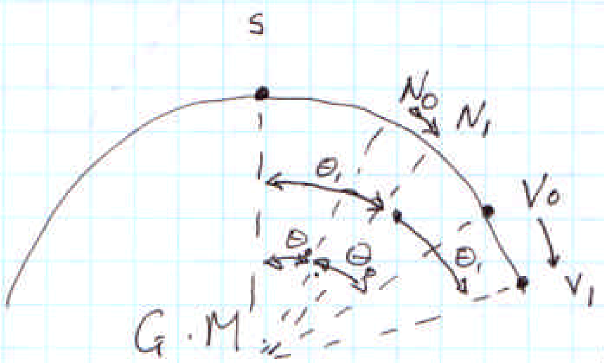
⑥ ~~What~~ ? The behaviour of specularities

recall: a specularity will occur when

- V, S, N are coplanar
- $V \cdot S = S \cdot N$

(N = normal; V = view direction; S = source direction)

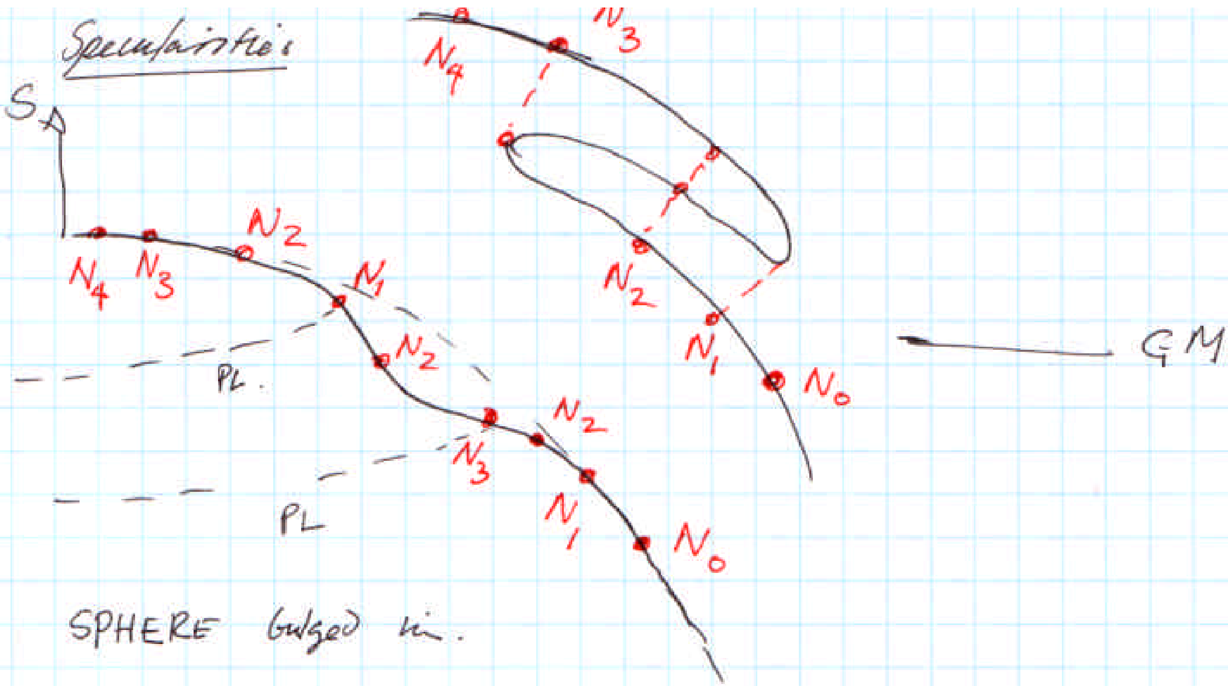
now, fix S (source at infinity) and move V
— how does the specularity behave?



- When V goes from V_0 to V_1 specularity moves from points with $N = N_0$ to $N = N_1$.

$\therefore$ 1 spec $\longrightarrow$ 3 !
or 3 $\longrightarrow$ 1 !

⑤ Specularities



SPHERE bulged in.

We will see more of specularities later.

GLOBAL RESULTS.

GAUSS-BONNET thm.

$$\int_S K \, dA = 4\pi(1-g)$$

6 ~~Where~~ ? The behaviour of specularities

recall: a specularity will occur when

- V, S, N are coplanar
- $V \cdot S = S \cdot N$

(N = normal; V = view direction; S = source direction)

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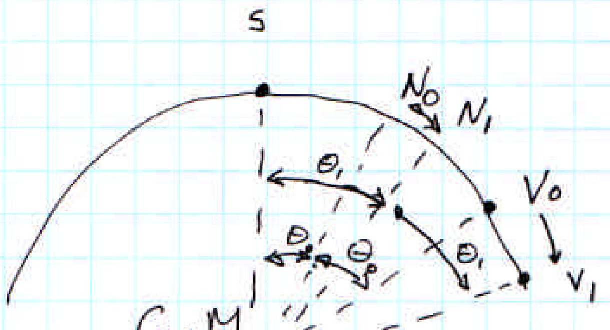
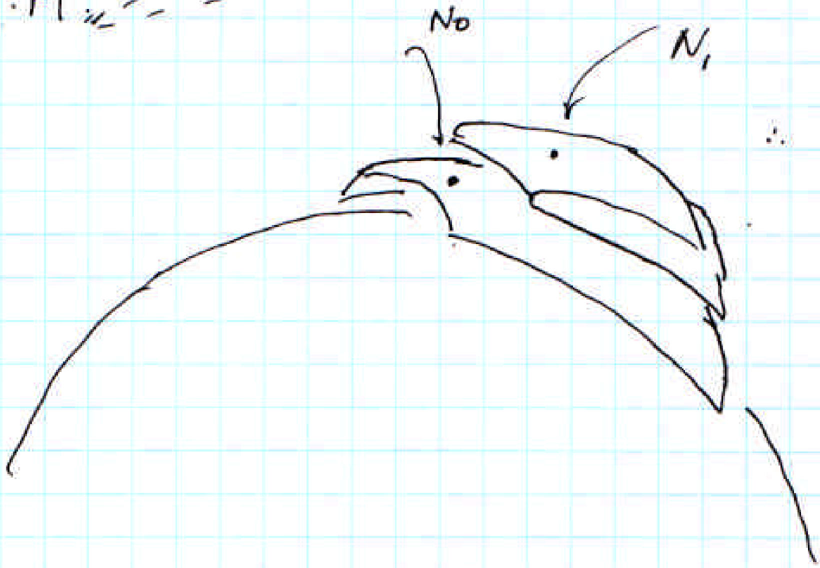


- When V goes from V_0 to V_1 specularity moves from points with $N = N_0$ to $N = N_1$.



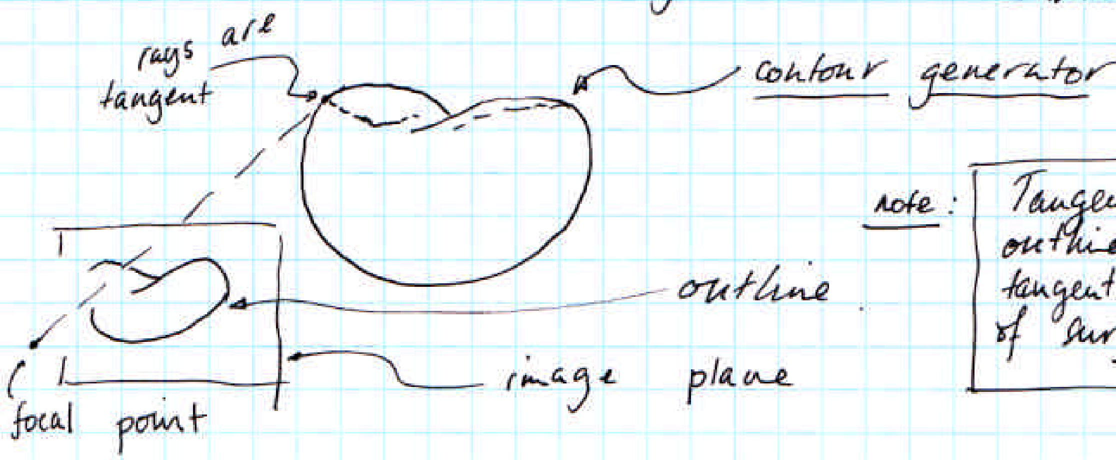
$\therefore$ 1 spec $\longrightarrow$ 3 !
or 3 $\longrightarrow$ 1 !

⑦

Contours, outlines, etc.

The points where a surface turns away from the eye are interesting

- hard to ray-trace
- sudden change in what is visible



note: Tangent to outline is tangent plane of surface

for geometric purposes, we think of the surface as translucent; c.g. moves w/ focal point

e.g. torus: