# 61A Lecture 7

Monday, September 15

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- No lab or office hours on Tuesday 9/23 and Wednesday 9/24 (staff will be grading exams)

# Recursive Functions

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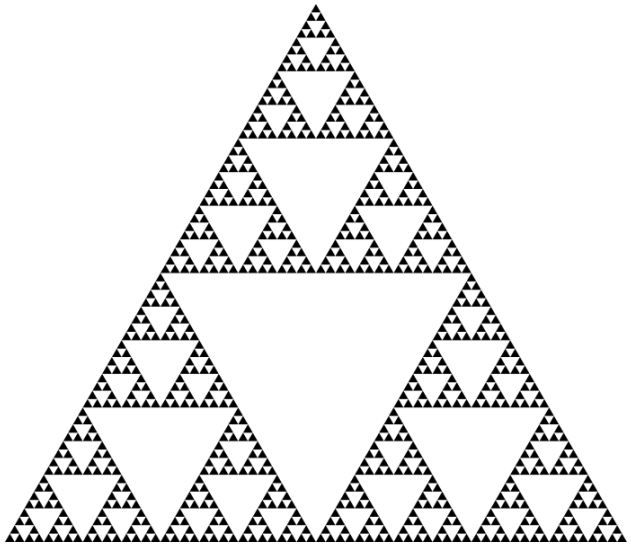

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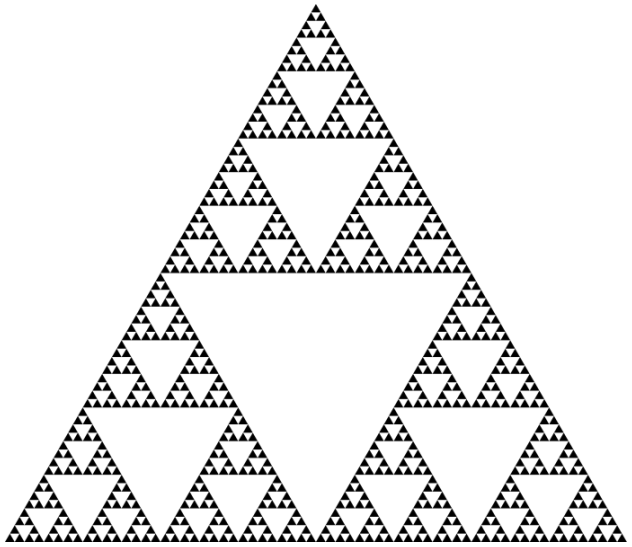

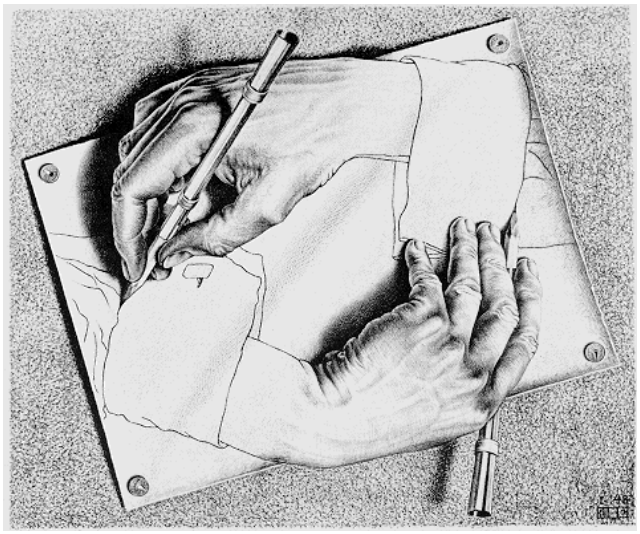

Drawing Hands, by M. C. Escher (lithograph, 1948)

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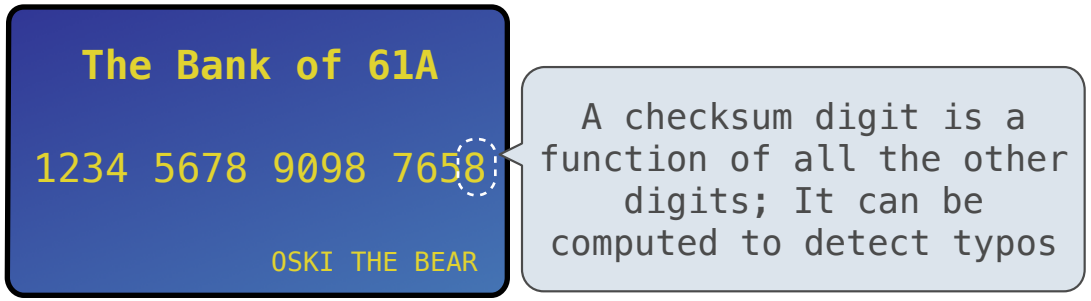

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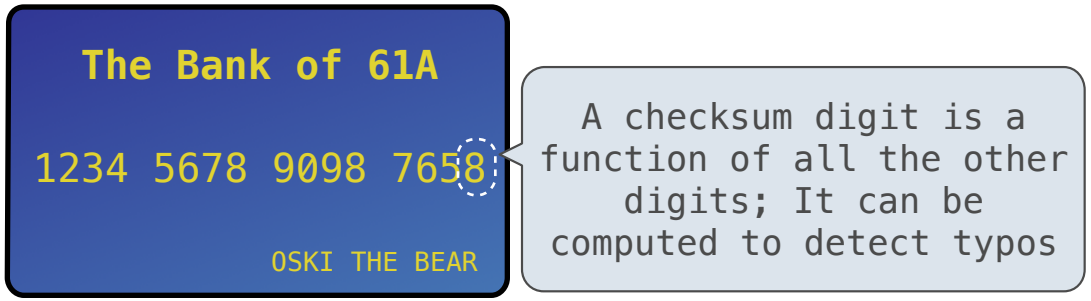


- Credit cards actually use the Luhn algorithm, which we'll implement after digit_sum.

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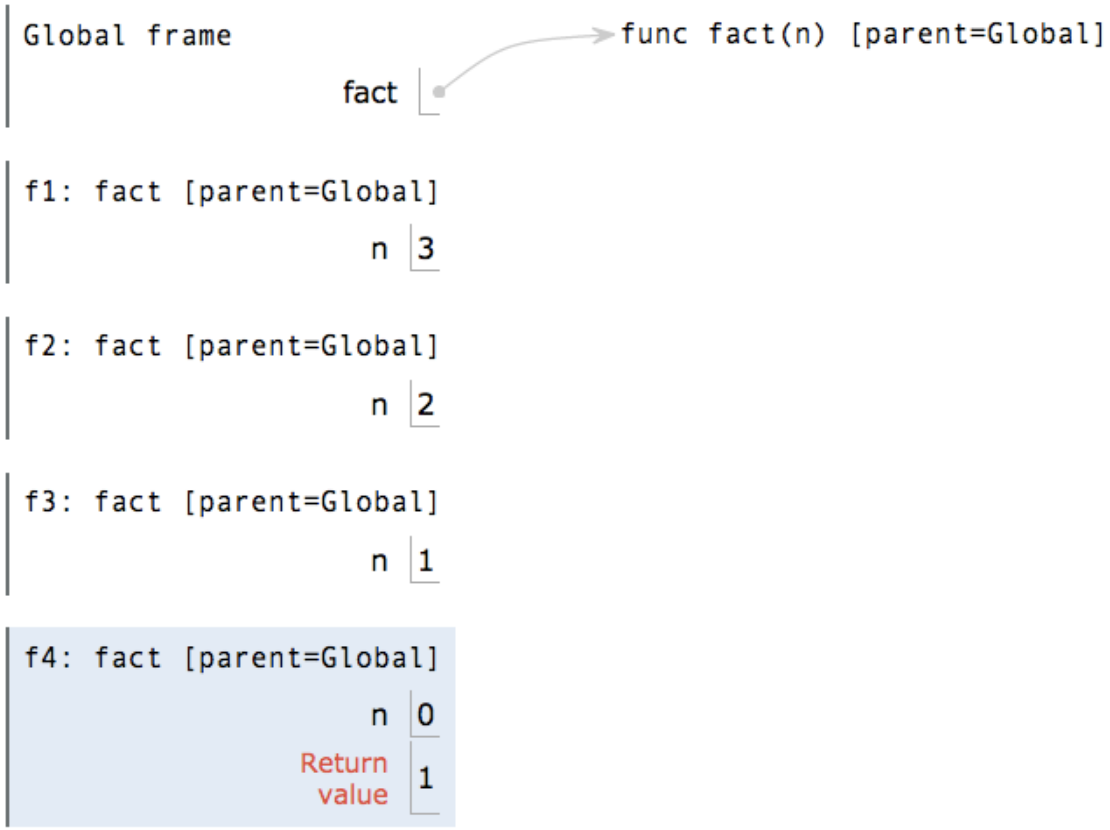


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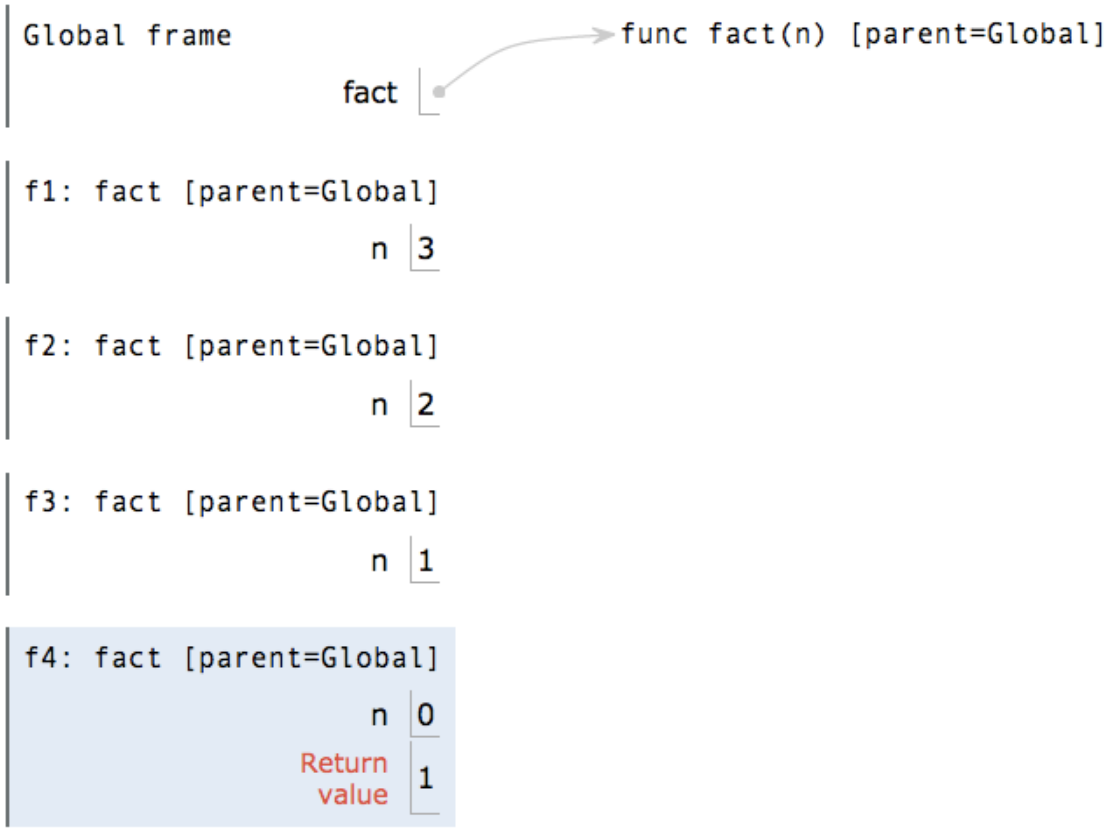


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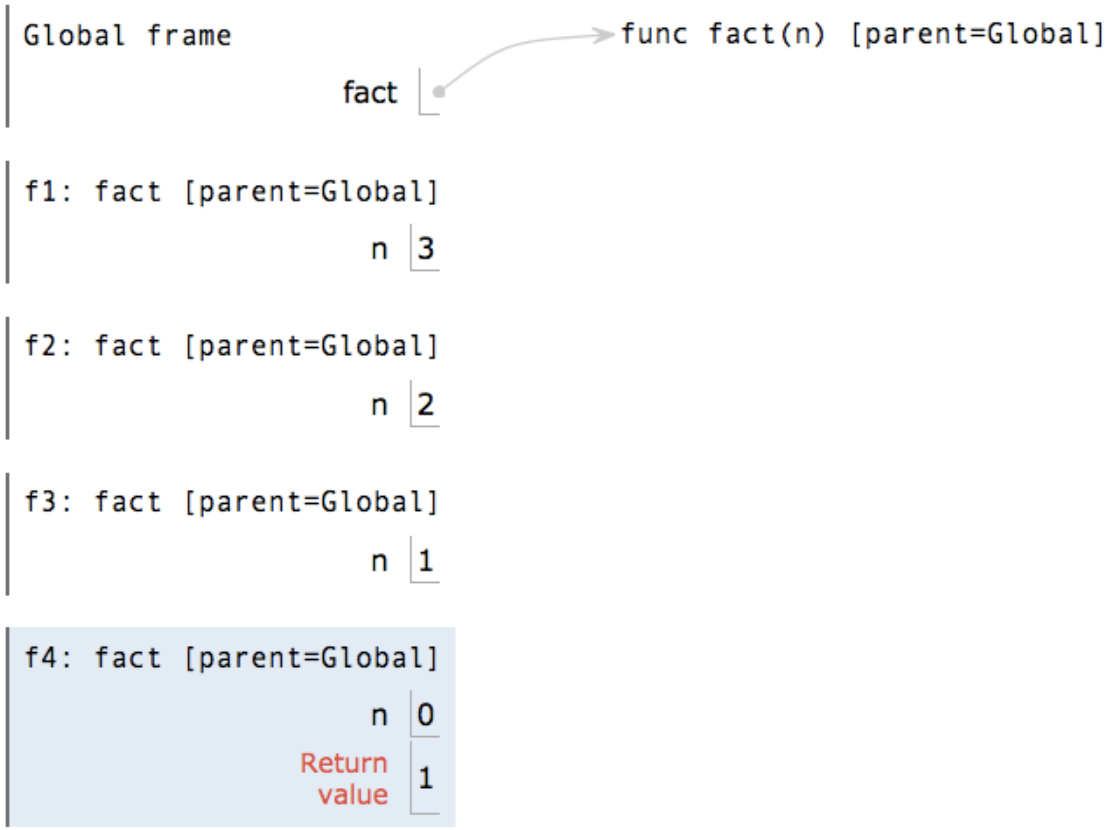


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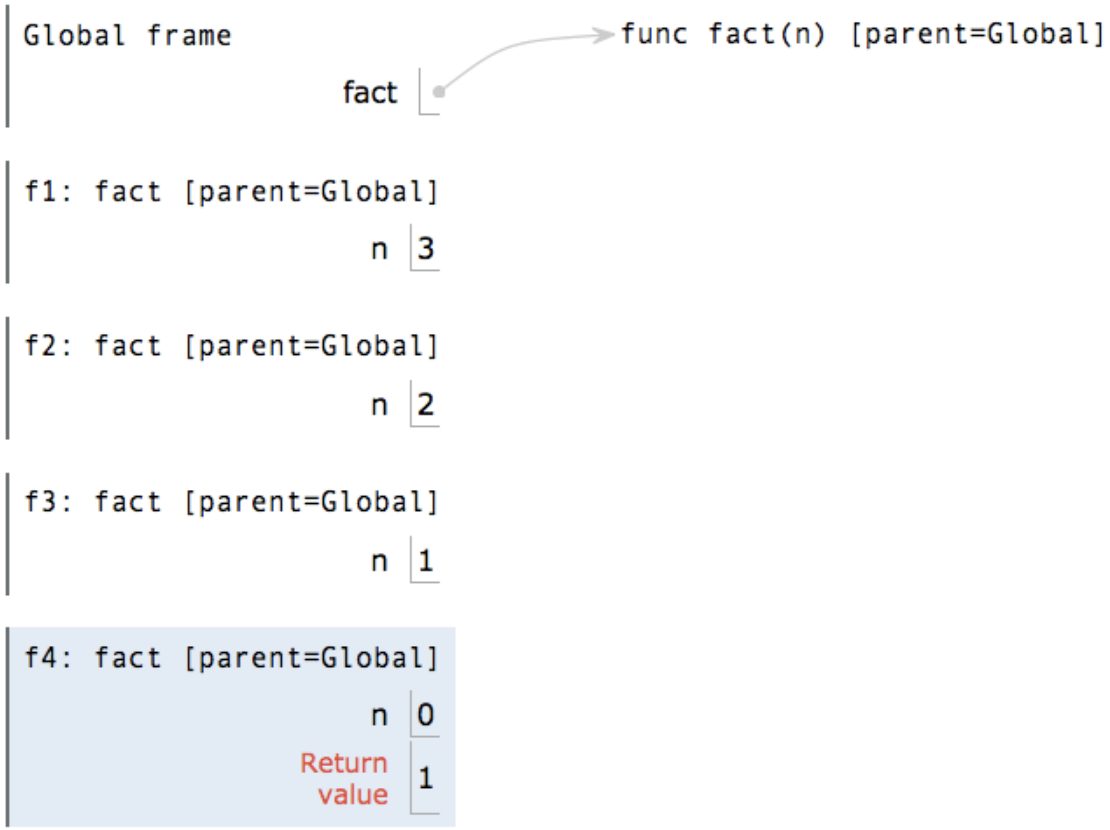


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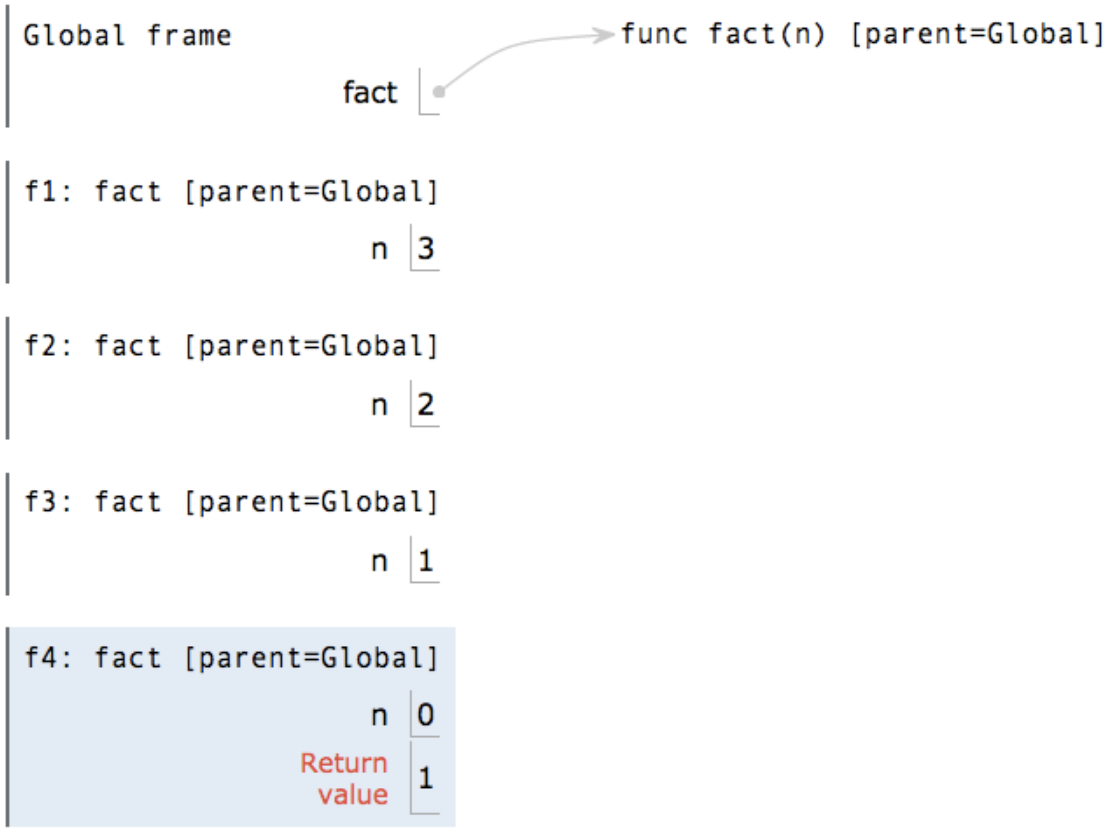


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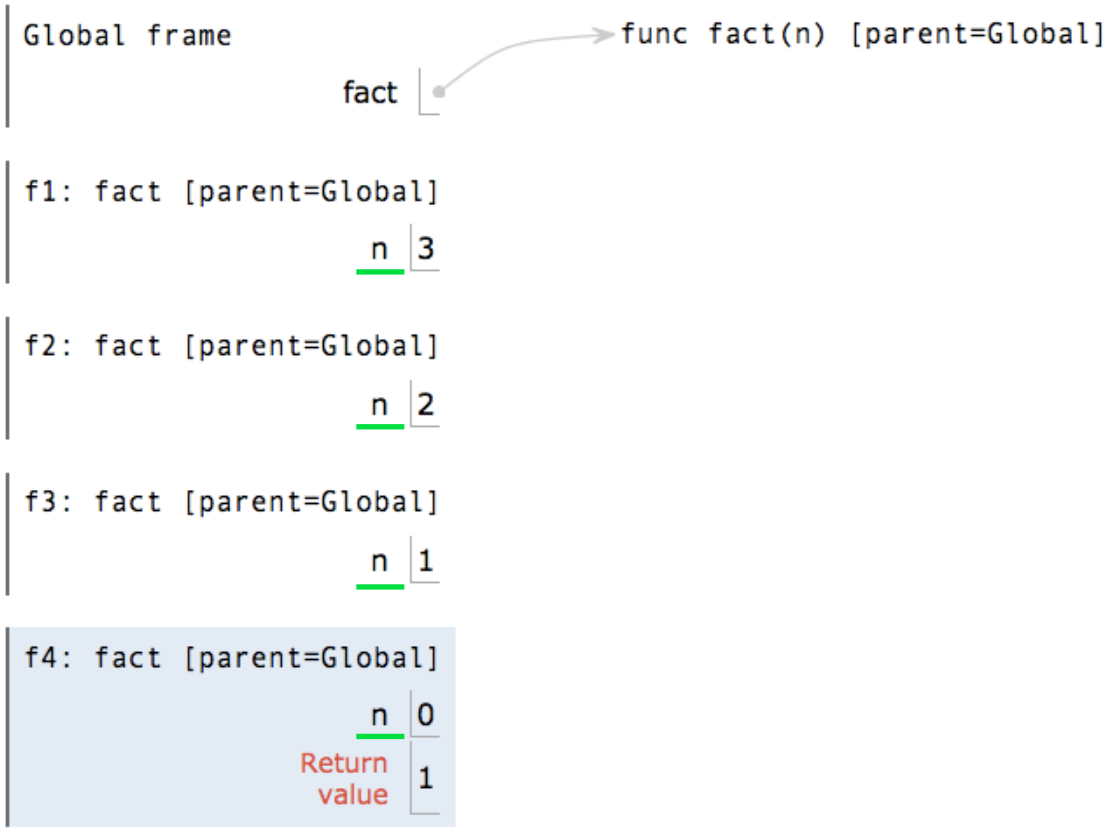


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- Each call to fact solves a simpler problem than the last: smaller n.

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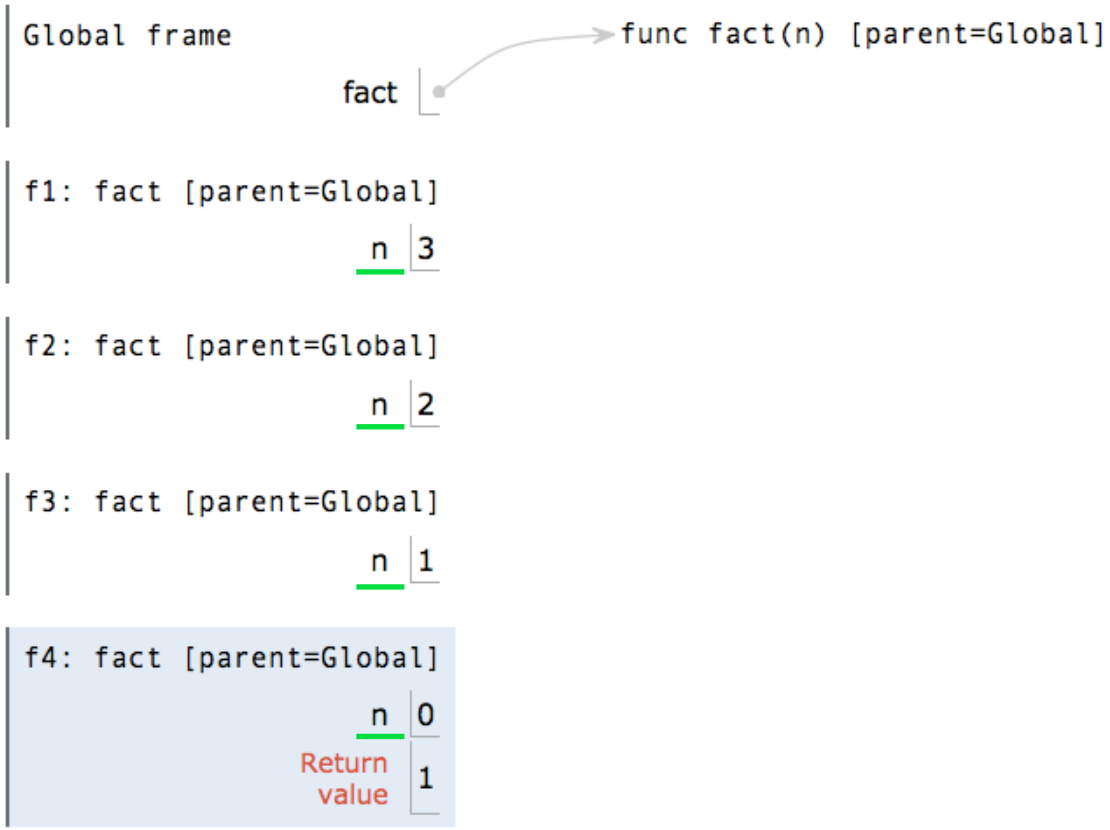


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`n, fact`

# Verifying Recursive Functions

# The Recursive Leap of Faith

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Photo by Kevin Lee, Preikestolen, Norway

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4. Verify that fact(n) is correct,
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# Mutual Recursion

# The Luhn Algorithm

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# Recursion and Iteration

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def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
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        return n
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...arguments to a recursive call