61A Lecture 10

Wednesday, September 24

Announcements

- Homework 3 due Wednesday 10/1 @ 11:59pm
  - Homework party on Monday evening, details TBD
- Optional Hog Contest entries due Wednesday 10/1 @ 11:59pm
- Composition scores for Project 1 will mostly be assigned this week
  - 3/3 is unusual on the first project
  - You can gain back composition points you lost on Project 1 by revising it (in November)
- Midterm 1 should be graded by Friday
  - Solutions to Midterm 1 will be posted after lecture
- Guerrilla section this Saturday 12-2 *and* 2:30-5 on recursion

Data

Data Types

Every value has a type

(demo)

Properties of native data types:

1. There are primitive expressions that evaluate to values of these types.

2. There are built-in functions, operators, and methods to manipulate those values.

Numeric Types in Python:

```
>>> type(2)
<class 'int'>
```

Represents integers exactly

```
>>> type(1.5)
<class 'float'>
```

Represents real numbers approximately

```
>>> type(1+1j)
<class 'complex'>
```

Objects

(Demo)

- Objects represent information.
- They consist of data and behavior, bundled together to create abstractions.
- Objects can represent things, but also properties, interactions, & processes.
- A type of object is called a class; classes are first-class values in Python.
- Object-oriented programming:
  - A metaphor for organizing large programs
  - Special syntax that can improve the composition of programs
- In Python, every value is an object.
  - All objects have attributes.
  - A lot of data manipulation happens through object methods.
  - Functions do one thing; objects do many related things.

Data Abstraction

Data Abstraction

- Compound objects combine objects together
  - A date: a year, a month, and a day
  - A geographic position: latitude and longitude
- An abstract data type lets us manipulate compound objects as units
- Isolate two parts of any program that uses data:
  - How data are represented (as parts)
  - How data are manipulated (as units)
- Data abstraction: A methodology by which functions enforce an abstraction barrier between ***representation*** and ***use***

All Programmers

Great Programmers

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost!

Assume we can compose and decompose rational numbers:

Constructor

- rational(n, d) returns a rational number x

Selectors

- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5} = \frac{21}{10}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

General Form

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

These functions implement an abstract data type for rational numbers

Pairs

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
2
```

Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

Element selection function

More lists next lecture

Representing Rational Numbers

```
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

Select item from a list

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

$$\frac{2}{5} + \frac{1}{10} = \frac{1}{2}$$

$$\frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

```
from fractions import gcd
```

Greatest common divisor

```
def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
    return [n//g, d//g]
```

Abstraction Barriers

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	add_rational, mul_rational rationals_are_equal, print_rational
Create rationals or implement rational operations	numerators and denominators	rational, numer, denom
Implement selectors and constructor for rationals	two-element lists	list literals and element selection

Implementation of lists

Violating Abstraction Barriers

Does not use constructors

Twice!

```
add_rational( [1, 2], [1, 4] )
```

```
def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
```

No selectors!

And no constructor!

Data Representations

What is Data?

- We need to guarantee that constructor and selector functions work together to specify the right behavior.
- Behavior condition: If we construct rational number x from numerator n and denominator d, then numer(x)/denom(x) must equal n/d.
- An abstract data type is some collection of selectors and constructors, together with some behavior condition(s).
- If behavior conditions are met, then the representation is valid.

You can recognize abstract data types by their behavior, not by their class

Behavior Conditions of a Pair

To implement our rational number abstract data type, we used a two-element list.

But is that the only way to make pairs of values? No!

Constructors, selectors, and behavior conditions:

> If a pair p was constructed from elements x and y, then
>
> - select(p, 0) returns x, and
> - select(p, 1) returns y.

Together, selectors are the inverse of the constructor

Generally true of container types.

Not true for rational numbers because of GCD

(Demo)

Functional Pair Implementation

```
def pair(x, y):
    """Return a function that represents a pair."""
    def get(index):
        if index == 0:
            return x
        elif index == 1:
            return y
    return get
```

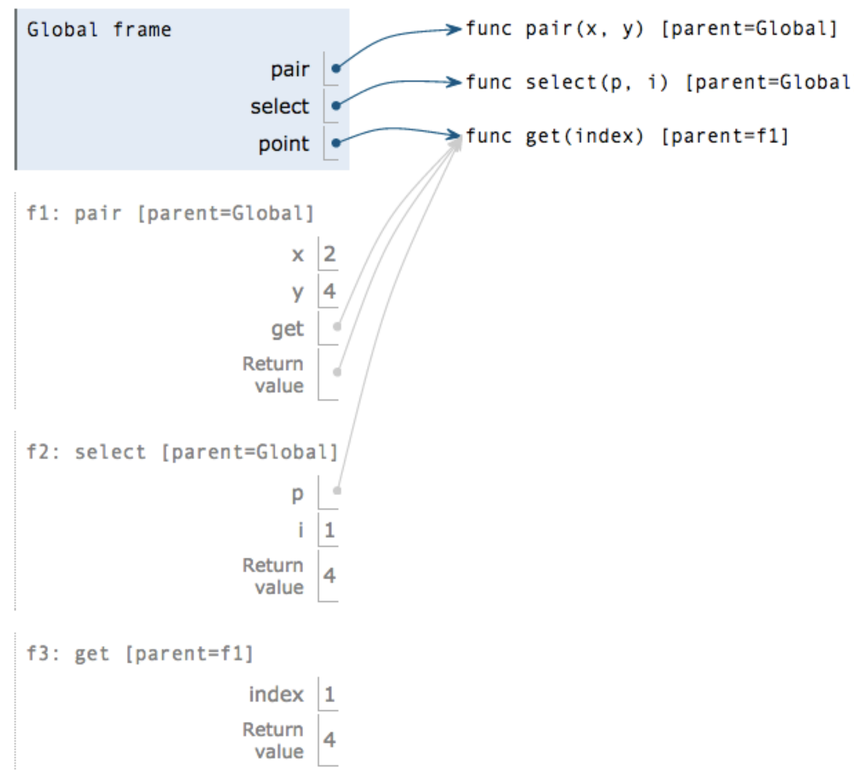
This function represents a pair

Constructor is a higher-order function

```
def select(p, i):
    """Return the element at index i of pair p."""
    return p(i)
```

Selector defers to the object itself

```
point = pair(2, 4)
select(point, 1)
```



Interactive Diagram

Using a Functionally Implemented Pair

```
>>> p = pair(1, 2)

>>> select(p, 0)
1

>>> select(p, 1)
2
```

As long as we do not violate the abstraction barrier, we don't need to know that pairs are just functions

If a pair p was constructed from elements x and y, then

- select(p, 0) returns x, and
- select(p, 1) returns y.

This pair representation is valid!