

## 61A Lecture 6

Friday, September 12

## Announcements

- Homework 2 due Monday 9/15 @ 11:59pm
- Project 1 due Wednesday 9/17 @ 11:59pm
- Optional Guerrilla section Saturday 9/13 @ 12:30pm in 306 Soda about higher-order functions
  - Organized by Andrew Huang and the readers
  - Work in a group on a problem until everyone in the group understands the solution
- Project party on Monday 9/15, 3pm-4pm in Wozniak Lounge and 6pm-8pm in 2050 VLSB
- Midterm 1 on Monday 9/22 from 7pm to 9pm
  - Details and review materials will be posted next week
  - There will be a web form for students who cannot attend due to a conflict
- There's a pinned Piazza thread to find partners

## Lambda Expressions

(Demo)

## Lambda Expressions

```
>>> x = 10
>>> square = x * x
>>> square = lambda x: x * x
>>> square(4)
16
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function with formal parameter x that returns the value of `x * x`

Must be a single expression

Lambda expressions are not common in Python, but important in general  
Lambda expressions in Python cannot contain statements at all!

## Lambda Expressions Versus Def Statements

`square = lambda x: x * x` VS `def square(x): return x * x`

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the frame in which they were defined.
- Both bind that function to the name `square`.
- Only the `def` statement gives the function an intrinsic name.



## Currying

## Function Currying

```
def make_adder(n):
    return lambda k: n + k
```

```
>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

There's a general relationship between these functions

(Demo)

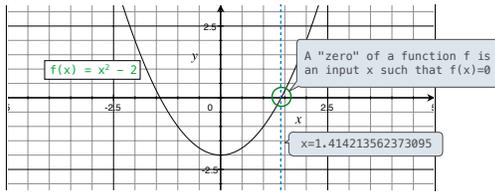
Curry: Transform a multi-argument function into a single-argument, higher-order function.  
Currying was discovered by Moses Schönfinkel and re-discovered by Haskell Curry.

Schönfinkeling?

## Newton's Method

## Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!



Application: a method for computing square roots, cube roots, etc.

The positive zero of  $f(x) = x^2 - a$  is  $\sqrt{a}$ . (We're solving the equation  $x^2 = a$ .)

## Newton's Method

Given a function  $f$  and initial guess  $x$ ,

Repeatedly improve  $x$ :

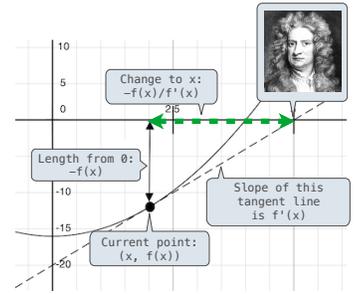
Compute the value of  $f$  at the guess:  $f(x)$

Compute the derivative of  $f$  at the guess:  $f'(x)$

Update guess  $x$  to be:

$$x - \frac{f(x)}{f'(x)}$$

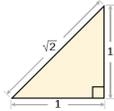
Finish when  $f(x) = 0$  (or close enough)



<http://en.wikipedia.org/wiki/File:NewtonIteration Ani.gif>

## Using Newton's Method

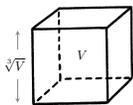
How to find the square root of 2?



```
>>> f = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

Applies Newton's method

How to find the cube root of 729?



```
>>> g = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

$g(x) = x^3 - 729$   
 $g'(x) = 3x^2$

Iterative Improvement

## Special Case: Square Roots

How to compute `square_root(a)`

**Idea:** Iteratively refine a guess  $x$  about the square root of  $a$

Update: 
$$x = \frac{x + \frac{a}{x}}{2}$$

Babylonian Method

**Implementation questions:**

What guess should start the computation?

How do we know when we are finished?

## Special Case: Cube Roots

How to compute `cube_root(a)`

**Idea:** Iteratively refine a guess  $x$  about the cube root of  $a$

Update: 
$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

**Implementation questions:**

What guess should start the computation?

How do we know when we are finished?

## Implementing Newton's Method

(Demo)