# 61A Lecture 21

Wednesday, October 23

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- Respond to lecture questions: http://goo.gl/FZKvgm

# Generic Functions of Multiple Arguments

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**What's different?** Today's generic functions apply to multiple arguments that *don't share a common interface*.

# Representing Numbers

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def add_rational(x, y):
    nx, dx = x.numer, x.denom
    ny, dy = y.numer, y.denom
    return Rational(nx * dy + ny * dx, dx * dy)
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def add_rational(x, y):
    nx, dx = x.numer, x.denom
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    return Rational(nx * dy + ny * dx, dx * dy)

def mul_rational(x, y):
    return Rational(x.numer * y.numer, x.denom * y.denom)
```

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# Complex Numbers: the Rectangular Representation

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```
class ComplexRI:

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

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def add_complex(z1, z2):
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Might be either ComplexMA or ComplexRI instances

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def add_complex(z1, z2):
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# Special Methods for Arithmetic

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http://getpython3.com/diveintopython3/special-method-names.html

http://docs.python.org/py3k/reference/datamodel.html#special-method-names

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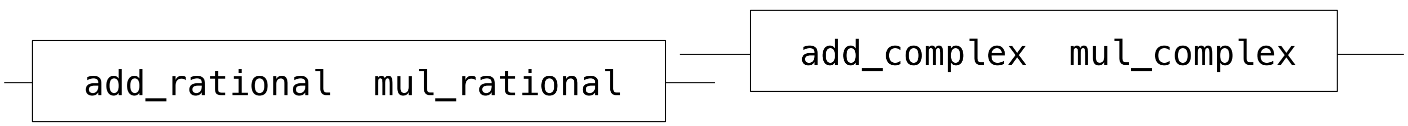


*Rational numbers as numerators & denominators*

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*How do we add a complex number and a rational number together?*

`add_rational  mul_rational`

*Rational numbers as numerators & denominators*

`add_complex  mul_complex`

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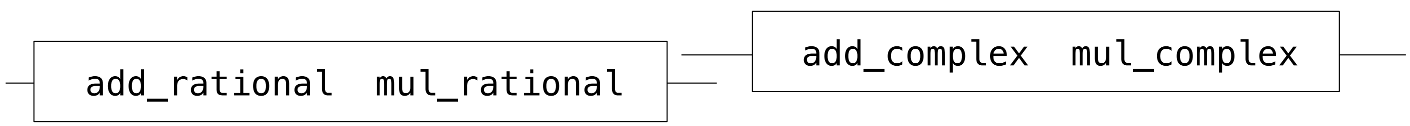


*Rational numbers as numerators & denominators*

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There are many different techniques for doing this!

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    else:
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| Arg 1 | Arg 2 | Add | Multiply |
|---|---|---|---|
| Complex | Complex | | |
| Rational | Rational | | |
| Complex | Rational | | |
| Rational | Complex | | |

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def apply(operator_name, x, y):
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    key = (operator_name, tags)
    return apply_implementations[key](x, y)
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**Question:** Have we been repeating ourselves with data-directed programming?

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(Demo)

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