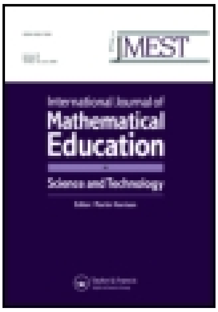

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An experimental study on the effectiveness of metacognitive and heuristic problem solving techniques on computational performance of students in mathematics

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An experimental study on the effectiveness of metacognitive and heuristic problem solving techniques on computational performance of students in mathematics

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The study focused on the effects of the application of two problem solving techniques—metacognitive (MPST) and heuristic (HPST)—on the achievement of students in the computation of creative mathematics problems. 245 (SS1) students were randomly selected from six senior secondary schools in Ibadan. The students were arranged in a $3 \times 3 \times 2$ pre-test–post-test quasi-experimental design. The students were taught for eight weeks using MPST, HPST and the traditional/conventional approach.

ANCOVA, ANOVA and multiple classification analysis (MCA) were the statistics used to analyse results. Results of the three null hyopotheses tested, showed that:

(i) There was a significant difference in the achievement of experimental and control groups.
(ii) There was a significant difference in the achievement of high, average, and low ability students.
(iii) There was no significant difference in the achievement of male and female students in the computation of creative problems in mathematics.

1. Introduction

Structural characteristics of problems contribute greatly to the difficulties compounding problem solving in mathematics. Problems with different characteristics normally require different skills and stategies. Some of the difficulties emanating from the characteristics of problems can arise from the nature, structure, context and representation of the problem [1].

Two broad categories of mathematics problems can be identified. There are creative (or ill-structured) problems and routine (or well-structured) problems [2, 3]. Creative problems are those types of problems that are not explicitly stated and full information to make them implicit is not made available. Examples include problems we meet in real life such as in political, social, economic and scientific situations that require deep and abstract thinking. On the other hand routine problems are presented with detailed information. Enough hints and procedures exist which, when properly followed, will lead to attainment of their solution. Examples are text book exercises, problems from the lower level of the cognitive domain, and the majority of problems we encounter during teaching/learning processes in the classroom, e.g. evalute $\int_1^4 (3x^3 + x^2)\,dx$.

In order to be a better problem solver and to be able to handle mathematics problems efficiently, students must be informed of and exposed to a variety of problem conceptions. A student may have the difficulty of conceptualizing the

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techniques to apply that would lead to the required solution. At times it can be the difficulty of computational procedures. As Turner [4] pointed out, a student has a problem, if he has a target, and know what should be achieved, but does not know how to achieve the target.

It is important that a problem solver should be in control of the information acquired during learning and should be able to direct applications of those learning tactics. More especially, a problem-solver should be able to evaluate, reorganize and regulate resources in order to achieve goals. Metacognitive psychologists [5–9] emphasize learners being aware of their own learning proceses as well as the ability to monitor and control those learning processes. Baron [10] argued that training of executive control will assist learners to control and direct their stylistic potentials.

Efficient computational ability is essential in mathematical problem solving. Kilpatrick [11] emphasized the importance of computations in solving mathematics problems. Meyer [12] asserted that mathematical problem solving performance depends on computational abilities of students. Martins [13], Akpan [14] and Oladunni [1] have shown that the ability of secondary school students to compute correlated significantly with their mathematical problem solving performance.

However, Chapman-Taylor [15] observed that Nigerian students failed their examinations in mathematics because they were too slow and made too many mistakes in computation.

This study therefore intended to examine mathematical computational ability of Nigerian students in mathematics after being exposed to two probem solving techniques. Specifically, the study intended to find out whether metacognitive and heuristic problem solving techniques can enhance the computational ability of students working on creative mathematics problems.

Hypotheses. The following null hypotheses were formulated and tested.

- HO_1 There is no significant difference between the achievement of experimental and control groups in the computaton of creative mathematics problems.
- HO_2 There is no significant difference in the achievement of low, average and high ability students in the computation of creative mathematics problems.
- HO_3 There is no significant difference between the achievement of male and female students in the computation of creative mathematics problems.

Sample. The 245 subjects for this study were drawn from six secondary schools in Ibadan. About 40 (SS1) students were randomly selected from each of the schools. Results of an analysis of variance (ANOVA) performed on an aptitude test given to the selected subjects indicated an F-ratio of 0.84, which was not significant at the 0.05 level. Thus, the homogeneity of the six groups was confirmed. A school was randomly assigned one treatment. The subjects were classified into three groups based on scores they obtained in the aptitude test. The groups were high achievers, average achievers and low achievers. The ages of the subjects ranged from 14 to 17 with a mean of 15.22.

2. Experimental design

The subjects were assigned to a 3 × 3 × 2 factorial design. The factors were treatments, achievement levels and sex:

$$\begin{array}{lllll} E_1 & : & O_1 & X_1 & O_2 \\ E_2 & : & O_3 & X_2 & O_4 \\ C & O & O_5 & & O_6 \end{array}$$

X_1 and X_2 were treatments
X_1 = metacognitive problem solving technique (MPST)
X_2 = heuristic problem solving technique (HPST)
$O_1 = O_3 = O_5$ = Pre-test
$O_2 = O_4 = O_6$ = Post-test
E_1 = Experimental group with MPST
E_2 = Experimental group with HPST

2.1. *Treatment procedure*

The second test (computation of creative mathematics problems) was administered after the subjects had been instructed for seven weeks using the two different problem solving techniques. Each model was accompanied by a package containing the learning instructions and problem tasks.

2.2. *Metacognitive problem solving technique (MPST)*

MPST was adapted from Schoenfeld's [16] Managerial Problem Solving Model. The technique emphasizes the ability of the problem solver to articulate a problem solving method and make plans for problem solving procedures. The steps mapped out in the plan should be followed systematically. The problem solver has the responsibility to consciously monitor and regulate each stage of the computational procedure, just as a business manager would monitor any decision taken among many alternatives. Solution procedures are checked periodically to ensure their continued utility.

The model has six steps and each step is a prerequisite for the next. The steps are:

(1) Identify the problem.
(2) Interpret the symbols, signs, concepts, etc.
(3) Form a suitable diagram or model.
(4) Choose an appropriate solution technique.
(5) Solve, using the chosen technique consciously.
(6) Does the solution satisfy the problem?

2.3. *Heuristic Problem Solving Technique (HPST)*

HPST was adapted from Polya's (1957) General Heuristic Problem Solving Procedure. HPST was centred on guided discovery. The model has four stages, with each stage accompanied by heuristic questions. The group (76 students) was instructed on how to respond to the questions and consequently solve problems.

The stages were:

(1) Understanding the problem.
(2) Plan an attack.

(3) Carry out the plan.
(4) Check over.

The control group (85 students) received no formal training in problem solving techniques. The students were taught by their usual teachers in the usual traditional/conventional manner the same curriculum context taught to the other two groups.

Three topics, quadratic equations, trigonometrical ratios and circular measures were selected for learning, from the curriculum designed for SS1 students. Each group met for two sessions per week, each session lasting one hour. The experiment lasted eight weeks.

2.4. *Instrumentation*

The two instruments used to collect data for the study were:

(i) aptitude test M;
(ii) test on computation of creative mathematics problems.

Aptitude test M. This test consisted of 35 non-verbal, standardized multiple choice items of abstract reasoning. The test items were constructed by Taylor and Brandshow [18]. The test was a good measure of students' scholastic aptitude as well as a good predictor of mathematics ability. The test has a reliability index of 0.82 and a validity index of 0.75 [19].

Test on computation of creative mathematics problems. The test consisted mainly of easy questions since it was intended to measure computation ability of the students. The test was made up of three creative problems in mathematics (see Appendix). The items would not require particular formulae but intelligence and articulate problem solving techniques. The items were adapted from Kulm *et al.* [20]. The content validity of the items was considered right for senior Secondary School One (SS1) students by three mathematics teachers who had taught for more than 4 years in secondary schools. The reliability of the test was determined (using a test–retest method) to be 0.87.

3. Analysis and results

Scores obtained from the subjects on test of computation of creative mathematics problems were analysed using three statistical techniques. These were analyses of variance (ANOVA), analyses of covariance (ANCOVA) and multiple classification analysis (MCA). ANOVA and ANCOVA were used to test the hypotheses formulated while MCA was used to show the relative effectiveness of the categories.

Analyses of the various results on the computation of creative mathematics problems are presented in tables 1–4.

The analysis of variance (ANOVA) shown in table 1, reveals that the main effects, treatments and ability were significant at the 0.05 level. The analyses indicated that there was a significant difference in the achievement of experimental and control groups on computation of creative mathematics problems. The ability effects were also found significant ($F = 43.33$) at the 0.05 level of significance.

In other words, there was a significant difference among the performance of high, average, and low ability students, on computation of creative mathematics

Source	SS	DF	MS	F	
Main effects	5 169.76	5	1033.95	22.60	XX
Treatment	1 249.94	2	624.97	13.66	XX
Ability	3 964.37	2	1982.19	43.33	XX
Sex	2.41	1	2.41	0.05	NS
2-way interactions	356.29	8	44.54	0.97	NS
Treatment × ability	173.04	4	43.26	0.94	NS
Treatment × sex	119.27	2	59.64	1.30	NS
Ability × sex	103.55	2	51.77	1.13	NS
3-way interactions	70.79	4	17.67	0.39	NS
Treatment × ability × sex	70.69	4	17.67	0.39	NS
Explained	5 596.73	17	329.22	7.20	XX
Residual	10 384.51	227	45.75		
Total	15 981.24	244	65.50		

Table 1. Analysis of variance (ANOVA) for scores on test of computation creative mathematics problems.

Source	SS	DF	MS	F
Explained	5 150.42	3	1716.81	38.20**
Residual	10 830.82	241	44.94	
Total	15 981.24	244	65.49	

** Significant at 0.05 level.

Table 2. ANCOVA for experimental and control groups on computation of creative problems with APTTEST as covariate.

Source	SS	DF	MS	F
Explained	4 385.05	3	1416.68	30.38**
Residual	11 596.20	241	48.12	
Total	15 981.24	244	65.49	

** Significant at 0.05 level.

Table 3. ANCOVA for high, average and low ability groups on computation of creative problems with APTTEST as covariate.

problems. There was no significant differences in the scores of male and female students. Thus the first two hypotheses of this study were rejected, but the third was upheld.

Analyses of covariance (ANCOVA, tables 2 and 3) performed on the scores of the students confirmed the former assertions. For instance, information on ANCOVA (table 2) shows that using aptitude test (APTTEST) as covariate, the treatment effects were significant ($F = 38.20$) at 0.05 level. ANCOVA provided evidence that when students were reduced to equal bases after the removal of aptitude effects, the results were still significant.

Var. + Category		N	Unadjusted deviation η		Adjusted for independence deviation β	
Treatment	1. MPST	84	1.95		2.18	
	2. HPST	76	1.18		1.02	
	3. Control	85	−2.99		−3.07	
				0.27		0.28
Ability	1. Low	39	−4.75		−4.97	
	2. Average	167	−0.90		−0.85	
	3. High	39	8.61		8.61	
				0.49		0.50
Sex	1. Male	152	0.02		−0.08	
	2. Female	93	−0.03		−0.13	
				0.00		0.01
Multiple R-squared					0.323	
Multiple R					0.569	

Grand mean = 17.11.

Table 4. Multiple classification analysis for computation of creative problems.

Table 3 also reveals that there was a significant difference (F = 30.38) among the performance of high, average, and low ability students, on computation of creative problems when aptitude test scores were used as covariates.

Multiple classification analysis (MCA) was carried out in order to identify the relative effectiveness of each category. MPST, HPST and the control groups had deviations 1.95, 1.18 and −2.99 units from the grand mean of 17.11. This is an indication that MPST was the most effective of the problem solving treatments. While the HPST group scored above average, the control group scored below the general average. The value of η^2 showed that the treatment factor contributed 7.29% of variations in computation of creative mathematics problems.

Table 4 reveals further that only high ability students performed brilliantly well with a deviation of 8.61 above the grand mean. The average and low ability students had poor performances, with deviations −0.90 and −4.75 units below the grand mean. However, the ability factor accounted for as much as 24.01% of the total 32.3% variations jointly contributed by treatment, ability and sex.

4. Discussion

Tables 1 and 2 show that there was a significant difference between the performances of the experimental and control group on computation of creative mathematics problems. This indicates that there was a significant differential treatment effect in computational skills between students who were exposed to metacognitive and heuristic problem solving techniques and those students who used the conventional approach. This is an indication that learning problem solving techniques could enhance performance in the computation of mathematics problems.

This study supports many similar studies carried out using either metacognitive or heuristic problem solving strategies [2, 12, 16, 21, 22]. For instance

Schoenfeld's experiment [16] showed that the experimental group had an improved performance over those in the control group.

The poor performance of the control group in this study stemmed from the poor computational techniques employed in solving problems. This exposed the poor system of computation employed in our classrooms. During this study, it was observed that students in the control group had no particular procedure or system to follow during their solution processes. They started computations immediately questions were given to them. They were too anxious to even pinpoint the demands of the questions. They merely 'run through' solution processes. They had no thinking phase nor planning phase. These findings were similar to those of Clements [23], whose control group students were in too much of a hurry to formulate correctly sequential mathematics steps.

Computation of mathematics problems involves much more than performing habitual operations of addition and subtraction. Computations in mathematics demand that the problem solver should make plans, evaluate and re-organize available information, formulate necessary sequential steps and then monitor consciously solution procedures.

There is no gain saying that a fundamental prerequisite in computation of creative mathematics problems is the acquisition of relevant cognitive knowledge by the problem solver. A successful problem solver needs to acquire relevant conceptual and procedural knowledge. Intellectual ability of the problem solver acts on these requisites. This probably accounts for the better performance of high ability students over those students with average and low abilities. Having acquired the necessary procedural problem solving techniques, they were able to manoeuvre the computations more intelligently than the lower ability students. According to Cross and Paris (1988) this is the self-management aspect of metacognitive techniques.

5. Conclusion

This study has identified that the metacognitive problem solving technique is effective and could enhance computational achievement in mathematics. Thus it is essential in problem solving situations to comprehend, interpret, analyse and transform problems to a familiar level before starting to solve them. Students could become better problem solvers and consequently better achievers if they learn to plan, regulate and take intelligent computational procedures.

Appendix. Test on computation of creative mathematics problems

1. A farmer has hens and rabbits. These animals have 21 heads and 70 feet. How many hens and rabbits has the farmer?
2. 12 trucks are parked in a parking lot and the park is completely full. Each truck can take the space of 3 cars, if there are 4 rows in the parking lot how many cars could be parked in each row?
3.

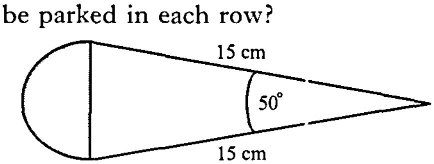

Figure 1

From figure 1 shown above calculate:

(i) the shaded part of the circle;
(ii) the distance all around the figure.

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