Sources, shadows and shading

- But how bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- similar to radiosity: a source can have both
  - radiosity, because it reflects
  - exitance, because it emits

- General idea:

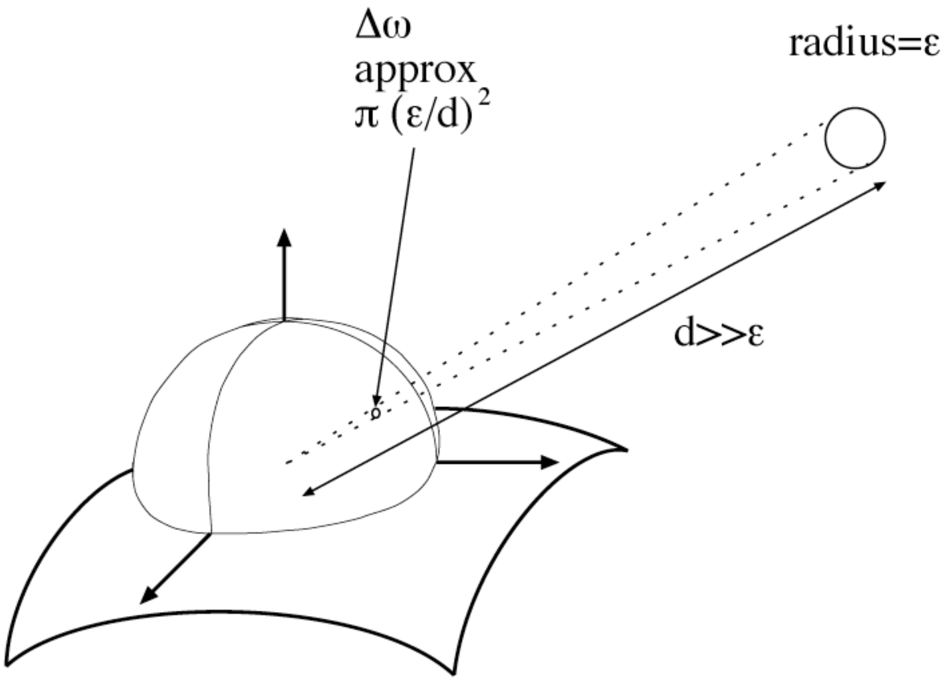
$$B(x) = E(x) + \int_{\Omega} \left(\begin{array}{c} \text{radiosity due to} \\ \text{incoming radiance} \end{array} \right) d\omega$$

- But what aspects of the incoming radiance will we model?

Shading models

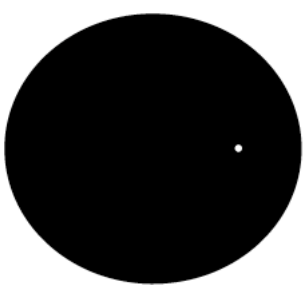
- Local shading model
  - Surface has radiosity due only to sources visible at each point
  - Advantages:
    - often easy to manipulate, expressions easy
    - supports quite simple theories of how shape information can be extracted from shading
- Global shading model
  - surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
  - Advantages:
    - usually very accurate

Radiosity due to point sources



- small, distant sphere radius ϵ and exitance E, which is far away subtends solid angle of about

$$\pi\left(\frac{\epsilon}{d}\right)^2$$



Constant radiance patch due to source

Radiosity due to a point source

- Radiosity is

$$
\begin{aligned}
B(x) &= \pi L_o(x) \\
&= \rho_d(x) \int_{\Omega} L_i(x,\omega)\cos\theta_i \, d\omega \\
&= \rho_d(x) \int_{D} L_i(x,\omega)\cos\theta_i \, d\omega \\
&\approx \rho_d(x)(\text{solid angle})(\text{Exitance term})\cos\theta_i \\
&= \frac{\rho_d(x)\cos\theta_i}{r(x)^2}(\text{Exitance term and some constants})
\end{aligned}
$$

Standard nearby point source model

$$\rho_d(x)\left(\frac{N(x)\bullet S(x)}{r(x)^2}\right)$$

- N is the surface normal
- rho is diffuse albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine

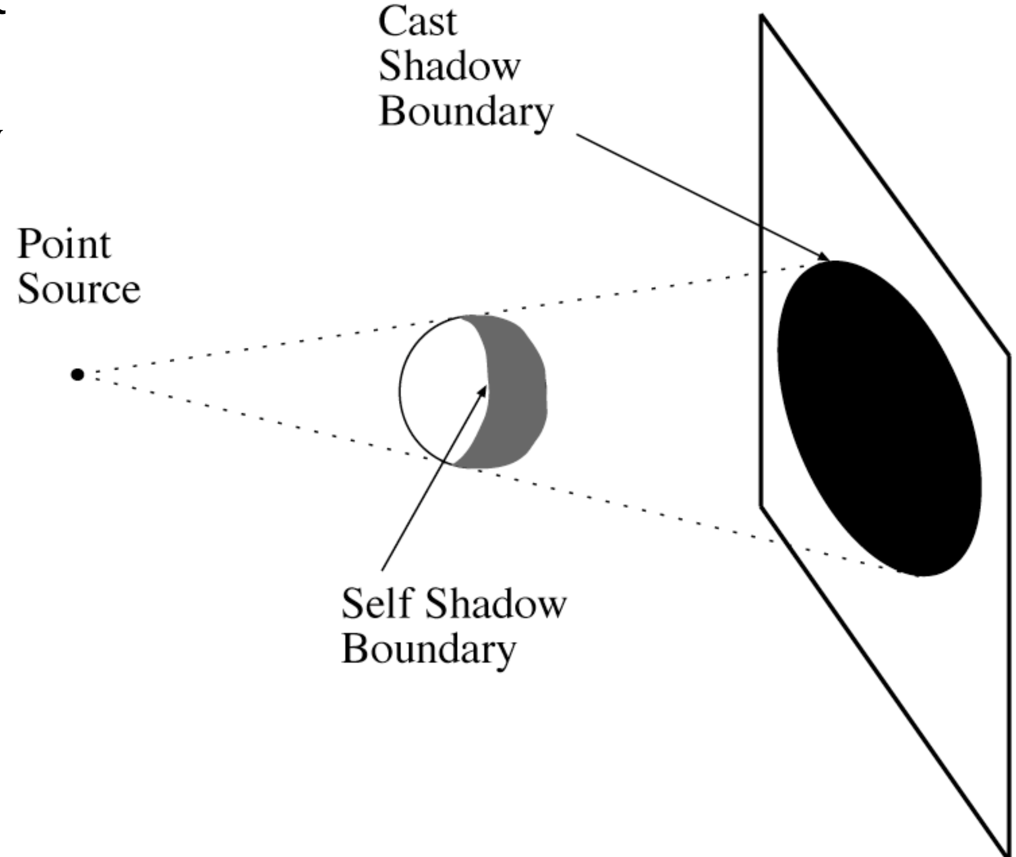
Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn't for any reasonable binding of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:

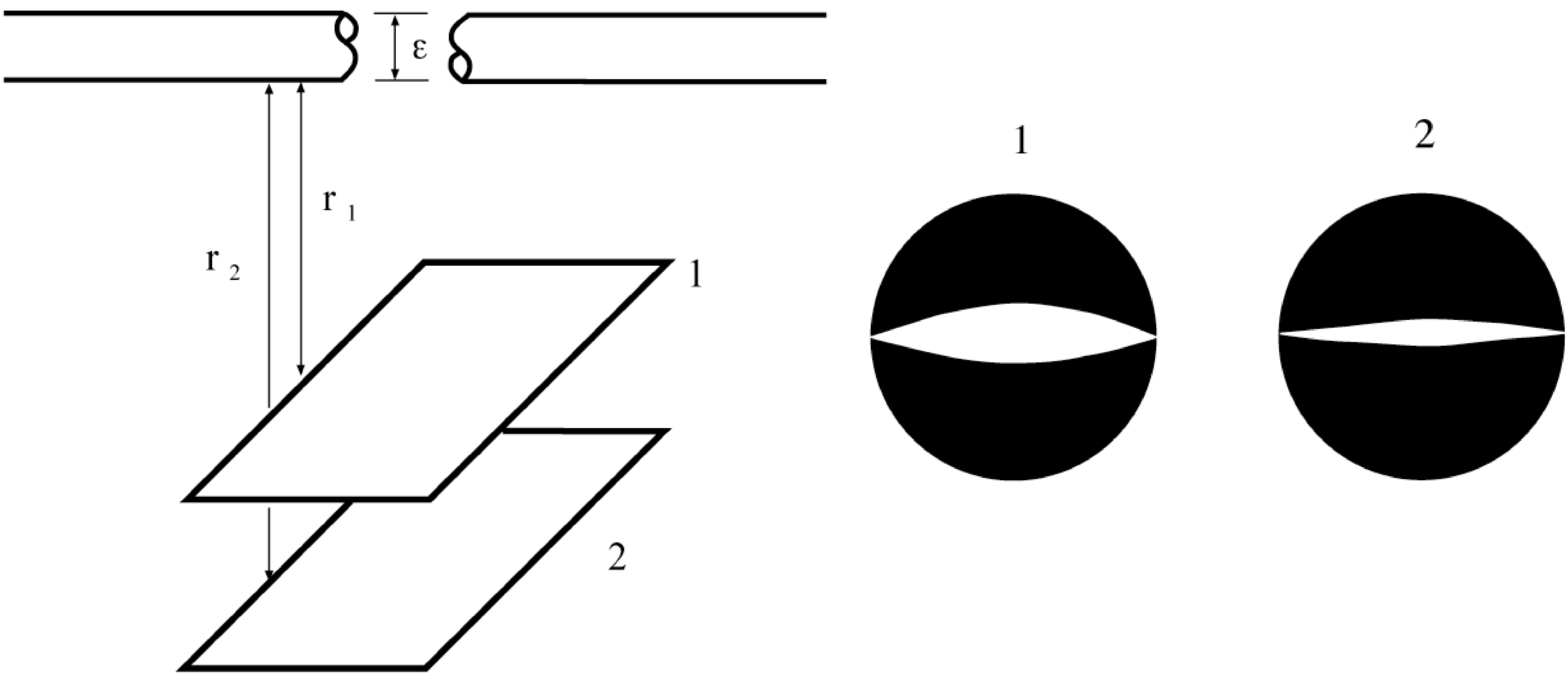
$$\rho_d(x)\left(N(x)\bullet S_d(x)\right)$$

Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple



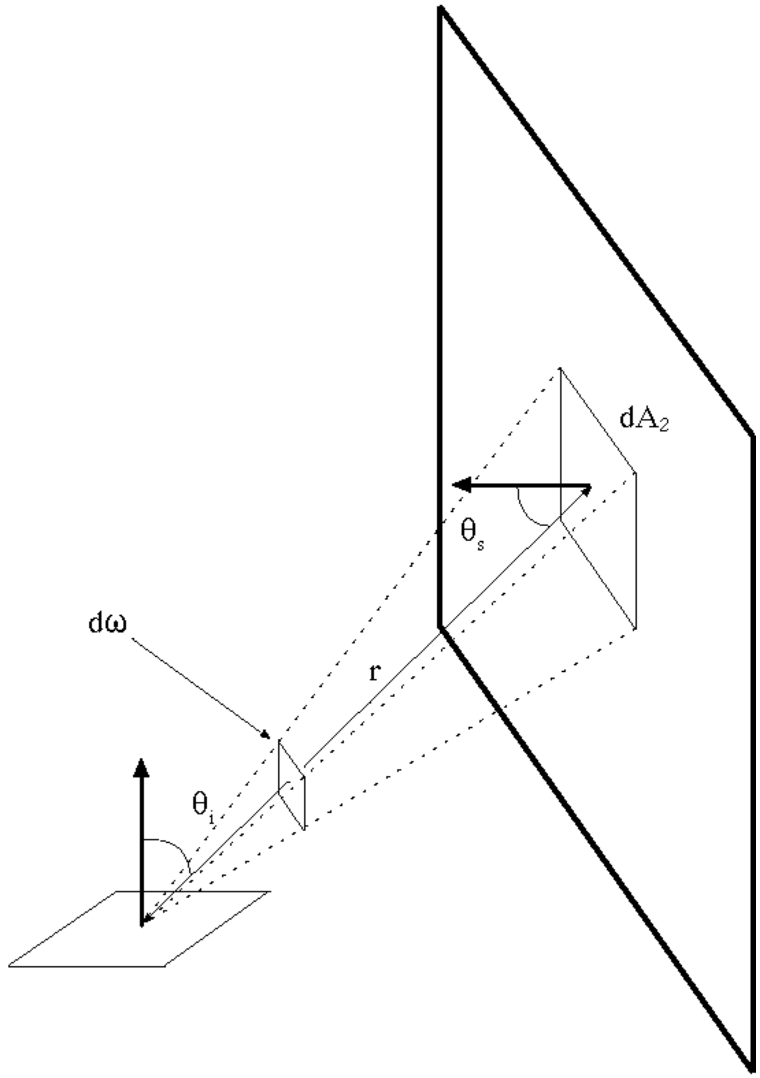
Line sources



radiosity due to line source varies with inverse distance, if the source is long enough

Area sources

- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
  - change variables and add up over the source



Radiosity due to an area source

- rho is albedo
- E is exitance
- r(x, u) is distance between points
- u is a coordinate on the source

$$B(x) = \rho_d(x) \int_{\Omega} L_i(x, u \to x) \cos\theta_i \, d\omega$$

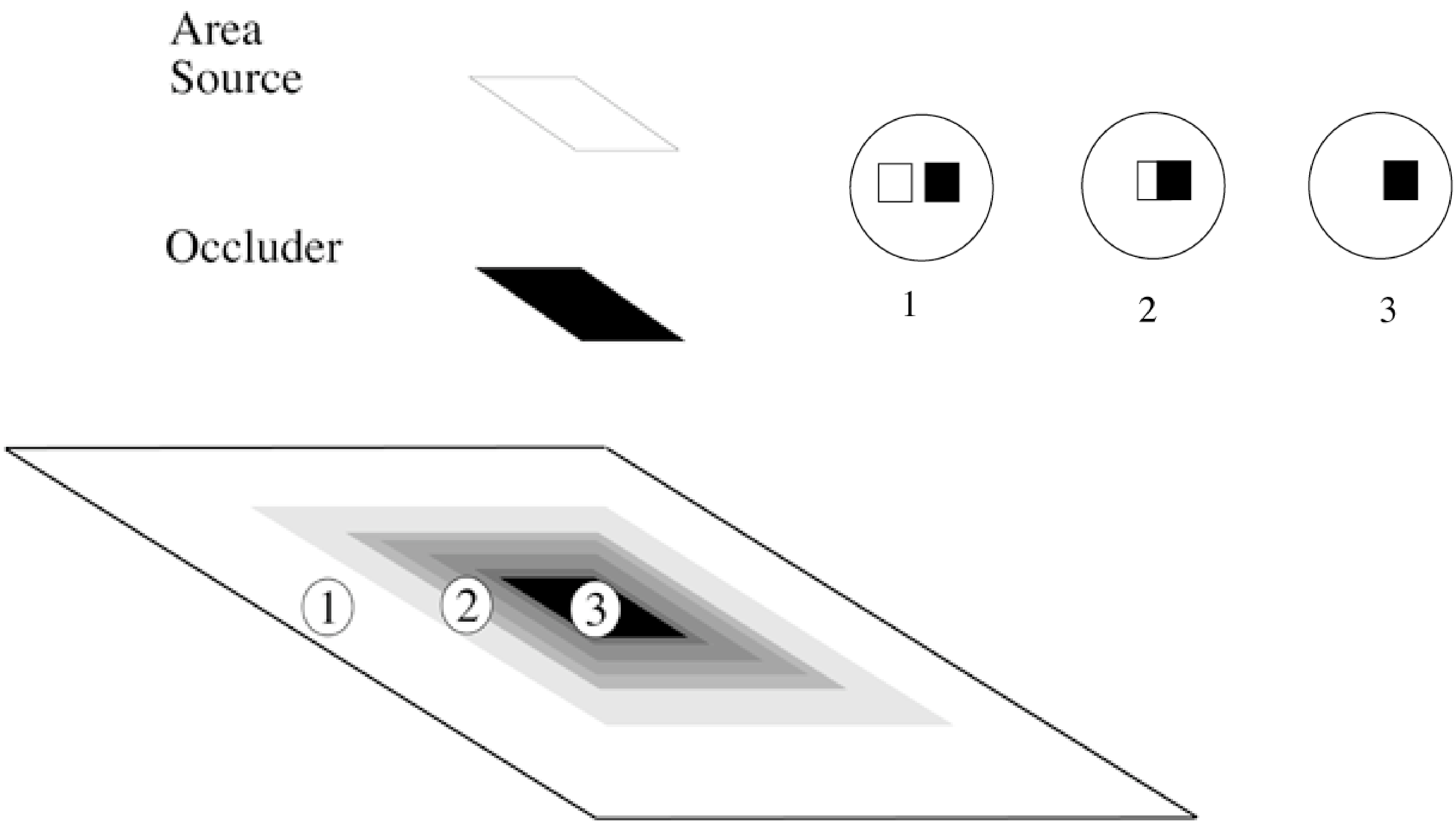
$$= \rho_d(x) \int_{\Omega} L_e(x, u \to x) \cos\theta_i \, d\omega$$

$$= \rho_d(x) \int_{\Omega} \left(\frac{E(u)}{\pi} \right) \cos\theta_i \, d\omega$$

$$= \rho_d(x) \int_{source} \left(\frac{E(u)}{\pi} \right) \cos\theta_i \left(\cos\theta_s \frac{dA_u}{r(x,u)^2} \right)$$

$$= \rho_d(x) \int_{source} E(u) \frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} dA_u$$

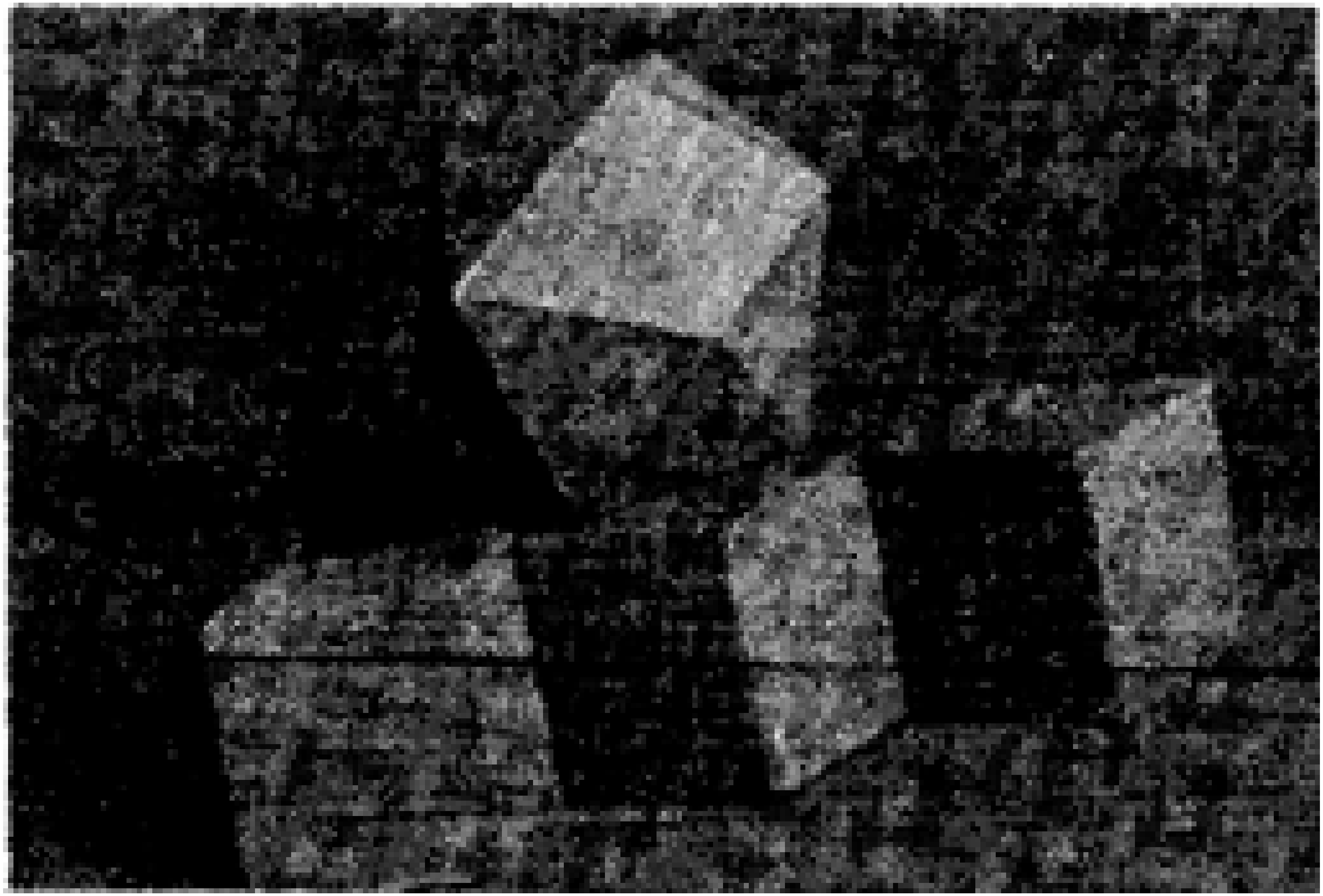
Area Source Shadows

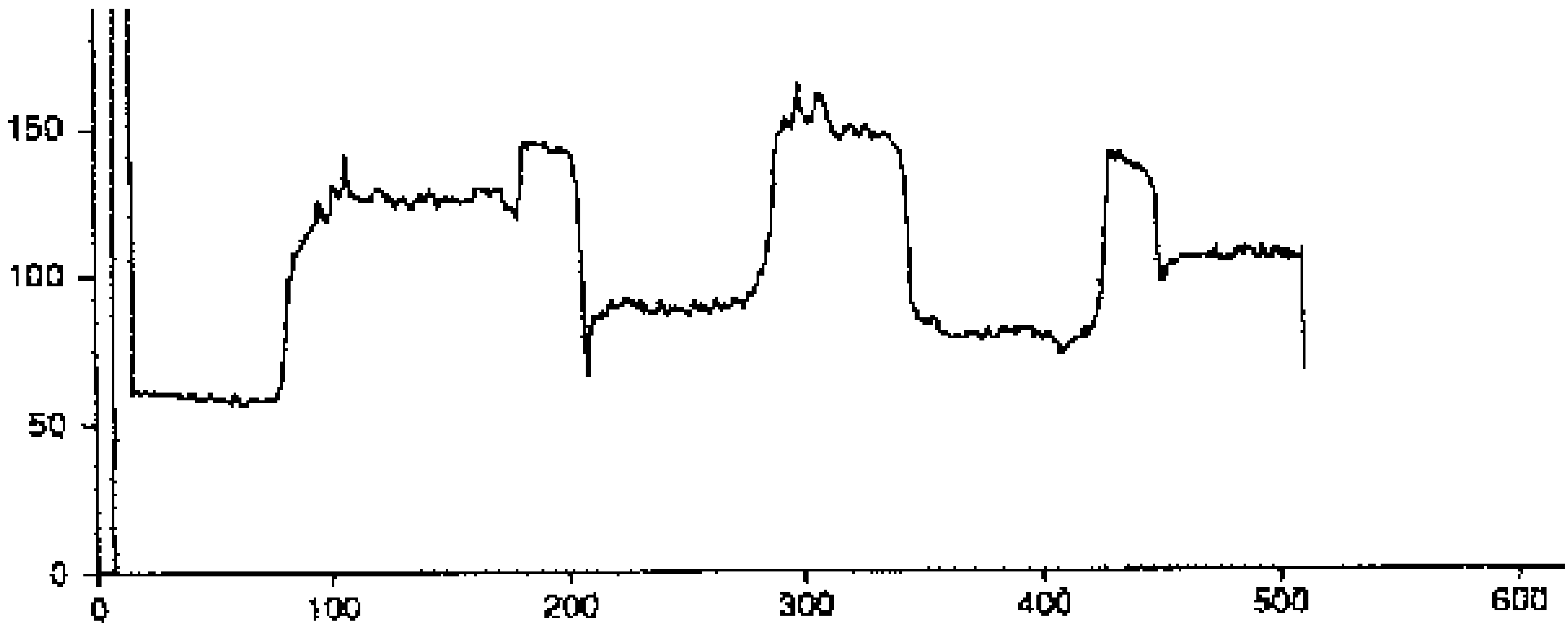


Curious Experimental Fact

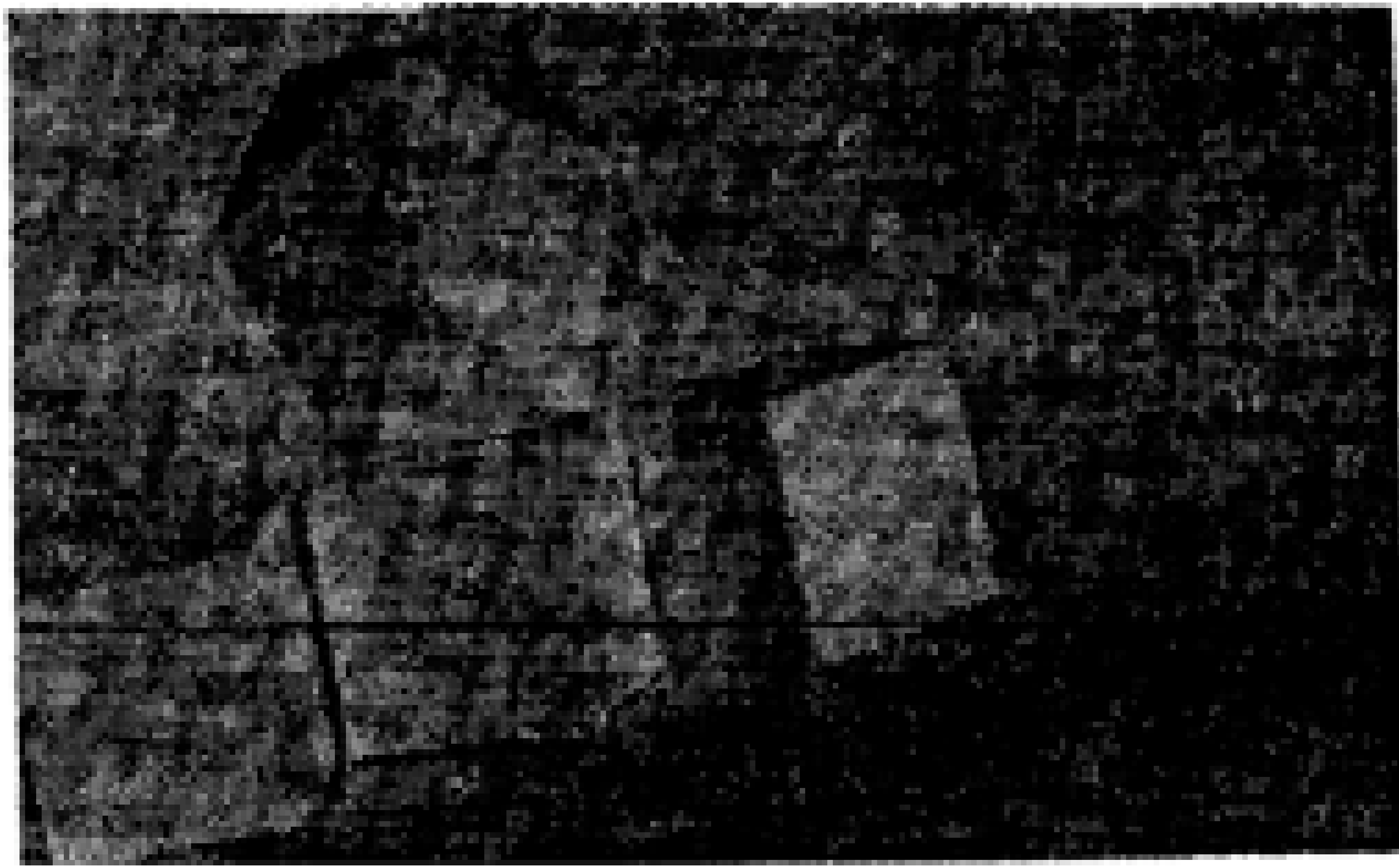
- Prepare two rooms, one with white walls and white objects, one with black walls and black objects
- Illuminate the black room with bright light, the white room with dim light
- People can tell which is which (due to Gilchrist)

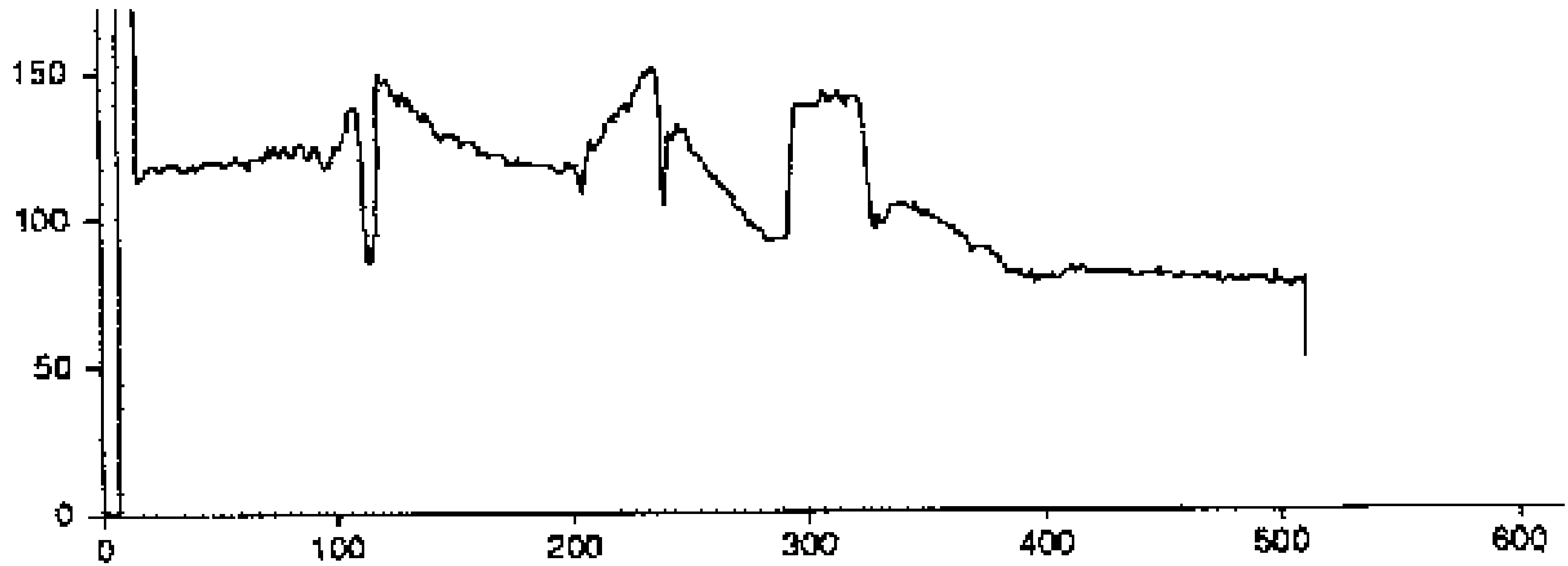
- Why? (a local shading model predicts they can't).





150
100
50
0
0
100
200
300
400
500
600

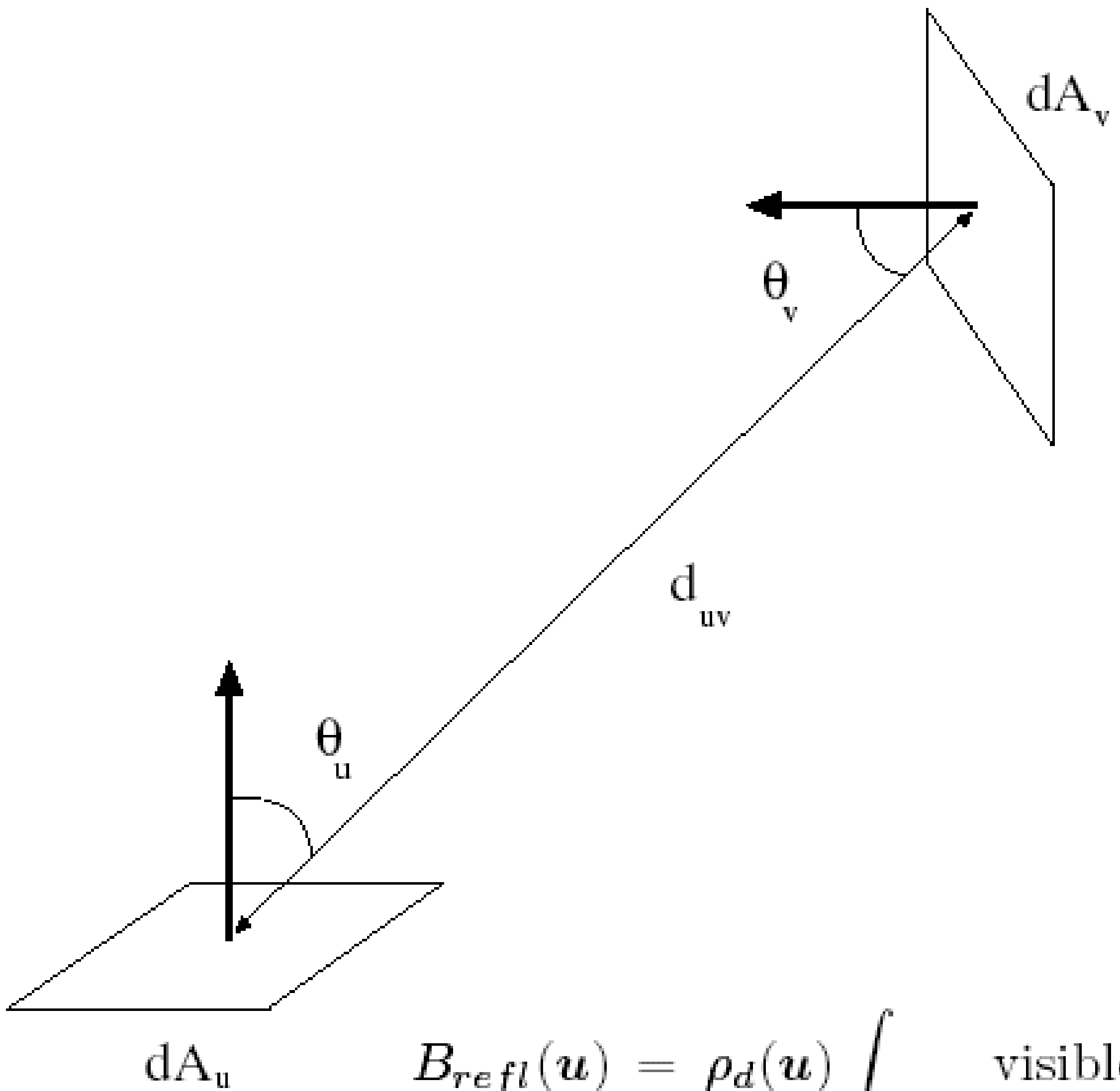




150
100
50
0
0
100
200
300
400
500
600

Interreflections

- Issue:
  - local shading model is a poor description of physical processes that give rise to images
    - because surfaces reflect light onto one another
  - This is a major nuisance; the distribution of light (in principle) depends on the configuration of every radiator; big distant ones are as important as small nearby ones (solid angle)
  - The effects are easy to model
  - It appears to be hard to extract information from these models

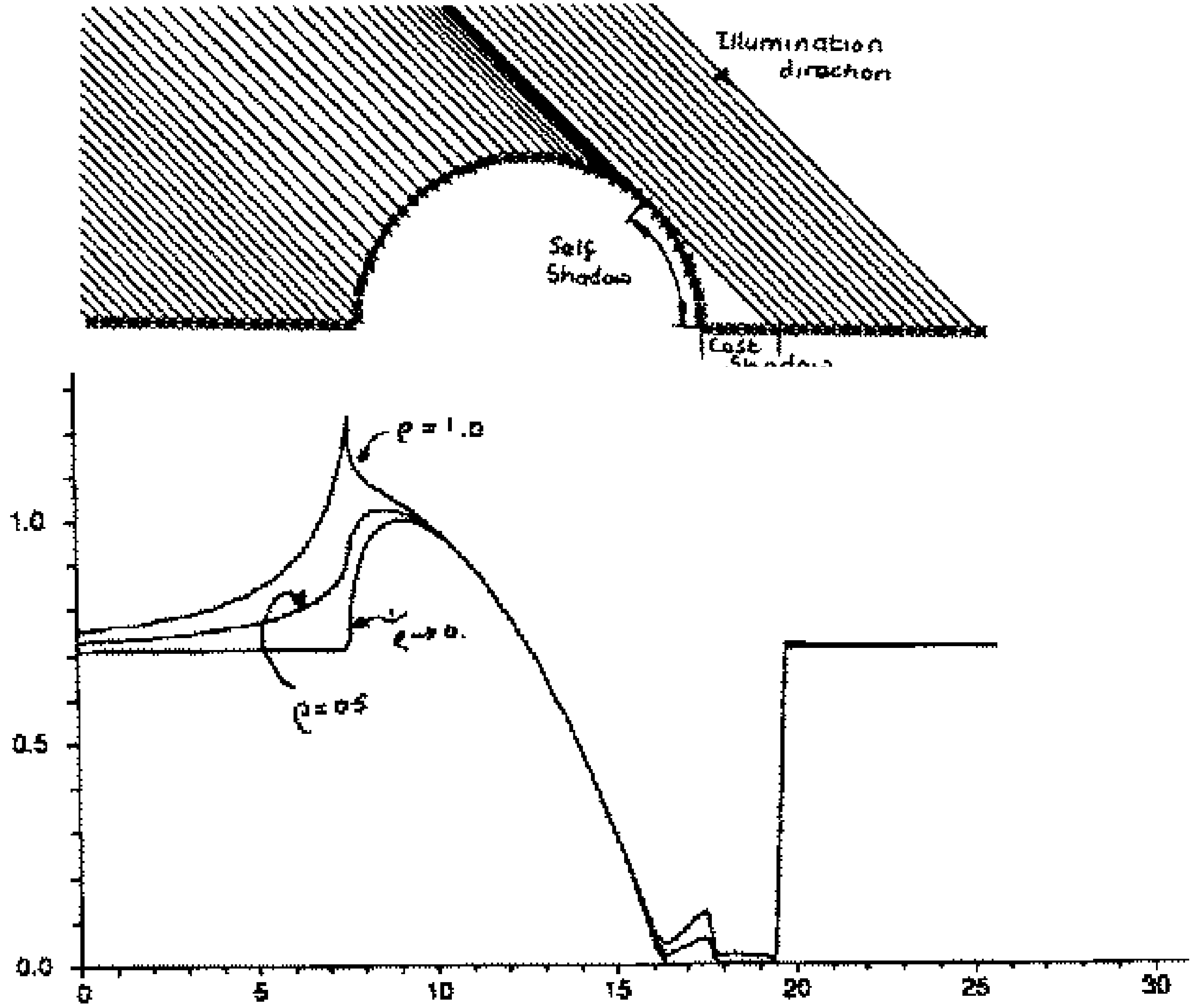


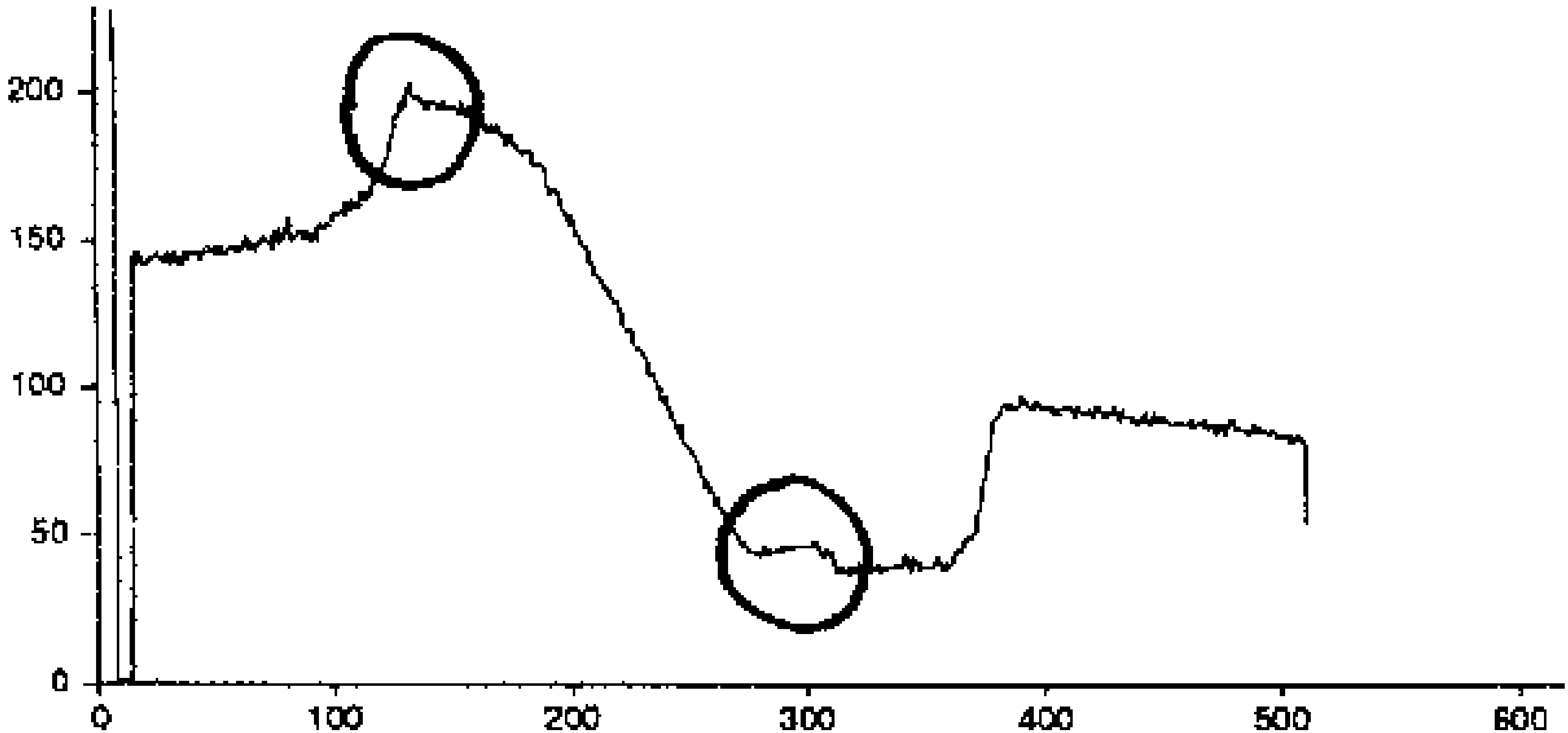
$$B(\boldsymbol{u}) = E(\boldsymbol{u}) + B_{refl}(\boldsymbol{u})$$

$$\begin{aligned} B_{refl}(\boldsymbol{u}) &= \rho_d(\boldsymbol{u}) \int_{world} \text{visible}(\boldsymbol{u}, \boldsymbol{v}) B(\boldsymbol{v}) \frac{\cos\theta_u \cos\theta_v}{\pi d_{uv}^2} dA_{\boldsymbol{v}} \\ &= \rho_d(\boldsymbol{u}) \int_{world} \text{visible}(\boldsymbol{u}, \boldsymbol{v}) K(\boldsymbol{u}, \boldsymbol{v}) B(\boldsymbol{v}) dA_{\boldsymbol{v}} \end{aligned}$$

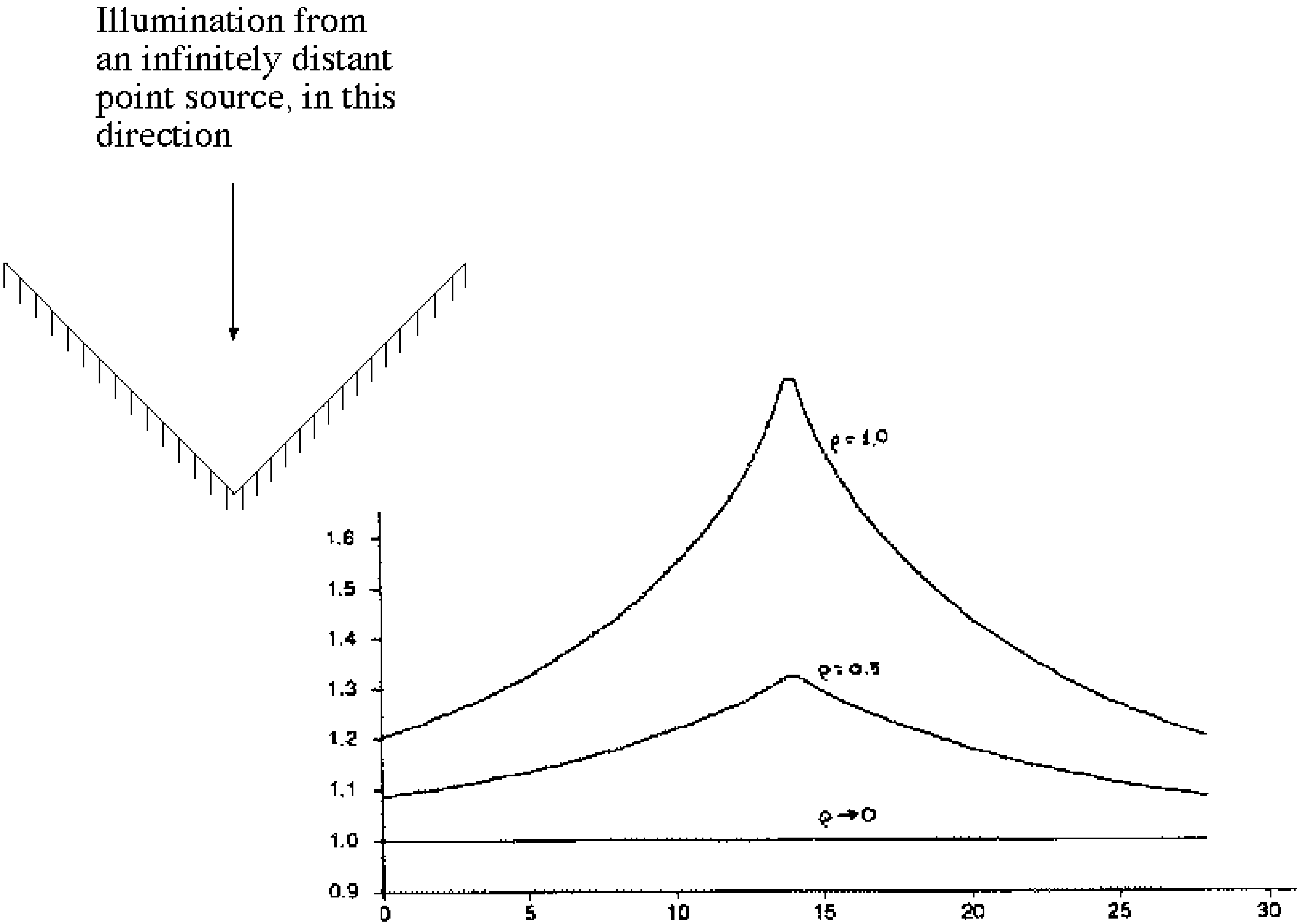
$$B(\boldsymbol{u}) = E(\boldsymbol{u}) + \rho_d(\boldsymbol{u}) \int_{world} \text{visible}(\boldsymbol{u}, \boldsymbol{v}) K(\boldsymbol{u}, \boldsymbol{v}) B(\boldsymbol{v}) dA_{\boldsymbol{v}}$$

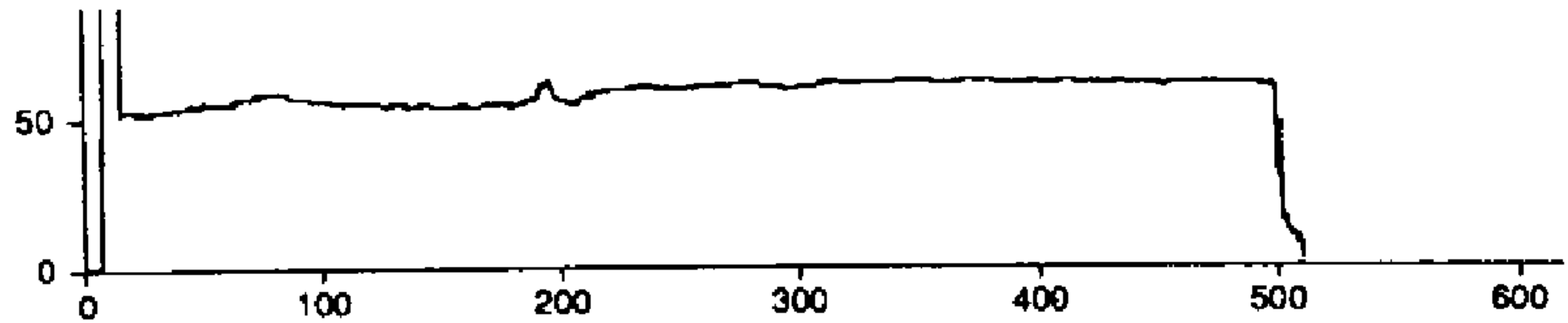
- Study qualitative effects
  - soft shadows
  - decreased dynamic range
  - reflexes
  - smoothing
- Build Approximations
  - ambient illumination
- Solve approximate versions


Illumination direction
Self Shadow
Cast Shadow
ρ = 1.0
ρ = 0.5
1.0
0.5
0.0
0
5
10
15
20
25
30

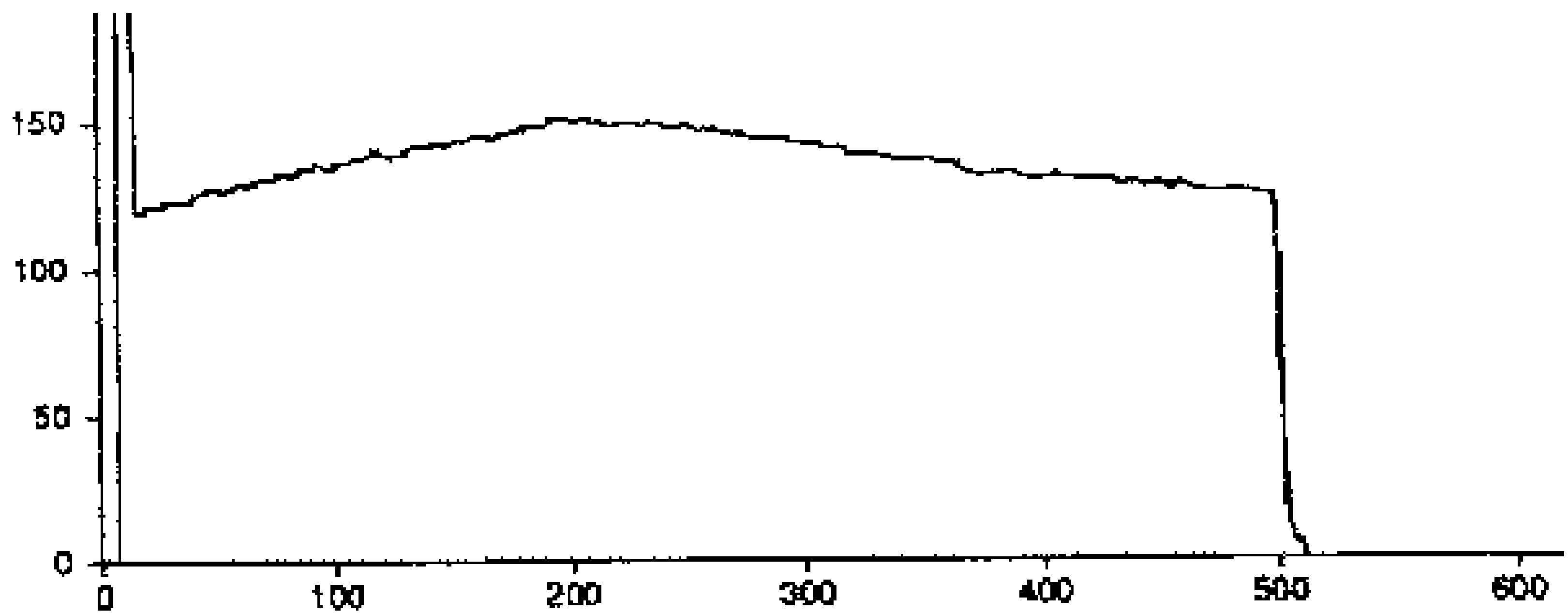


200
150
100
50
0
0
100
200
300
400
500
600

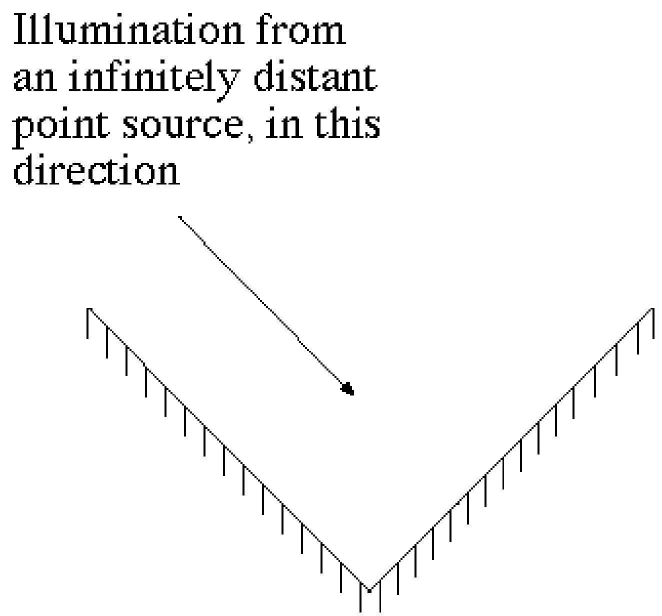

Illumination from an infinitely distant point source, in this direction
ρ = 1.0
ρ = 0.5
ρ → 0
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0
5
10
15
20
25
30

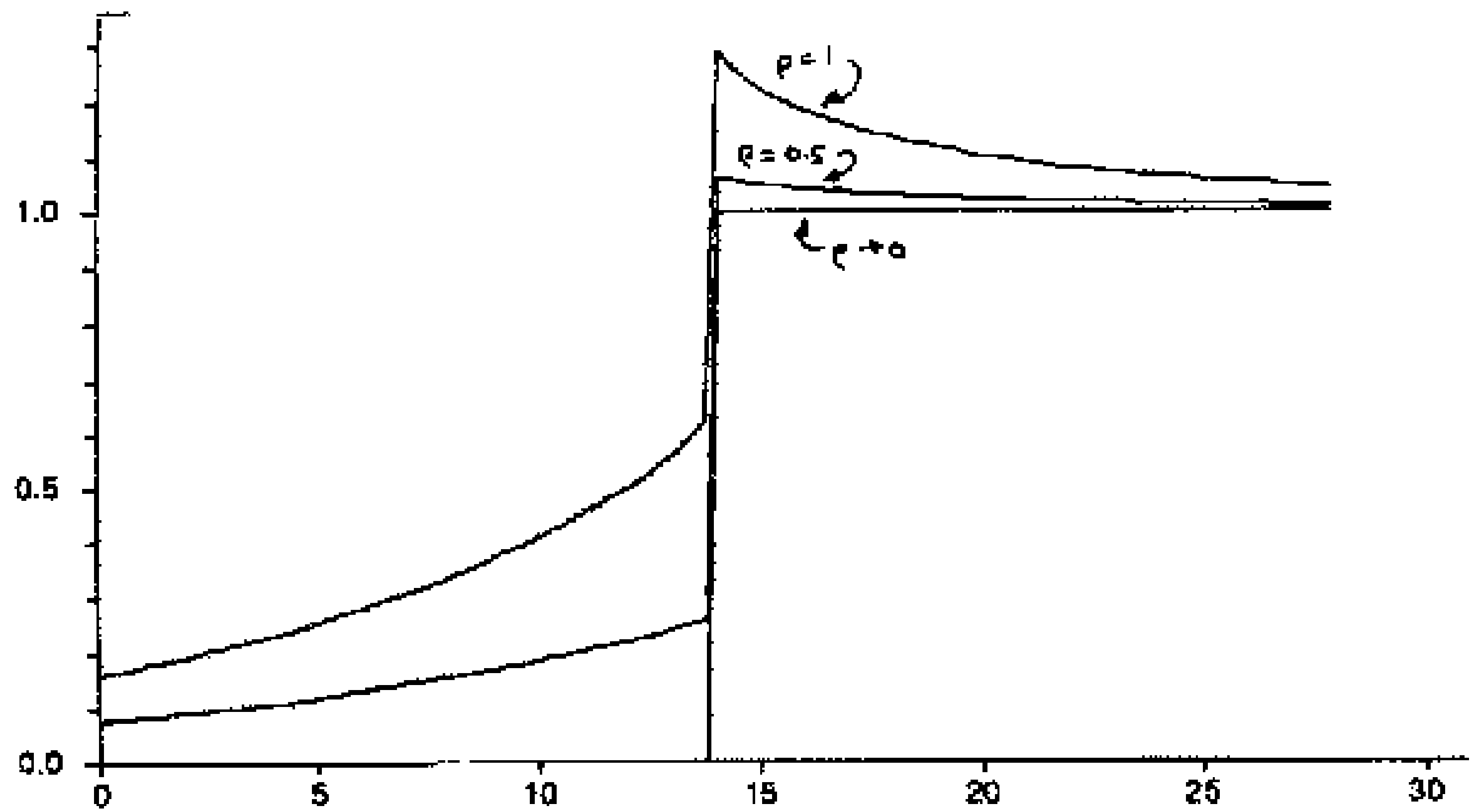


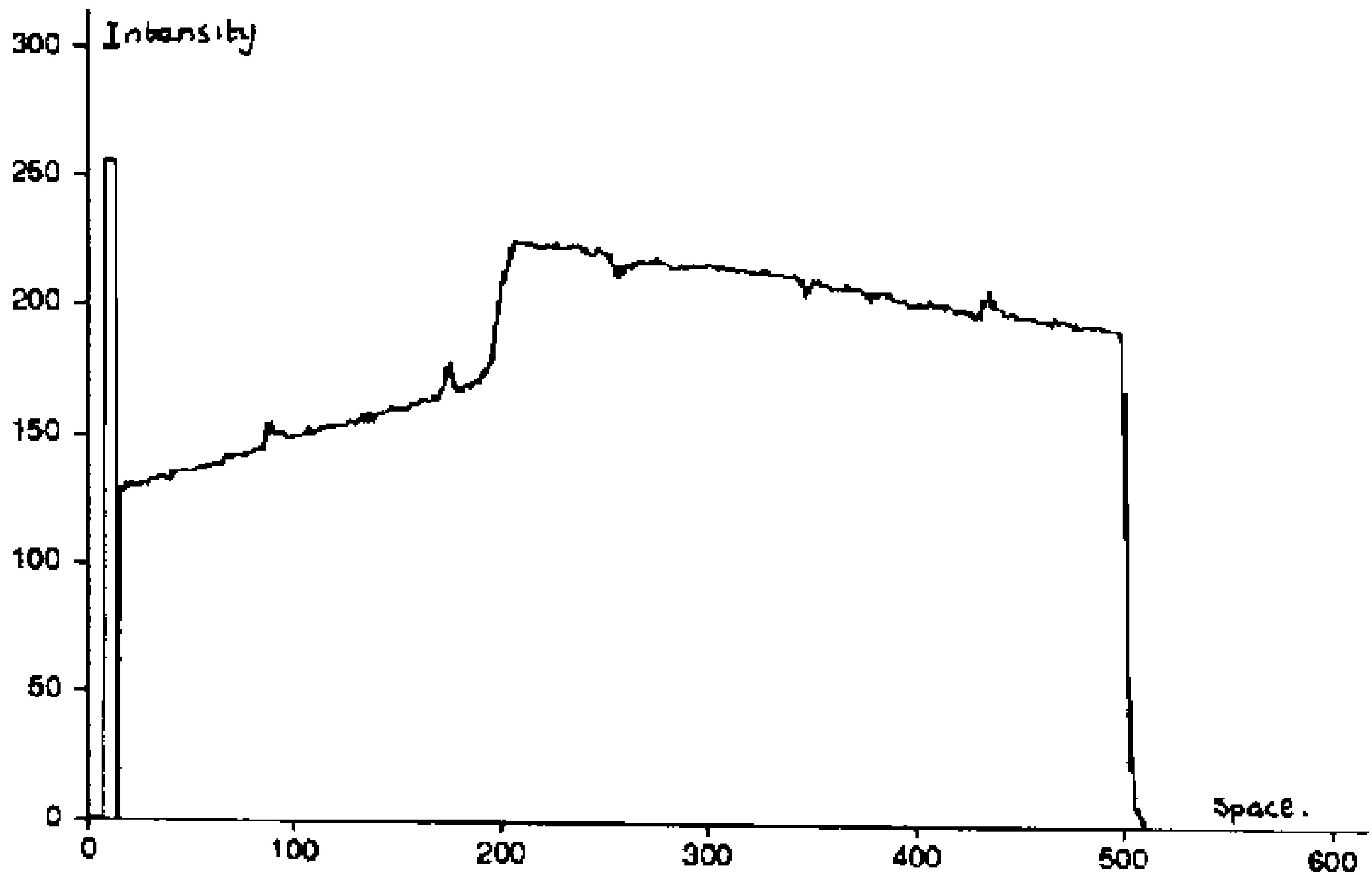
50
0
0
100
200
300
400
500
600

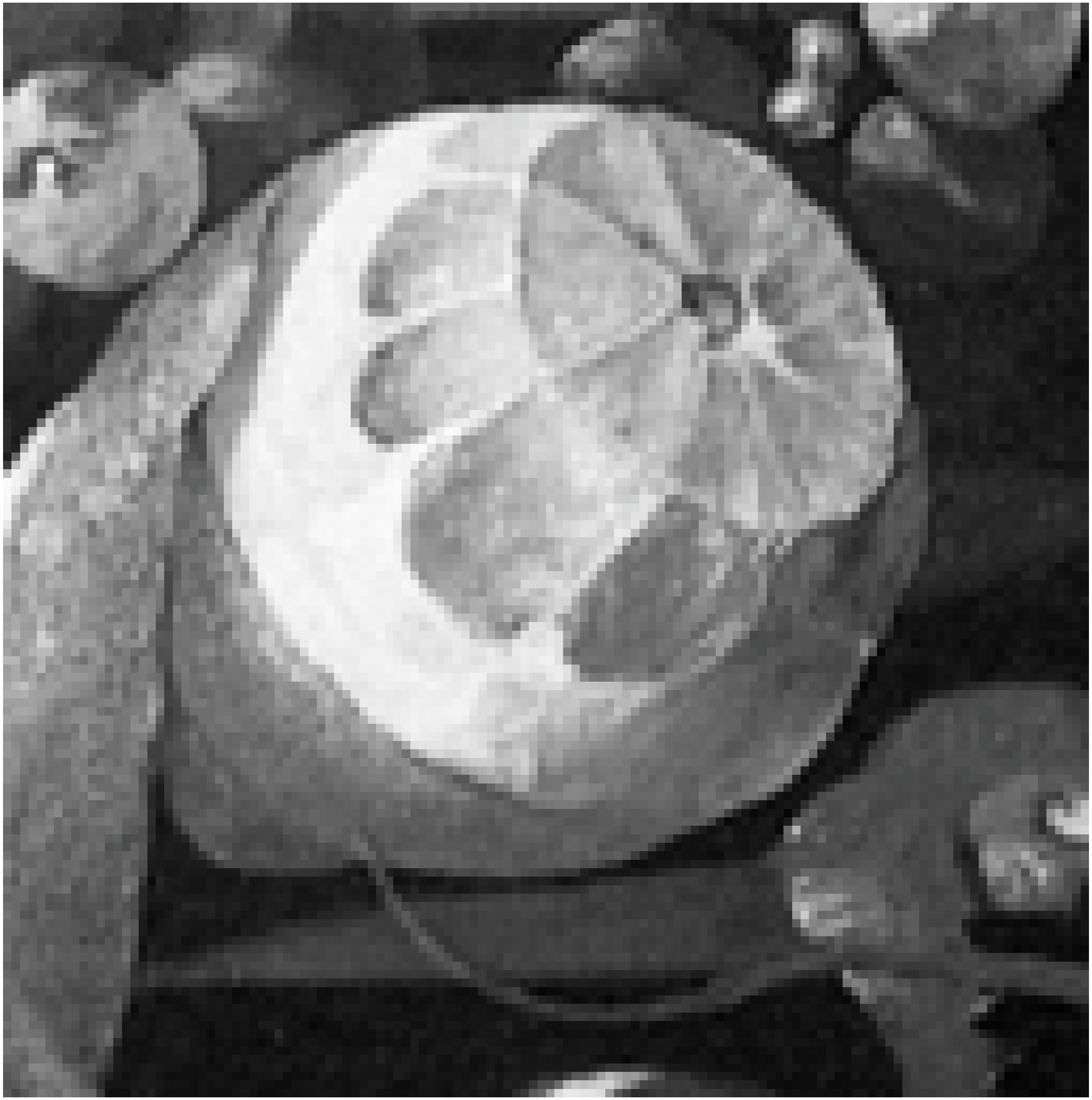


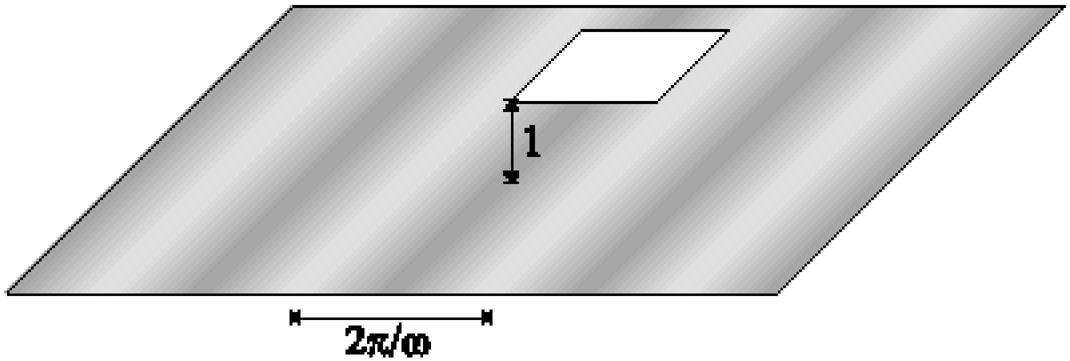
150
100
50
0
0
100
200
300
400
500
600

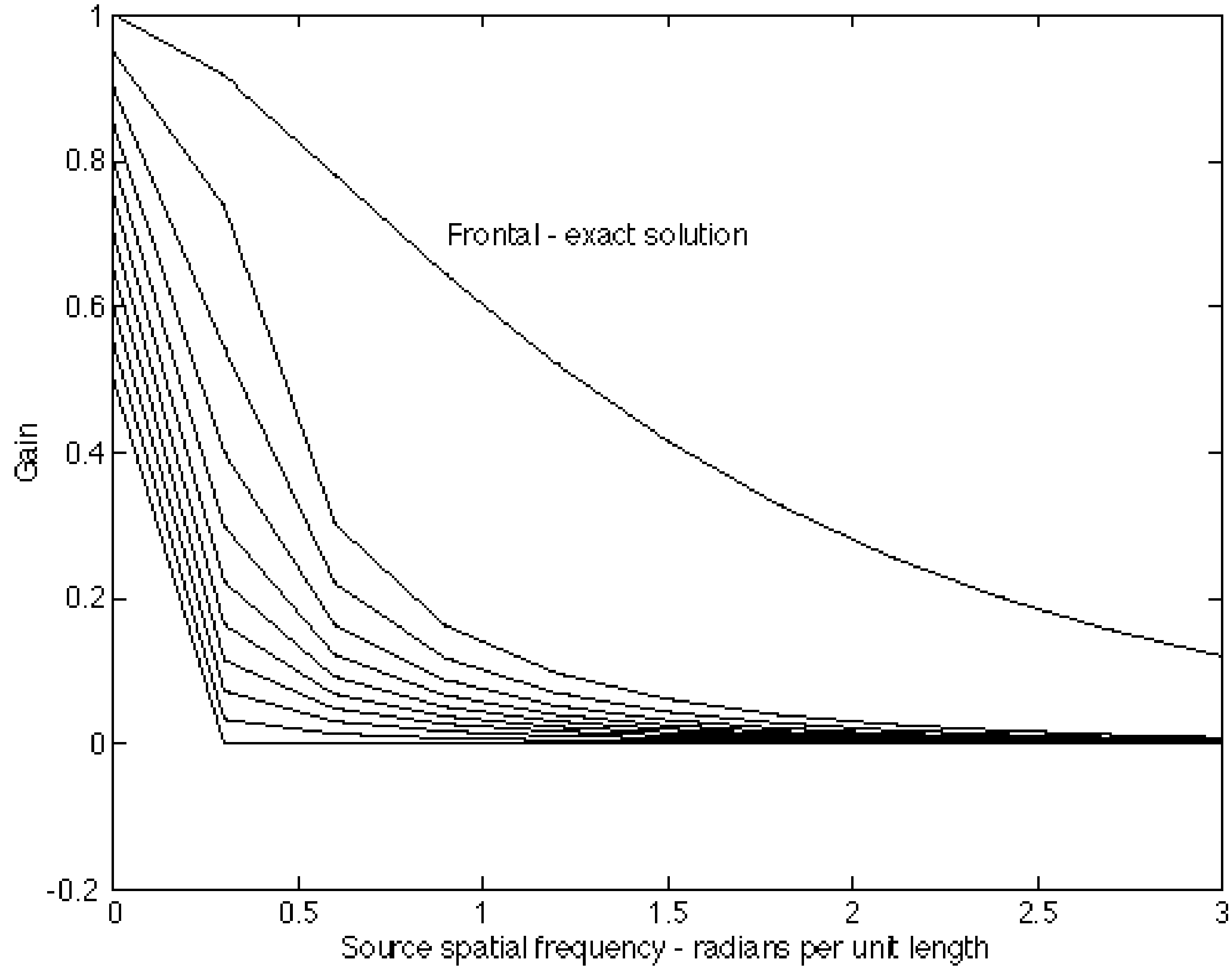

Illumination from
an infinitely distant
point source, in this
direction


ρ = 1
ρ = 0.5
ρ → 0
1.0
0.5
0.0
0
5
10
15
20
25
30


Intensity
300
250
200
150
100
50
0
0
100
200
300
400
500
600
Space.




1
2π/ω


Frontal - exact solution
Gain
Source spatial frequency - radians per unit length
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3

A simple finite element model

- Divide world into patches of constant albedo and represent radiosity, exitance as constant on each patch.
- We must now figure out patch-patch interactions.

$$B_{j\to i}(\boldsymbol{x}) = \rho_d(\boldsymbol{x}) \int_{\text{patch } j} \text{visible}(\boldsymbol{x}, \boldsymbol{v}) K(\boldsymbol{x}, \boldsymbol{v}) dA_{\boldsymbol{v}} B_j$$

where $\boldsymbol{x}$ is a coordinate on the i'th patch and $\boldsymbol{v}$ is a coordinate on the j'th patch. Now this expression is not a constant, and so we must average it over the i'th patch to get

$$\overline{B}_{j\to i} = \frac{1}{A_i} \int_{\text{patch } i} \rho_d(\boldsymbol{x}) \int_{\text{patch } j} \text{visible}(\boldsymbol{x}, \boldsymbol{v}) K(\boldsymbol{x}, \boldsymbol{v}) dA_{\boldsymbol{v}} dA_{\boldsymbol{x}} B_j$$

where A_i is the area of the i'th patch. If we insist that the exitance on each patch is constant, too, we obtain the model:

$$\begin{aligned} B_i &= E_i + \sum_{\text{all } j} B_{\text{average incoming at } i \text{ from } j} \\ &= E_i + \sum_{\text{all } j} K_{ij} B_j, \end{aligned}$$

where

$$K_{ij} = \frac{1}{A_i} \int_{\text{patch } i} \rho_d(\boldsymbol{x}) \int_{\text{patch } j} \text{visible}(\boldsymbol{x}, \boldsymbol{v}) K(\boldsymbol{x}, \boldsymbol{v}) dA_{\boldsymbol{v}} dA_{\boldsymbol{x}}$$

The two big technical problems

- Integration
  - computing form factors
  - penumbra
  - rendering
  - smoothing example
- Linear algebra issues