61A Lecture 8

Wednesday, September 17

Announcements

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- Optional Hog strategy contest ends Wednesday 10/1 @ 11:59pm

Hog Contest Rules

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- `Up to two people submit one entry;`
  `Max of one entry per person`

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Kaylee Mann
Yan Duan & Ziming Li
Brian Prike & Zhenghao Qian
Parker Schuh & Robert Chatham

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Fall 2014 Winners

YOUR NAME COULD BE HERE... FOREVER!

Order of Recursive Calls

The Cascade Function

(Demo)

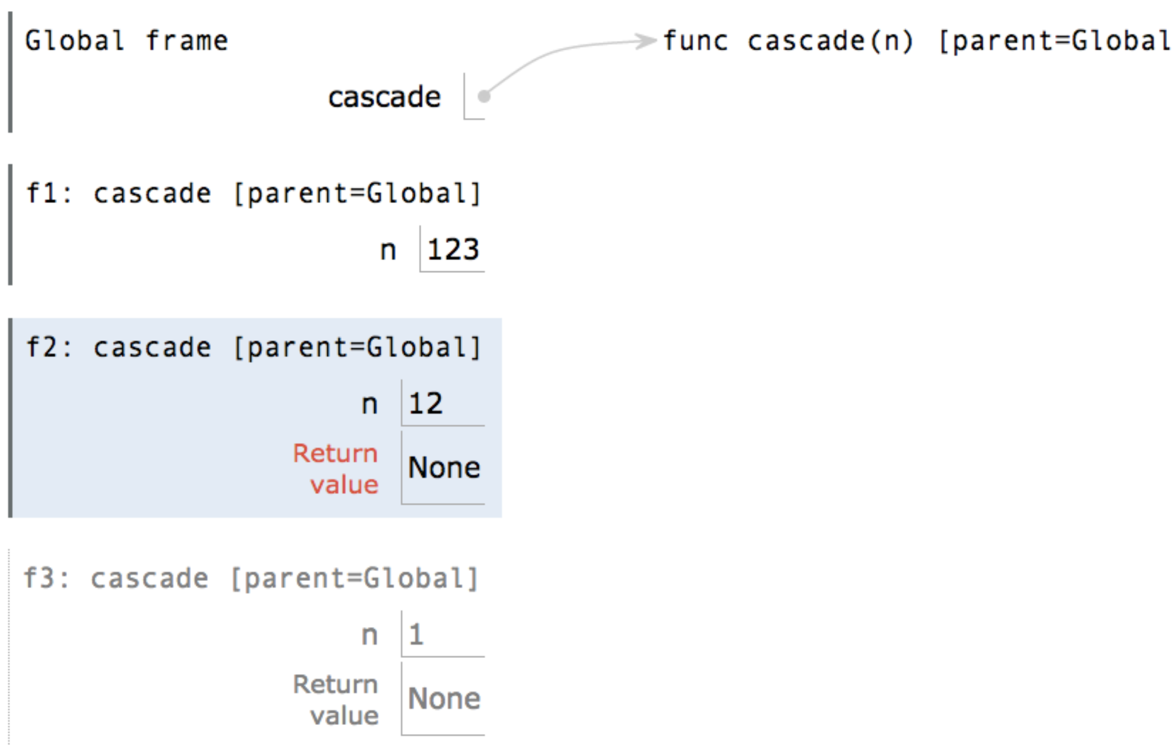
Interactive Diagram

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



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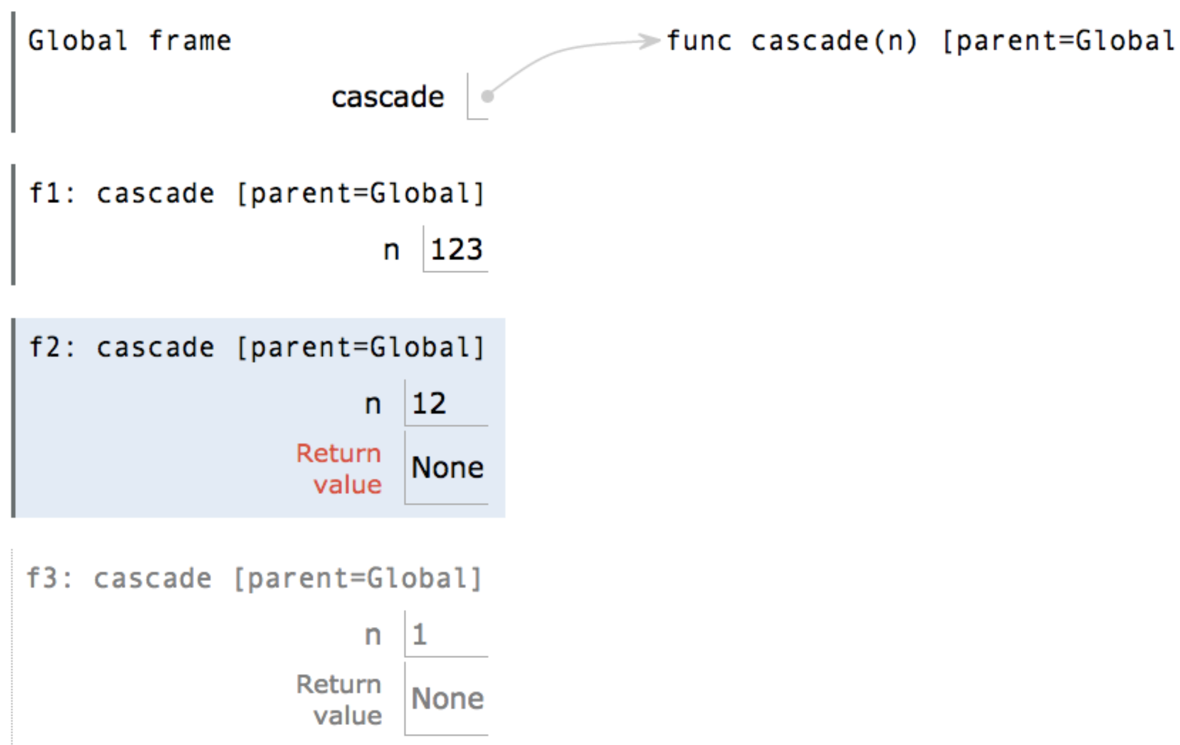

Program output:

```
123
12
1
12
```

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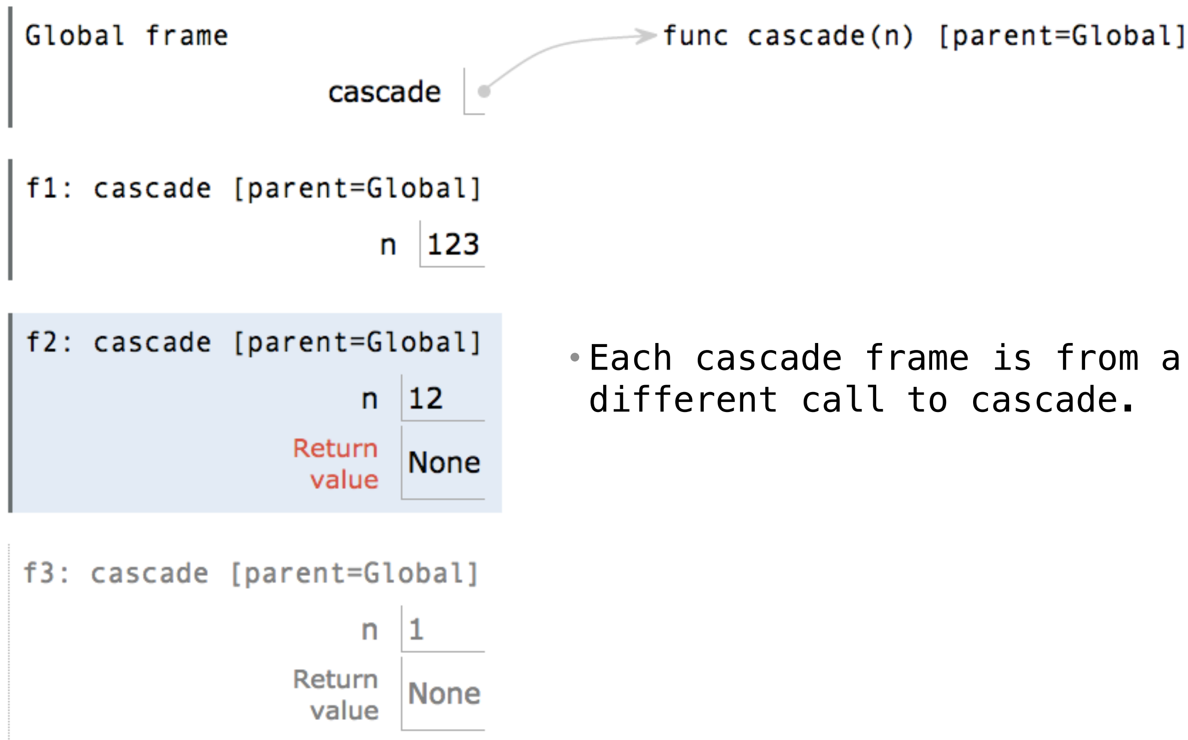

- Each cascade frame is from a different call to cascade.

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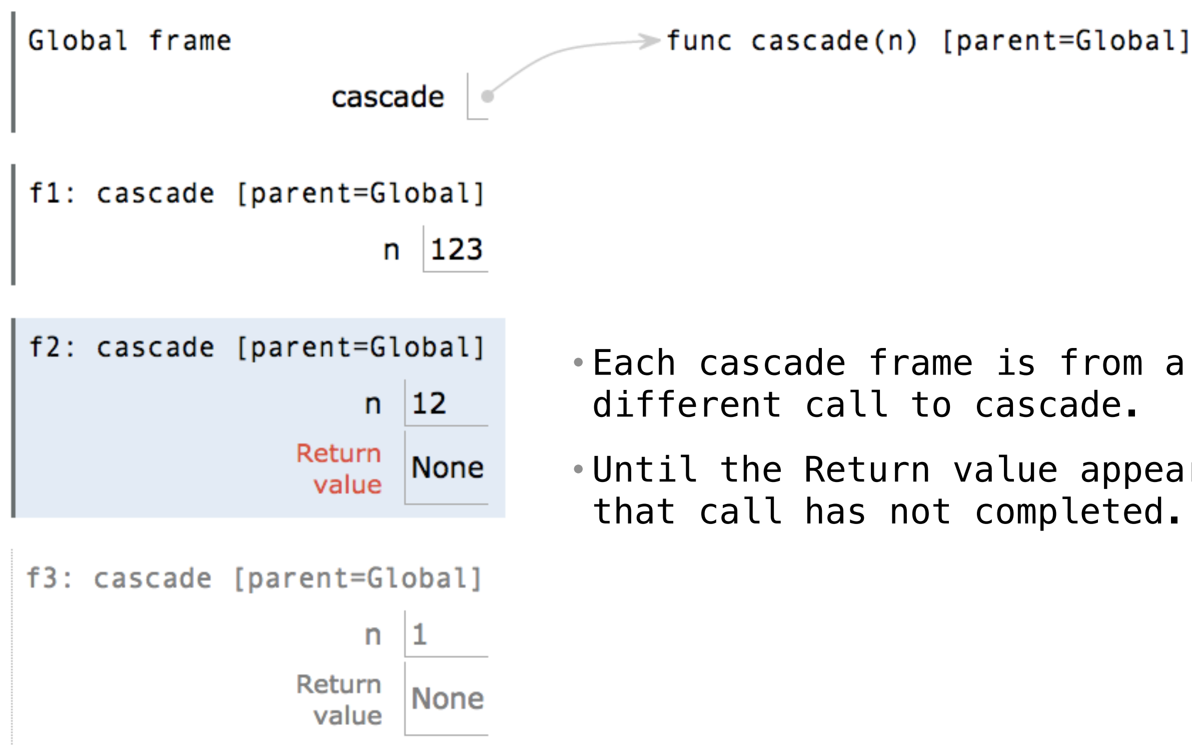

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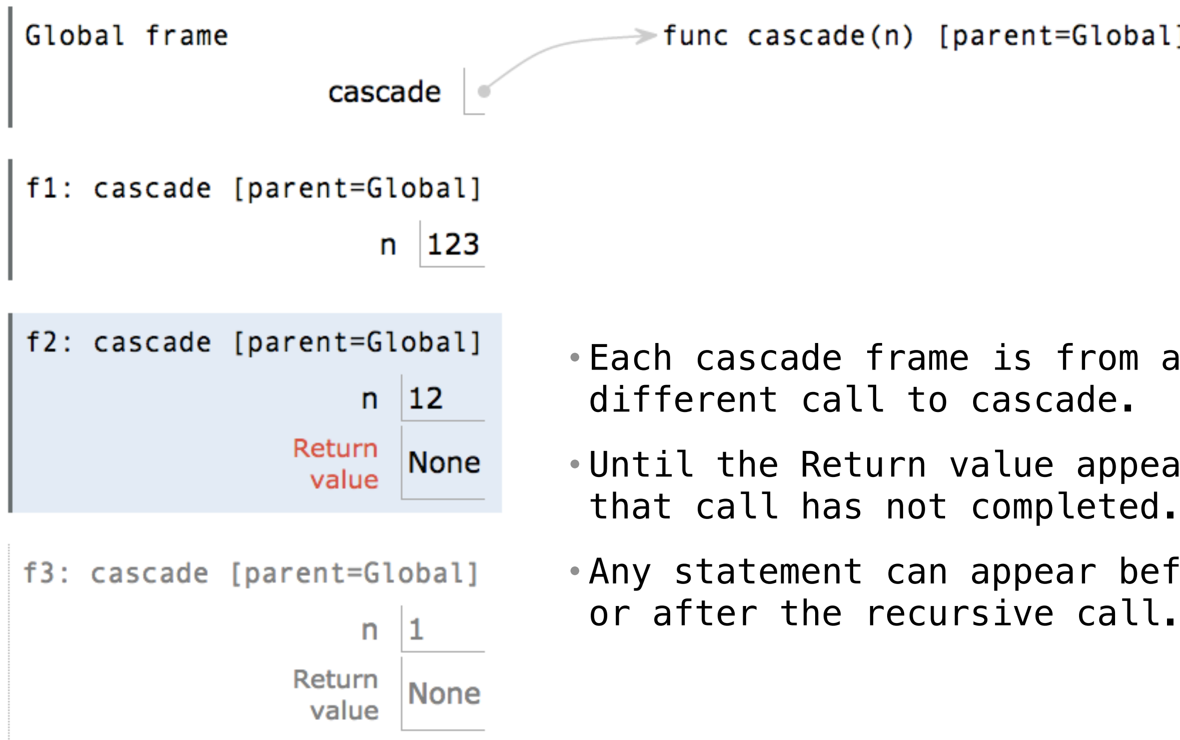

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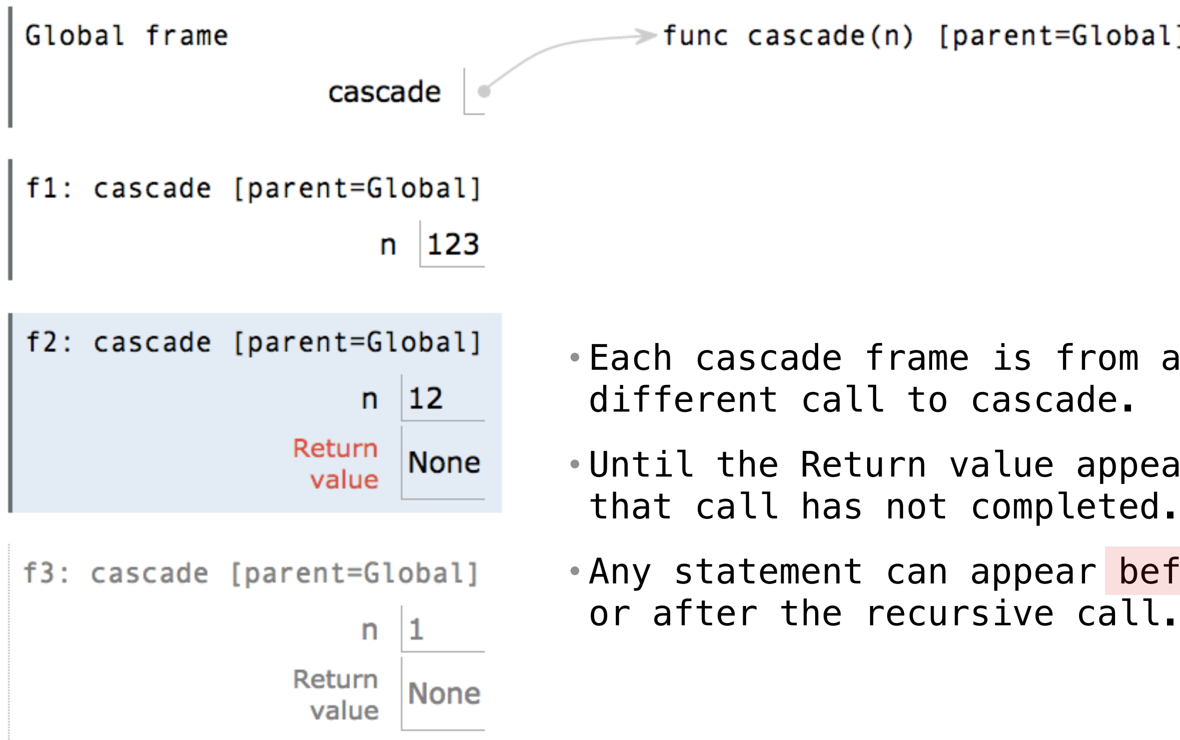

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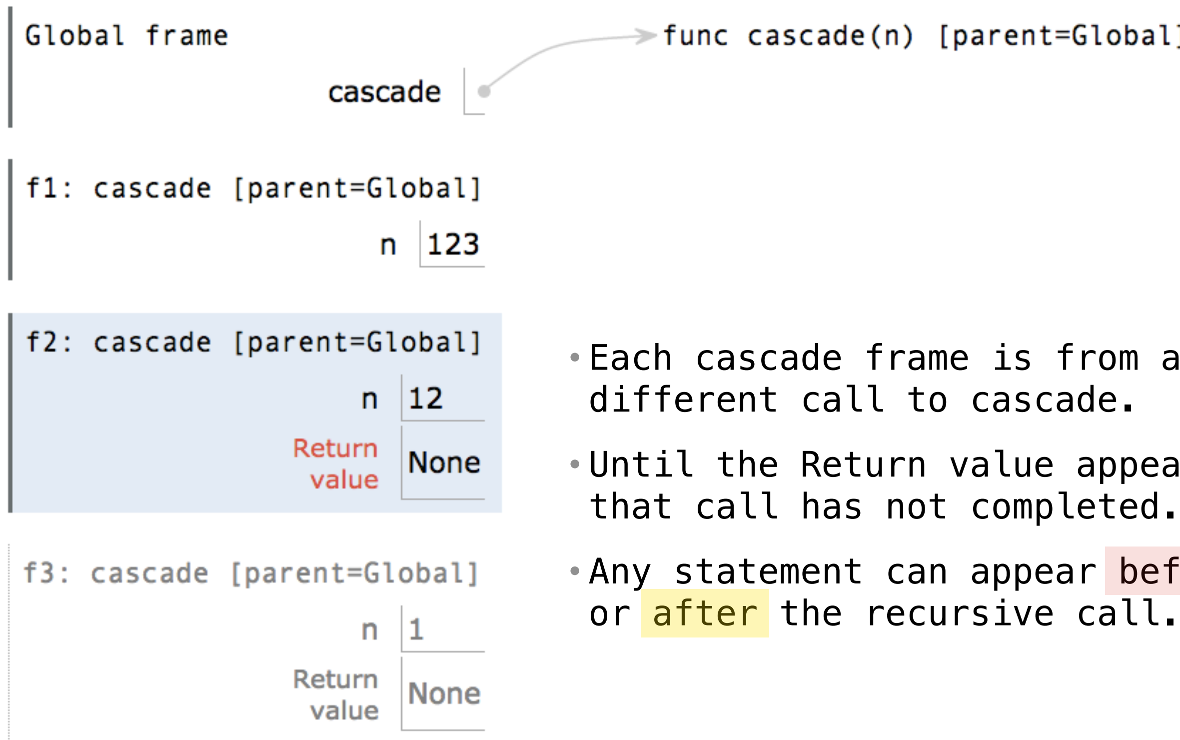

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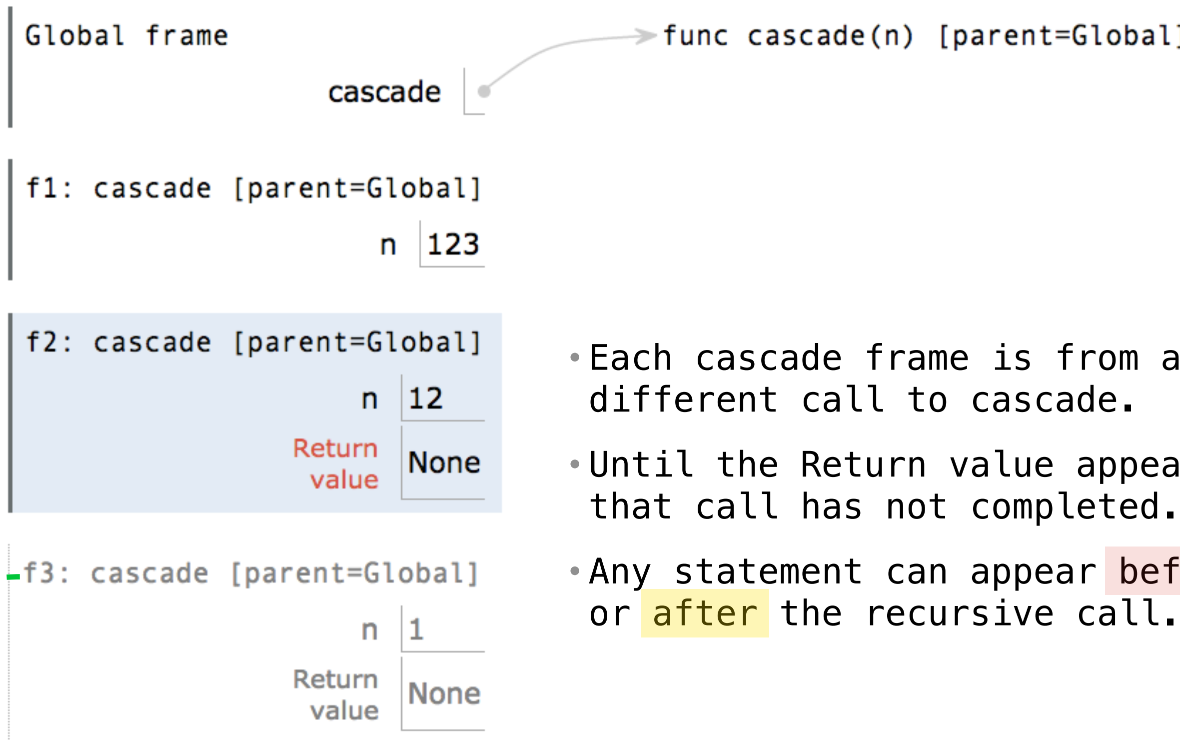

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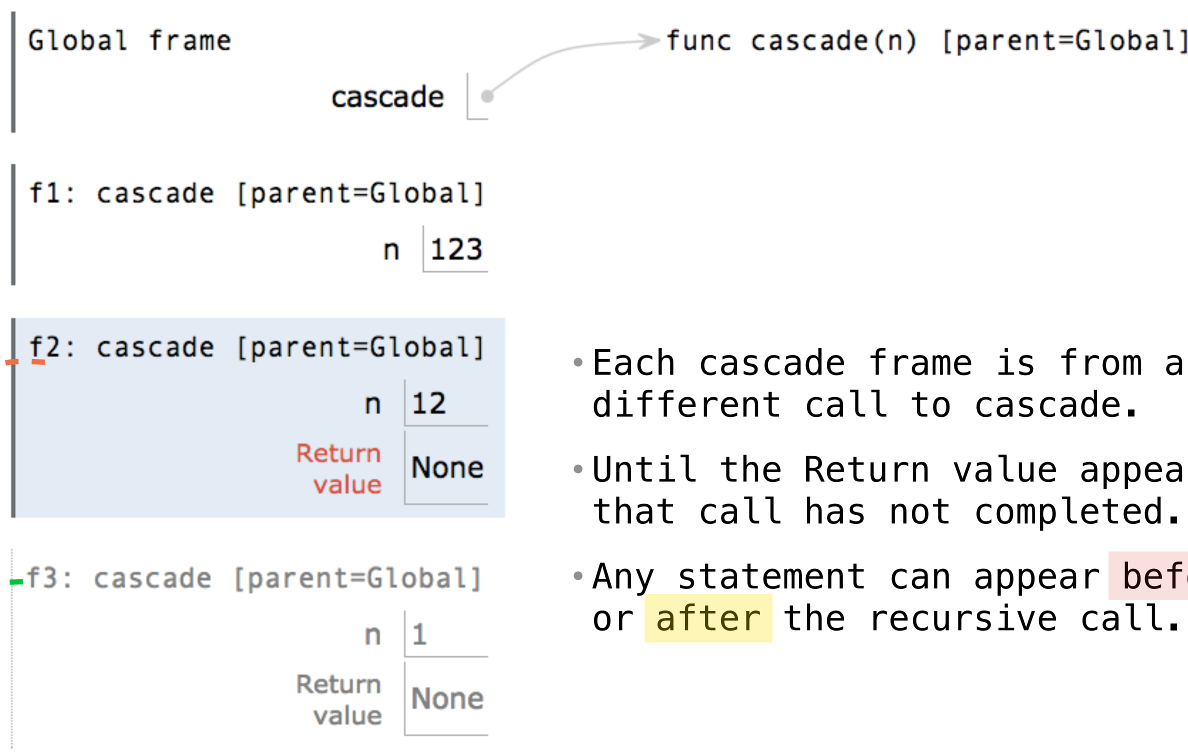

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Interactive Diagram

Two Definitions of Cascade

(Demo)

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def cascade(n):
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    else:
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```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

Two Definitions of Cascade

(Demo)

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def cascade(n):
    if n < 10:
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    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better

Two Definitions of Cascade

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    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

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- In this case, the longer implementation is more clear (at least to me)

Two Definitions of Cascade

(Demo)

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- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first

Two Definitions of Cascade

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def cascade(n):
    print(n)
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        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

Example: Inverse Cascade

Inverse Cascade

Write a function that prints an inverse cascade:

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(                    )
shrink = lambda n: f_then_g(                    )
```

Inverse Cascade

Write a function that prints an inverse cascade:

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12
123
1234
123
12
1
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def inverse_cascade(n):
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    if n:
        f(n)
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grow =   lambda n: f_then_g(grow,  print,  n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
```

Tree Recursion

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one call to that function.

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n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

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n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one call to that function.

n: 0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35

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n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

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```
def fib(n):
    if n == 0:
```

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def fib(n):
    if n == 0:
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    if n == 0:
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    elif n == 1:
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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

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A Tree-Recursive Process

The computational process of fib evolves into a tree structure

A Tree-Recursive Process

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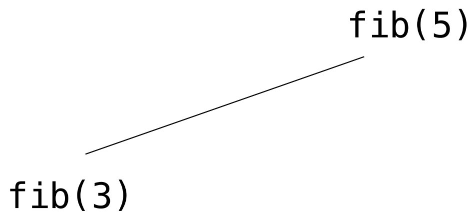
fib(5)

A Tree-Recursive Process

The computational process of fib evolves into a tree structure



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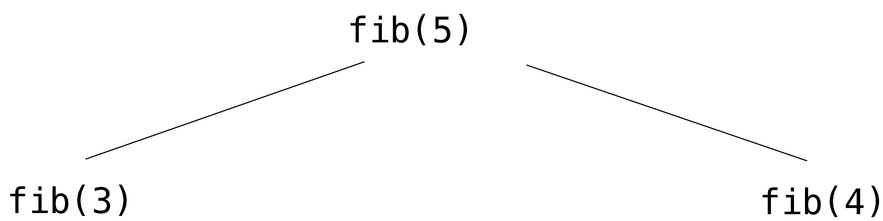
The computational process of fib evolves into a tree structure



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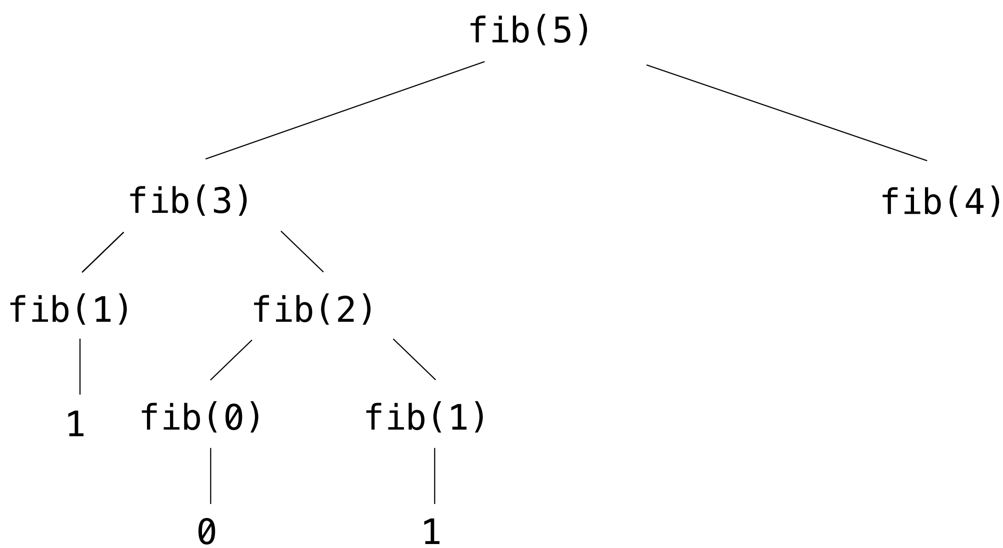

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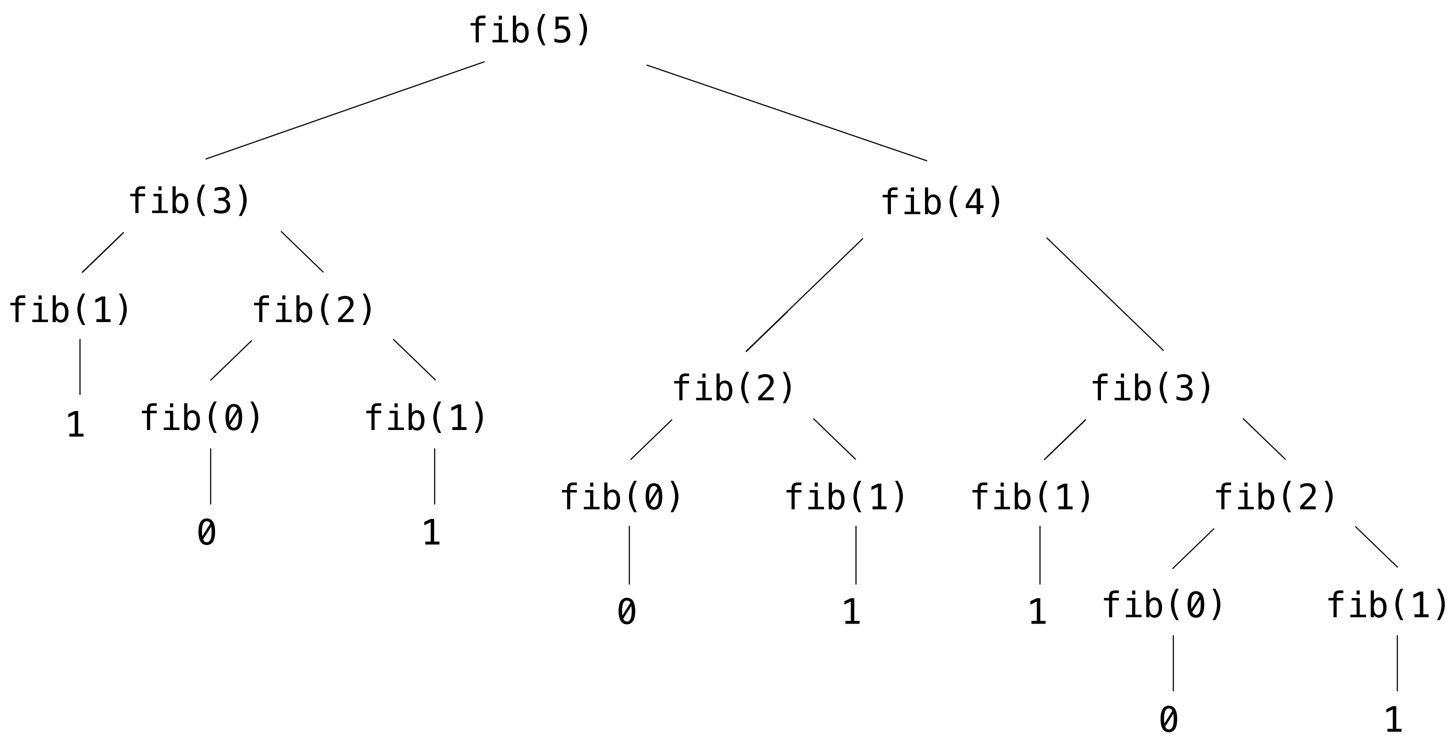
`The computational process of fib evolves into a tree structure`



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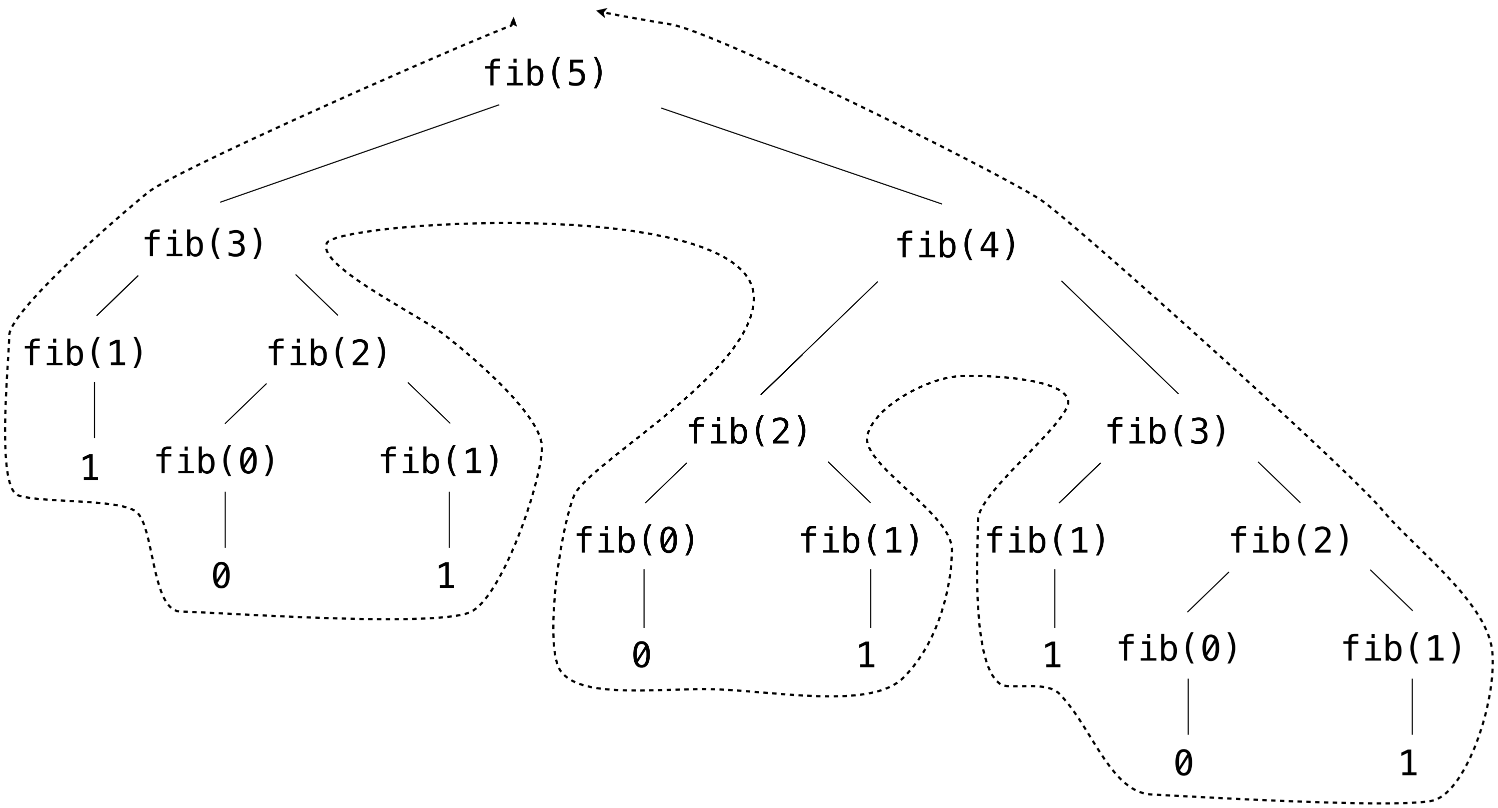

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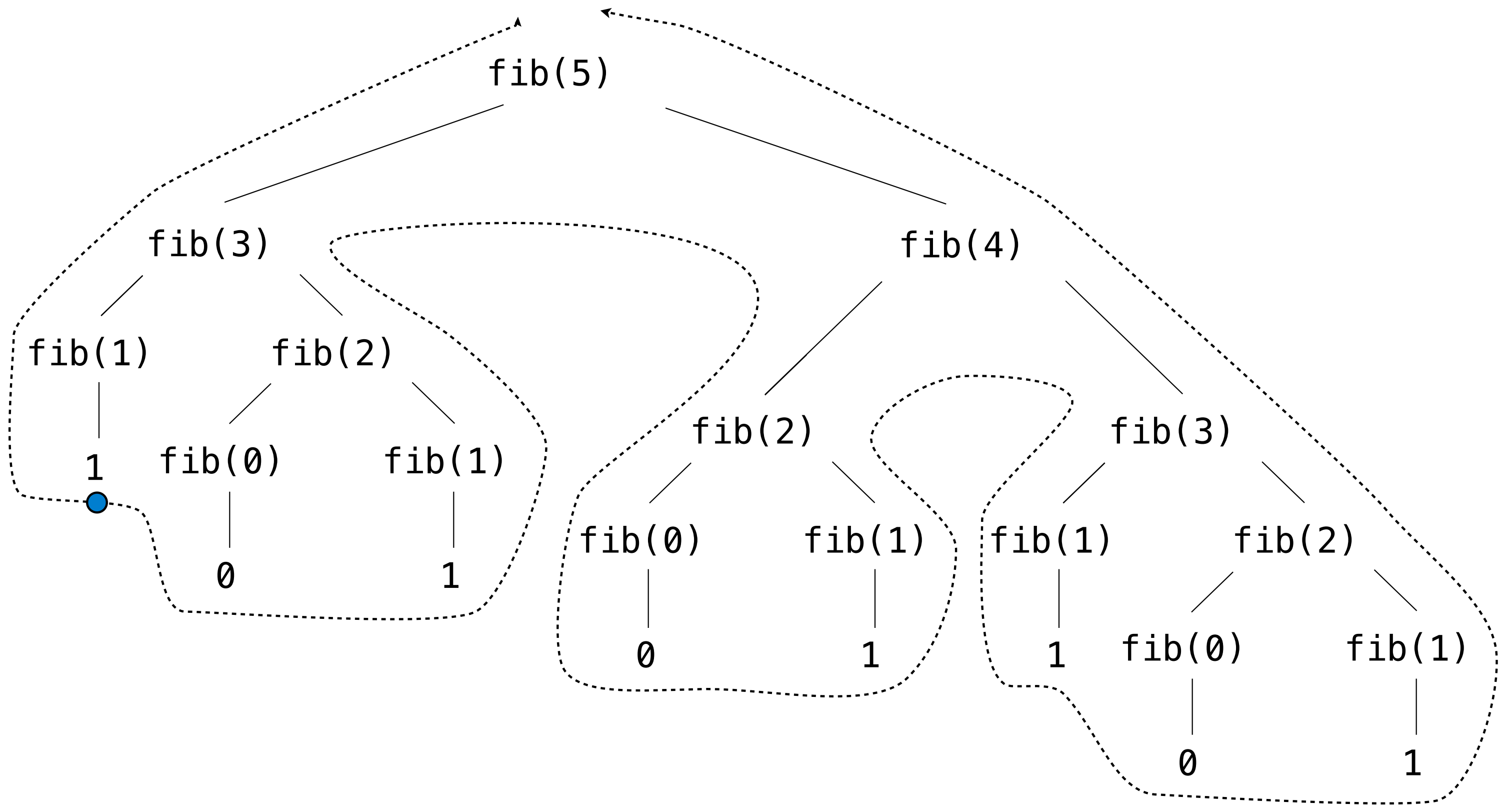

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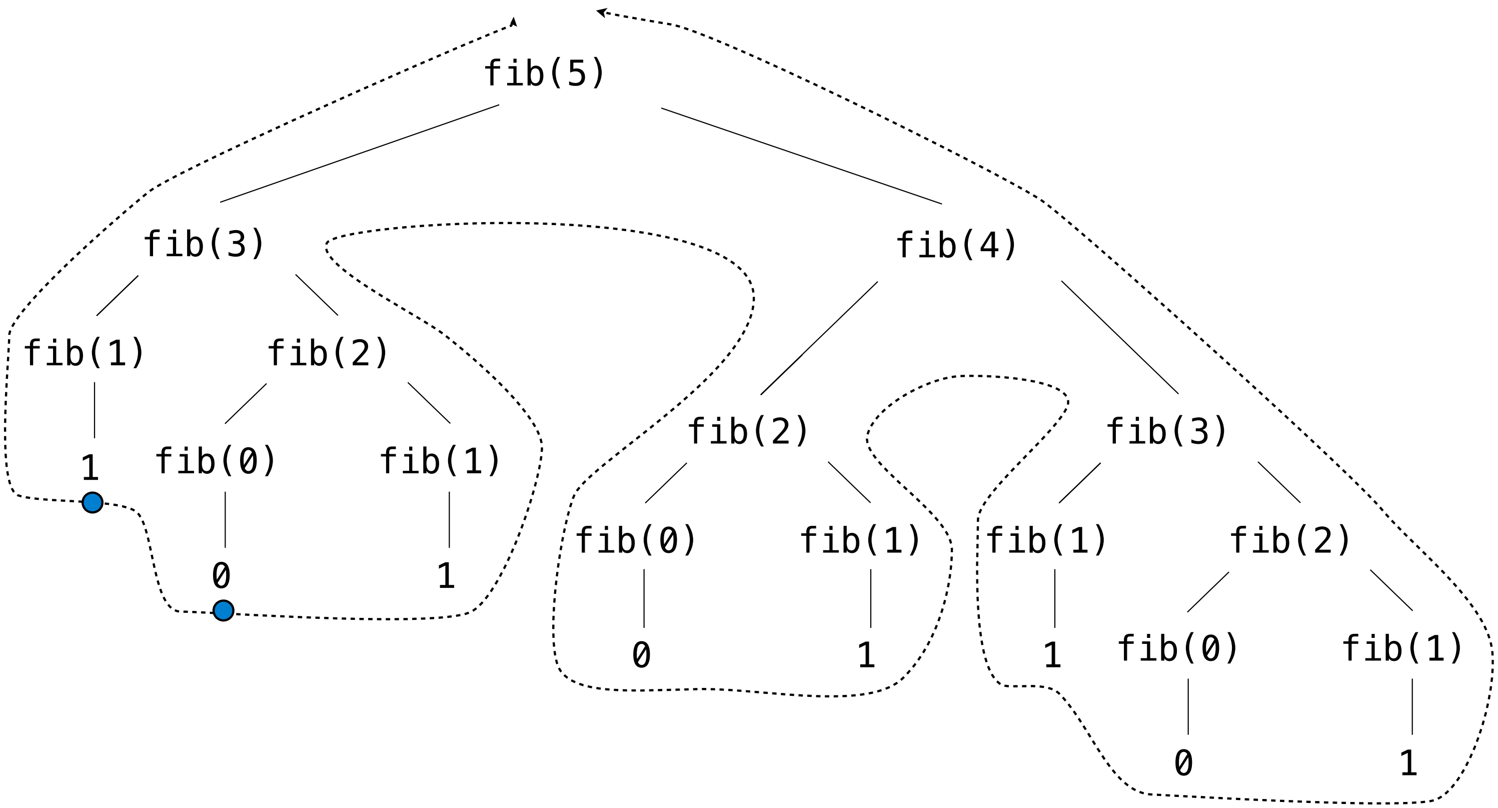

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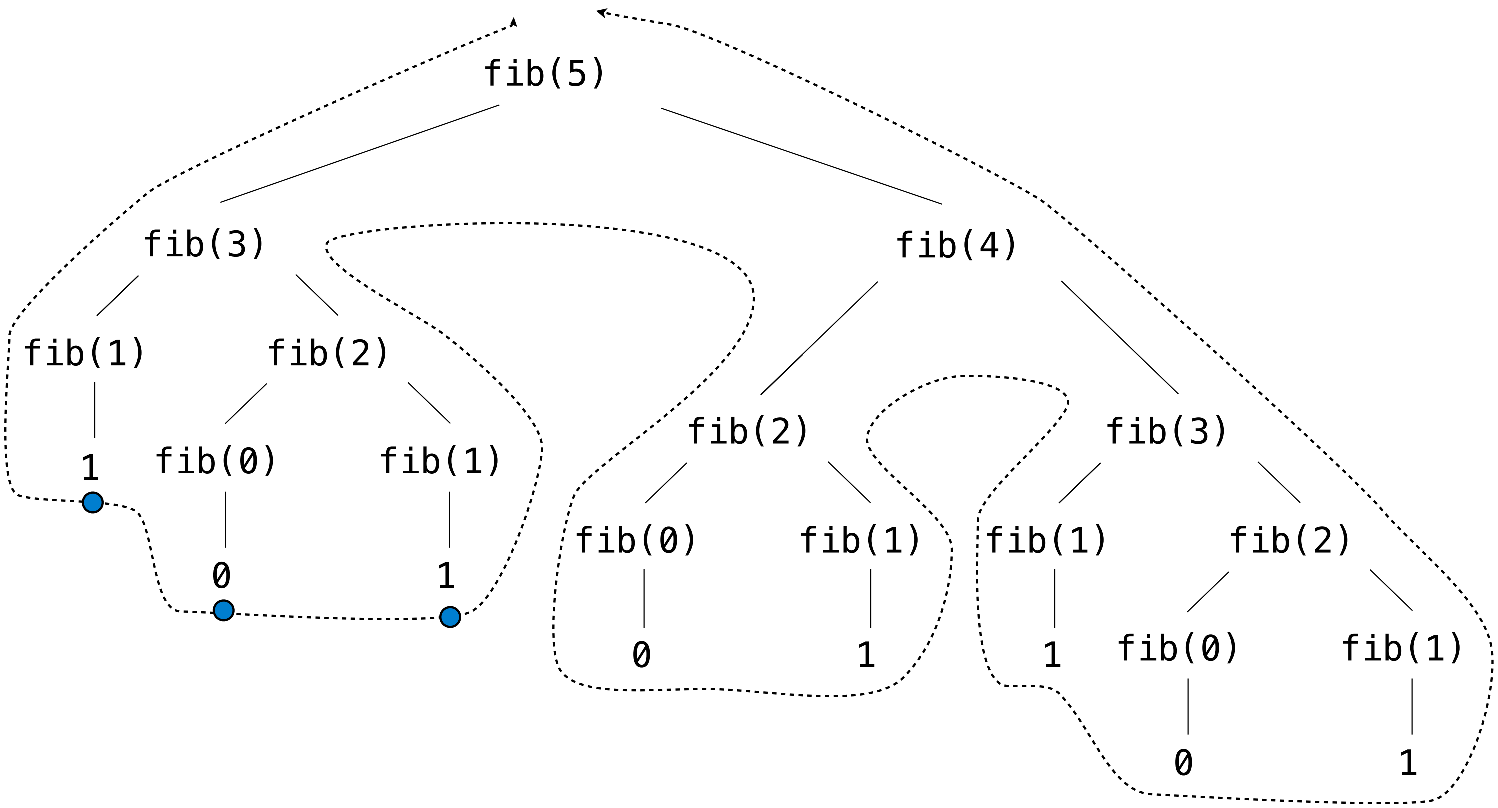

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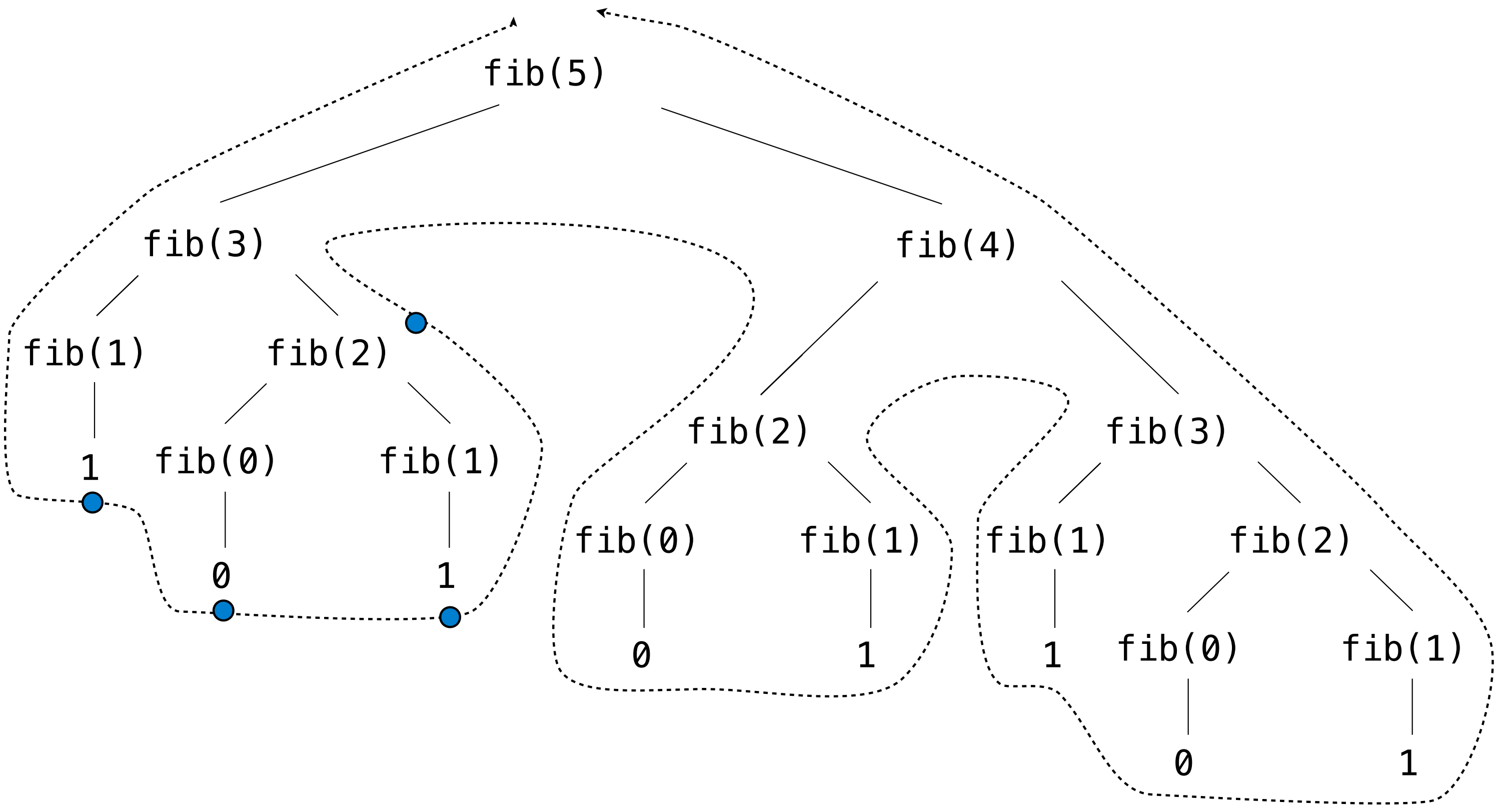

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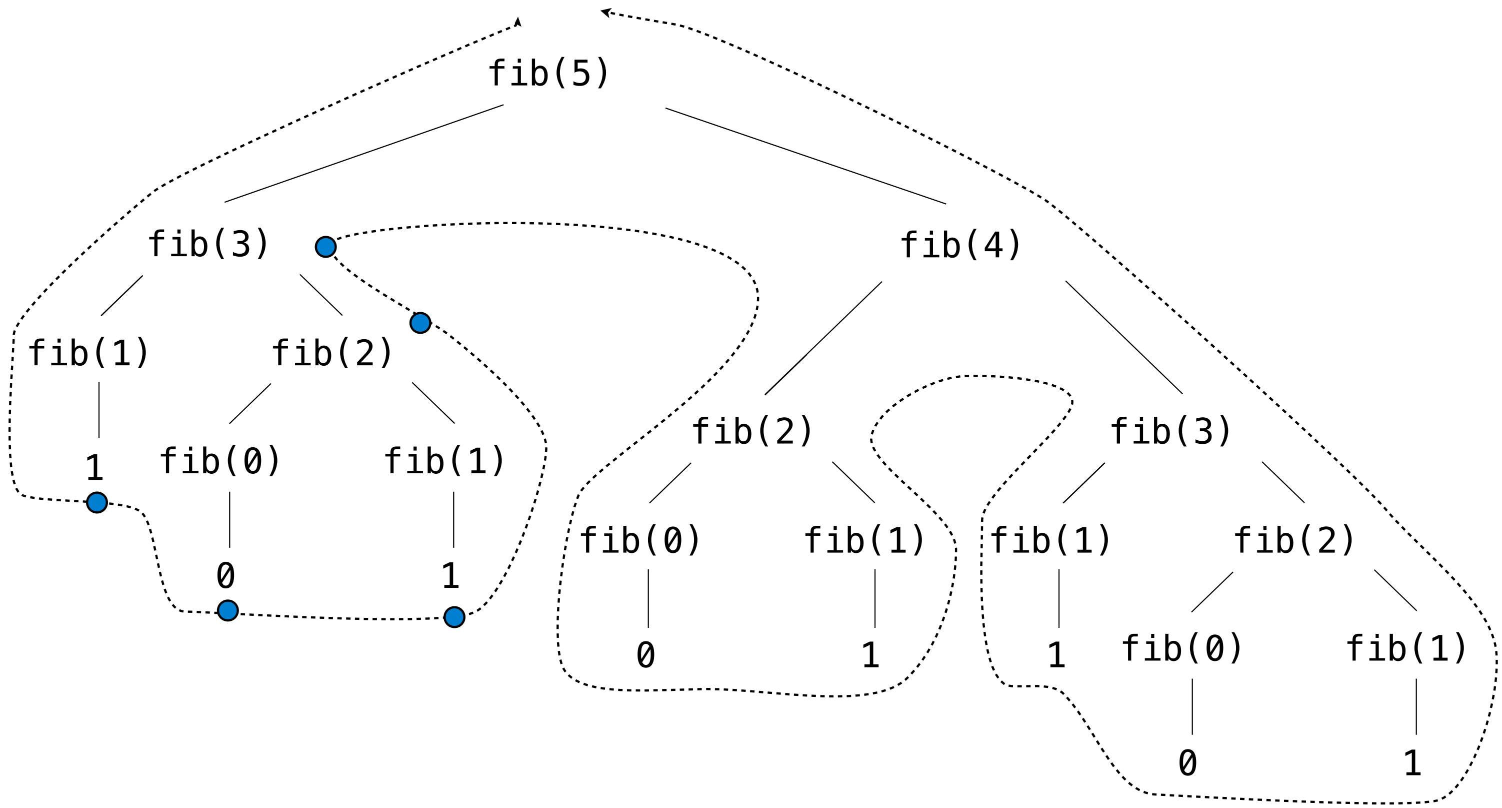

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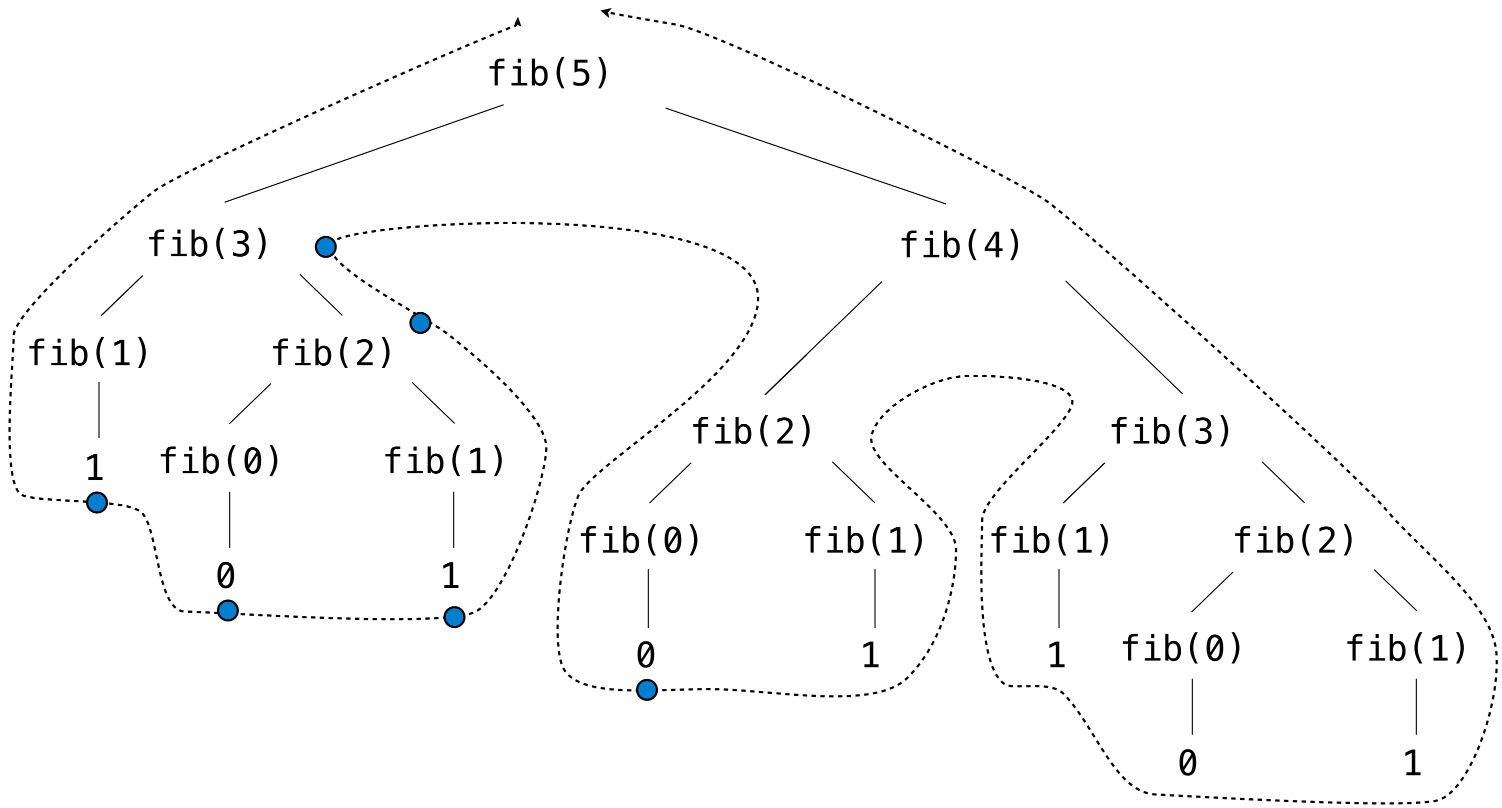

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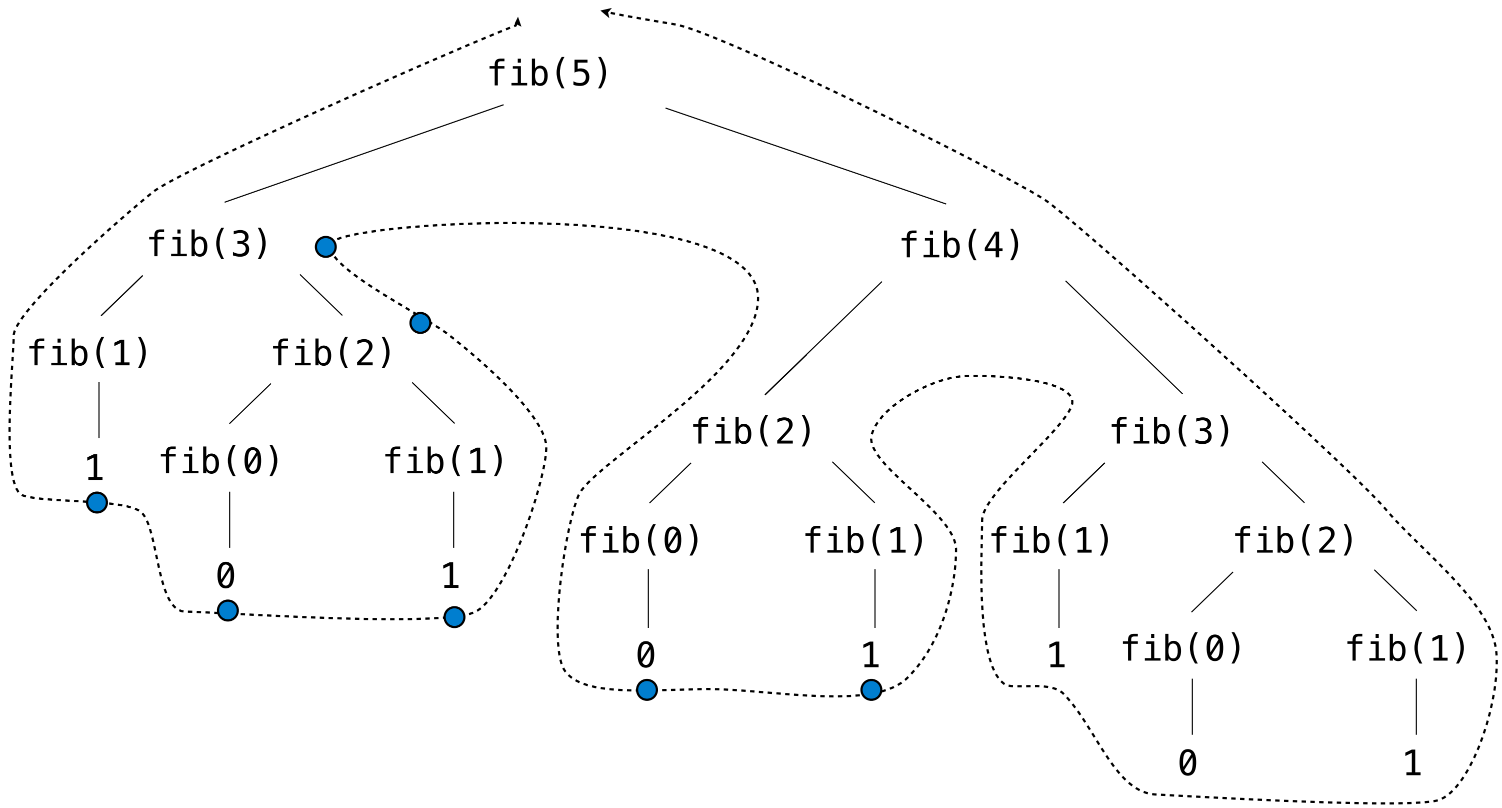

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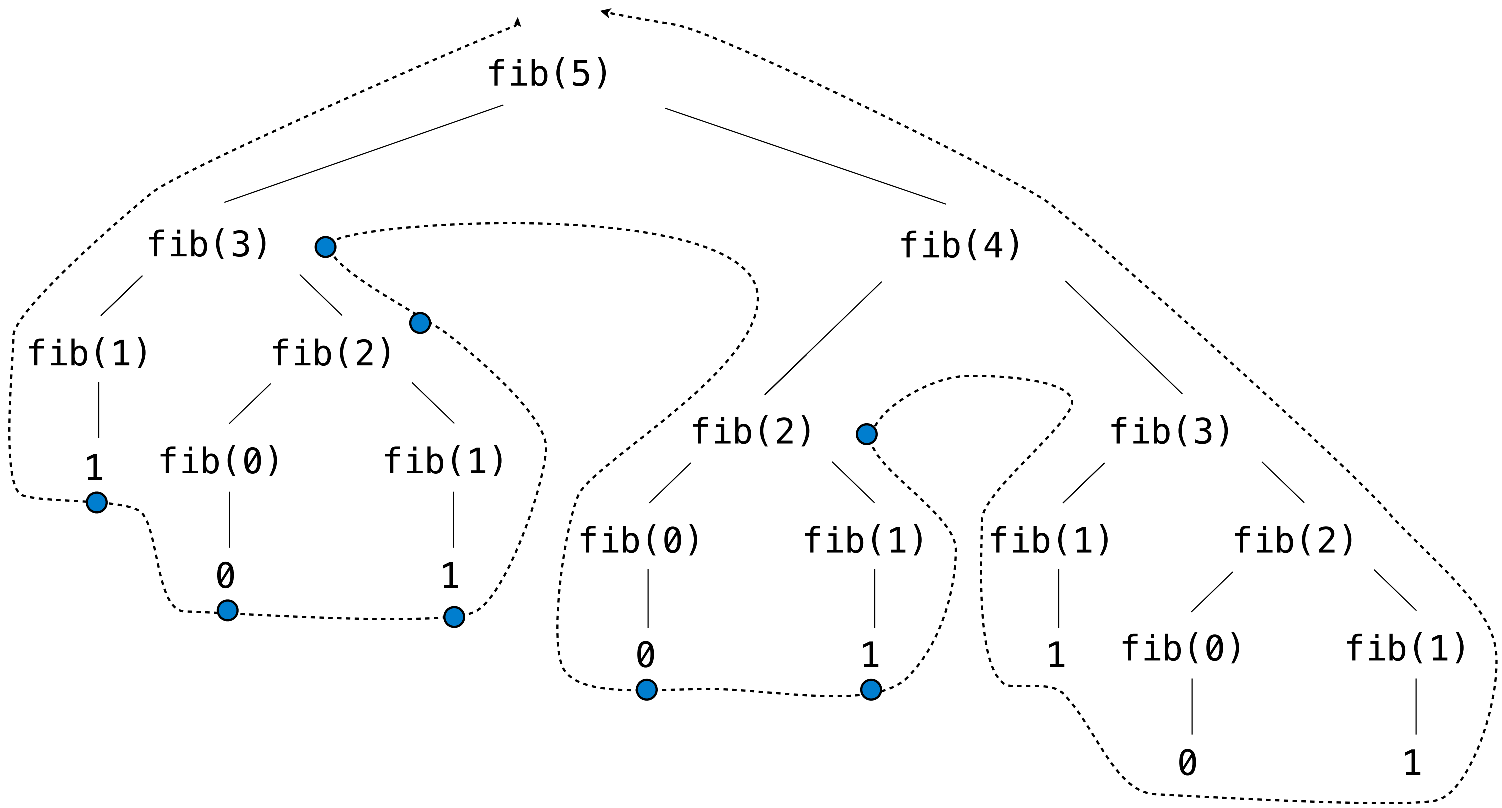

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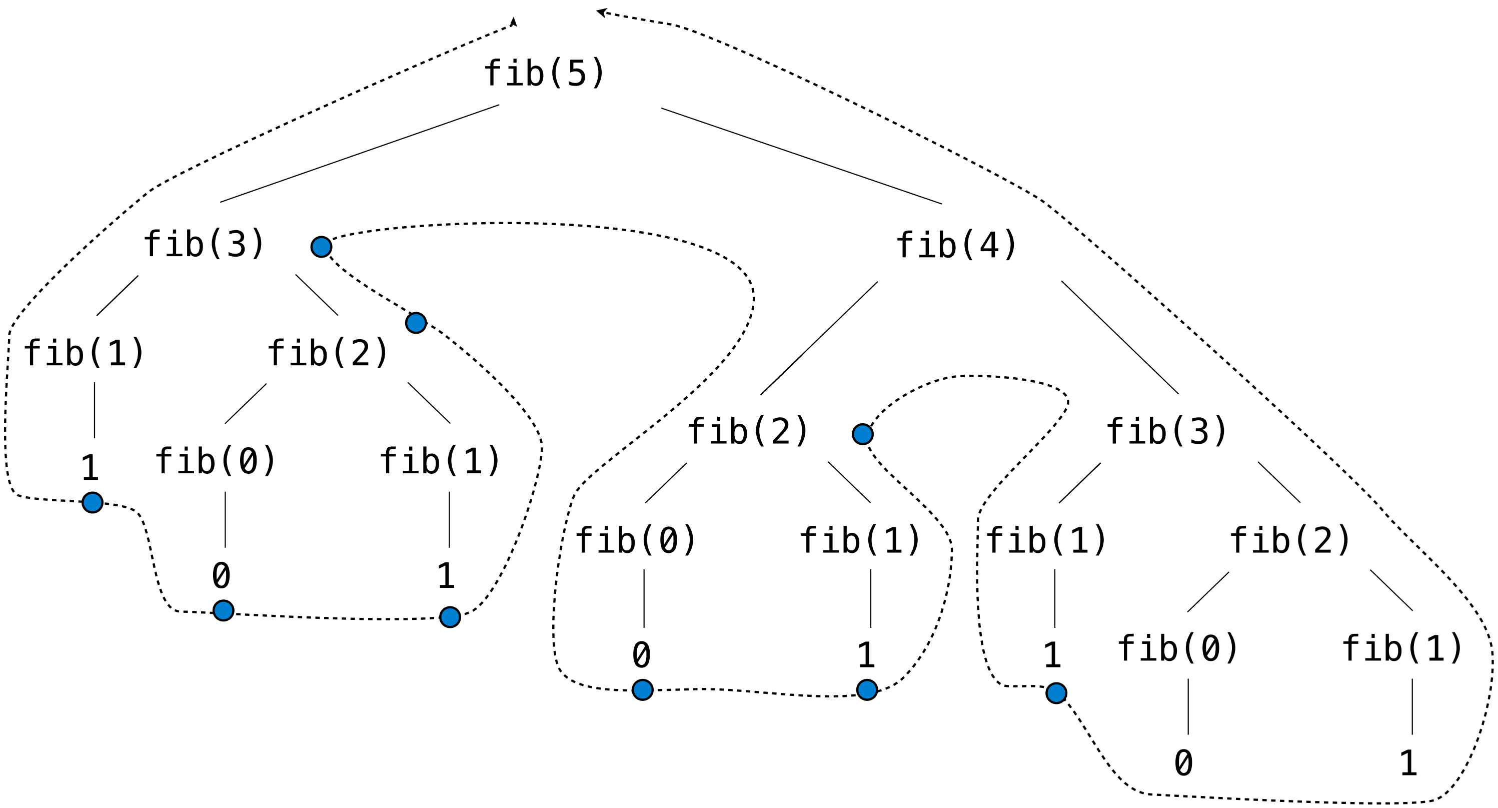

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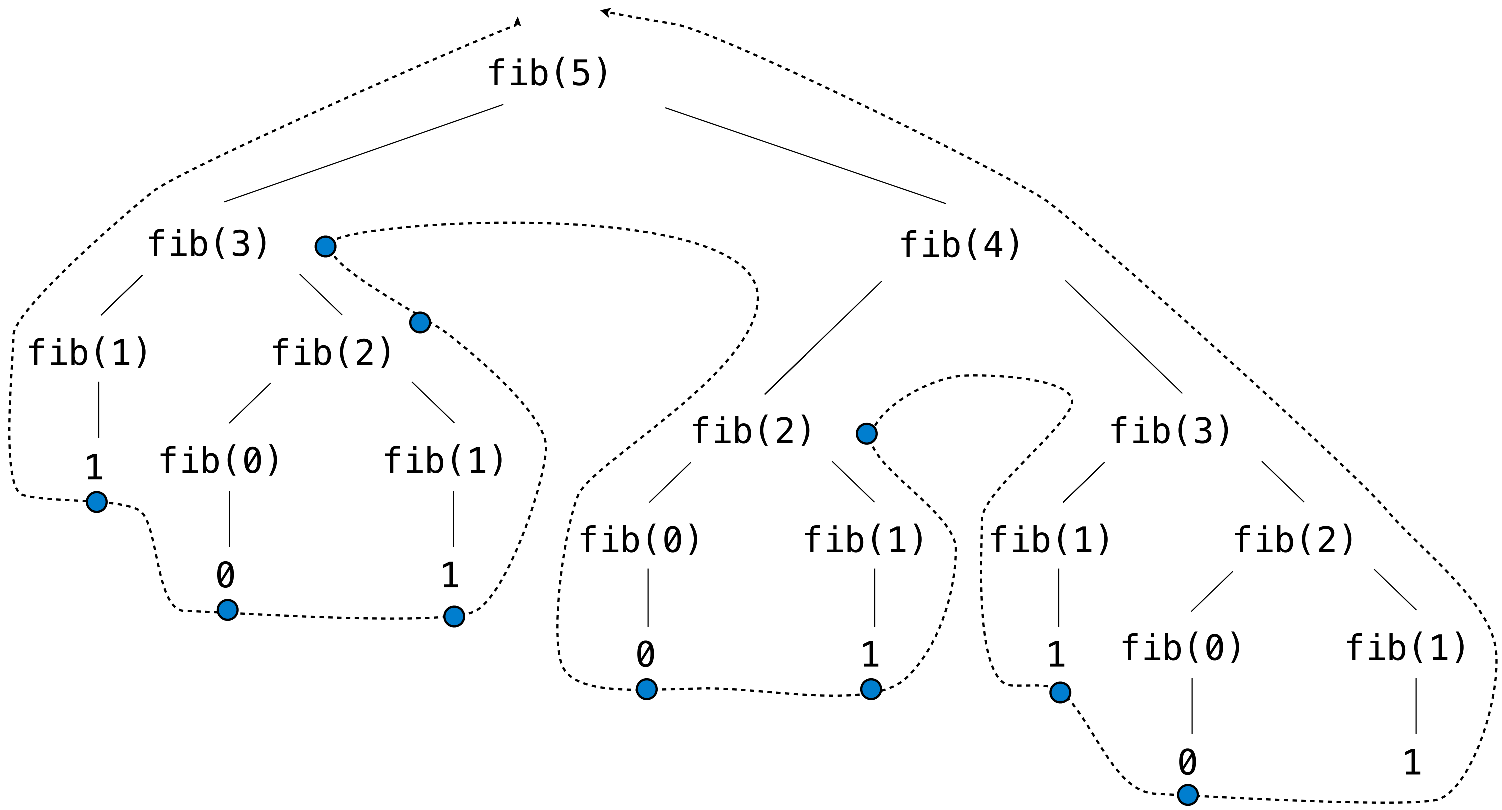

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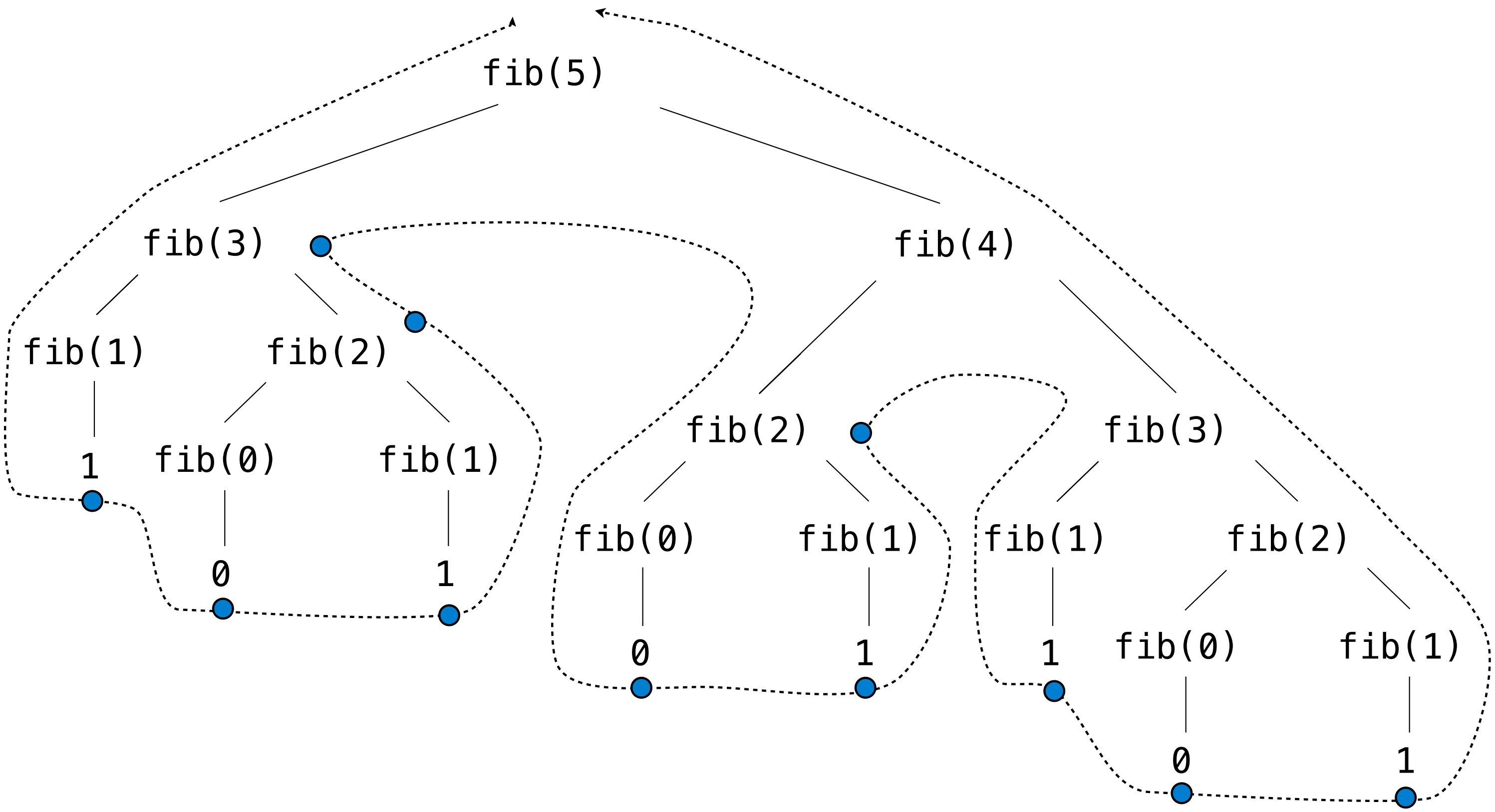

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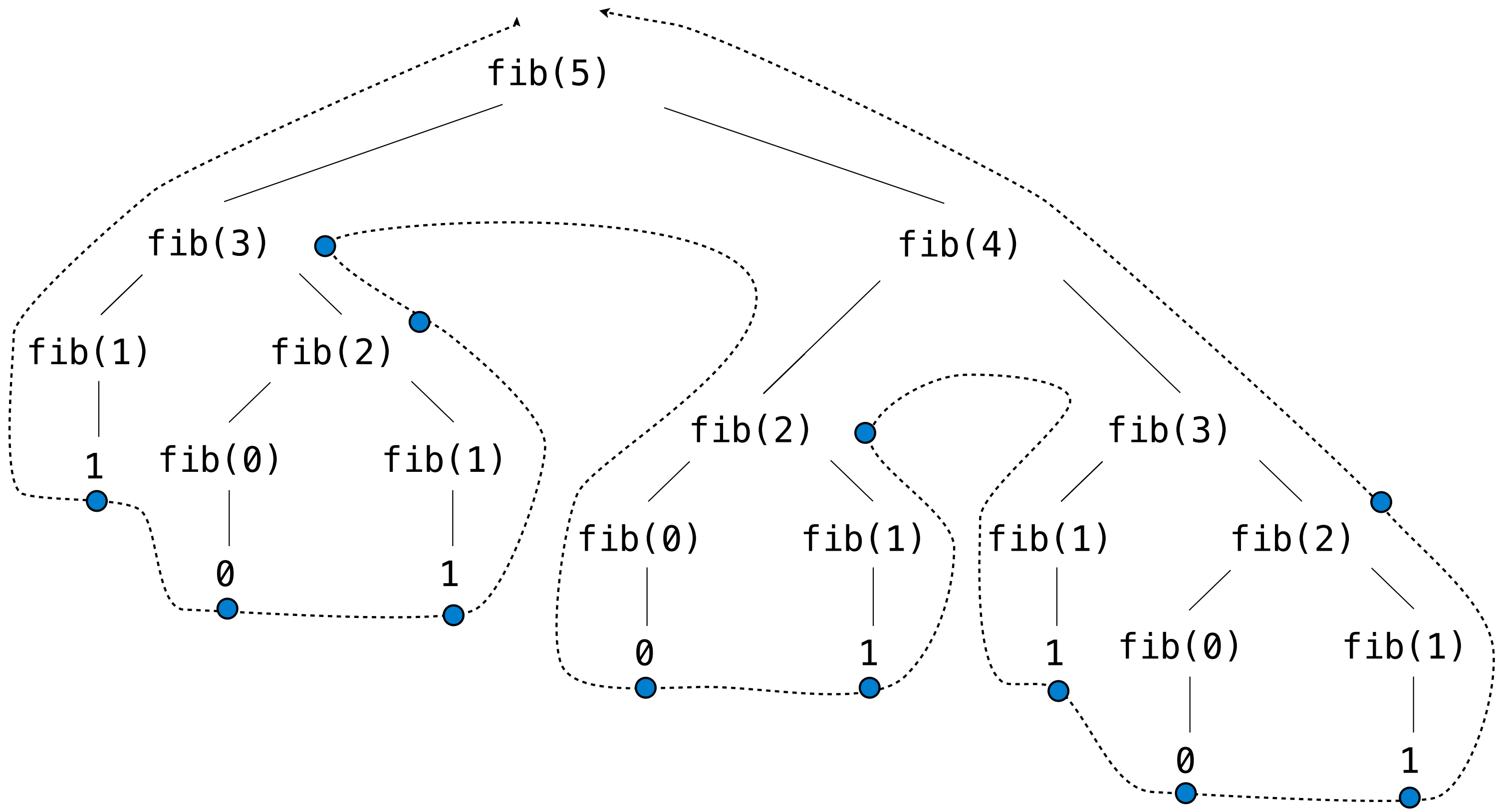

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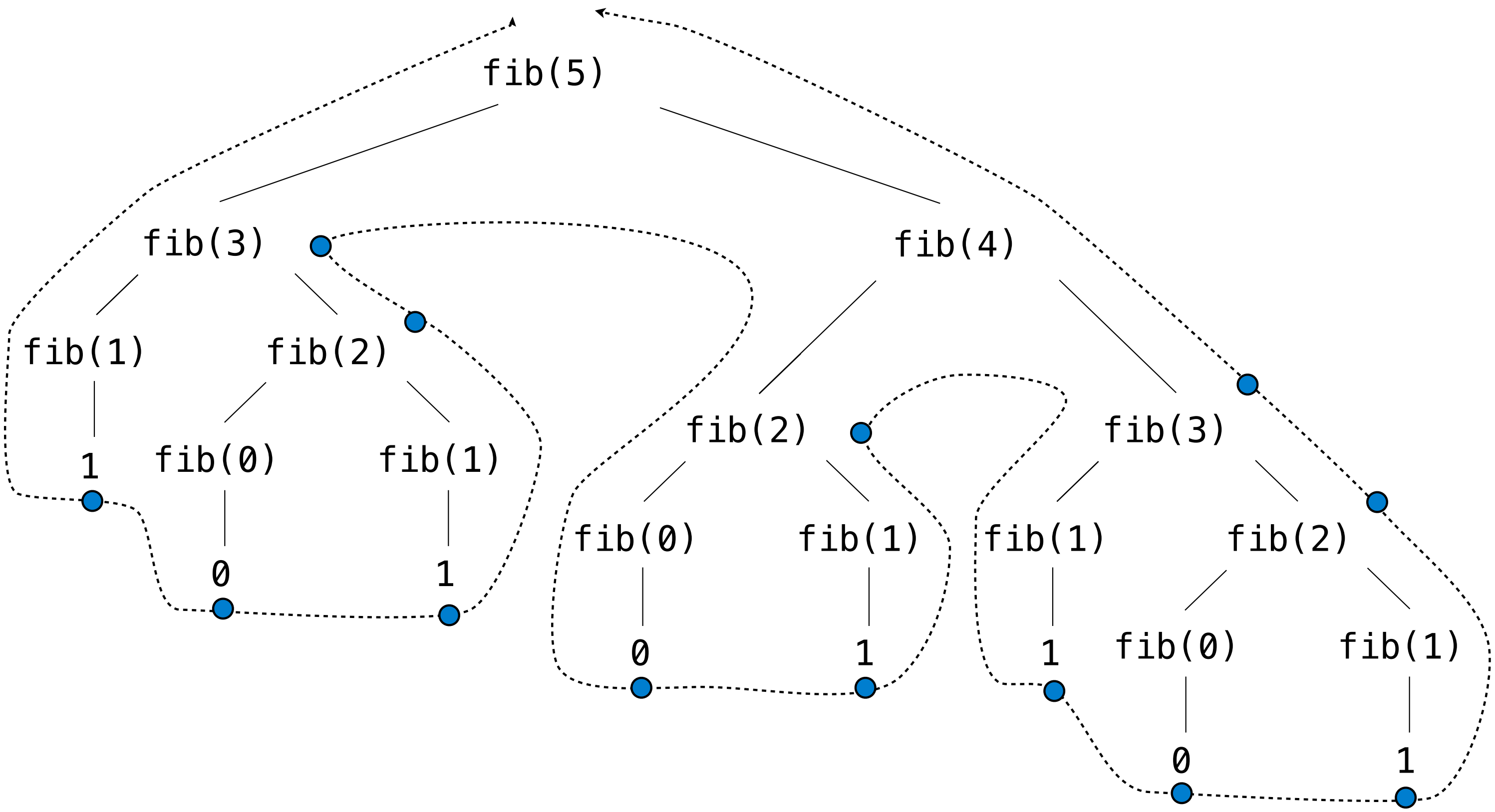

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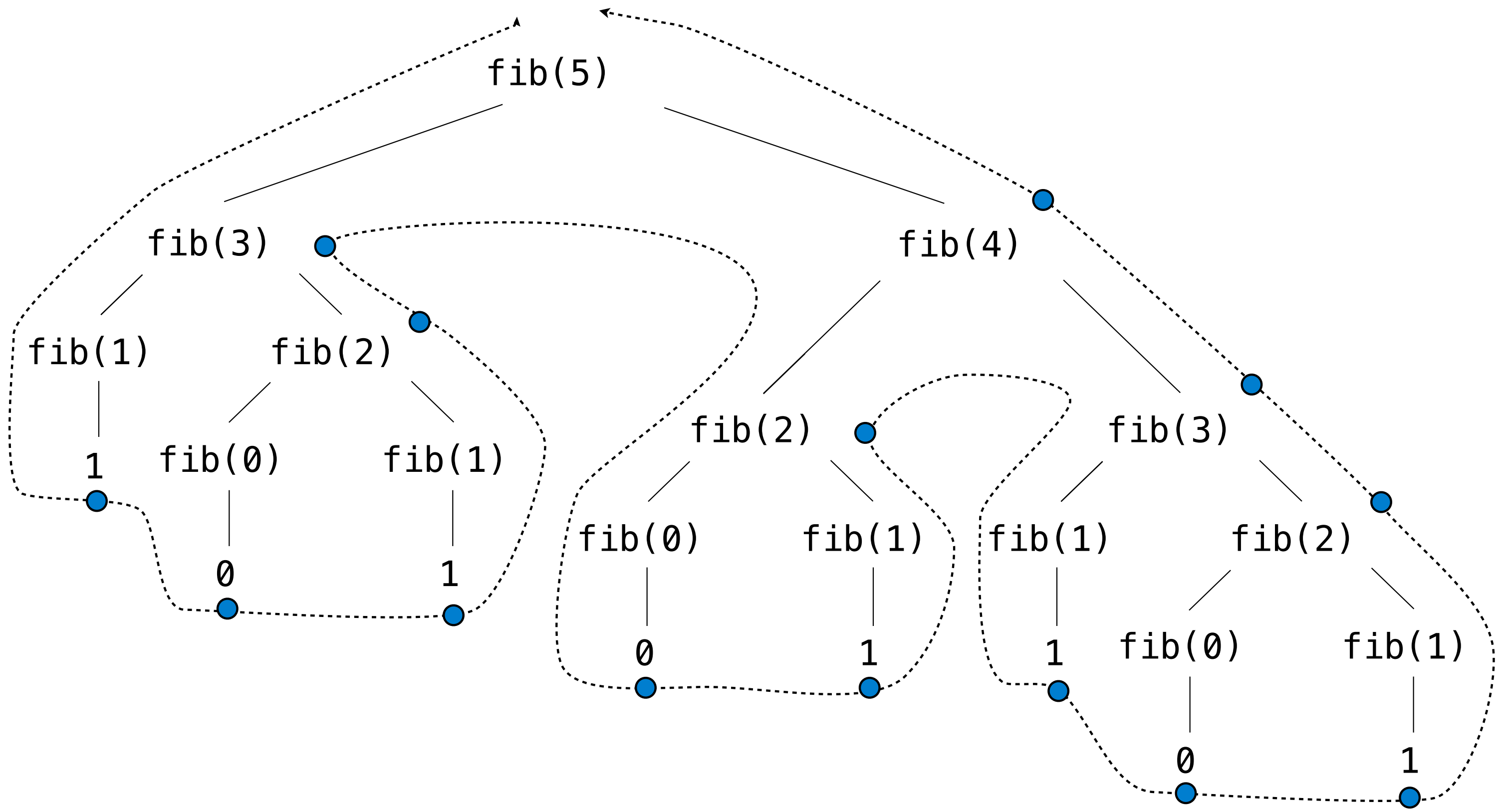

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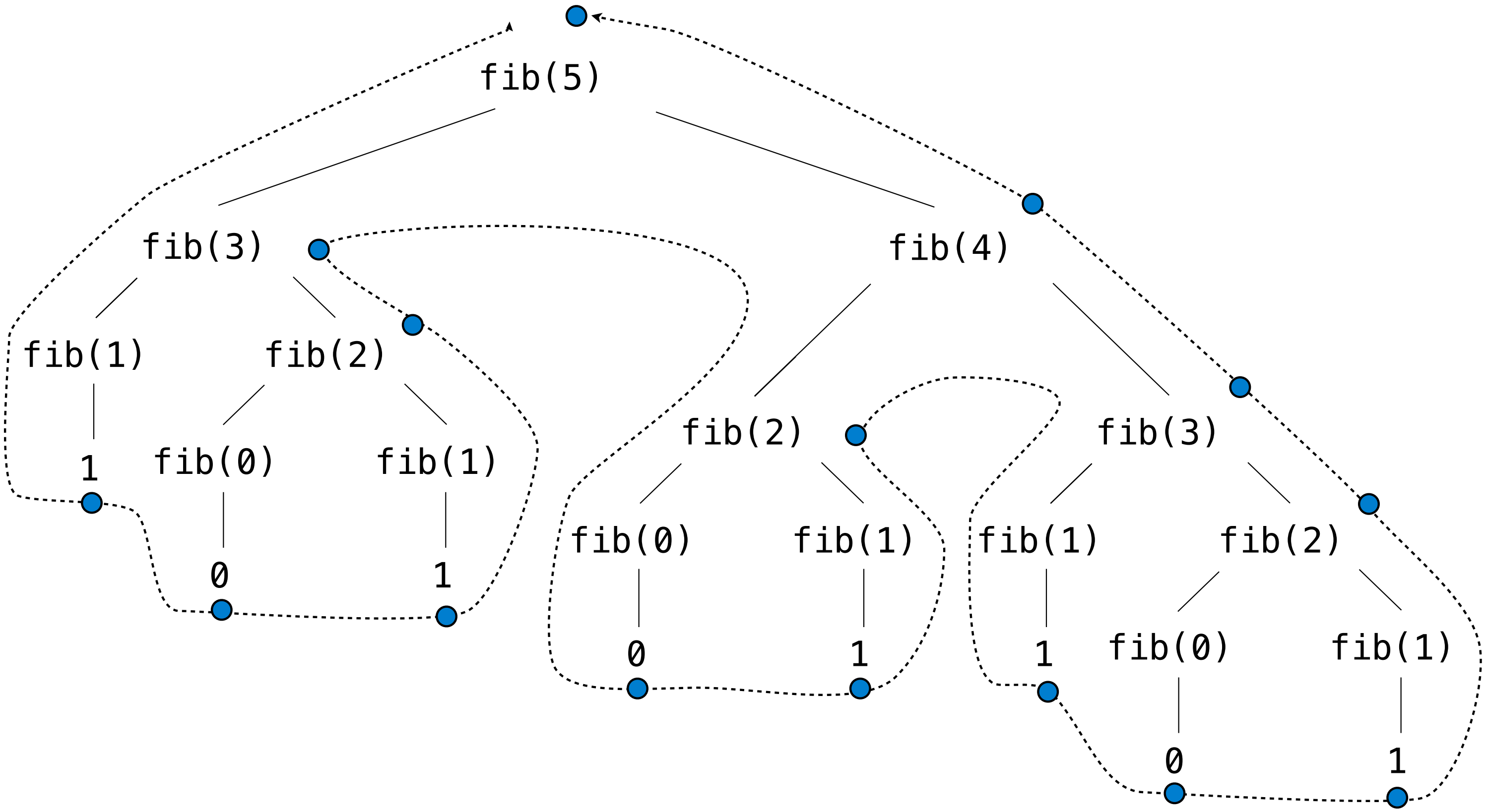

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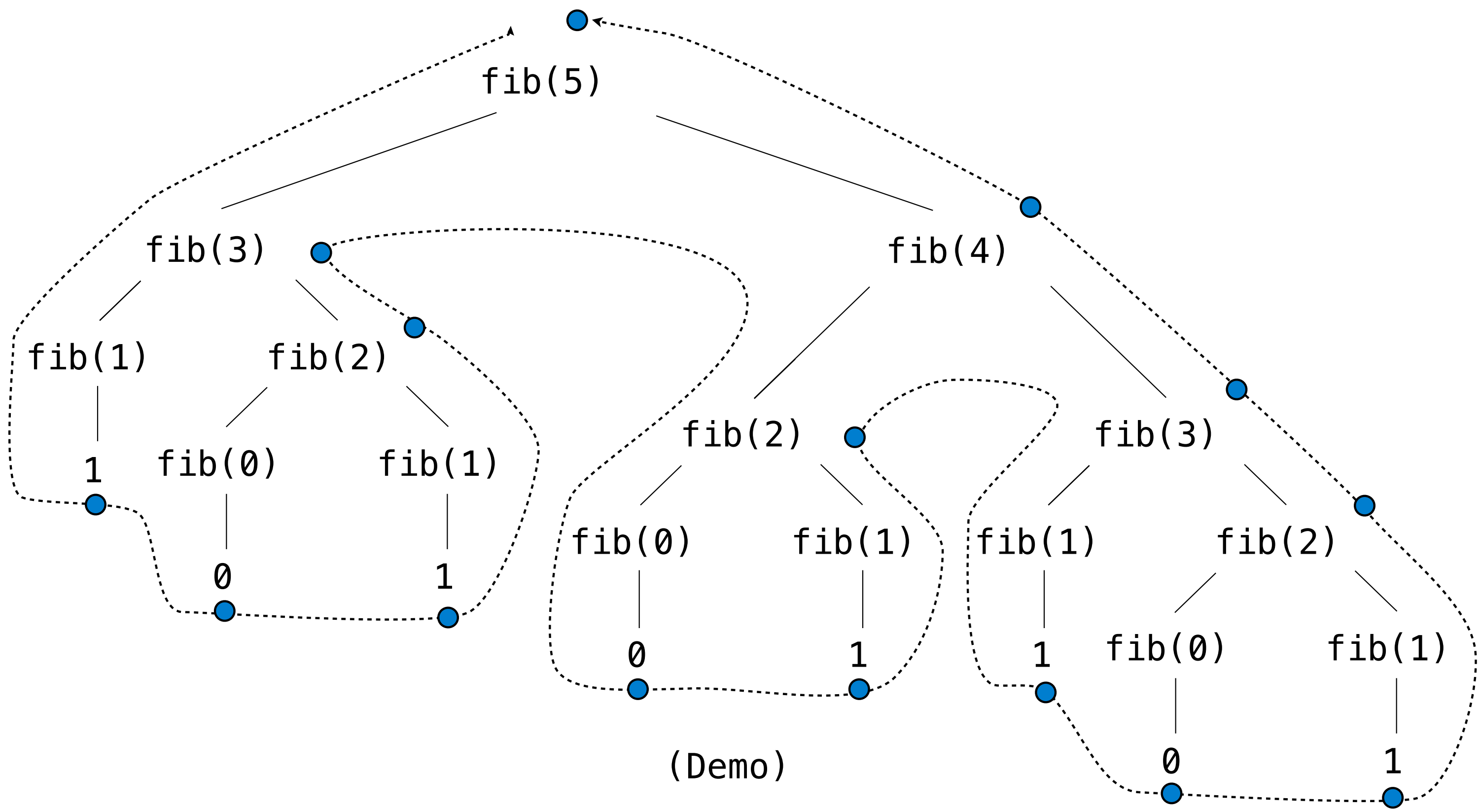

(Demo)

Repetition in Tree-Recursive Computation

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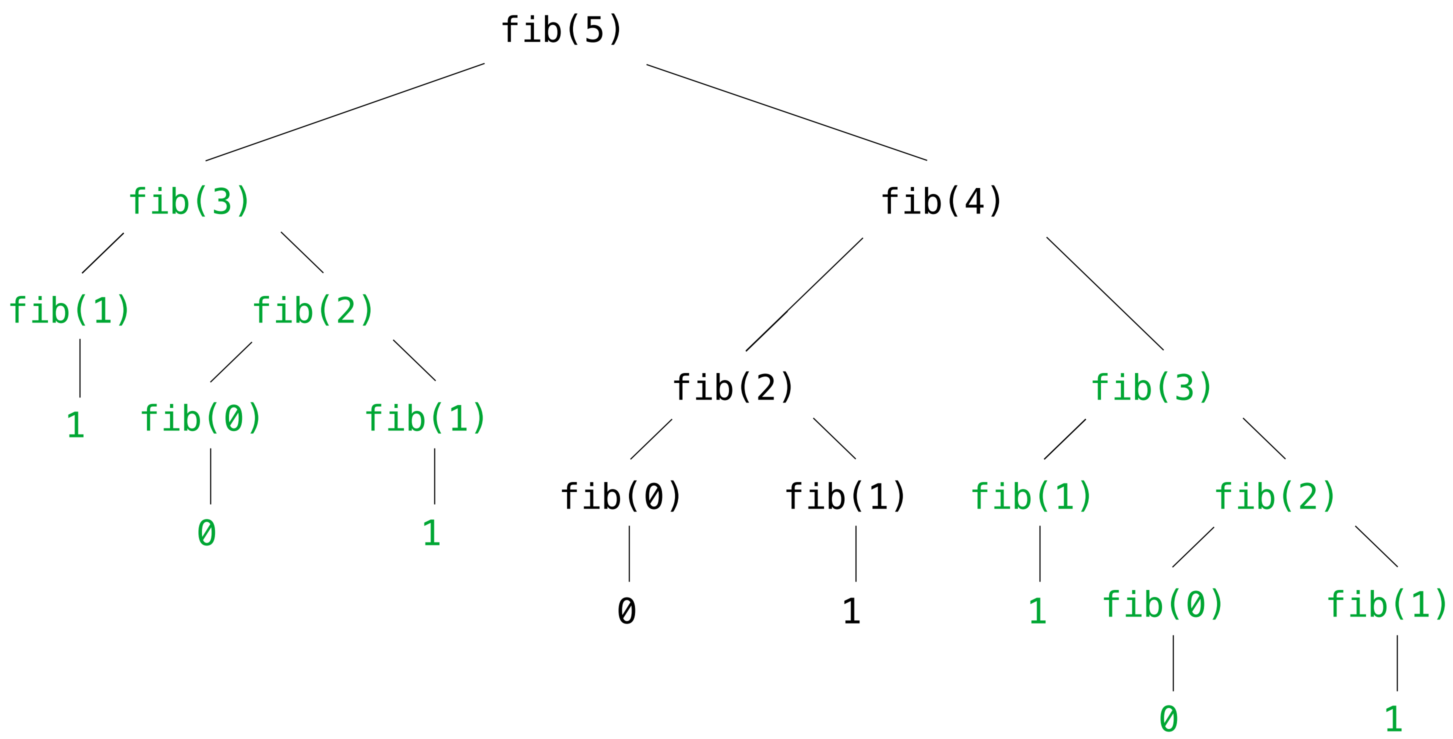

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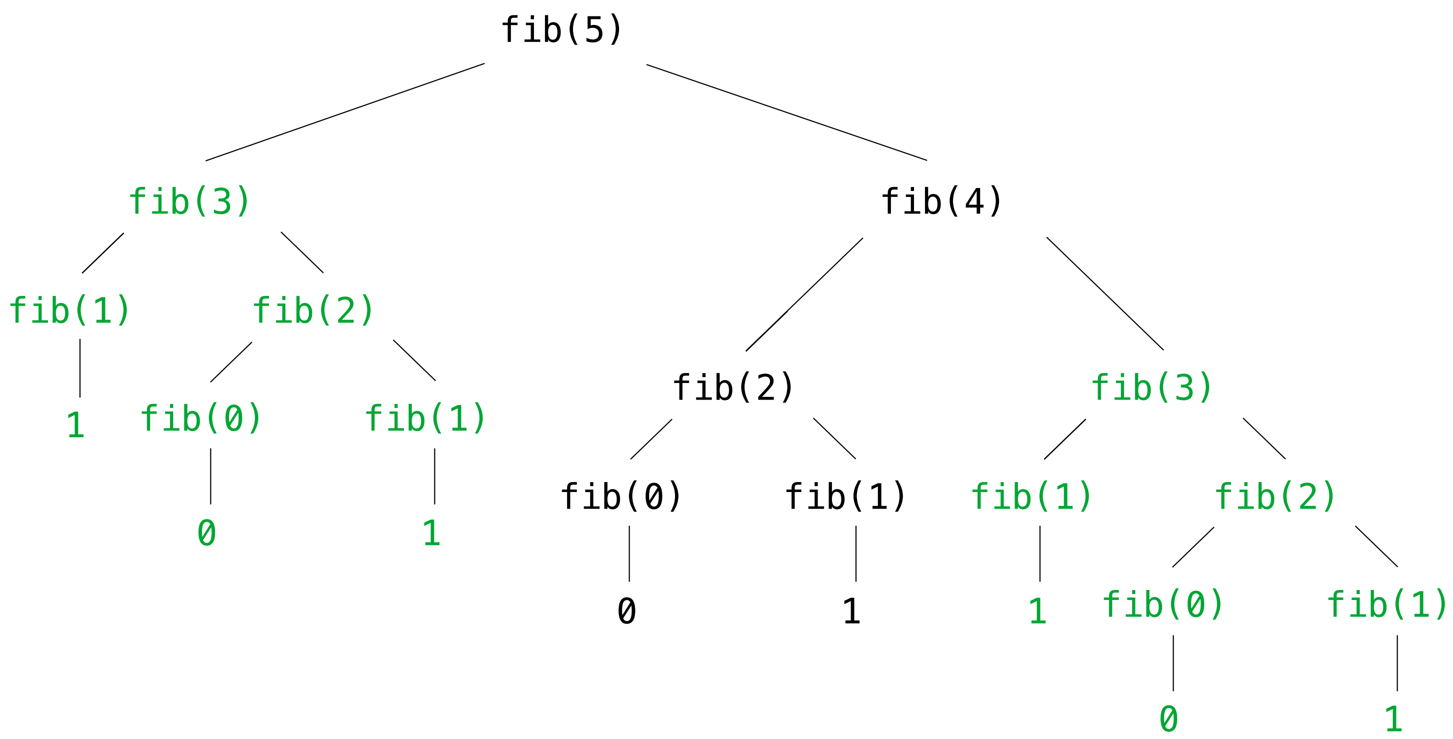

We can speed up this computation dramatically in a few weeks by remembering results.

Example: Counting Partitions

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The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

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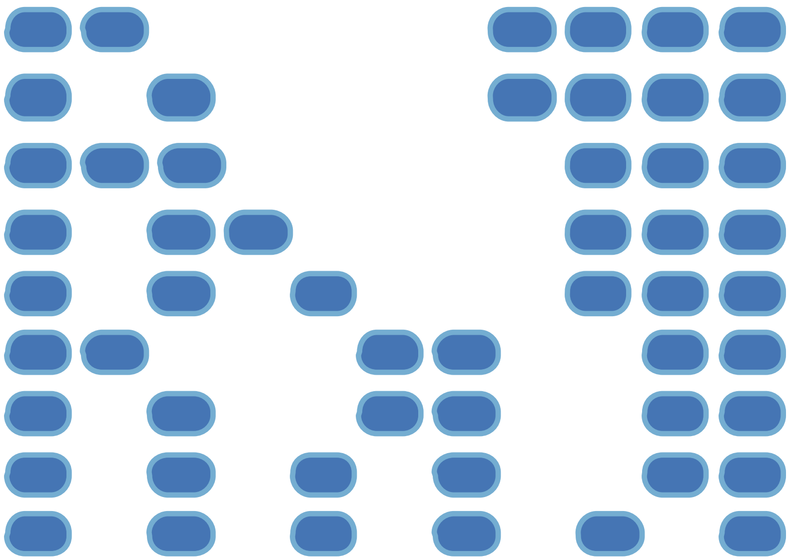

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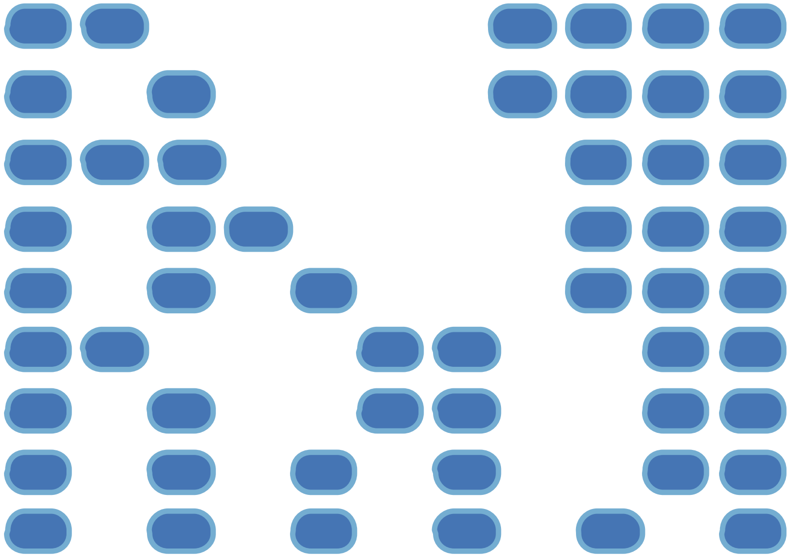

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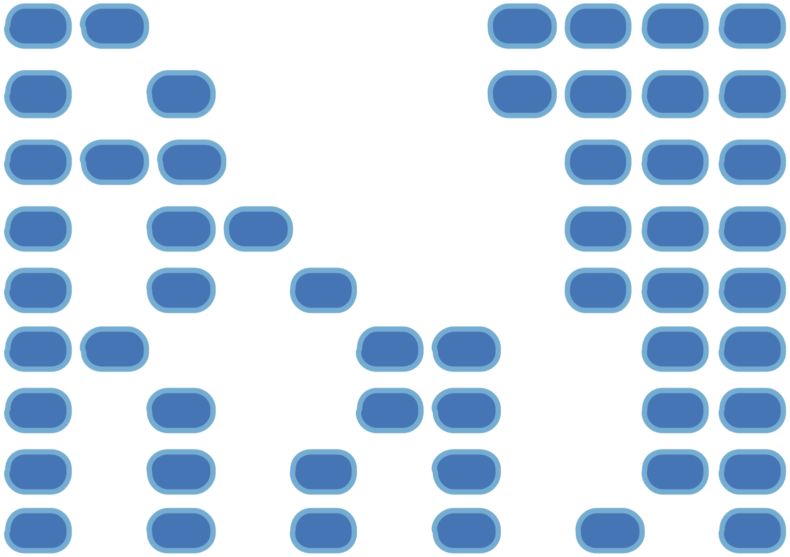

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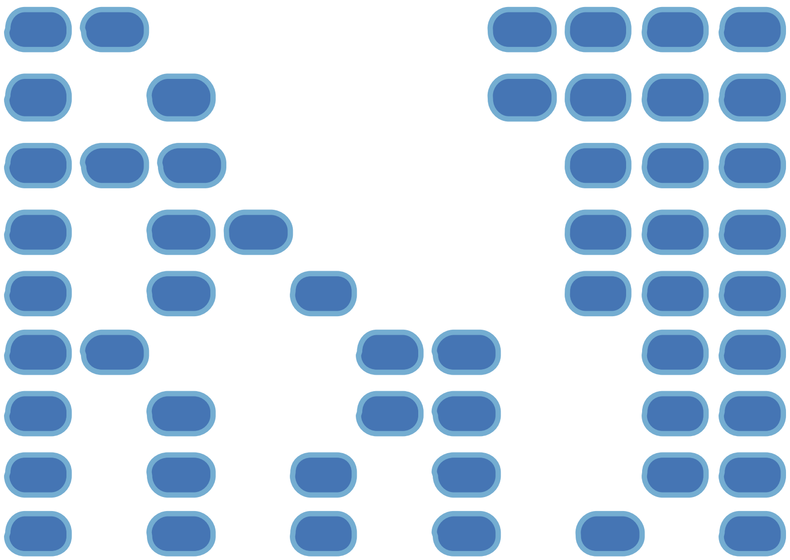

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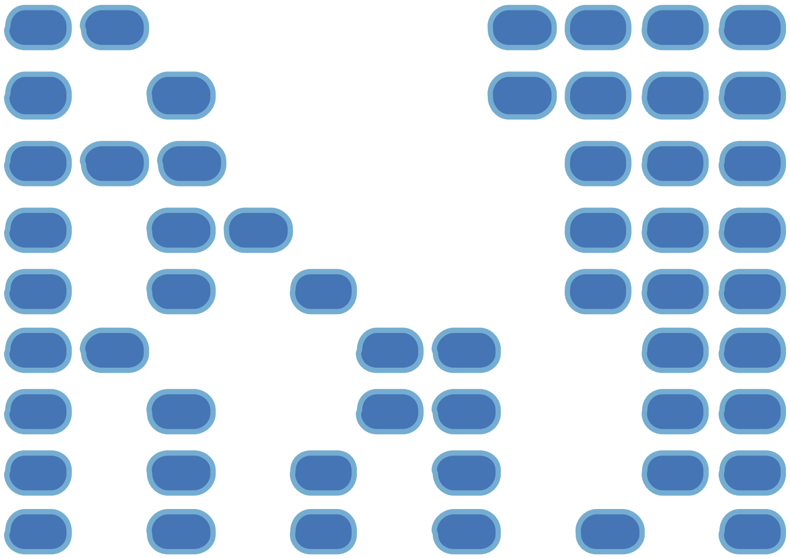

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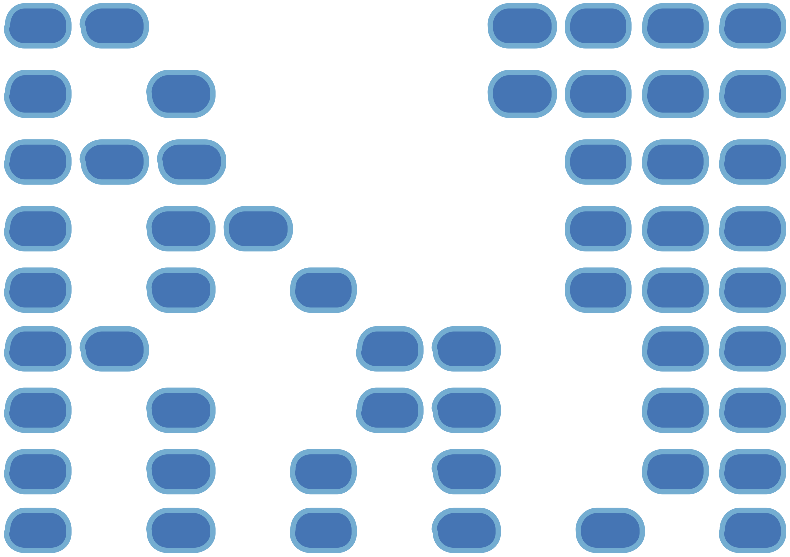

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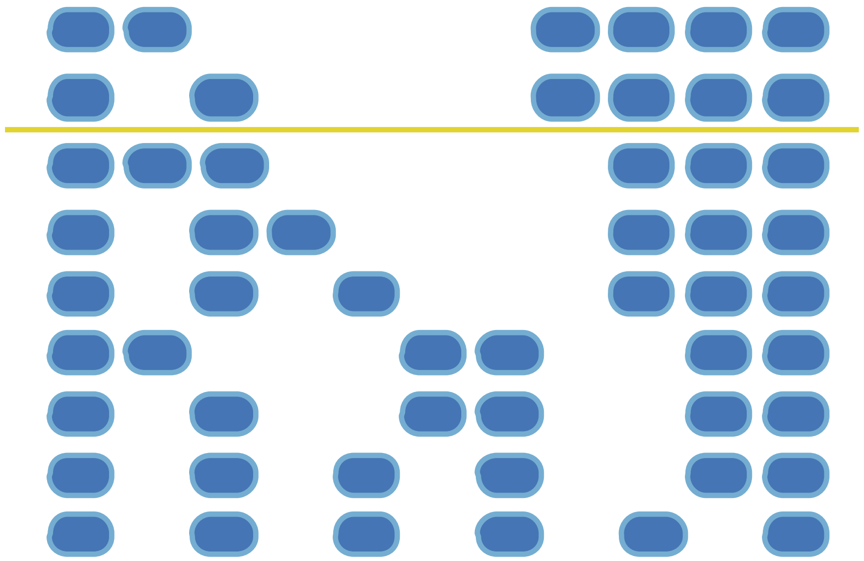

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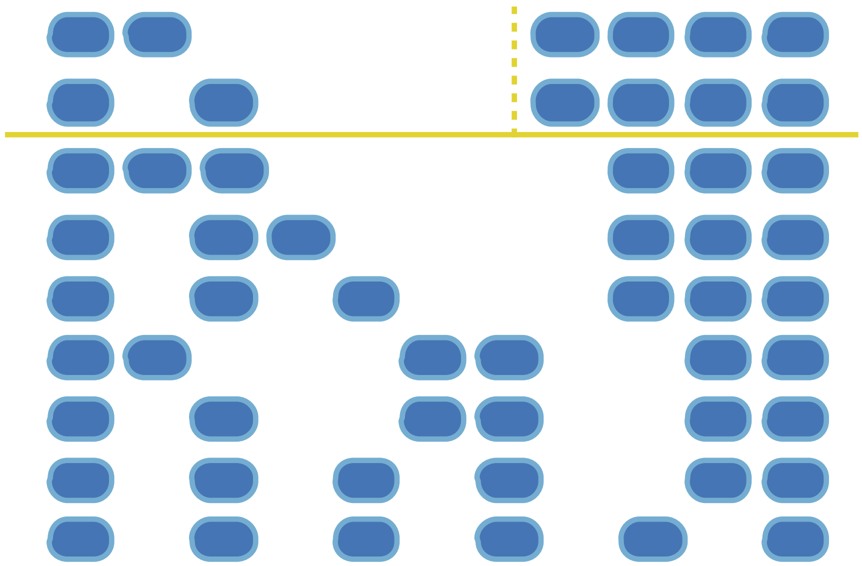

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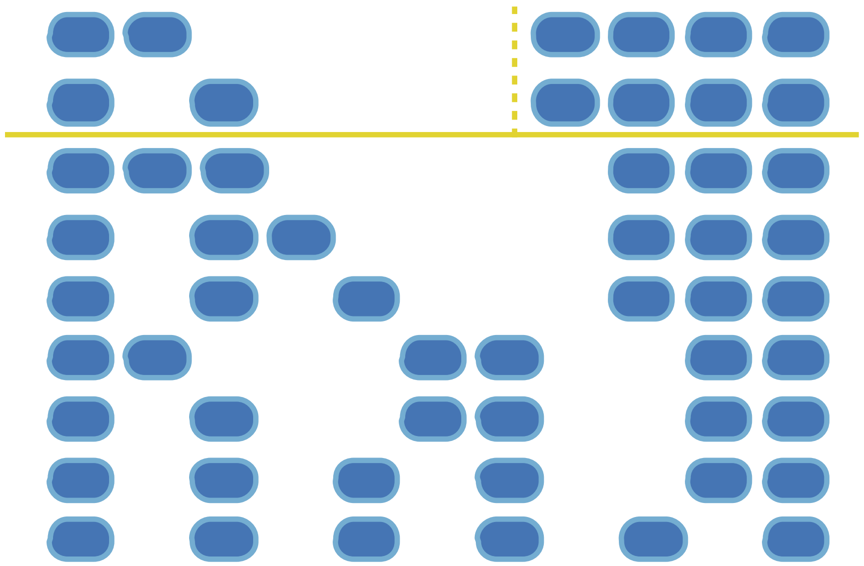

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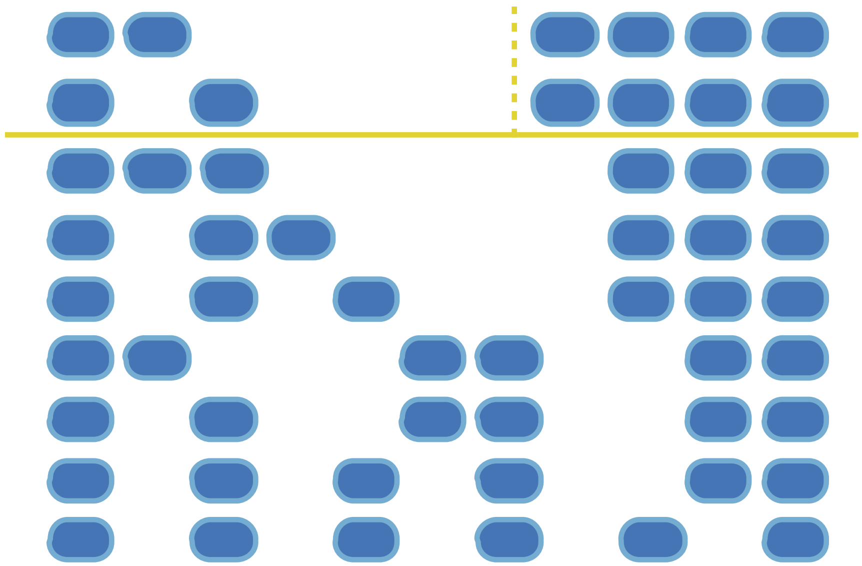

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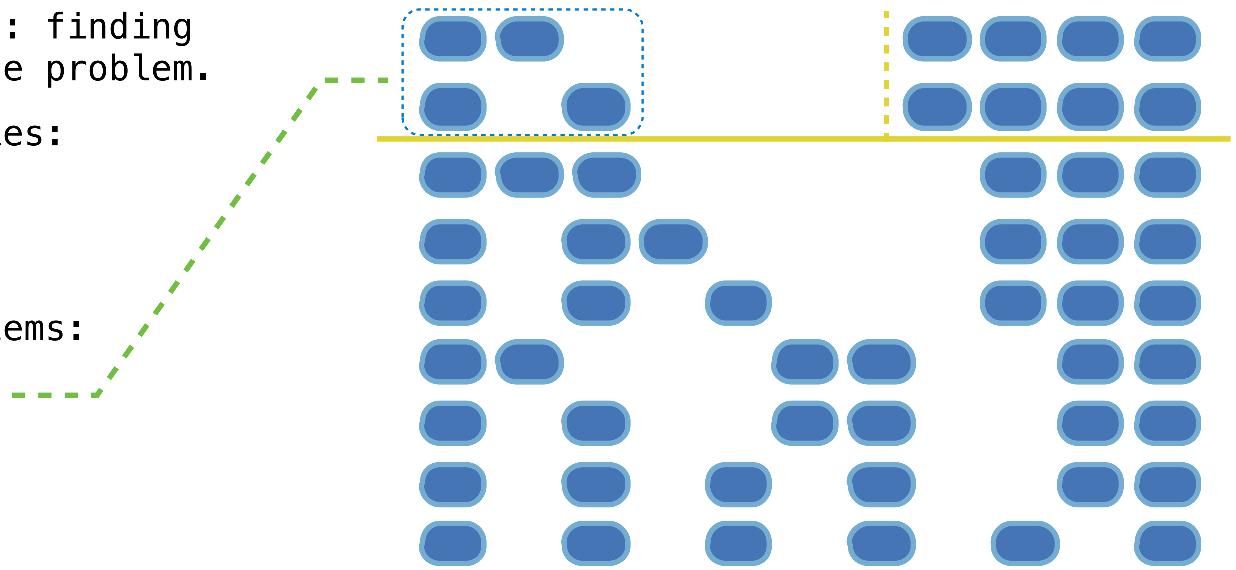

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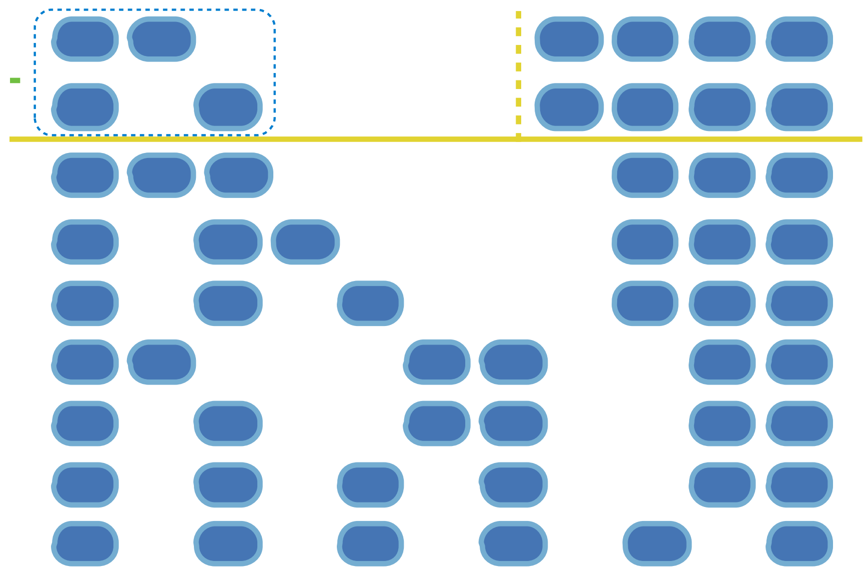

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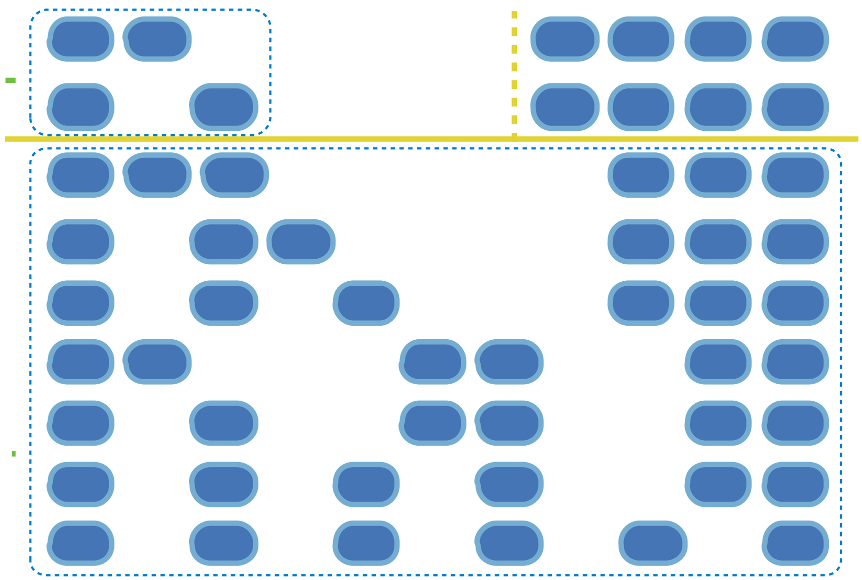

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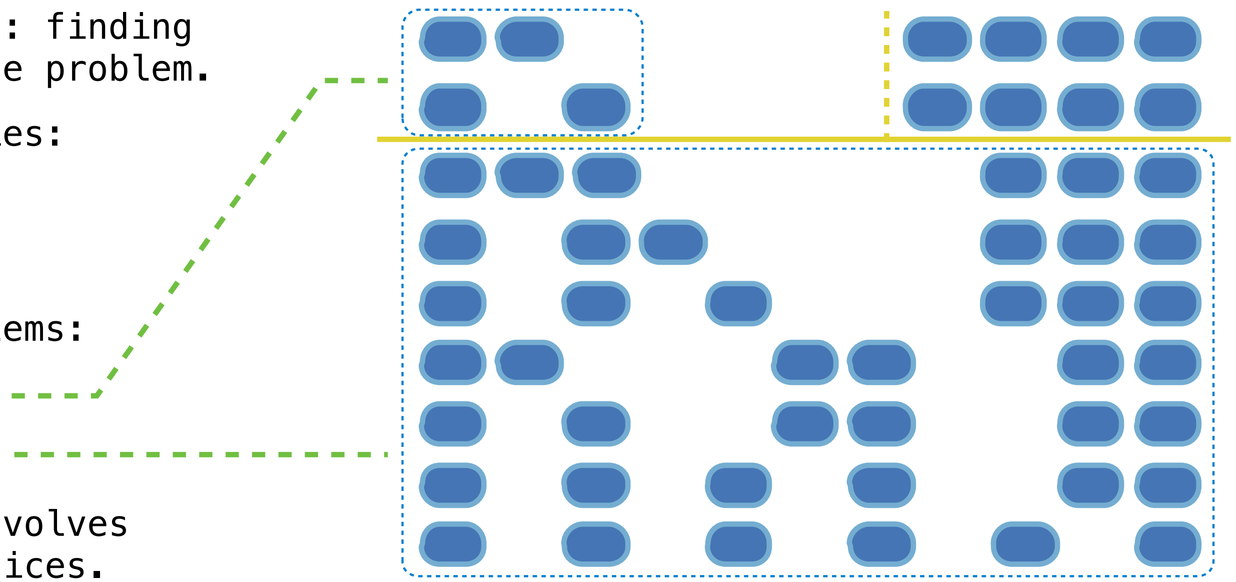

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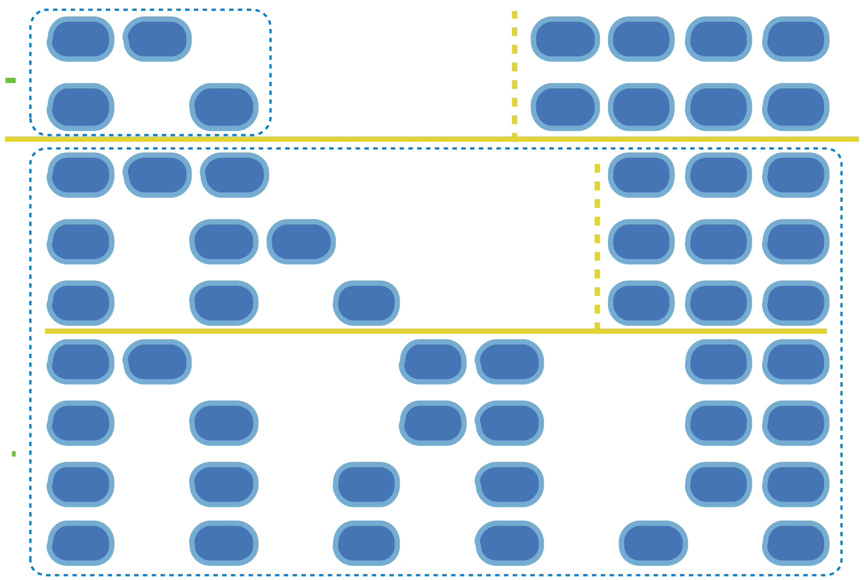

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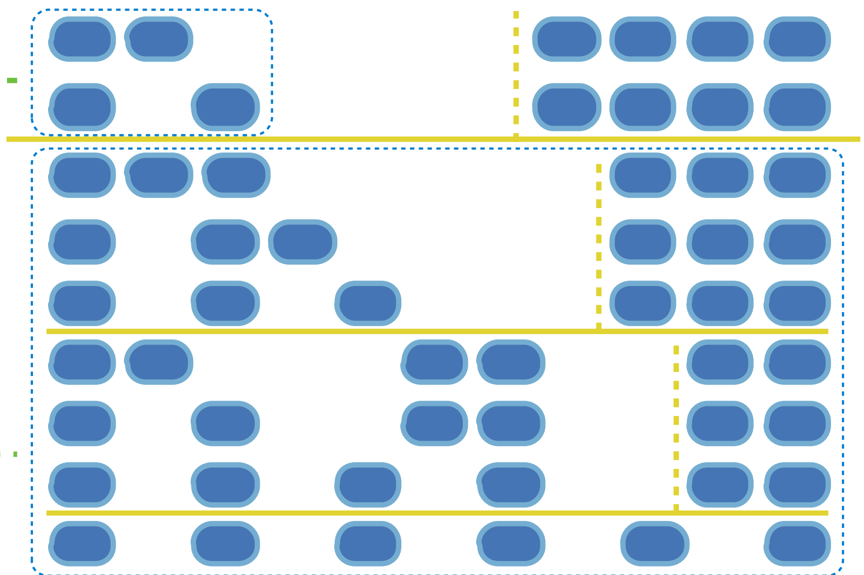

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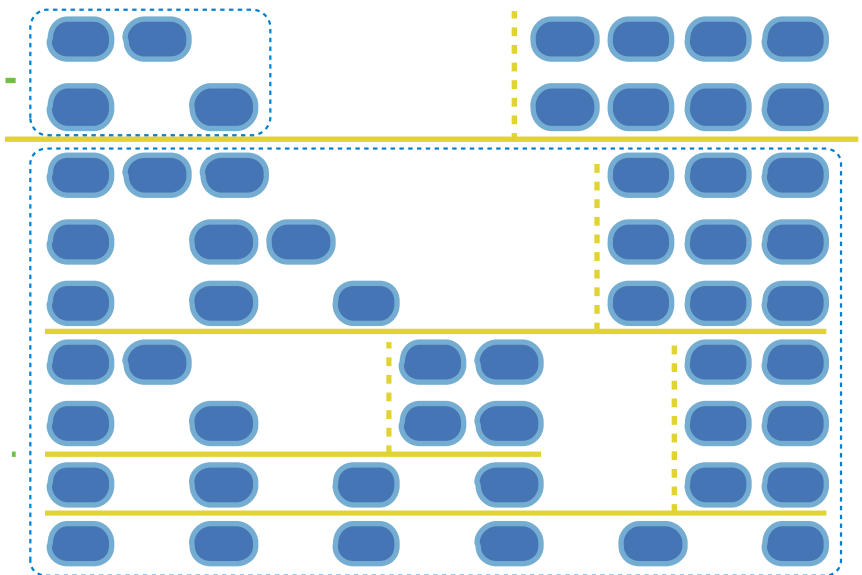

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    else:
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    else:
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def count_partitions(n, m):




    else:
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def count_partitions(n, m):



    else:
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```
def count_partitions(n, m):
    if n == 0:



    else:
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Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4) → with_m
  - count_partitions(6, 3) → without_m
- Tree recursion often involves exploring different choices.

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(Demo)

Interactive Diagram