

61A Lecture 6

Friday, September 13

Announcements

- Homework 2 due Tuesday 9/17 @ 11:59pm
- Project 2 due Thursday 9/19 @ 11:59pm
- Optional Guerrilla section next Monday for students to master higher-order functions
 - Organized by Andrew Huang and the readers
 - Work in a group on a problem until everyone in the group understands the solution
- Midterm 1 on Monday 9/23 from 7pm to 9pm
 - Details and review materials will be posted early next week
 - There will be a web form for students who cannot attend due to a conflict

Lambda Expressions

(Demo)

Lambda Expressions

```
>>> ten = 10
>>> square = x * x
>>> square = lambda x: x * x
>>> square(4)
16
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function with formal parameter x that returns the value of "x * x"

Must be a single expression

Lambda expressions are not common in Python, but important in general
Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

 `square = lambda x: x * x` **VS** `def square(x):`
`return x * x` 

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the environment in which they were defined.
- Both bind that function to the name `square`.
- Only the `def` statement gives the function an intrinsic name.



Example: <http://goo.gl/XH54uE>

Currying

Function Currying

```
def make_adder(n):
    return lambda k: n + k
```

```
>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

There's a general relationship between these functions

(Demo)

Newton's Method

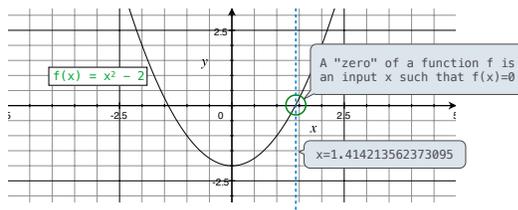
Currying: Transforming a multi-argument function into a single-argument, higher-order function.

Currying was discovered by Moses Schönfinkel and re-discovered by Haskell Curry.

⌵
Schönfinkeling?

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!



Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is \sqrt{a} . (We're solving the equation $x^2 = a$.)

Newton's Method

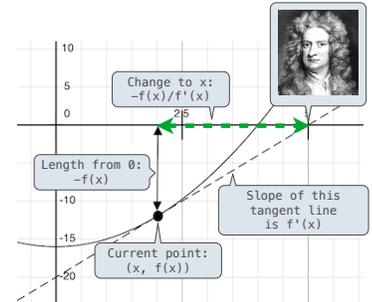
Given a function f and initial guess x ,

Repeatedly improve x :

1. Compute the value of f at the guess: $f(x)$
2. Compute the *derivative* of f at the guess: $f'(x)$
3. Update guess x to be:

$$x - \frac{f(x)}{f'(x)}$$

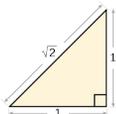
Finish when $f(x) = 0$ (or close enough)



http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

How to find the **square root** of 2?

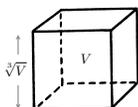


```
>>> f = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

$f(x) = x^2 - 2$
 $f'(x) = 2x$

Applies Newton's method until $|f(x)| < 10^{-15}$, starting at 1

How to find the **cube root** of 729?



```
>>> g = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

$g(x) = x^3 - 729$
 $g'(x) = 3x^2$

Iterative Improvement

Special Case: Square Roots

How to compute `square_root(a)`

Idea: Iteratively refine a guess x about the square root of a

Update:

$$x = \frac{x + \frac{a}{x}}{2}$$

Babylonian Method

Implementation questions:

What *guess* should start the computation?

How do we know when we are finished?

Special Case: Cube Roots

How to compute `cube_root(a)`

Idea: Iteratively refine a guess x about the cube root of a

Update:

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

Implementation questions:

What *guess* should start the computation?

How do we know when we are finished?

Implementing Newton's Method

(Demo)