CHECKMATE!

A Brief Introduction to Game Theory

Dan Garcia
UC Berkeley

The World

Kasparov

Game Theory: Economic or Combinatorial?

- Economic
  - ◊ von Neumann and Morgenstern's 1944 *Theory of Games and Economic Behavior*
  - ◊ Matrix games
  - ◊ Prisoner's dilemma
  - ◊ Incomplete info, simultaneous moves
  - ◊ Goal: Maximize payoff
- Combinatorial
  - ◊ Sprague and Grundy's 1939 *Mathematics and Games*
  - ◊ Board (table) games
  - ◊ Nim, Domineering
  - ◊ Complete info, alternating moves
  - ◊ Goal: Last move

Combinatorial Game Theory History

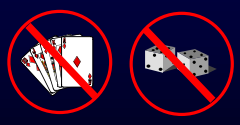
- Early Play
  - ◊ Egyptian wall painting of Senat (c. 3000 BC)
- Theory
  - ◊ C. L. Bouton's analysis of Nim [1902]
  - ◊ Sprague [1936] and Grundy [1939] Impartial games and Nim
  - ◊ Knuth *Surreal Numbers* [1974]
  - ◊ Conway *On Numbers and Games* [1976]
  - ◊ Prof. Elwyn Berlekamp (UCB), Conway, & Guy *Winning Ways* [1982]

What is a combinatorial game?

- Two players (Left & Right) move alternately
- No chance, such as dice or shuffled cards
- Both players have perfect information
  - ◊ No hidden information, as in Stratego & Magic
- The game is finite – it must eventually end
- There are no draws or ties
- Normal Play: Last to move wins!

What games are out, what are in?

- Out
  - ◊ All card games
  - ◊ All dice games



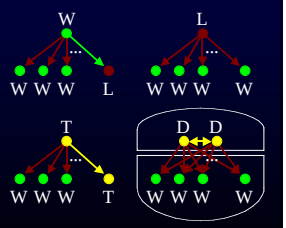
- In
  - ◊ Nim, Domineering, Dots-and-Boxes, Go, etc.
  - ◊ 1,2,…,10 , Kayles, Toads & Frogs, Snake, Tactix, Poison
- In, but not normal play
  - ◊ Chess, Checkers, Othello, Tic-Tac-Toe, etc.

"Computational" Game Theory (for non-normal play games)

- Large games
  - ◊ Can theorize strategies, build AI systems to play
  - ◊ Can study endgames, smaller version of original
    - Examples: Quick Chess, 9x9 Go, 6x6 Checkers, etc.
- Small-to-medium games
  - ◊ Can have computer solve and teach us strategy
  - ◊ GAMESMAN does exactly this
    - It can solve BOTH normal and non-normal play games

Computational Game Theory

- Simplify games / value
  - ◊ Store turn in position
  - ◊ Each position is (for player whose turn it is)
    - Winning (∃ losing child)
    - Losing (All children winning)
    - Tieing (!∃ losing child, but ∃ tieing child)
    - Drawing (can't force a win or be forced to lose)



Exciting Game Theory Research at Berkeley

- Combinatorial Game Theory Workshop
  - ◊ MSRI July 24-28th, 2000: Son of Games of No Chance
  - ◊ 1994 Workshop book: Games of No Chance
- Prof. Elwyn Berlekamp
  - ◊ Dots & Boxes, Go endgames
  - ◊ Economist's View of Combinatorial Games
- Dr. Dan Garcia
  - ◊ Undergraduate Game Theory Research Group
    `http://www.cs.berkeley.edu/~ddgarcia/research/gametheory/current/`