

Function Currying

```
def make_adder(n):
    return lambda k: n + k
```

```
>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

There's a general relationship between these functions

(Demo)

Newton's Method

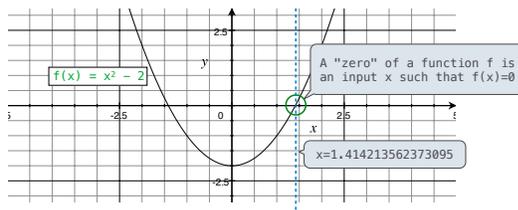
Currying: Transforming a multi-argument function into a single-argument, higher-order function.

Currying was discovered by Moses Schönfinkel and re-discovered by Haskell Curry.

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Schönfinkeling?

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!



Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is \sqrt{a} . (We're solving the equation $x^2 = a$.)

Newton's Method

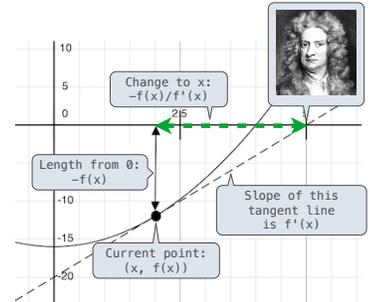
Given a function f and initial guess x ,

Repeatedly improve x :

1. Compute the value of f at the guess: $f(x)$
2. Compute the *derivative* of f at the guess: $f'(x)$
3. Update guess x to be:

$$x - \frac{f(x)}{f'(x)}$$

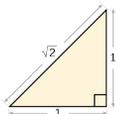
Finish when $f(x) = 0$ (or close enough)



http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

How to find the **square root** of 2?

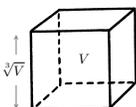


```
>>> f = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

$f(x) = x^2 - 2$
 $f'(x) = 2x$

Applies Newton's method until $|f(x)| < 10^{-15}$, starting at 1

How to find the **cube root** of 729?



```
>>> g = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

$g(x) = x^3 - 729$
 $g'(x) = 3x^2$

Iterative Improvement

Special Case: Square Roots

How to compute `square_root(a)`

Idea: Iteratively refine a guess x about the square root of a

Update:

$$x = \frac{x + \frac{a}{x}}{2}$$

Babylonian Method

Implementation questions:

What *guess* should start the computation?

How do we know when we are finished?

Special Case: Cube Roots

How to compute `cube_root(a)`

Idea: Iteratively refine a guess x about the cube root of a

Update:

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

Implementation questions:

What *guess* should start the computation?

How do we know when we are finished?

Implementing Newton's Method

(Demo)