

61A Lecture 15

Monday, October 6

Announcements

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- Homework 4 due Tuesday 10/7 @ 11:59pm

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- Tips for approaching computer science problems

Mutable Functions

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of \$100

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>>> withdraw(25)
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Second withdrawal of
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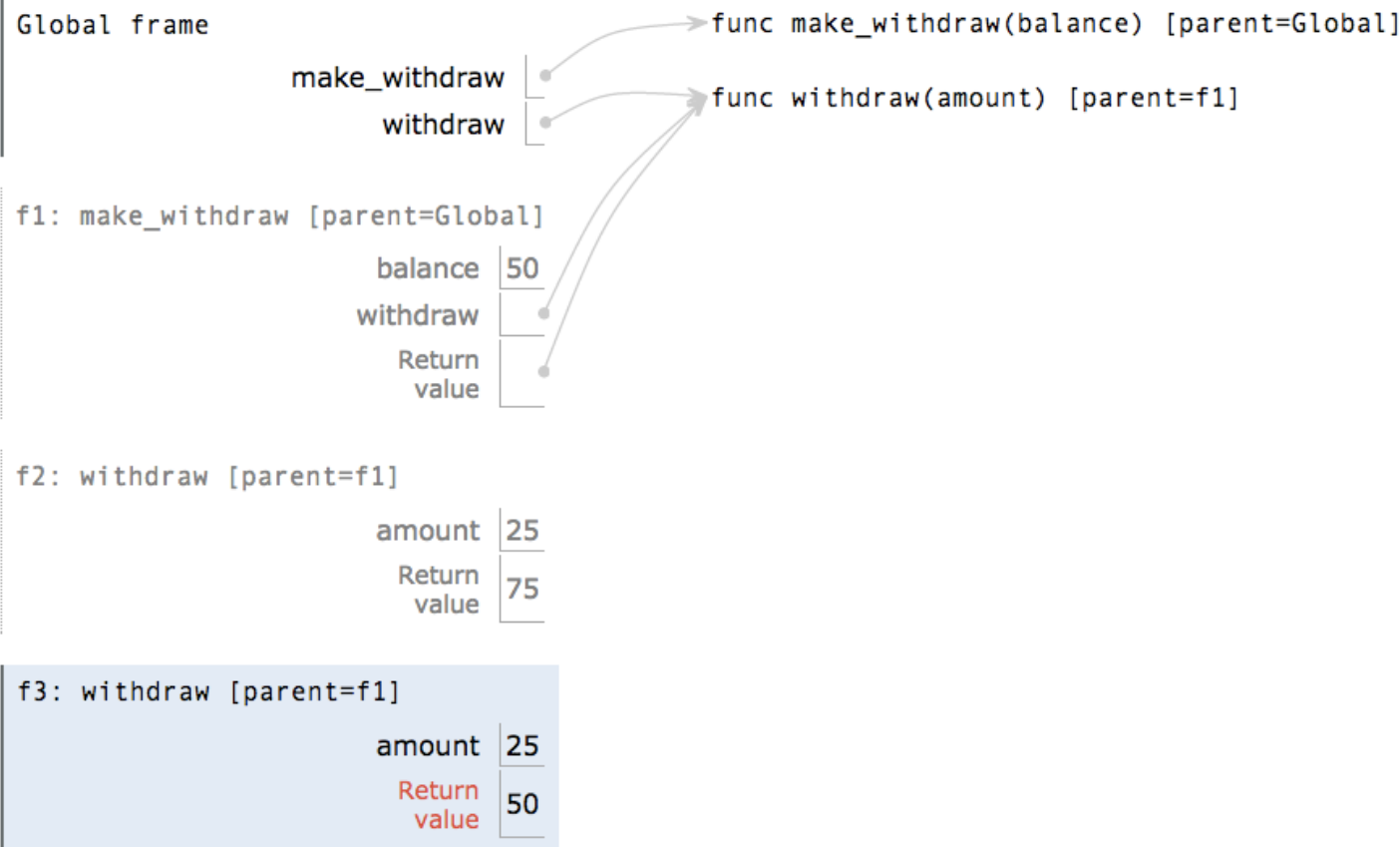
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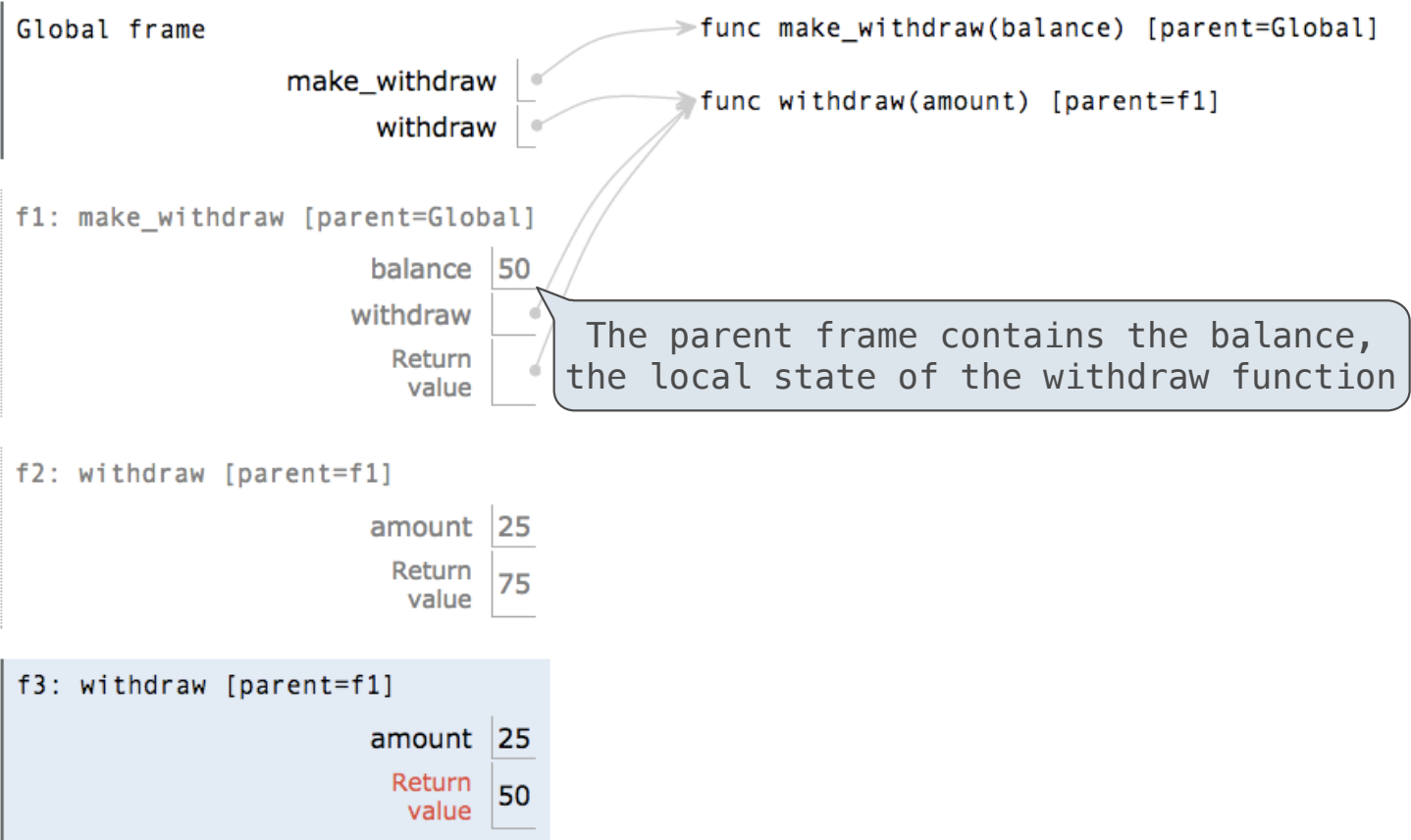
Within the parent frame
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A function has a body and
a parent environment

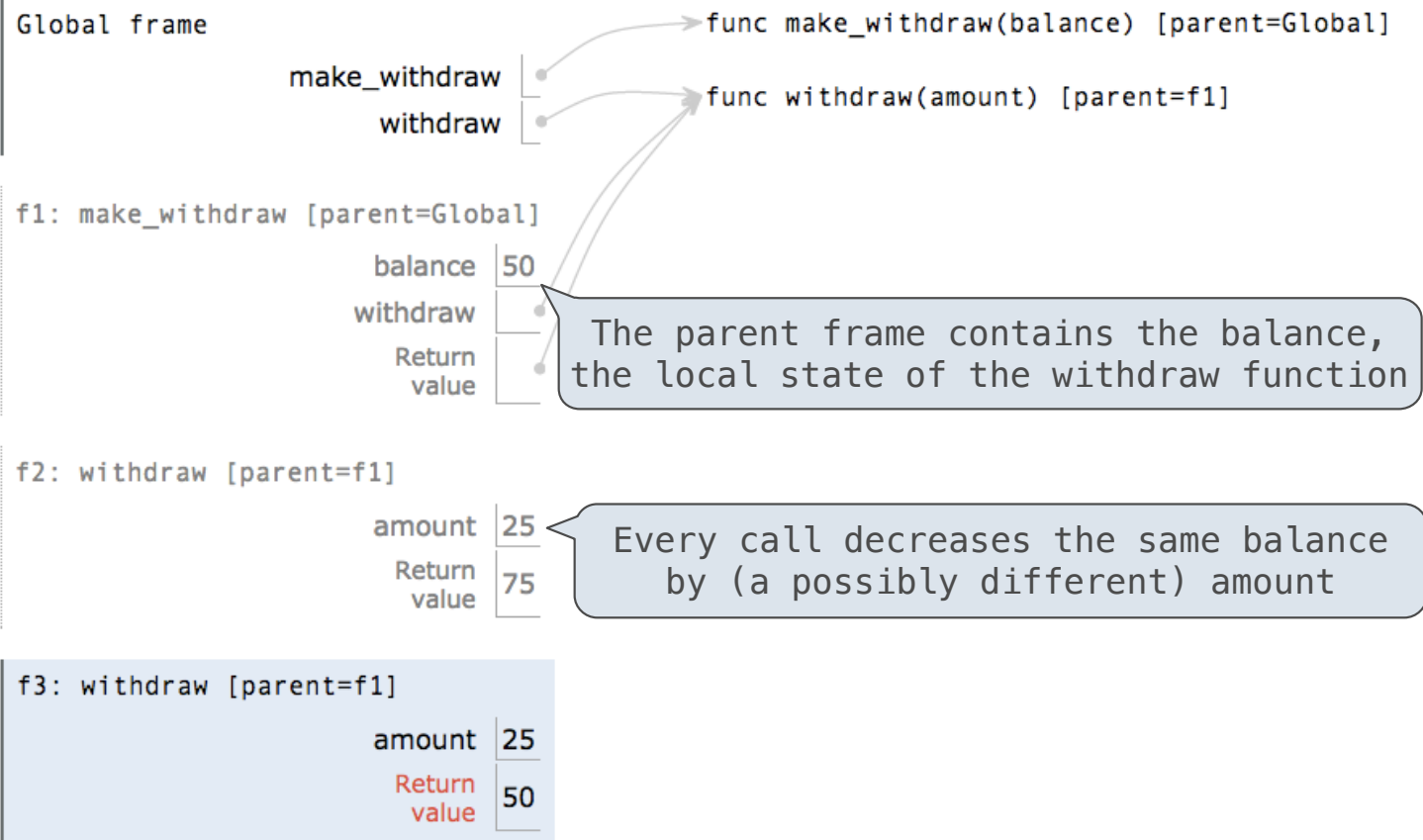
Persistent Local State Using Environments



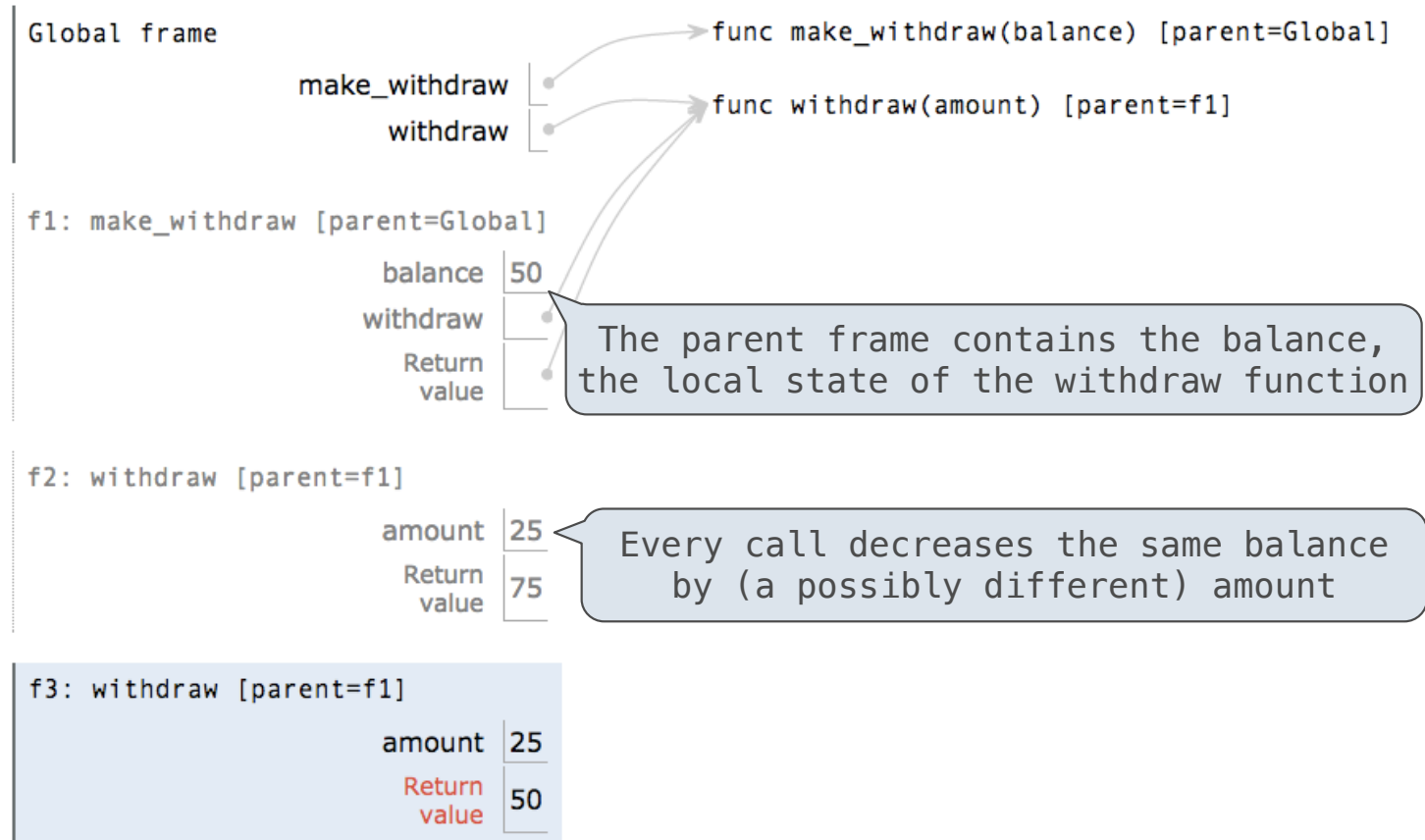
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Persistent Local State Using Environments



Interactive Diagram


Reminder: Local Assignment

```
def percent_difference(x, y):  
    difference = abs(x-y)  
    return 100 * difference / x  
diff = percent_difference(40, 50)
```

Global frame

percent_difference

func percent_difference(x, y) [parent=Global]



f1: percent_difference [parent=Global]

x	40
y	50
difference	10

Interactive Diagram

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Assignment binds name(s) to value(s) in the first frame of the current environment

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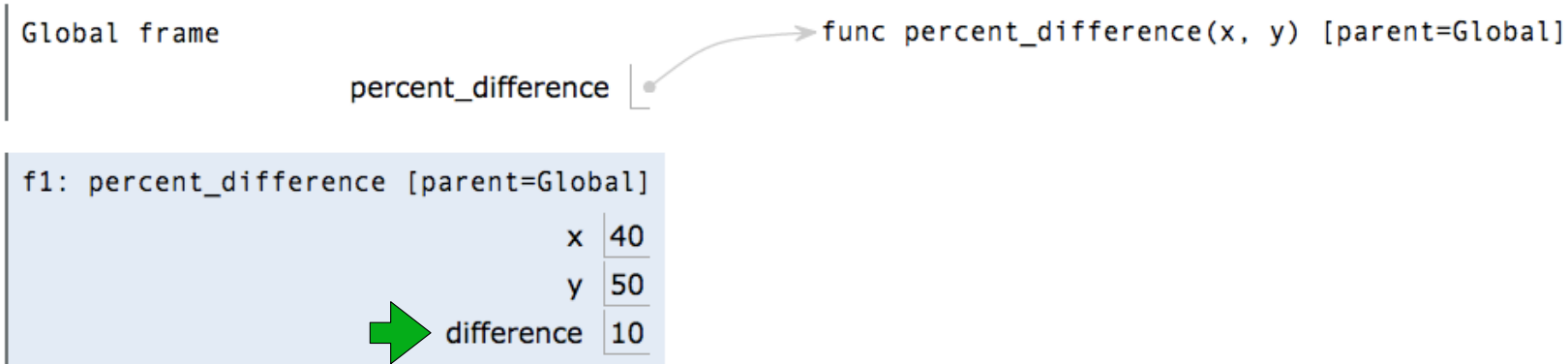
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Execution rule for assignment statements:

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Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right
2. Bind the names on the left to the resulting values in the **current frame**

Interactive Diagram

Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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def make_withdraw(balance):  
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    def withdraw(amount):  
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```

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```

Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):  
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Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned

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```
        if amount > balance:
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Re-bind balance in the first non-local frame in which it was bound previously

```
        return balance
```

```
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(Demo)

Non-Local Assignment

The Effect of Nonlocal Statements

`nonlocal <name>`

The Effect of Nonlocal Statements

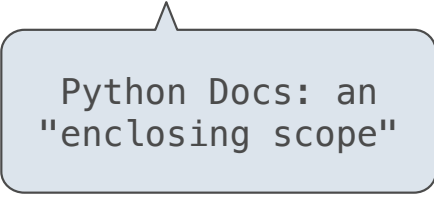
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Effect: Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

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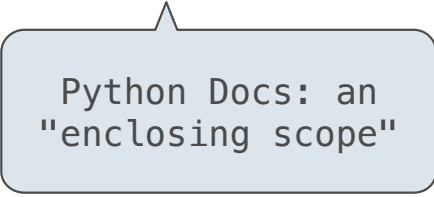


Python Docs: an
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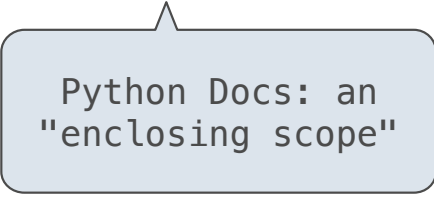


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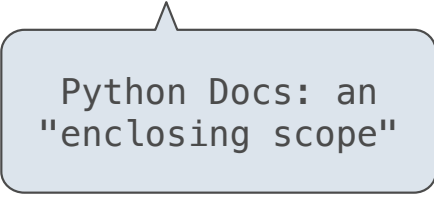
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<http://www.python.org/dev/peps/pep-3104/>

The Many Meanings of Assignment Statements

$$x = 2$$

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Status

$$x = 2$$

Effect

The Many Meanings of Assignment Statements

<code>x = 2</code>

Status

Effect

- No nonlocal statement
 - "x" **is not** bound locally
-
-
-
-
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Create a new binding from name "x" to object 2 in the first frame of the current environment

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SyntaxError: name 'x' is parameter and nonlocal

Python Particulars

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def make_withdraw(balance):  
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        if amount > balance:  
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        balance = balance - amount  
        return balance  
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wd = make_withdraw(20)  
wd(5)
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Local assignment

[Interactive Diagram](#)

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Local assignment

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UnboundLocalError: local variable 'balance' referenced before assignment

[Interactive Diagram](#)

Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

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```
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
withdraw(25)
```

Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

Name bound
outside of
withdraw def

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def make_withdraw_list(balance):
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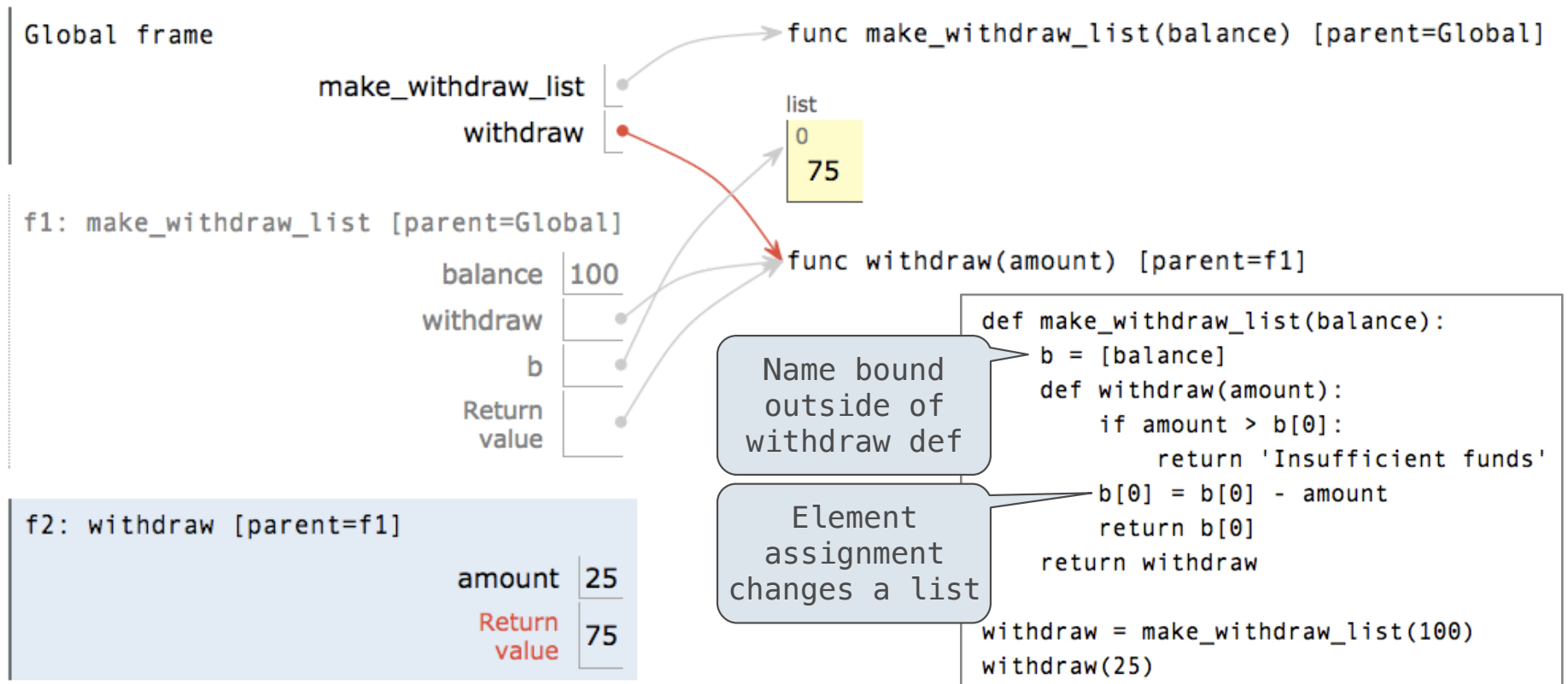
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Element
assignment
changes a list

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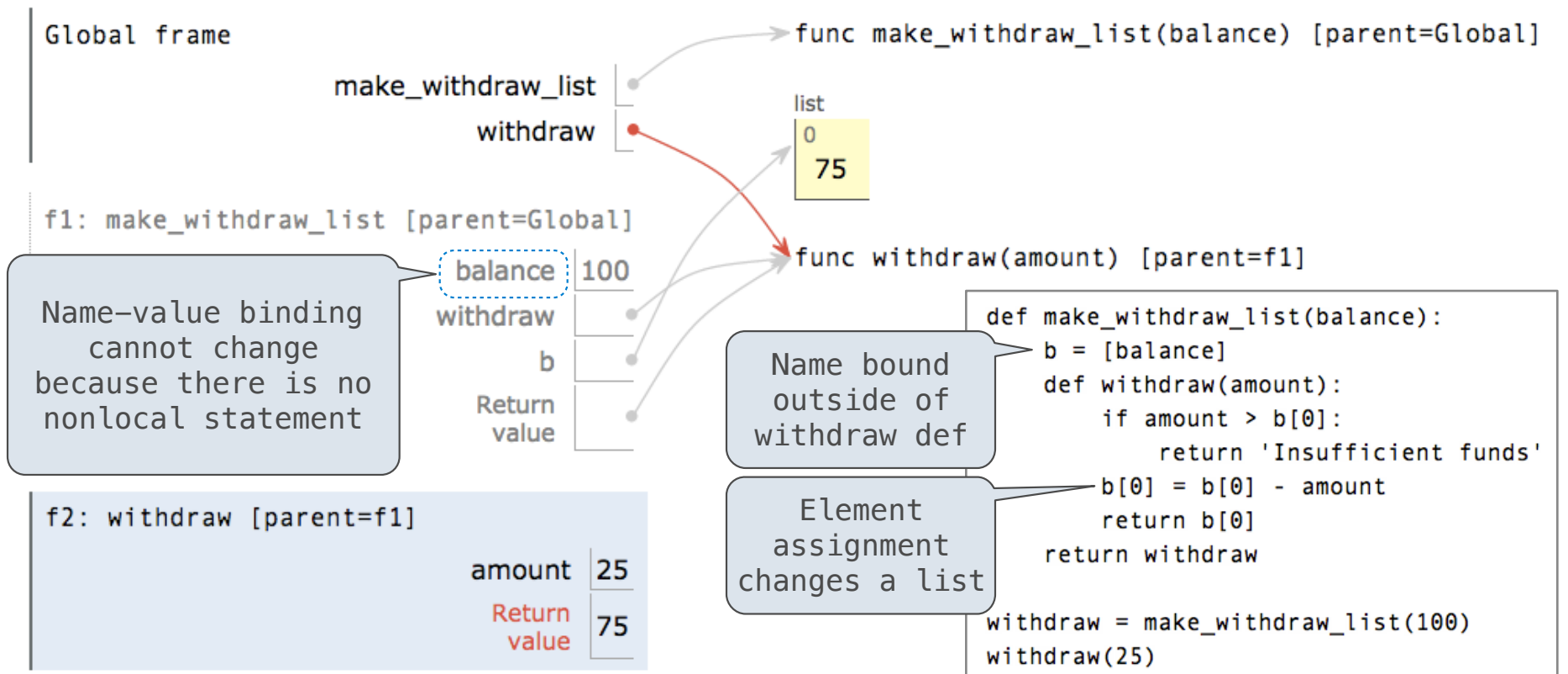
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Interactive Diagram

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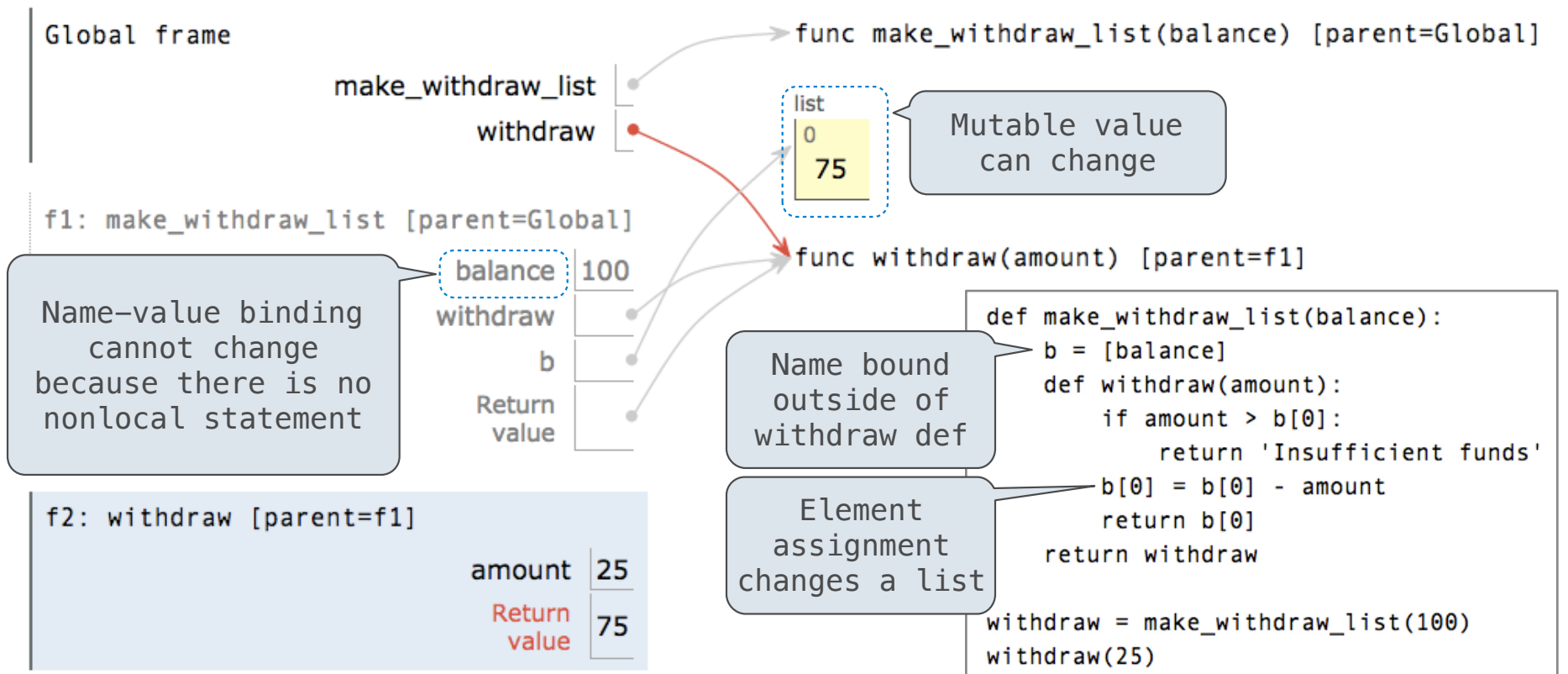
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Interactive Diagram

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Interactive Diagram

Multiple Mutable Functions

(Demo)

Referential Transparency, Lost

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