# 61A Lecture 19

Wednesday, October 15

# Announcements

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- Guerrilla Section 4 on Sunday 10/19: Object-oriented programming and recursive data

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  - Emphasis: mutable data, object-oriented programming, recursion, and recursive data
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  - Review session on Saturday 10/26 3pm–4:30pm and 4:30pm–6pm in 2050 VLSB

# Generic Functions of Multiple Arguments

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*What's different*? Today's generic functions apply to multiple arguments that don't share a common interface.

# Rational Numbers

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```
class Rational:
    """A rational number represented as a numerator and denominator."""
    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numer = numer // g
        self.denom = denom // g

    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)
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    def add(self, other):
        nx, dx = self.numer, self.denom
        ny, dy = other.numer, other.denom
        return Rational(nx * dy + ny * dx, dx * dy)

    def mul(self, other):
        numer = self.numer * other.numer
        denom = self.denom * other.denom
        return Rational(numer, denom)
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(Demo)

# Complex Numbers

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class Complex:
    def add(self, other):
        return ComplexRI(self.real + other.real,
                         self.imag + other.imag)
    def mul(self, other):
        return ComplexMA(self.magnitude * other.magnitude,
                         self.angle + other.angle)
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class ComplexRI(Complex):
    """A rectangular representation."""
    def __init__(self, real, imag):
        self.real = real
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    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

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```
class ComplexMA(Complex):
    """A polar representation."""
    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
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    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)
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class Complex:
    def add(self, other):
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# Special Method Names

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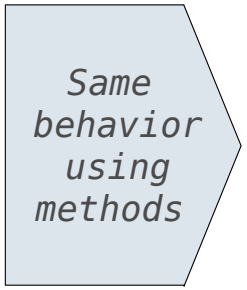

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```
>>> zero, one, two = 0, 1, 2
>>> one + two
3
>>> bool(zero), bool(one)
(False, True)
```

*Same behavior using methods*

```
>>> zero, one, two = 0, 1, 2
```

# Special Method Names in Python

Certain names are special because they have built-in behavior

These names always start and end with two underscores

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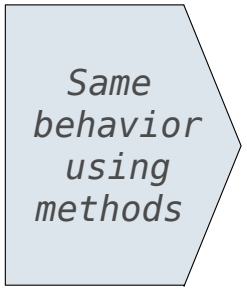


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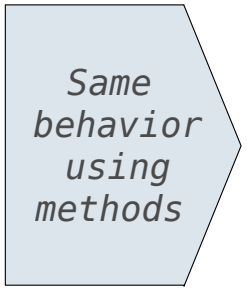


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# Special Methods

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Adding instances of user-defined classes invokes the `__add__` method

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class Number:
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class Complex(Number):
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http://getpython3.com/diveintopython3/special-method-names.html

http://docs.python.org/py3k/reference/datamodel.html#special-method-names

# Type Dispatching

# The Independence of Data Types

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Complex numbers as two-dimensional vectors

```
def add_complex_and_rational(c, r):
    """Return c + r for complex c and rational r."""
    return ComplexRI(c.real + r.numer/r.denom, c.imag)
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(Demo)

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$$m \cdot (m-1) \cdot n$$

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| Rational | Rational | | |
| Complex | Rational | | |
| Rational | Complex | | |

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Question: Is coercion exact?

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| Type | Add | Multiply |
|---|---|---|
| Complex | | |
| Rational | | |