61A Lecture 19

Wednesday, October 10

Generic Functions, Continued

A function might want to operate on multiple data types

Last time:

- Polymorphic functions using message passing
- Interfaces: collections of messages with a meaning for each
- Two interchangeable implementations of complex numbers

Today:

- An arithmetic system over related types
- Type dispatching instead of message passing
- Data-directed programming
- Type coercion

What's different? Today's generic functions apply to multiple arguments that don't share a common interface

Rational Numbers

Rational numbers represented as a numerator and denominator

```
class Rational(object):

    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numer = numer // g
        self.denom = denom // g

    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)


def add_rational(x, y):
    nx, dx = x.numer, x.denom
    ny, dy = y.numer, y.denom
    return Rational(nx * dy + ny * dx, dx * dy)

def mul_rational(x, y):
    return Rational(x.numer * y.numer, x.denom * y.denom)
```

Greatest common divisor

Complex Numbers: the Rectangular Representation

```
class ComplexRI(object):

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

    def __repr__(self):
        return 'ComplexRI({0}, {1})'.format(self.real,
                                            self.imag)
```

Might be either ComplexMA or ComplexRI instances

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)
```

Special Methods

Adding instances of user-defined classes with `__add__`.

Demo

```
>>> ComplexRI(1, 2) + ComplexMA(2, 0)
ComplexRI(3.0, 2.0)
>>> ComplexRI(0, 1) * ComplexRI(0, 1)
ComplexMA(1.0, 3.141592653589793)
```

http://getpython3.com/diveintopython3/special-method-names.html

http://docs.python.org/py3k/reference/datamodel.html#special-method-names

The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

How do we add a complex number and a rational number together?

add_rational mul_rational

Rational numbers as numerators & denominators

add_complex mul_complex

Complex numbers as two-dimensional vectors

There are many different techniques for doing this!

Type Dispatching

Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

```
def iscomplex(z):
    return type(z) in (ComplexRI, ComplexMA)

def isrational(z):
    return type(z) is Rational

def add_complex_and_rational(z, r):
    return ComplexRI(z.real + r.numer/r.denom, z.imag)

def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational."""
    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
        add_rational(z1, z2)
```

Converted to a real number (float)

Demo

Tag-Based Type Dispatching

Idea: Use dictionaries to dispatch on type

```
def type_tag(x):
    return type_tag.tags[type(x)]

type_tag.tags = {ComplexRI: 'com',
                 ComplexMA: 'com',
                 Rational:  'rat'}

def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add.implementations[types](z1, z2)

add.implementations = {}
add.implementations[('com', 'com')] = add_complex
add.implementations[('rat', 'rat')] = add_rational
add.implementations[('com', 'rat')] = add_complex_and_rational
add.implementations[('rat', 'com')] = add_rational_and_complex
```

Declares that ComplexRI and ComplexMA should be treated uniformly

`lambda r, z: add_complex_and_rational(z, r)`

Type Dispatching Analysis

Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types

Extensible: Any new numeric type can "install" itself into the existing system by adding new entries to various dictionaries

```
def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add.implementations[types](z1, z2)
```

Question: How many cross-type implementations are required to support *m* types and *n* operations?

integer, rational, real, complex

$$m \cdot (m-1) \cdot n$$

add, subtract, multiply, divide

$$4 \cdot (4-1) \cdot 4 = 48$$

Type Dispatching Analysis

Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types

Extensible: Any new numeric type can "install" itself into the existing system by adding new entries to various dictionaries

Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

Type Dispatching

Message Passing

Data-Directed Programming

There's nothing addition-specific about add_by_type

Idea: One dispatch function for (operator, types) pairs

```
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply.implementations[key](x, y)
```

Demo

Coercion

Idea: Some types can be converted into other types

Takes advantage of structure in the type system

```
>>> def rational_to_complex(x):
        return ComplexRI(x.numer/x.denom, 0)

>>> coercions = {('rat', 'com'): rational_to_complex}
```

Question: Can any numeric type be coerced into any other?

Question: Have we been repeating ourselves with data-directed programming?

Applying Operators with Coercion

1. Attempt to coerce arguments into values of the same type

2. Apply type-specific (not cross-type) operations

```
def coerce_apply(operator_name, x, y):
    tx, ty = type_tag(x), type_tag(y)
    if tx != ty:
        if (tx, ty) in coercions:
            tx, x = ty, coercions[(tx, ty)](x)
        elif (ty, tx) in coercions:
            ty, y = tx, coercions[(ty, tx)](y)
        else:
            return 'No coercion possible.'
    assert tx == ty
    key = (operator_name, tx)
    return coerce_apply.implementations[key](x, y)
```

Demo

Coercion Analysis

Minimal violation of abstraction barriers: we define cross-type coercion as necessary, but use abstract data types

Requires that all types can be coerced into a common type

More sharing: All operators use the same coercion scheme

Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

From	To	Coerce
Complex	Rational	
Rational	Complex	

Type	Add	Multiply
Complex		
Rational		