# Teaching Problem Solving

James E. Stice
Chemical Engineering
The University of Texas at Austin

When I was an undergraduate student, many of my professors would derive an equation during lecture, and then would proceed to work an example problem. They would outline the situation, invoke the equation, plug in the numbers and arrive at a solution. What they did always seemed very logical and straightforward, I'd get it all down in my notes, and I'd leave the class feeling that I had understood what they had done. Later I often was chagrined to find that I couldn't work a very similar problem for homework. The professor had made it all seem so easy in class . . . what had I missed?

Today many of my students are like I was then. They "know" a lot of things (the "knowledge" level of Bloom's (1) cognitive taxonomy), and they "understand" a lot of it (Bloom's "comprehension" level). More, they can apply it to cases with which they are familiar (Bloom's "application" level), that is, they are good at using familiar routines in situations they have seen before. They have trouble applying what they know to novel situations, however, cases that are somewhat different from what they have experienced. In the past I have called these students "memorizers," but that's not quite fair; they are more than that. But no one has taught them (and they haven't learned on their own) to function very well at the analysis level, the fourth level of Bloom's taxonomy.

So part of my problem then (and theirs now) was that I didn't have good analysis skills. Another part was that most of my professors didn't know how to help me develop them, even though they might have had those skills themselves. More will be said about this later.

A sizeable literature has appeared on the subject of "teaching problem solving" because some of the research on analytical thinking has been done by physics, mathematics and engineering professors. These professors have a particular interest in the solution of mathematical or numerical problems, but the scope of problem solving is much wider than that. I view problem solving as a subset of the entire domain of analytical thinking, and perhaps we need a definition of what we are talking about. Donald R. Woods and his colleagues at McMaster University (8,9) offer this one:

> "Problem solving is the process of obtaining a satisfactory solution to a novel problem, or at least a problem which the problem solver has not seen before."

William Brownell (3) suggested a more general definition:

> "Problem solving refers
>
> (a) only to perceptual and conceptual tasks,
>
> (b) the nature of which the subject, by reason of original nature, of previous learning, or of organization of the task, is able to understand, but
>
> (c) for which he knows no direct means of satisfaction at the time.
>
> (d) The subject experiences perplexity in the problem situation, but he does not experience utter confusion. From this he is saved by the condition described above under (b). Then problem solving becomes the process by which the subject extricates himself from the problem."

I want to emphasize the shift of focus from looking at the product of problem solving to the importance of viewing problem solving as a process. Woods et al. (8) suggest that successful problem solvers employ some or all of the following elements:

1. An awareness that a problem exists.

2. Prerequisite skills
   a. Basic knowledge pertaining to the problem area
   b. The learning skills necessary to obtain other information required for the solution
   c. Motivation to want to solve the problem
   d. Memorized experience factors that provide "feelings" about what assumptions might be made and how reasonable an answer is
   e. Ability to communicate the result
   f. Group skills, if a team approach is used

3. An overall, organized strategy

4. Alternatives for specific steps in the strategy (contradiction, reasoning by analogy, working backwards, solving a simpler problem first, etc.)

5. Knowledge of heuristics or "rules of thumb" that offer suggestions about what to do next.

6. Ability to create, to generalize and to simplify.

A *strategy* is a set of sequential steps used by a problem solver in arriving at a solution. Many strategies may be found in the literature, and all have a number of similarities. A good strategy to teach students is the one suggested by the mathematician Gyorgy Polya (5), because it is simple and straightforward, and it sounds right. Polya's strategy is recognized by many people as the sort of thing they have done when they have solved some problem successfully. The steps in Polya's strategy are:

**DEFINE**
Identify the actual problem

**THINK ABOUT IT**
What are the attributes of the problem?
Identify the area of knowledge involved
Collect information

**PLAN**
Flowchart a solution
Think of alternate plans
Translate

**CARRY OUT PLAN**
Solve the problem

**LOOK BACK**
Verify that problem solved was the one originally defined.
Check reasonableness and math
Check criteria and constraints
Study related problems
Identify applications
Identify and memorize order-of magnitude numbers
Develop successive approximation strategies
Study the problem-solving skills learned
Communicate results

Polya's original strategy included four steps: DEFINE, DEVISE A PLAN, CARRY OUT PLAN, and LOOK BACK. Woods added the THINK ABOUT IT step, and I think this addition is helpful. There is a period of "getting acquainted" with a problem, during which the problem solver goes over what he/she knows about the sort of situation posed. What do I need to know? Has anyone already solved this problem, or something like it? Should I go to the library, or talk to a colleague? Can I draw a sketch, or make some sort of pictorial representation of this situation? Do I need to get more data, and if so, can I look it up someplace, or do I have to develop some numbers on my own? After some time spent in this kind of thinking, one may be ready to begin to plan, but not before.

After presenting Polya's strategy (or someone else's) to students, one must give them an opportunity to use it in solving problems. The students must practice the process consciously, and must receive constant feedback so they will know how they are doing, and so they can continue to improve their skills. The best way to do this is to have a good problem solver work one-to-one with a single student, but this is very time-consuming. Given the number of students who need help and the press of other duties, few of us can afford to provide such intensive one-to-one tutoring.

A technique is available, however, which provides problem-solving practice for an entire class. It is called *Thinking Aloud Pairs Problem Solving (TAPPS).* This method evidently was first explored by Claparede (described in Woodworth, 11), and was later used by Bloom and Broder (2) in their study of the problem-solving processes of college students. Art Whimbey and Jack Lochhead (6,7) have further expanded the technique in their attempts to improve the teaching of reading, mathematics, and physics. In the method a class is divided into a number of teams, each team consisting of two students, with one student being the Problem Solver (PS) and the other being the Listener (L). Each member of the team has a definite role to play, and both must adhere strictly to some rules.

In a recent article Lochhead (4) has elaborated some of these rules. PS reads the problem aloud and then continues to talk aloud as much as possible about everything he/she is thinking while attempting to solve the problem. L listens, and has the more difficult role. L must try to keep PS talking; a short silence should be met with, "Tell me what you're thinking." More, L must understand in detail every step made by PS. Thus L should ask questions whenever PS says anything that is in the least mysterious. "Why do you say that?" "I don't understand. Would you explain that to me?" "Run that by again." are some of the questions/comments L may use. L must avoid solving the problem herself, and must not ask questions which are actually intended as hints to PS. In fact, it isn't necessary that L be able to solve the problem; her role is to help PS solve it. When students are first learning the method L perhaps can point out that PS has made an error, but should not tell him where it is. With more advanced students it is probably better to let PS find the error on his own. PS and L should switch roles after every problem, but they should never change roles within a problem.

The teacher's role in early classes should be restricted to rule enforcement. This usually means sitting with a pair of students, monitoring their activity and paying particular attention to L. It is also useful to emphasize to PS that getting the "right" answer is not as important as verbalizing the route to the answer. If PS gets an incorrect result but understands how and why he reached it, then he is far less likely to repeat the error.

The instructions for PS and L are summarized below (Woods, 10):

**The Problem Solver:**

1. Adjust the chairs so that you and your partner are comfortably seated at a worktable.

2. Make sure that you have paper, pencil, a calculator, and anything else you may need to solve the problem.

3. There may be hints or suggestions given about how to

approach a particular problem. Discuss these with your partner before you start.

4. Read the problem aloud.

5. Start to solve the problem on your own. You are solving the problem; your partner is only listening to you and reacting to what you say, not collaborating in the solution.

6. Thinking aloud isn't easy. At first you may have trouble finding the right words; don't search for them, say whatever comes into your mind. You and your partner are trying to help each other, and no one is evaluating you.

7. Go back over any part of a problem you wish. Use such words as, "I'm stuck. I'd better start over." "No, that won't work. . .let's see. . .hmmm."

8. Try to solve the problem even if you think it trivial, or if you don't think you're learning anything. Most people don't realize the fantastic improvement that occurs when they engage in this process. When you complete a problem, record what you think you learned about the process so you can see your progress. Then have your partner add his/her ideas.

**The Listener**:

1. Establish as quickly as possible that you will be a questioner and not a critic, and that you are not criticizing when you ask questions like, "Please elaborate," "What are you thinking now?" "Can you check that?"

2. Your role is to:

    a. demand that PS keep talking, but don't keep interrupting when PS is thinking.

    b. make sure that PS follows the strategy and doesn't skip any of the steps.

    c. help PS improve his/her accuracy.

    d. help reflect the mental process PS is following.

    e. make sure that you understand each step that PS takes.

3. Do not turn away from PS and start to work the problem on your own. It may be better if you don't even pick up a pencil. Track PS's procedure actively.

4. Do not let PS continue if:

    a. you don't understand what he/she is doing. Say, "I don't understand," or "I can't follow that."

    b. you think a mistake has been made. Ask him/her "to check that," or "Does that sound right?"

5. Do not give PS hints. If he/she continues to make an error in thinking or in computation, then point out the error, but do not correct it.

This sounds sort of like Twenty Questions, not really serious stuff, right? It may be all right for high school students, but it's not dignified enough for college classes? Don't underestimate the value of games in motivating people to learn, and don't pass judgement until you know more about why the process might be expected to work!

Why does the method require PS to talk out loud? Well, if PS didn't talk, obviously L would have nothing to listen to, but there's more to it than that. When a person thinks quietly about something, his thoughts can be half-formed, poorly organized, and subject to a variety of random thoughts about other things which intrude themselves. When PS verbalizes those thoughts, however, he or she is obliged to concentrate harder and to structure the thoughts so they will make some kind of sense to L. PS is therefore forced to process his thinking at a deeper level. . . he literally has to think harder. (If there is no listener, writing down one's reasoning can have the same effect.) More, PS gets active feedback from L, whose responses let PS know whether his communication is making sense; fuzzy thinking, careless errors, unsubstantiated statements and incorrect approaches may get past PS's filter, but they are less likely to get past L's filter, if she is alert and doing her job. Further, most people can talk about 125 words per minute, whereas it is estimated that they can "think" at 800-1,000 words per minute. Thus, having to speak one's thoughts aloud slows one down, and makes sloppy thinking and careless errors less likely.

Perhaps an example which should be a familiar one to many teachers will serve to underscore the importance of thinking aloud. Many of us have had a student come by the office for help. "I can't figure out how to do Problem 3," he says.

"What have you done so far?" you ask.

The student proceeds to tell you what he has tried to do. Suddenly he gets a funny look in his eyes, breaks into a smile, and says, "Oh! I see!" Then he says, "Thank you, very much!" and goes on his way. You appreciate the big "Thank you!" but what did you do? You forced him to talk, talking forced him to concentrate and to "make sense," and he suddenly saw how to deal with whatever was puzzling him. It's far better to let the student figure things out for himself than to show him or tell him. A good tutor asks questions instead of giving answers.

Now to return to the fact that most of my professors didn't know how to help me develop problem-solving skills. What had I missed when I found myself unable to solve homework problems like the ones shown in class? It was the analysis process. If a professor is at all astute, he/she will have worked out those derivations and example problems, or thought through those discussion questions, ahead of time so that things go smoothly in class. We fumble around a bit, follow some blind alleys, and maybe make some mistakes before we get things nicely worked out in our minds. But what do the students see? They see a cleaned-up, sanitized presentation, a PRODUCTION, neat, even elegant! The professor states the problem (DEFINE) and then presents the solution (CARRY OUT PLAN). The students never see the second step (THINK ABOUT IT) or the third (PLAN). They also don't see the false starts or the mental thumb-twiddling, and maybe they miss the assumptions the professor made, or don't understand why he made them. The solution is a product, but it is the result of a process. The class saw only parts of the process, and not the most important parts, at that. Because their own attempts to solve problems seem more painful and often are unsuccessful, they may think that the professor is a genius (or a magician), or that they are dumb. Odds are that neither is the case.

If you want to begin to develop your students' analytical skills, I suggest you begin to teach them a strategy such as Polya's, and give them some practice with analysis-level questions and problems. When you're ready to present an example, state the problem briefly, or provide a short summary on the chalkboard or on an overhead transparency. Then have the class form pairs and work on it for five to ten minutes. After a reasonable time has elapsed, have a student outline a possible solution, or present one yourself. Everyone will have had time to think about the problem, and many will have made an attempt at a solution. They are ready to listen to an explanation. . .and everyone will listen! Most will leave the class understanding what happened.

The trouble with this procedure is that it takes time. I can state a problem and go through its solution in much less time than it takes for the class to work through it, which leaves more time to introduce new content. Of course, covering content is not quite the same as uncovering it for the students; "covering" can also mean "burying". At any rate, teaching the analysis process will probably require that you sacrifice some content.

I believe the trade-off is worthwhile. I am no longer naive enough to think that, just because I go over something in class, the students all understand (whatever we mean when we say "understand"). Many of the things students learn in the course of their education become outdated, or they never have occasion to use them. They all will have to think, however, and for the rest of their lives. Further, the problems with which they will have to deal are likely to be ambiguous, poorly defined, and dependent upon social and technological developments which haven't occurred yet. Time spent on something like the TAPPS method helps them to become better thinkers, and to better be able to use what they know.

## REFERENCES

1. Bloom, Benjamin S. (ed.). *Taxonomy of Educational Objectives, Handbook I: Cognitive Domain.* New York: David McKay Co., 1956.

2. Bloom, Benjamin S., and Broder, Lois J. "Problem Solving Processes of College Students." Chicago: Supplementary Educational Monograph No. 73, University of Chicago Press, 1950.

3. Brownell, William A. "Problem Solving." Chapter 12 in Nelson B. Henry (ed.), *The Psychology of Learning* , 41st Yearbook of the National Society for the Study of Education. Chicago: NSEE, 1942.

4. Lochhead, John. "Teaching Analytical Reasoning Through Thinking Aloud Pair Problem Solving." In James E. Stice (ed.), *Teaching Thinking Through Problem Solving* . New Directions for Teaching and Learning, No.30. San Francisco: Jossey-Bass (to be published in June 1987).

5. Polya, Gyorgy. *How To Solve It* (2nd ed.). Princeton: Princeton University Press, 1971.

6. Whimbey, Arthur E., and Lochhead, John. *Problem Solving and Comprehension* (3rd ed.). Hillsdale, NJ: Lawrence Erlbaum Assoc., 1982.

7. Whimbey, Arthur E., and Lochhead, John. *Beyond Problem Solving and Comprehension.* Hillsdale, NJ: Lawrence Erlbaum Assoc., 1984.

8. Woods, Donald R., et al. "Teaching Problem-Solving Skills." *Engineering Education* , *66* (No.3), 238-243 (December 1975).

9. Woods, Donald R. Problem Solving Workshop. Annual Conference of American Society for Engineering Education, University of Massachusetts, Amherst, June 22, 1980.

10. Woodworth, Robert S. *Experimental Psychology* . New York: Holt & Co., 1938.

## OTHER SOURCES

Besides the references cited here, there are many other sources of information for those interested in knowing more about problem solving. The following are recommended:

1. Rubinstein, Moshe F. *Tools for Thinking and Problem Solving.* Englewood Cliffs, NJ: Prentice-Hall, 1986.

2. Stice, James E. (ed.) *Developing Critical Thinking and Problem-Solving Abilities*. New Directions for Teaching and Learning, No. 30. San Francisco: Jossey-Bass, June 1987.

3. Woods, Donald R. "PS Newsletter." (This is a bimonthly newsletter which Woods began in 1979. It gives abstracts of articles and books on problem solving, short reports and discussions of things people have tried in teaching PS skills, and a calendar of events for upcoming workshops and conferences dealing with PS. Coverage is world-wide. The newsletter costs $8 per year. To subscribe, send check or money order to Dr. Donald R. Woods, Department of Chemical Engineering, McMaster University, Hamilton, Ontario, Canada L8S 4L7.)

4. Young, Robert E. (ed.). *Fostering Critical Thinking*. New Directions for Teaching and Learning, No. 3. San Francisco: Jossey-Bass, 1980.

5. Dover Publications. Dover publishes out-of-print classics from many fields, as well as original publications. One of their paperback series deals with mathematical puzzles and logic problems. Many of the problems in these books are useful for introducing students to the TAPPS process, before you begin to use problems more germane to your course. Three of the Dover books which I have used are:

   a. Wylie, C.R., Jr. *101 puzzles in Thought and Logic* . New York: Dover Publications, 1957. (Math not required.)

   b. Phillips, H. *My Best Problems in Logic and Reasoning* . New York: Dover Publications, 1961. (Wicked little puzzles, math not required.)

   c. Graham, L.A. *Ingenious Mathematical Problems and Methods.* New York: Dover Publications, 1959. (These problems are hard, and many require considerable mathematical sophistication.)