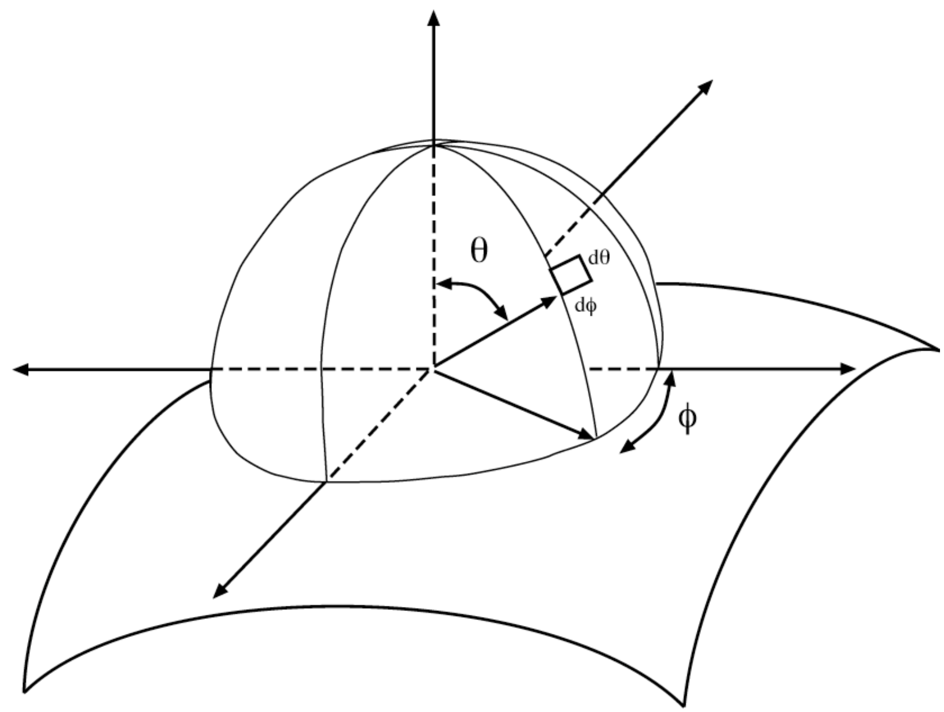
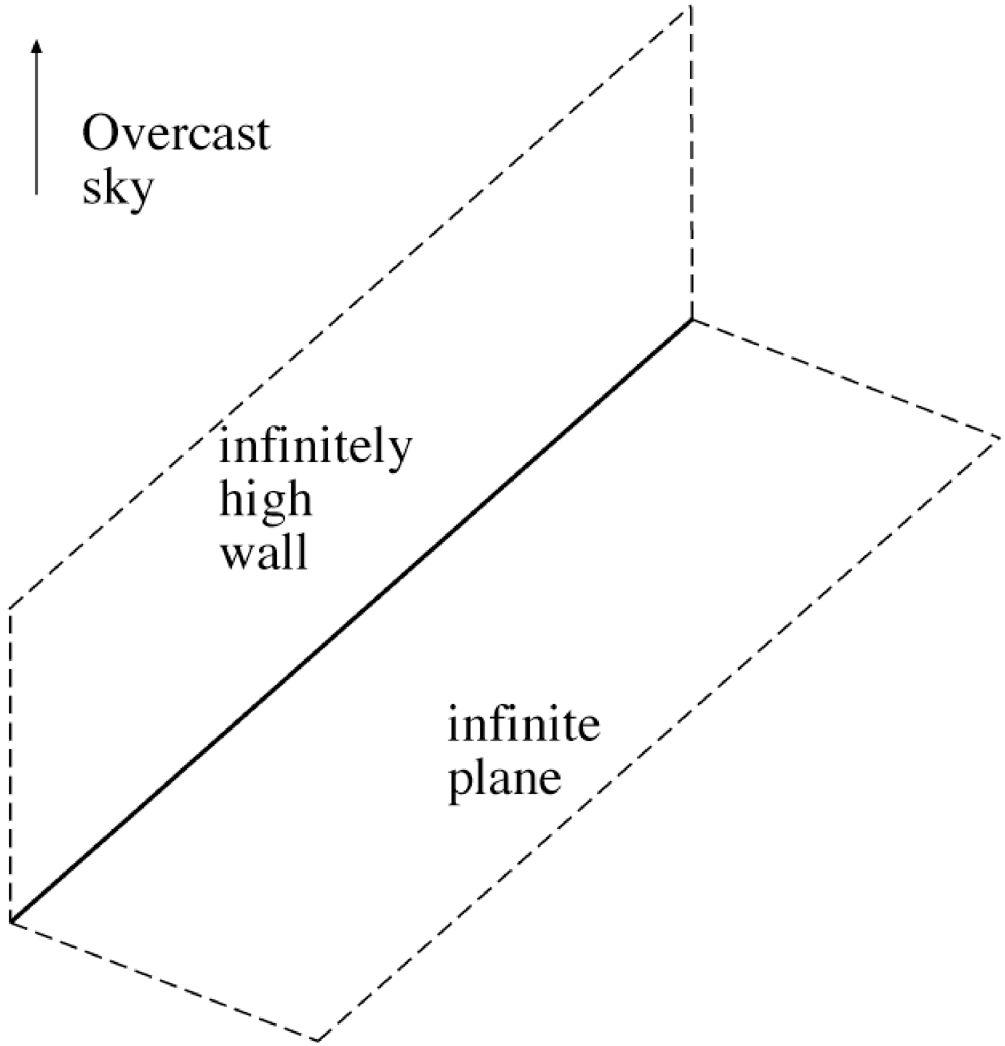
Radiometry

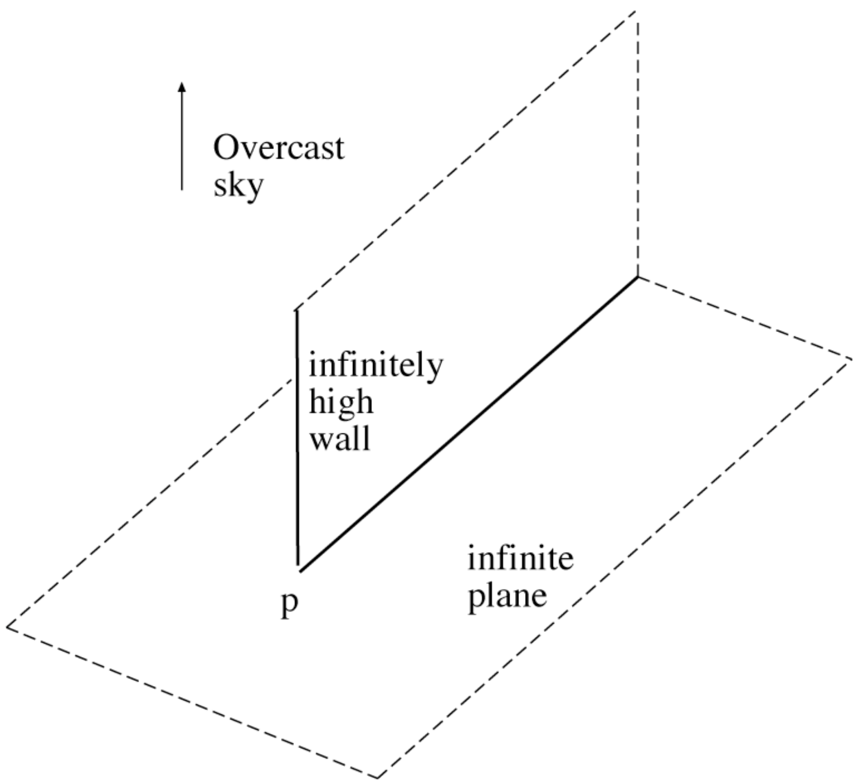
- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

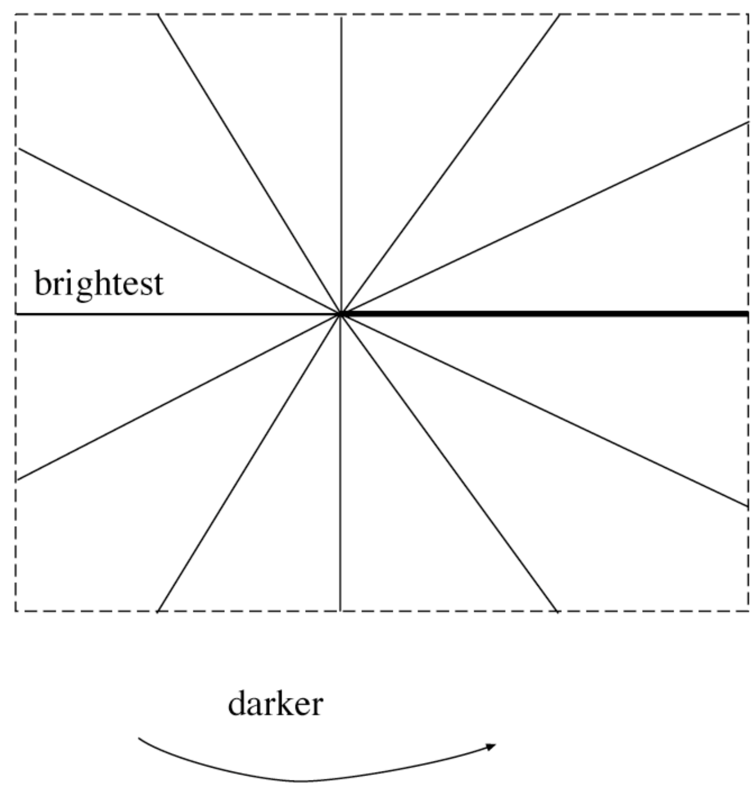


Lambert's wall



More complex wall





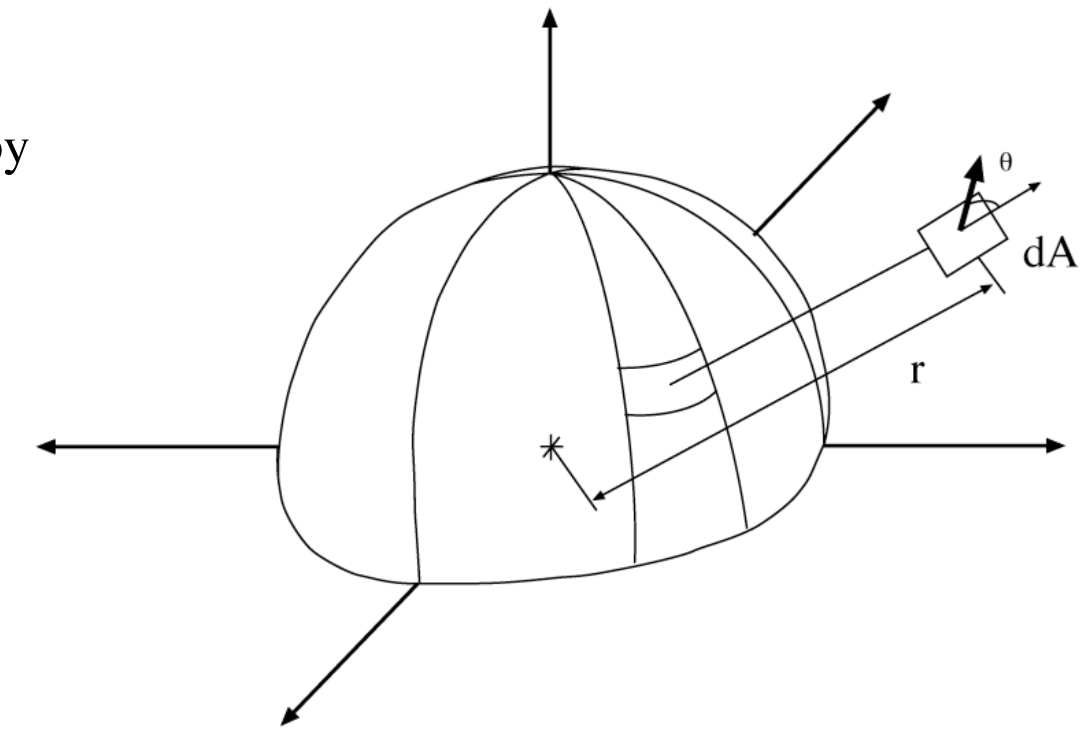
Solid Angle

- By analogy with angle (in radians)
- The solid angle subtended by patch area dA is given by

$$d\omega = \frac{dA\cos\theta}{r^2}$$

- Another useful expression:

$$d\omega = \sin\theta\left(d\theta\right)\left(d\phi\right)$$



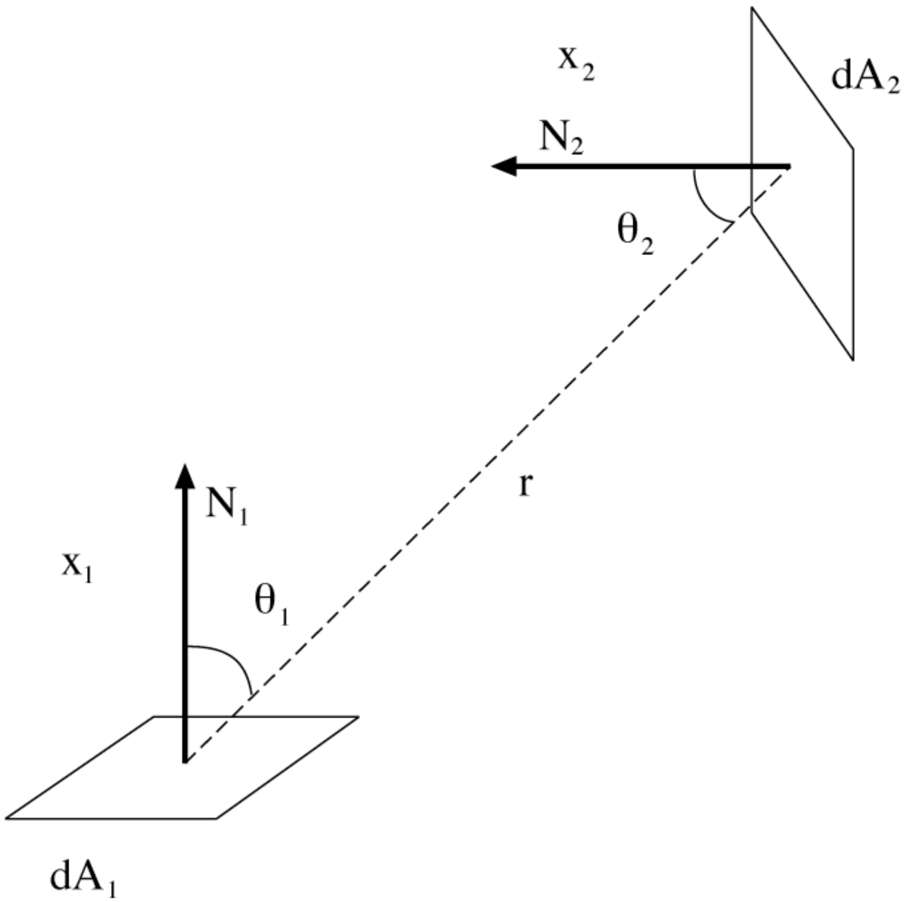
Radiance

- Measure the "amount of light" at a point, in a direction
- Property is:
  Radiant power per unit foreshortened area per unit solid angle
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

$$L\left(\underline{x},\square,\square\right)$$

- Crucial property:
  In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
  - hence the units

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1,\theta,\phi)(dA_1 \cos\theta_1)\left(\frac{dA_2 \cos\theta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\theta,\phi)(dA_2 \cos\theta_2)\left(\frac{dA_1 \cos\theta_1}{r^2}\right)$$

Irradiance

- How much light is arriving at a surface?
- Sensible unit is *Irradiance*
- Incident power per unit area *not foreshortened*
- This is a function of incoming angle.
- A surface experiencing radiance L(x,□,□) coming in from d□ experiences irradiance

$$L(\underline{x},\square,\square)\cos\square \, d\square$$

- Crucial property:
  Total power arriving at the surface is given by adding irradiance over all incoming angles --- this is why it's a natural unit
- Total power is

$$\square_{\square} L(\underline{x},\square,\square)\cos\square \sin\square \, d\square \, d\square$$

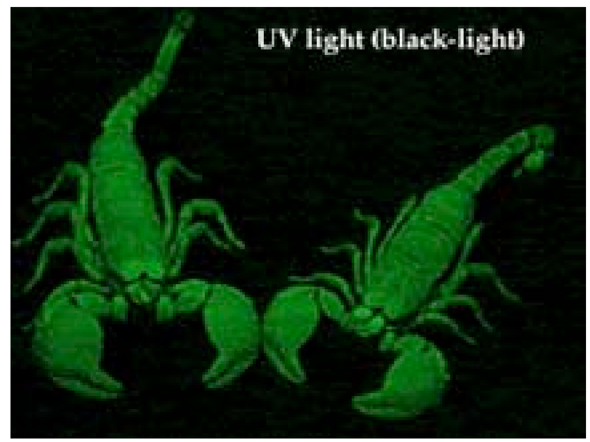
Surfaces and the BRDF

- Many effects when light strikes a surface -- could be:
  - absorbed
  - transmitted
  - reflected
  - scattered
- Assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point

- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- the ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}\left(\underline{x},\theta_o,\phi_o,\theta_i,\phi_i,\right) = \frac{L_o\left(\underline{x},\theta_o,\phi_o\right)}{L_i\left(\underline{x},\theta_i,\phi_i\right)\cos\theta_i d\omega}$$


White light (indoor)


UV light (black-light)

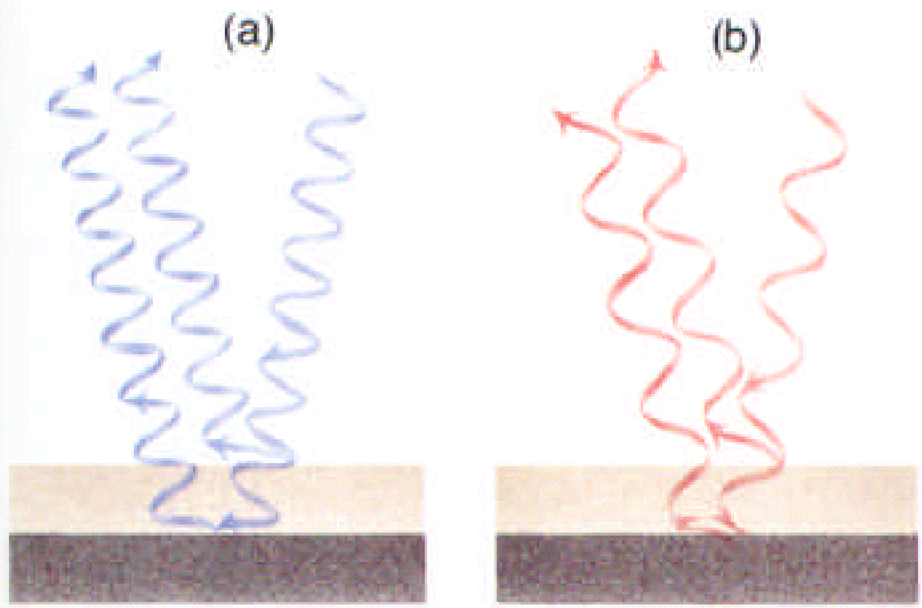
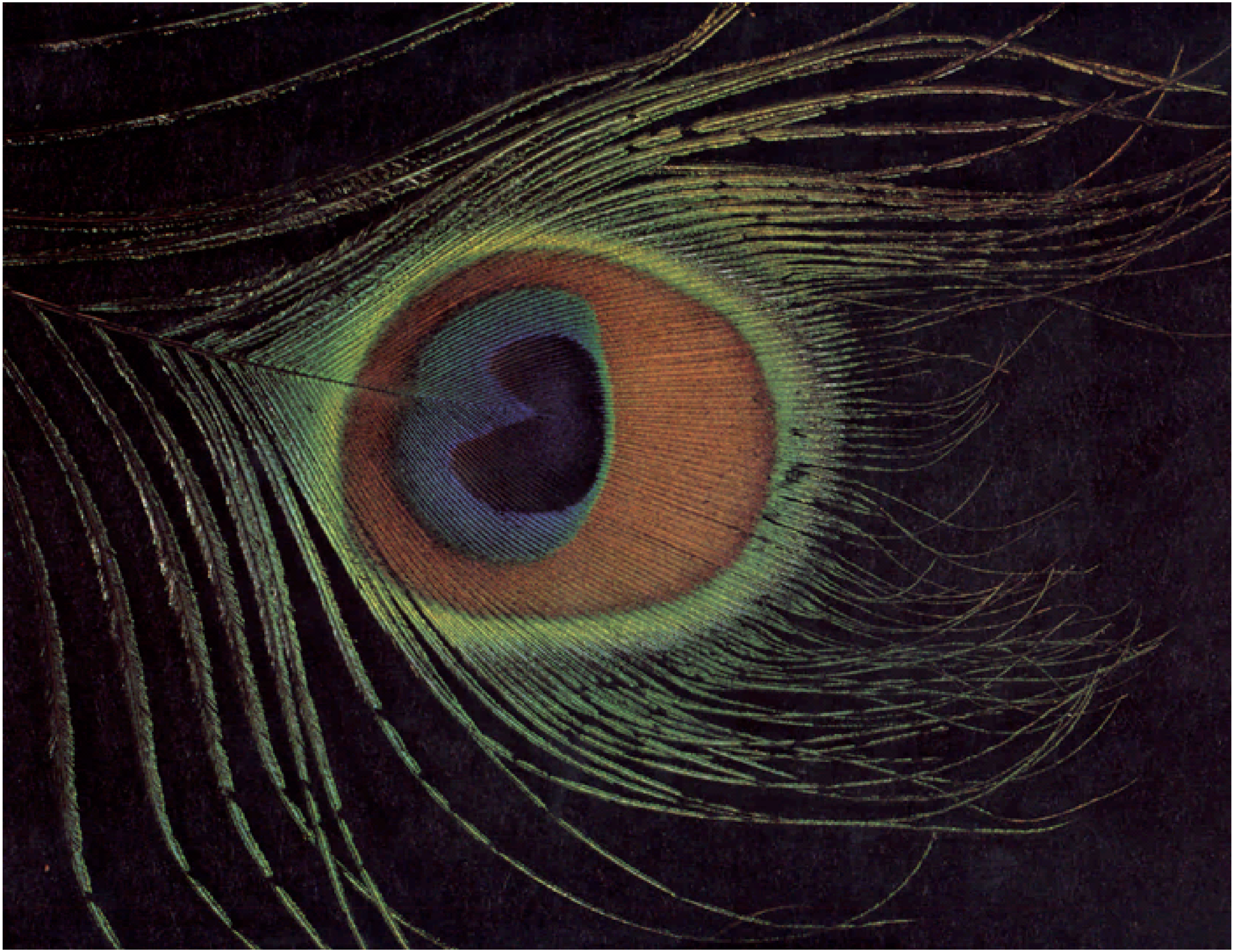
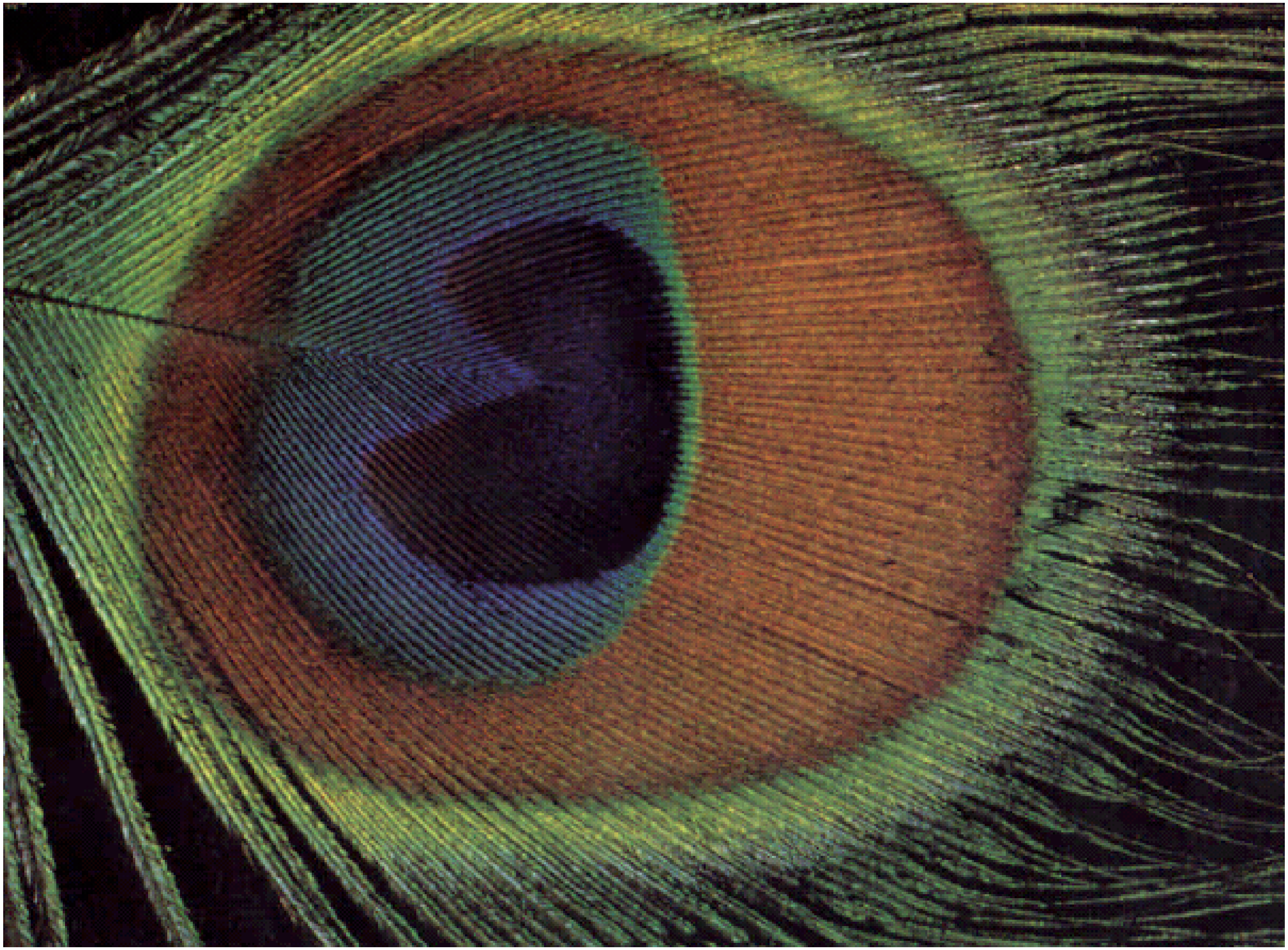
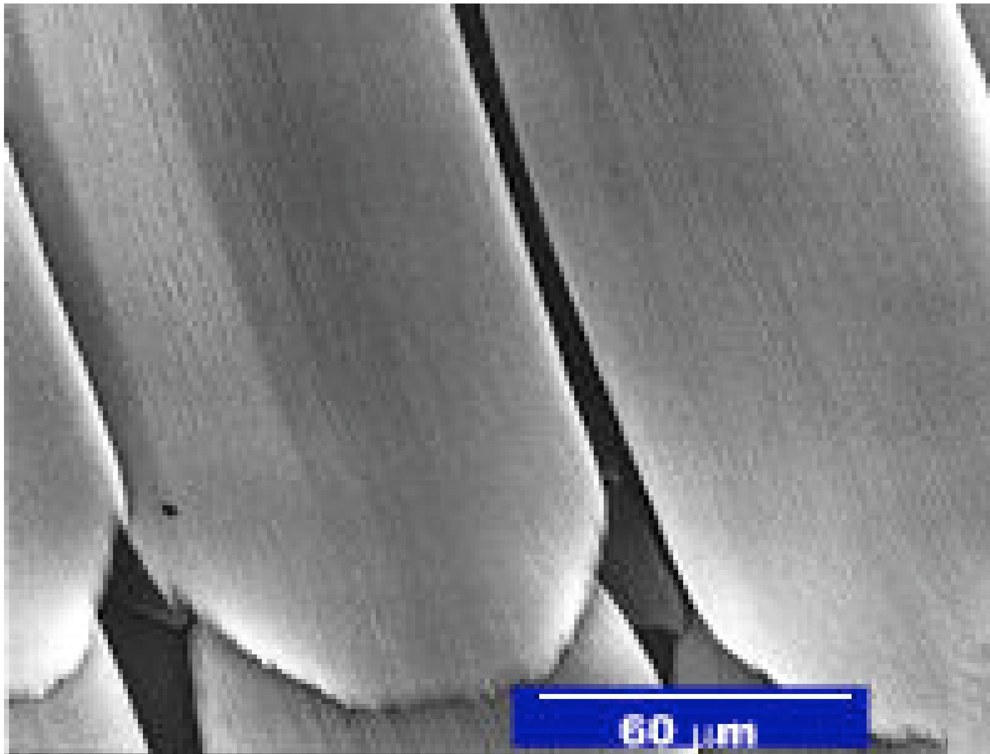
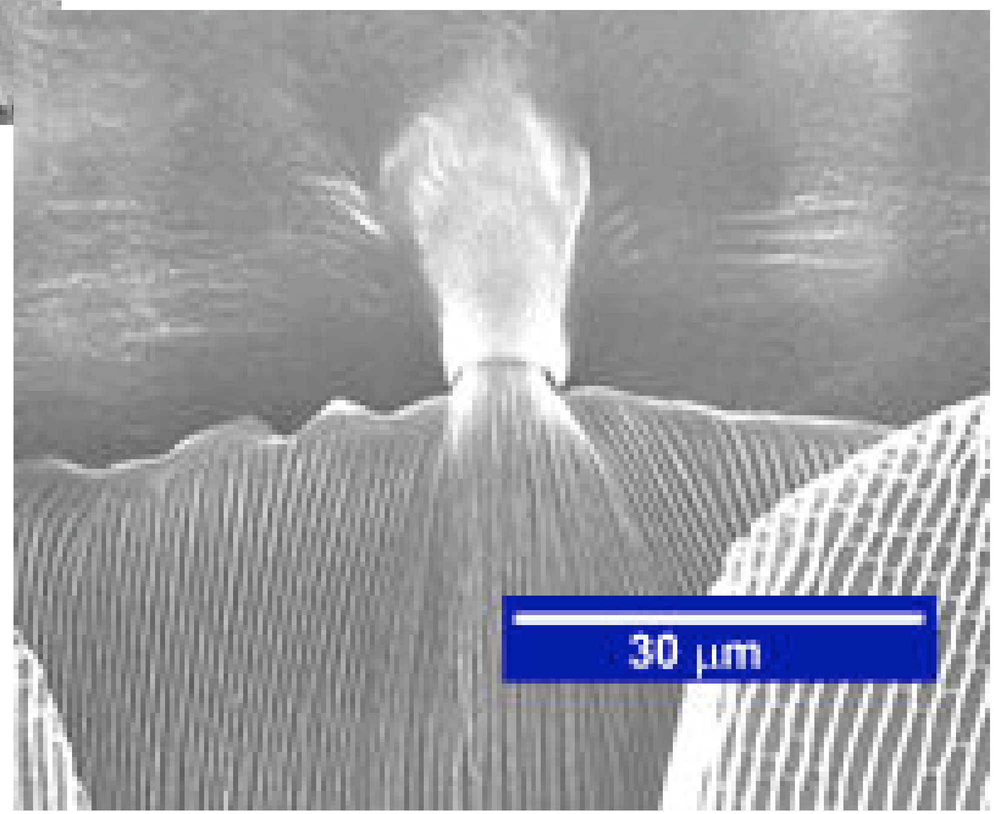


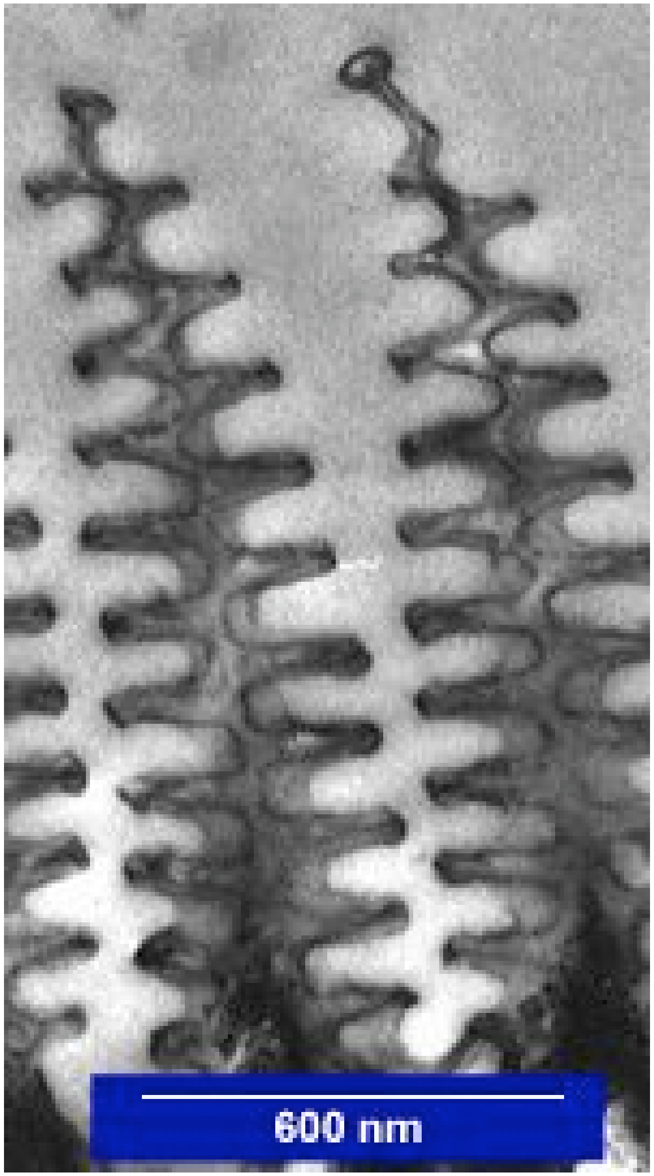
Fig. 1.22 Iridescence: when a light wave is partially reflected and partially transmitted at the surface of a thin layer of transparent material (e.g. a bubble), the two parts of the original wave may interfere with each other when the transmitted wave is reflected from a lower layer and re-emerges at the surface. In this case the blue waves are in phase and their colour is reinforced (a) but the red waves are out of phase and their colour is cancelled (b).






60 µm


30 µm


600 nm

BRDF

- Units: inverse steradians (sr^{-1})
- Symmetric in incoming and outgoing directions
- Radiance leaving in a particular direction:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{bd}\left(\underline{x},\theta_o,\phi_o,\theta_i,\phi_i,\right)L_i\left(\underline{x},\theta_i,\phi_i\right)\cos\theta_i\, d\omega_i$$

Suppressing Angles - Radiosity

- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm^{-2})

- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

$$B(\underline{x}) = \int_{\Omega} L_o(\underline{x}, \theta, \phi) \cos\theta \, d\omega$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

$$
\begin{aligned}
B(\underline{x}) &= \int_{\Omega} L_o(\underline{x},\theta,\phi)\cos\theta \, d\omega \\
&= L_o(\underline{x}) \int_{\Omega} \cos\theta \, d\omega \\
&= L_o(\underline{x}) \int_0^{\pi/2}\int_0^{2\pi} \cos\theta \sin\theta \, d\phi \, d\theta \\
&= \pi L_o(\underline{x})
\end{aligned}
$$

Directional hemispheric reflectance

- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure and very unstable
  - for many surfaces, light leaving the surface is largely independent of exit angle (surface roughness is one source of this property)
- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - unitless, range is 0-1

$$\rho_{dh}(\theta_i,\phi_i) = \frac{\int_{\Omega} L_o(\underline{x},\theta_o,\phi_o)\cos\theta_o \, d\omega_o}{L_i(\underline{x},\theta_i,\phi_i)\cos\theta_i \, d\omega_i}$$

$$= \int_{\Omega} \rho_{bd}(\underline{x},\theta_o,\phi_o,\theta_i,\phi_i)\cos\theta_o \, d\omega_o$$

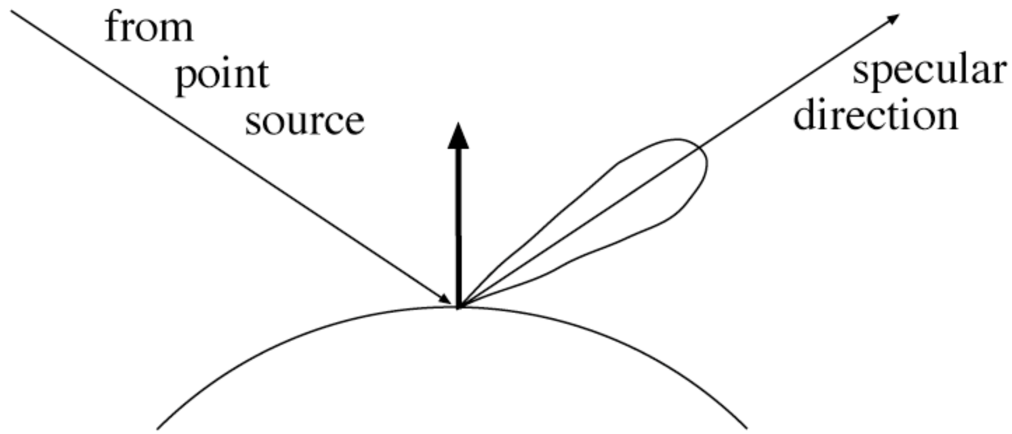
Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of direction
  - cotton cloth, carpets, matte paper, matte paints, etc.
  - radiance leaving the surface is independent of angle
  - Lambertian surfaces (same Lambert) or ideal diffuse surfaces
  - Use radiosity as a unit to describe light leaving the surface
  - DHR is often called diffuse reflectance, or albedo
- for a Lambertian surface, BRDF is independent of angle, too.
- Useful fact:

$$\rho_{brdf} = \frac{\rho_d}{\pi}$$

Specular surfaces

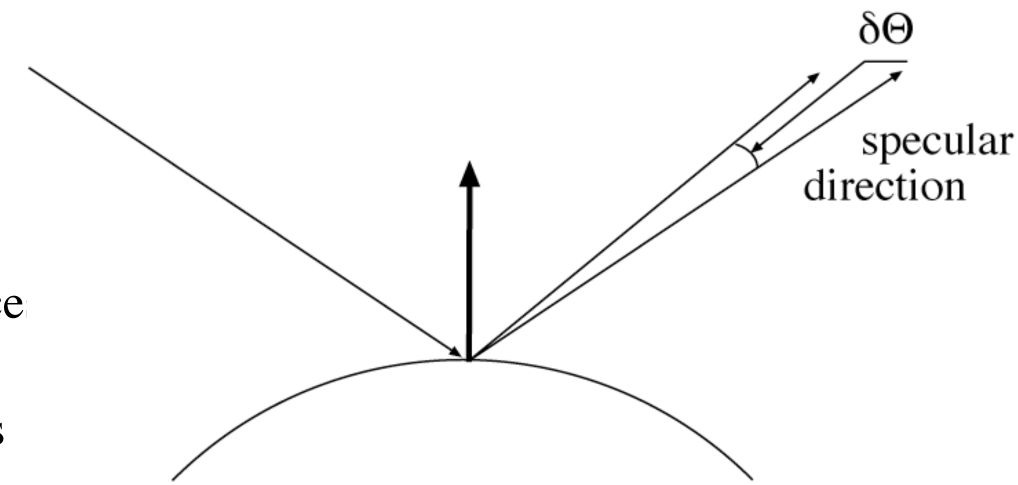
- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions



Phong's model

- There are very few cases where the exact shape of the specular lobe matters.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light source as "specularities"
  - very big -- faint specularities
- Phong's model
  - reflected energy falls off with

$$\cos^n(\square\square)$$



Lambertian + specular

- Widespread model
  - all surfaces are Lambertian plus specular component
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces