61A Lecture 20

Friday, October 17

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  - 10am - 11pm in Wozniak Lounge

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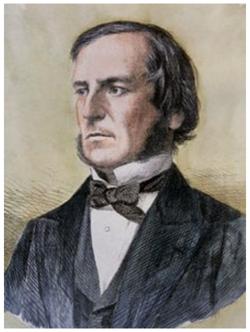

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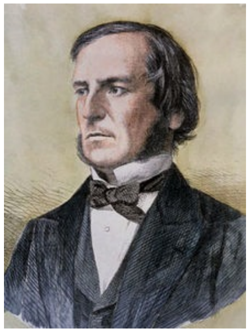

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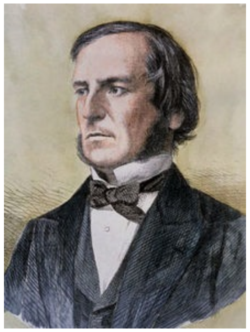

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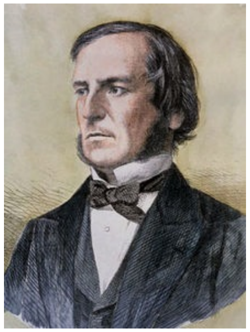

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cs61a.org/cohorts.html

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

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def fib(n):
    if n == 0:
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fib(5)

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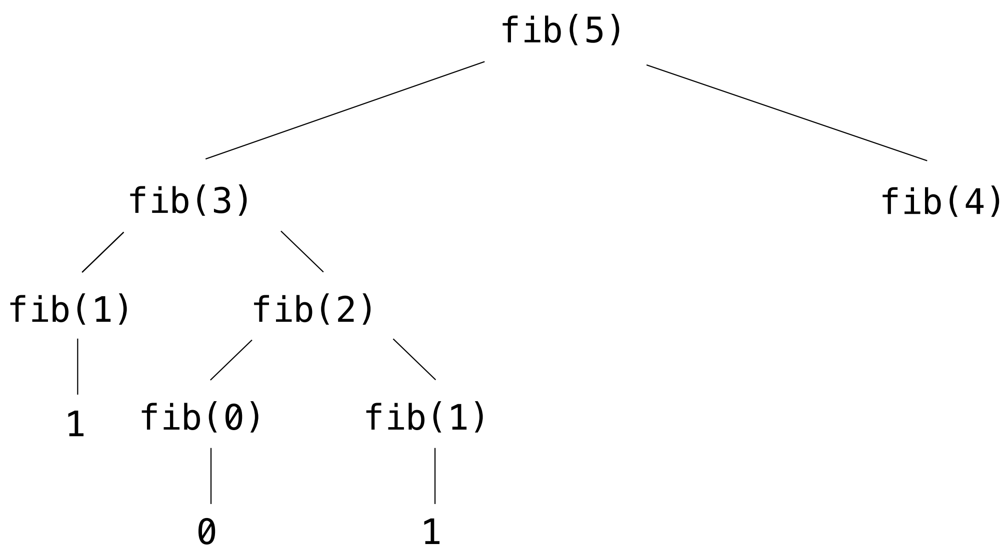


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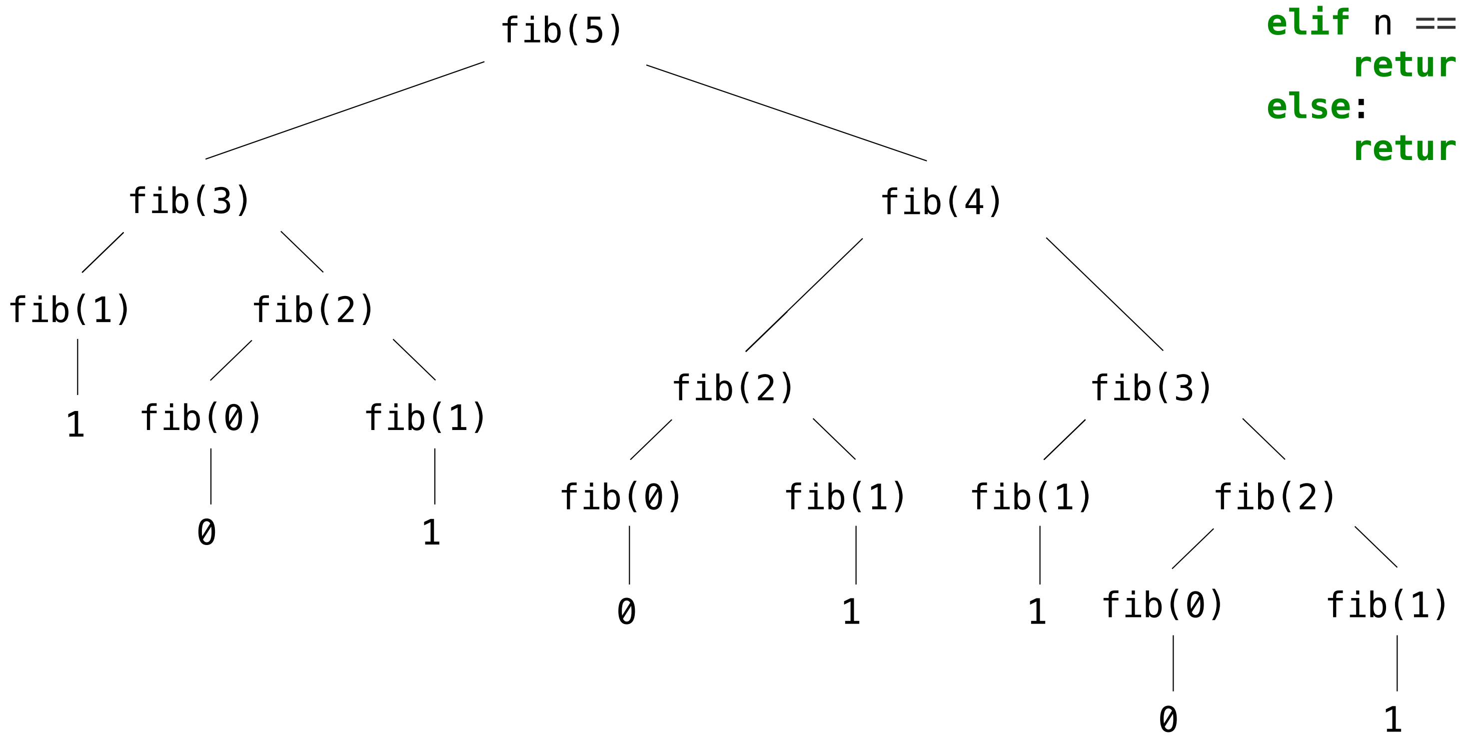

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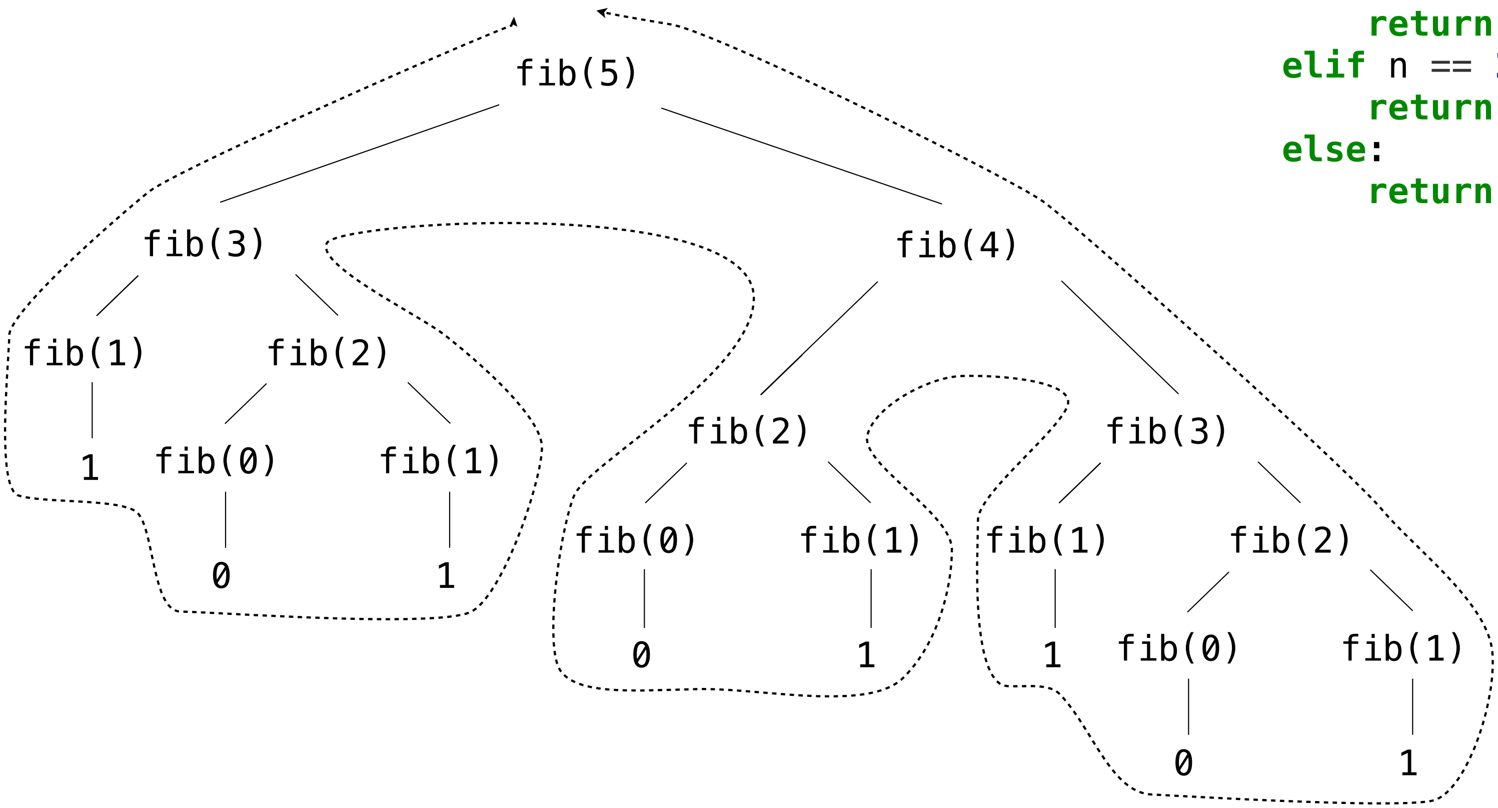

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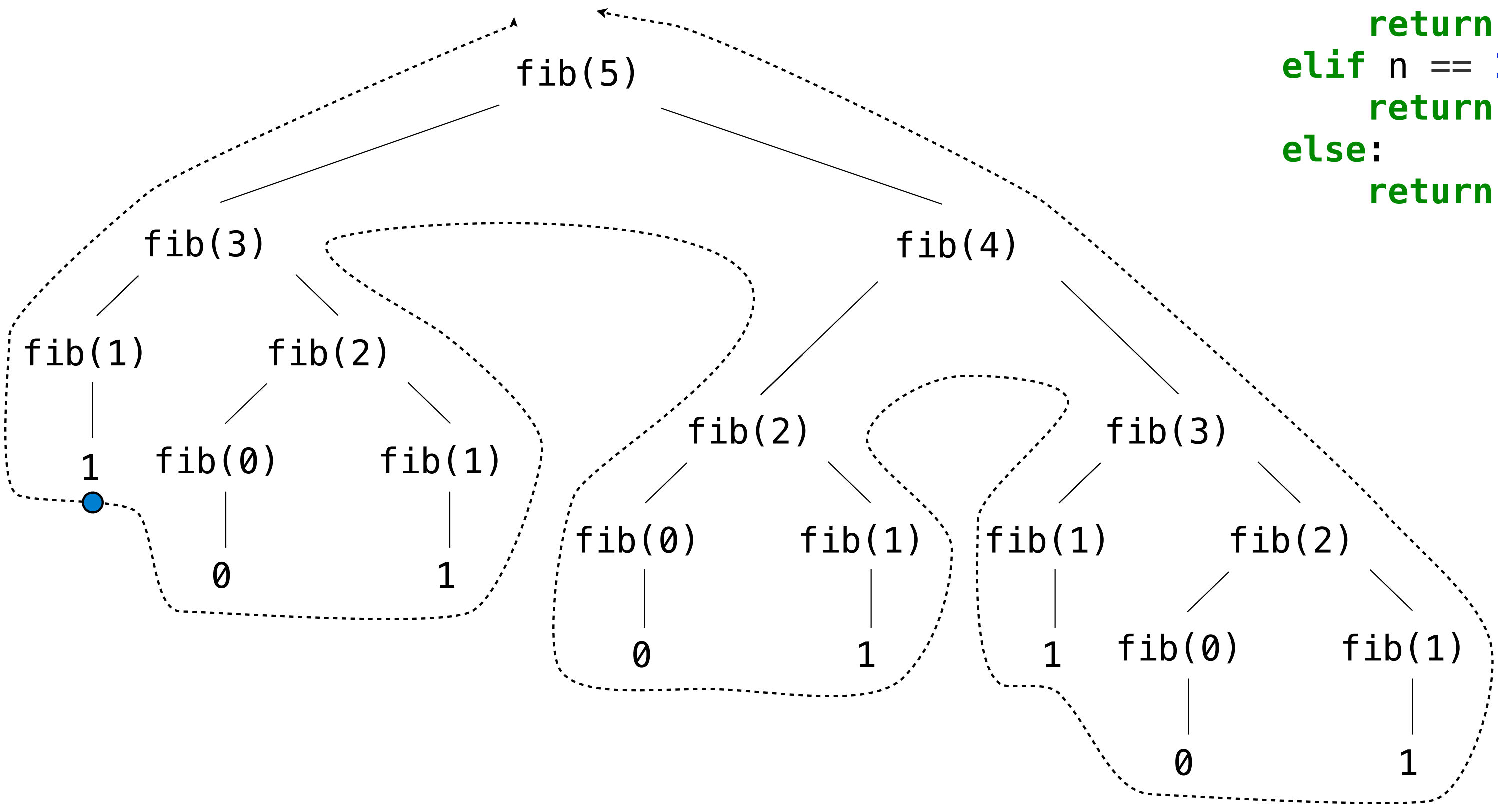

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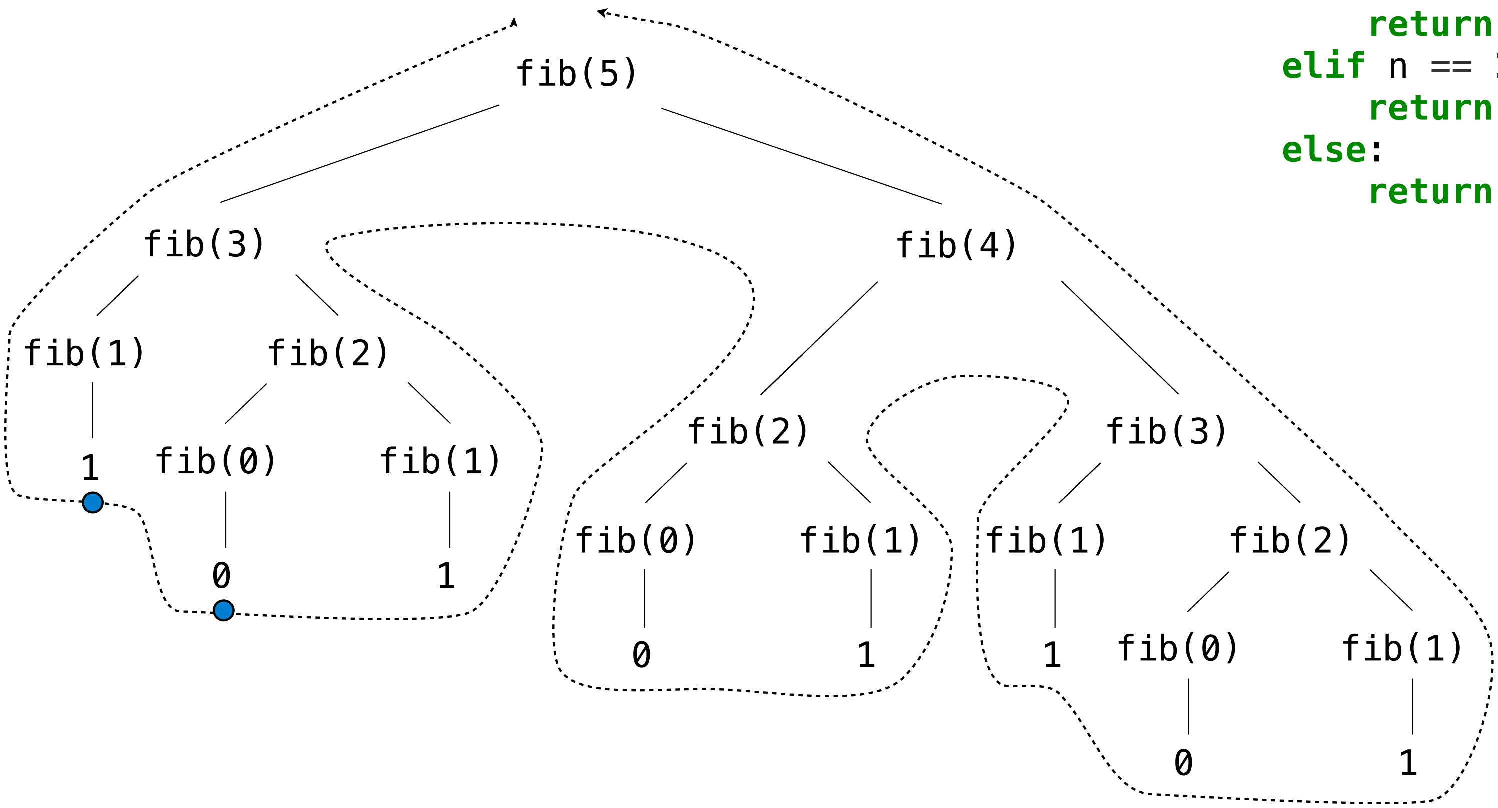

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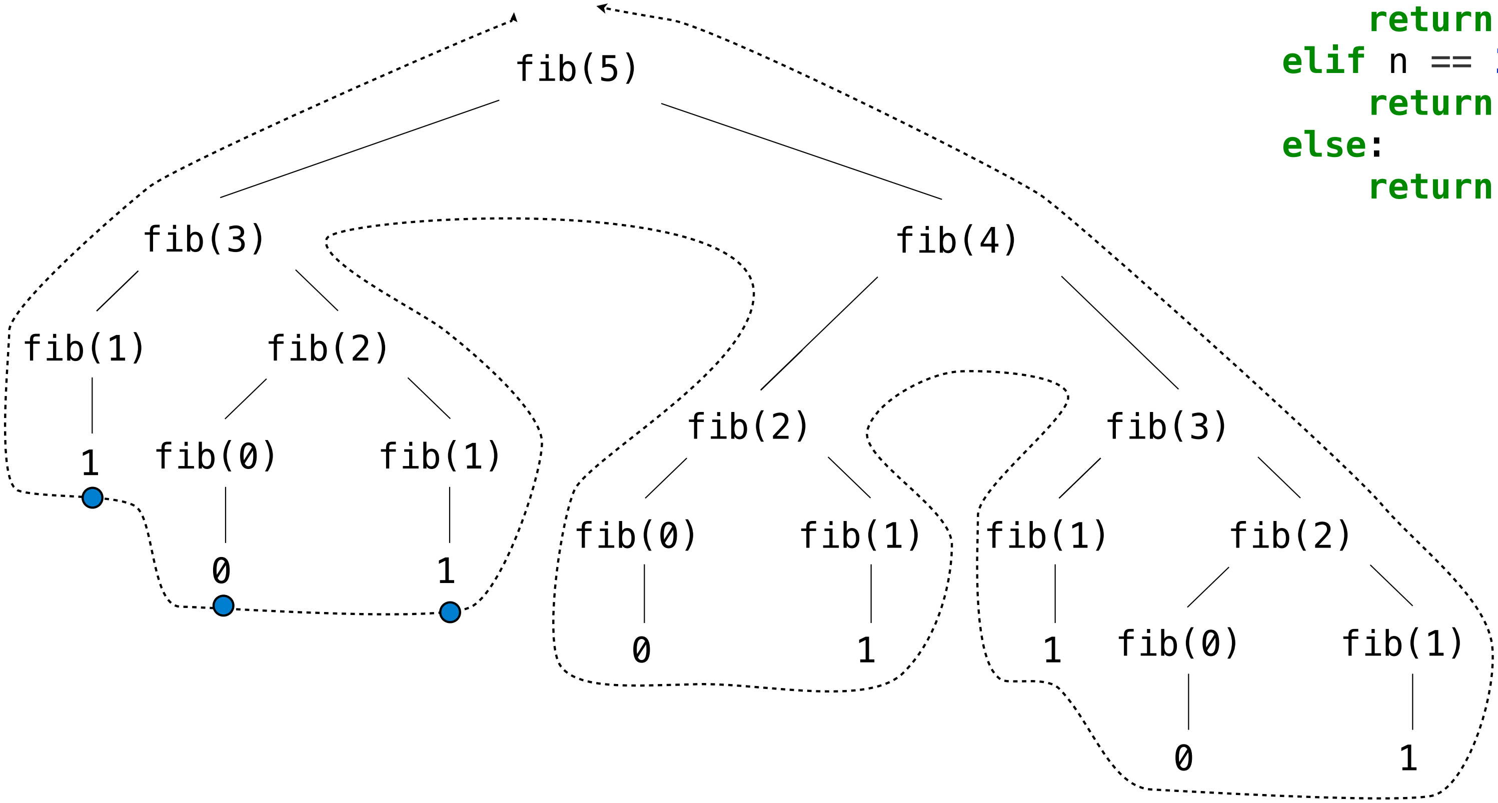

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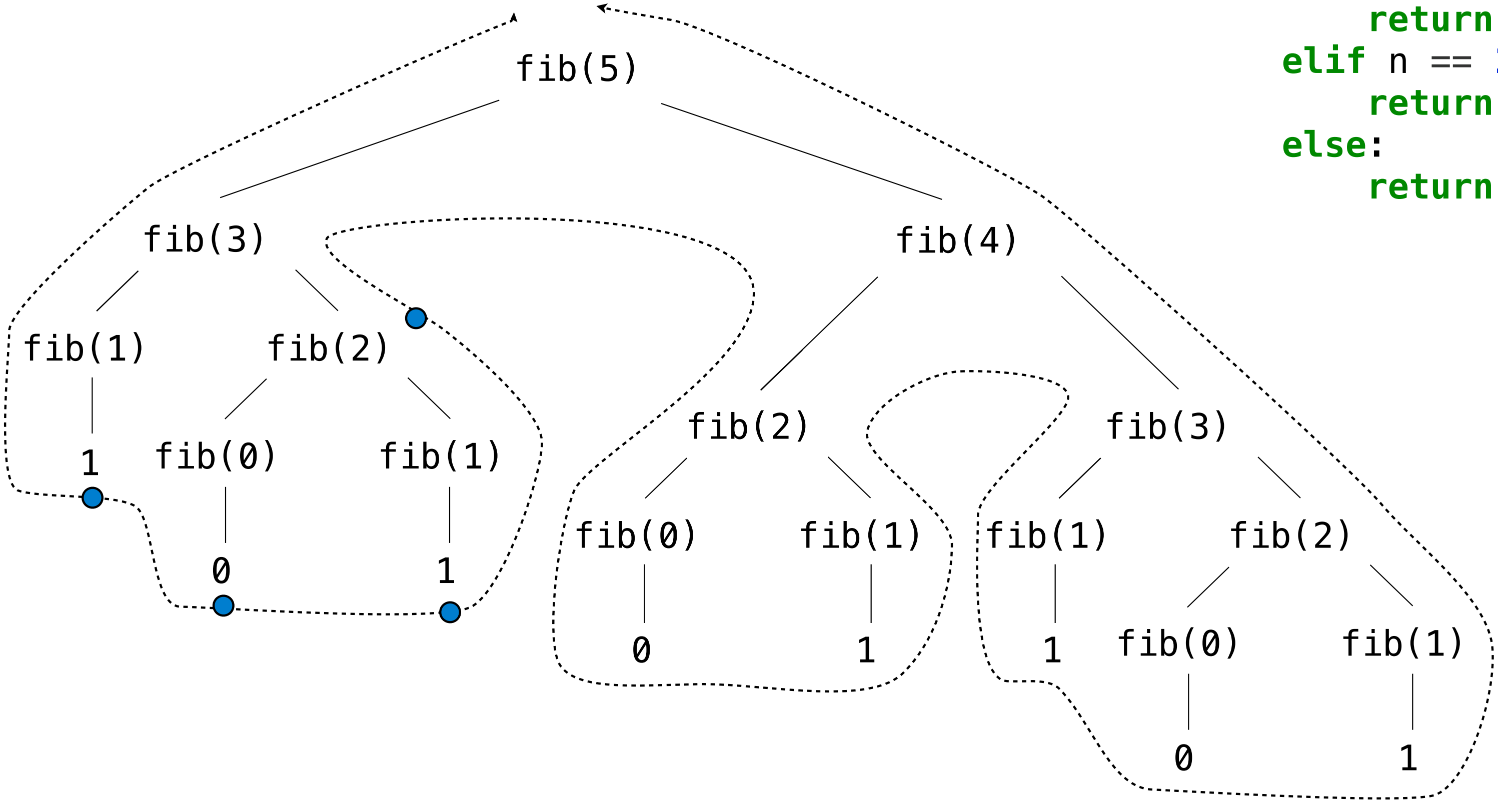

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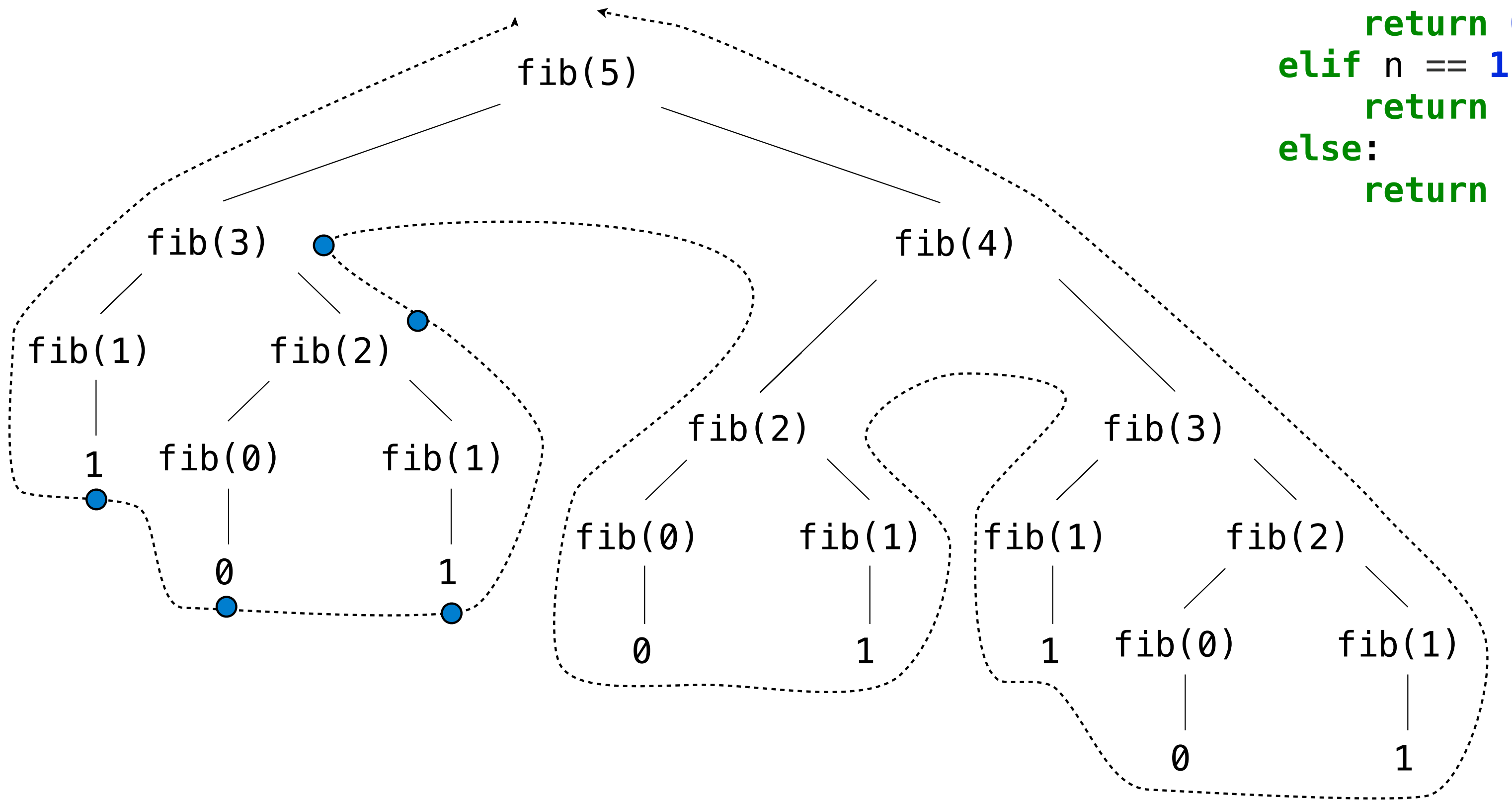

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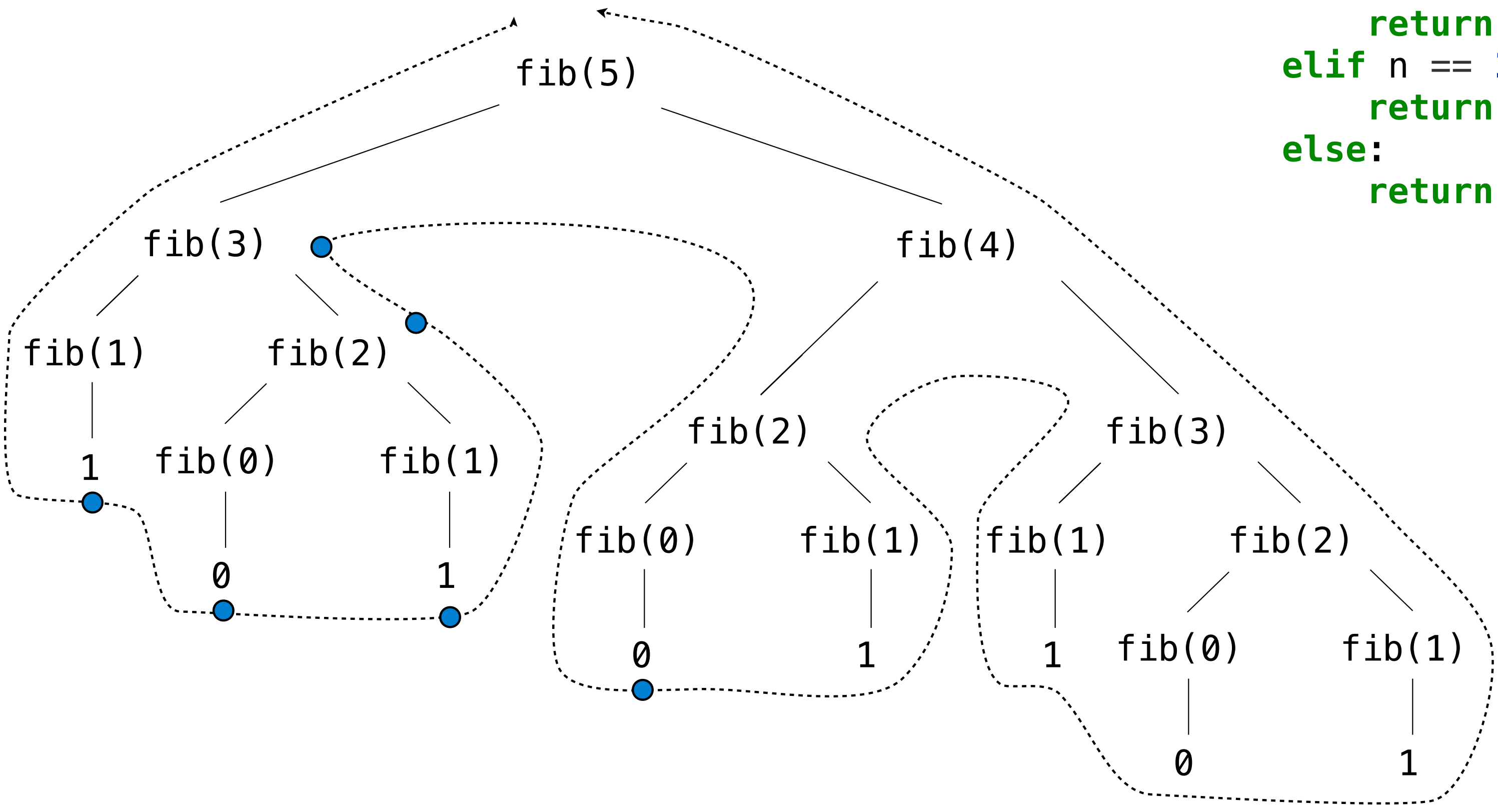

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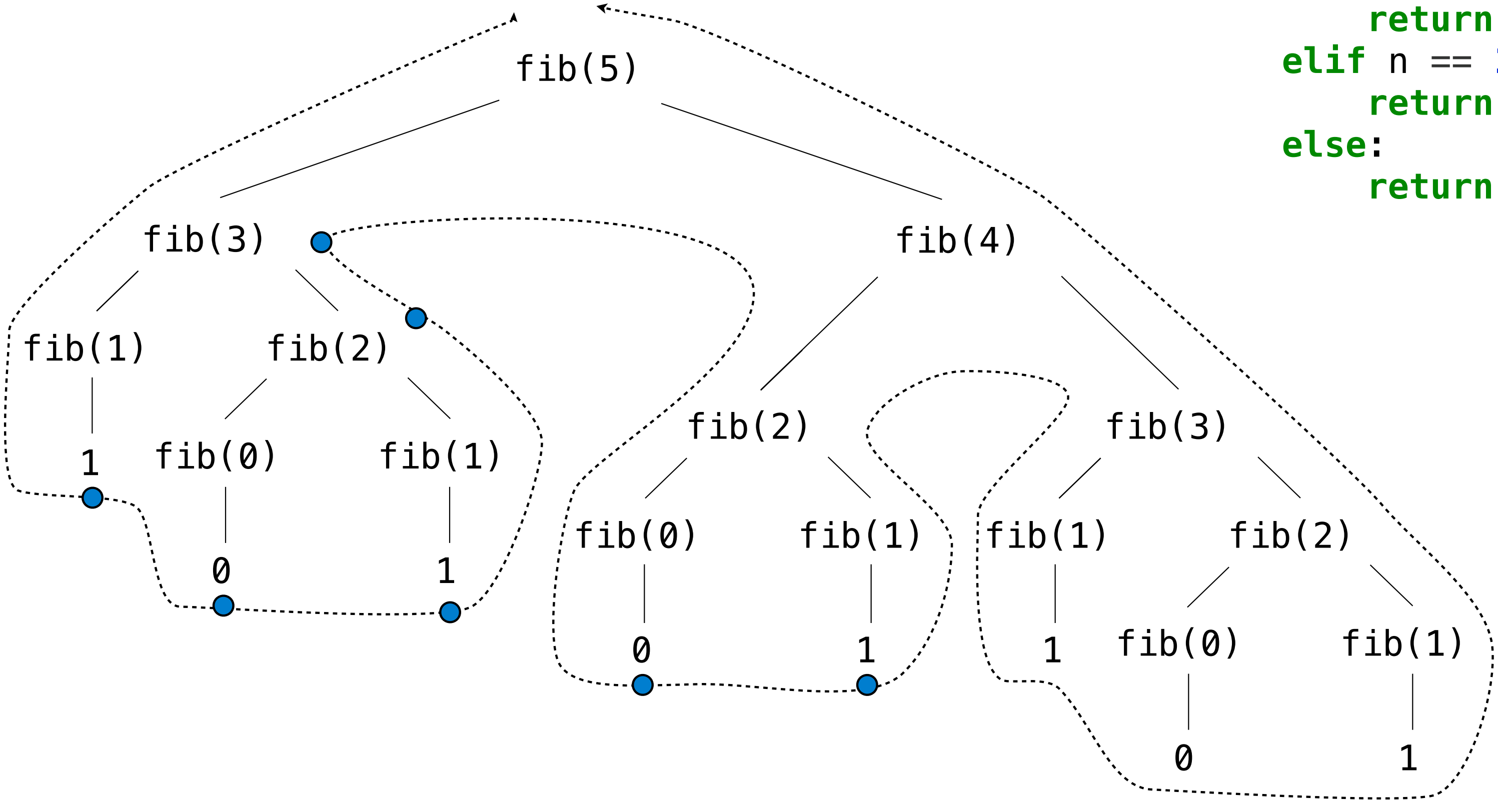

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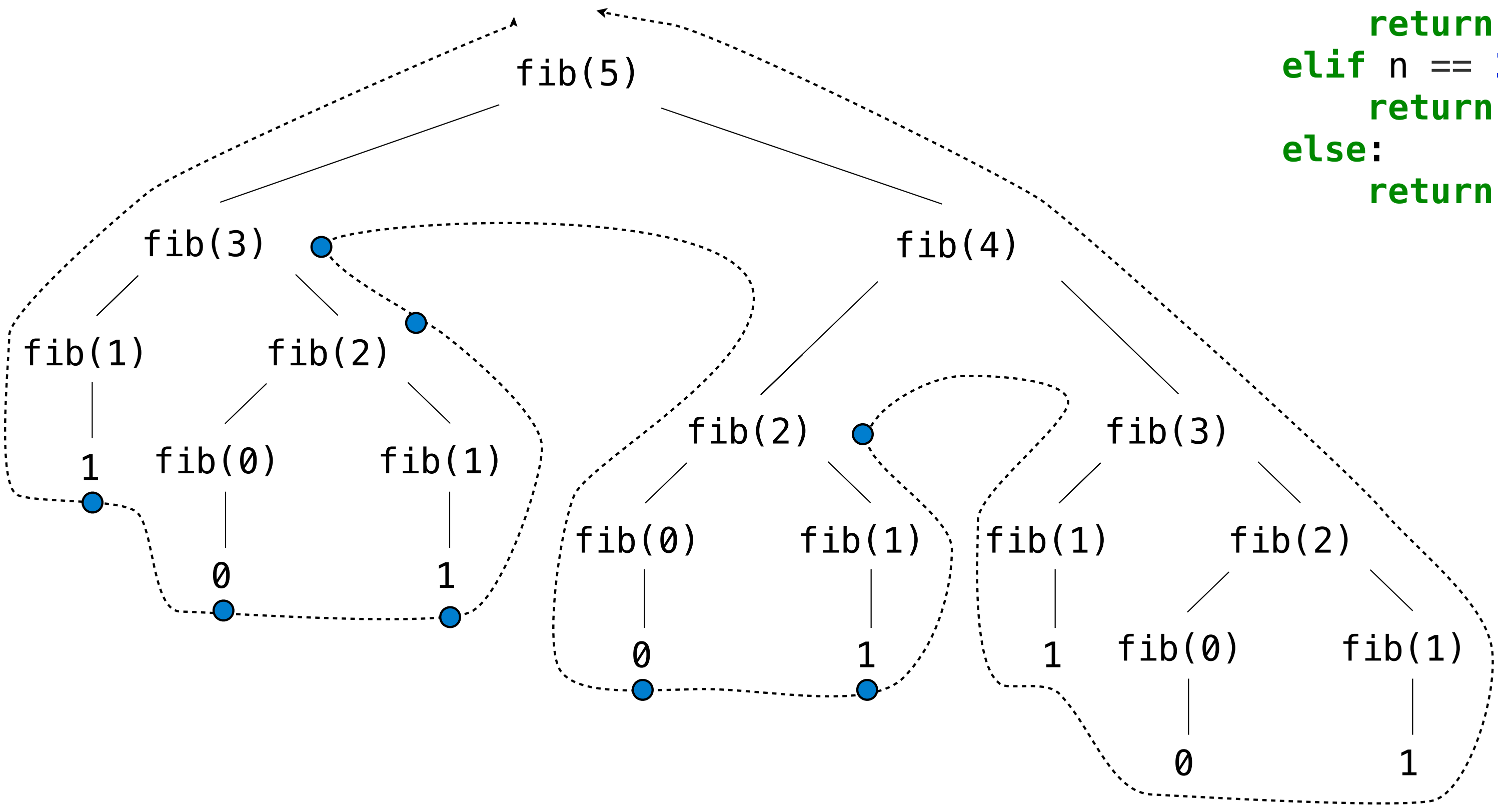

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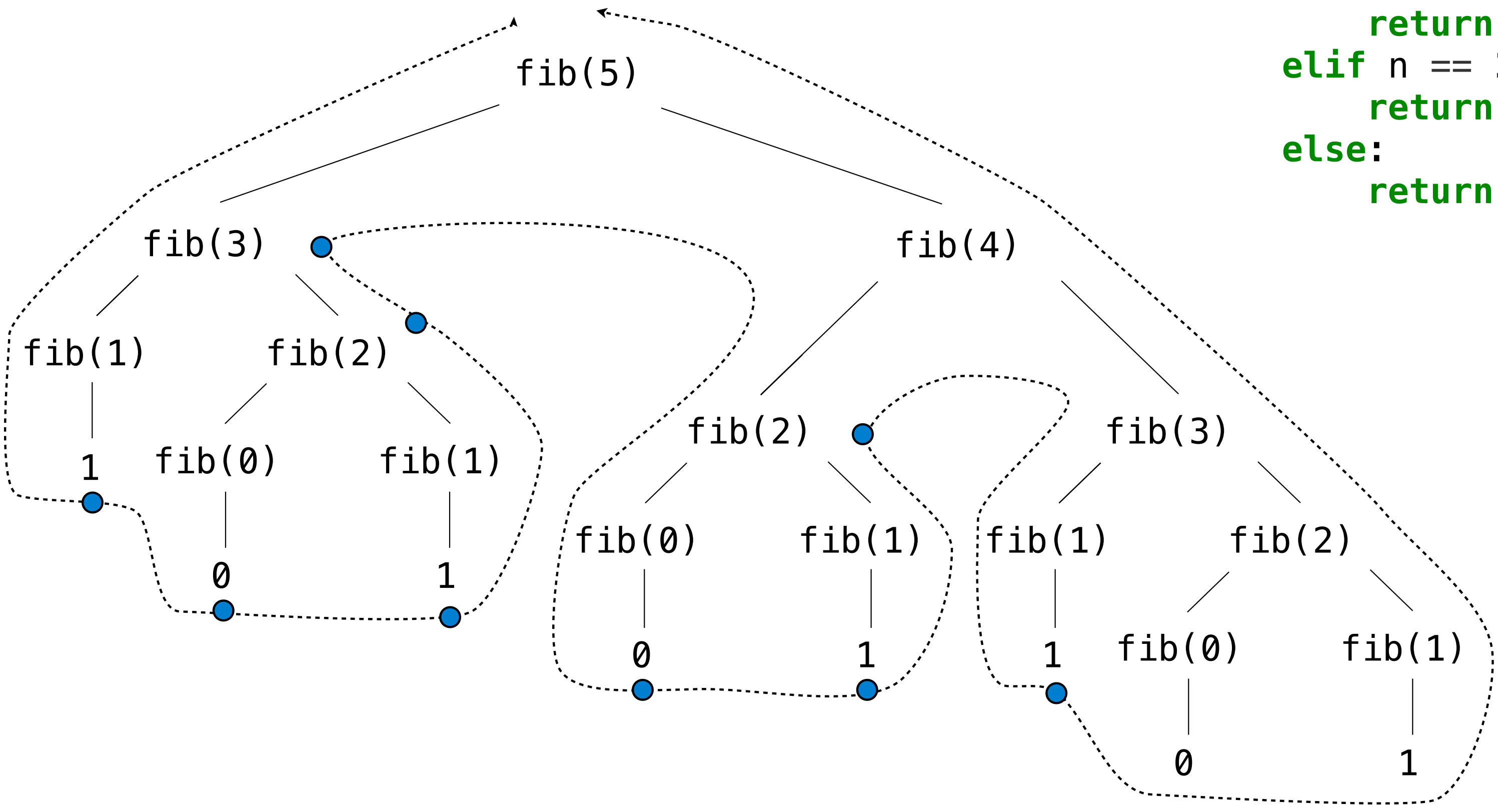

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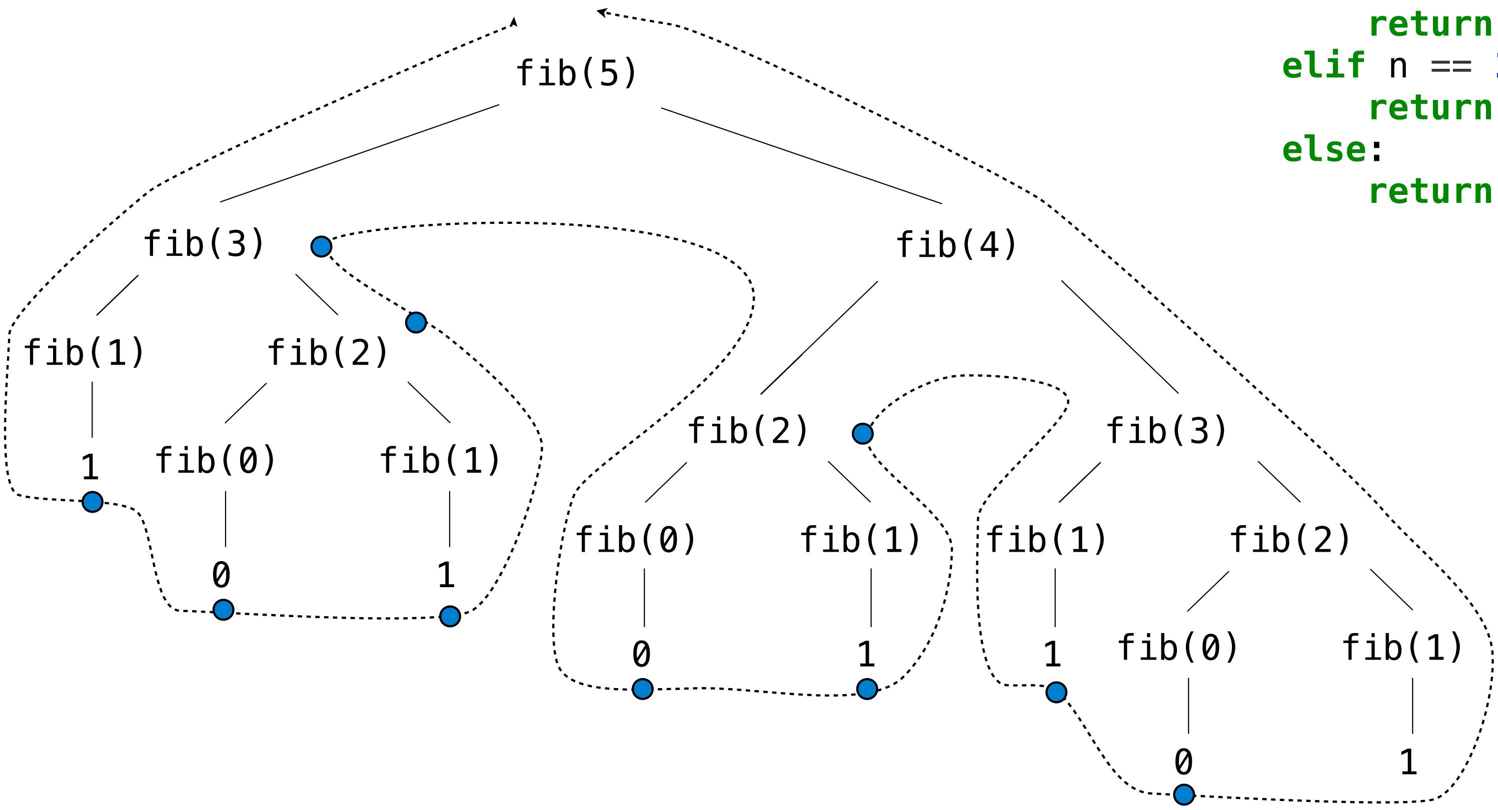


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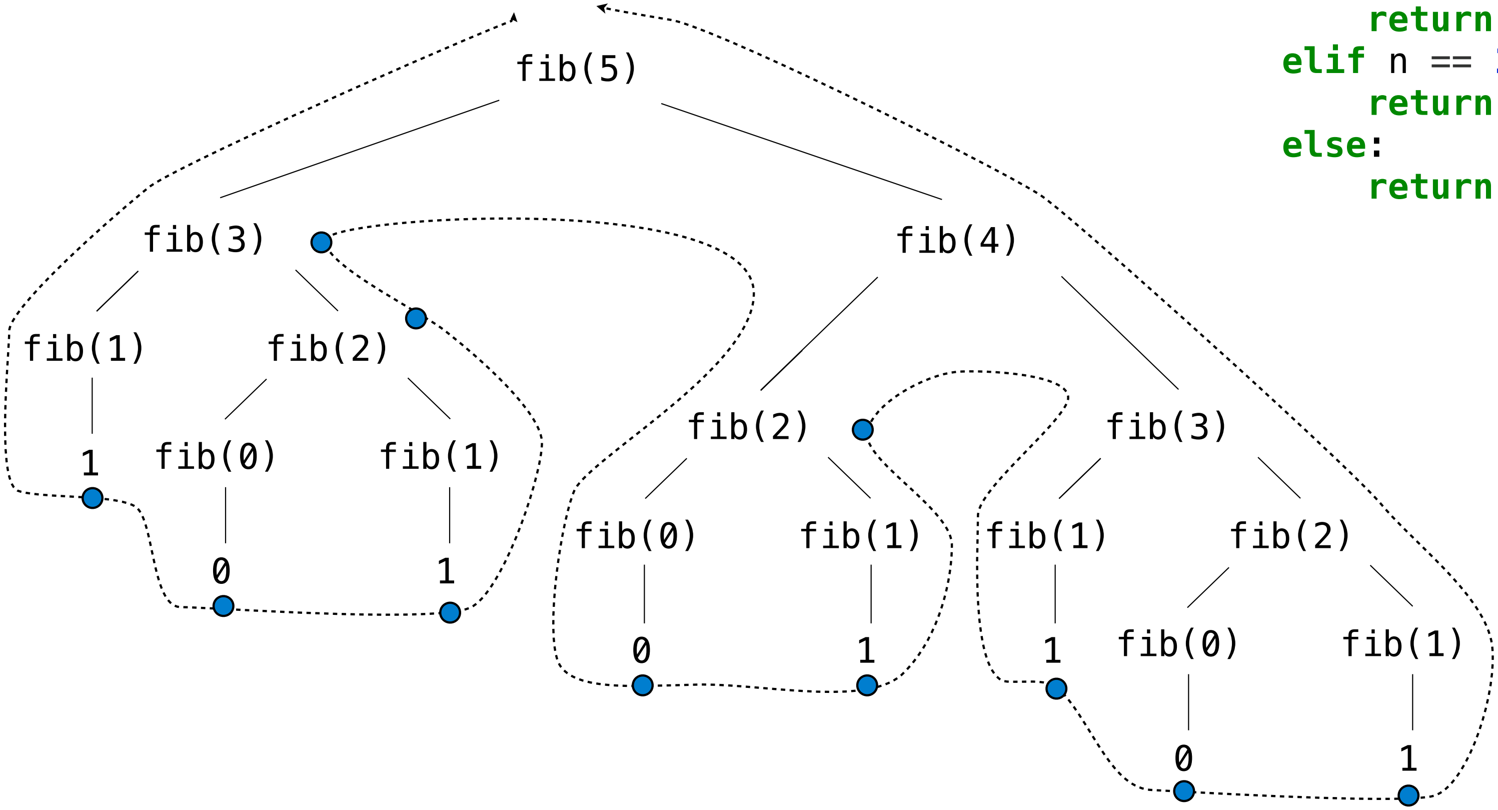

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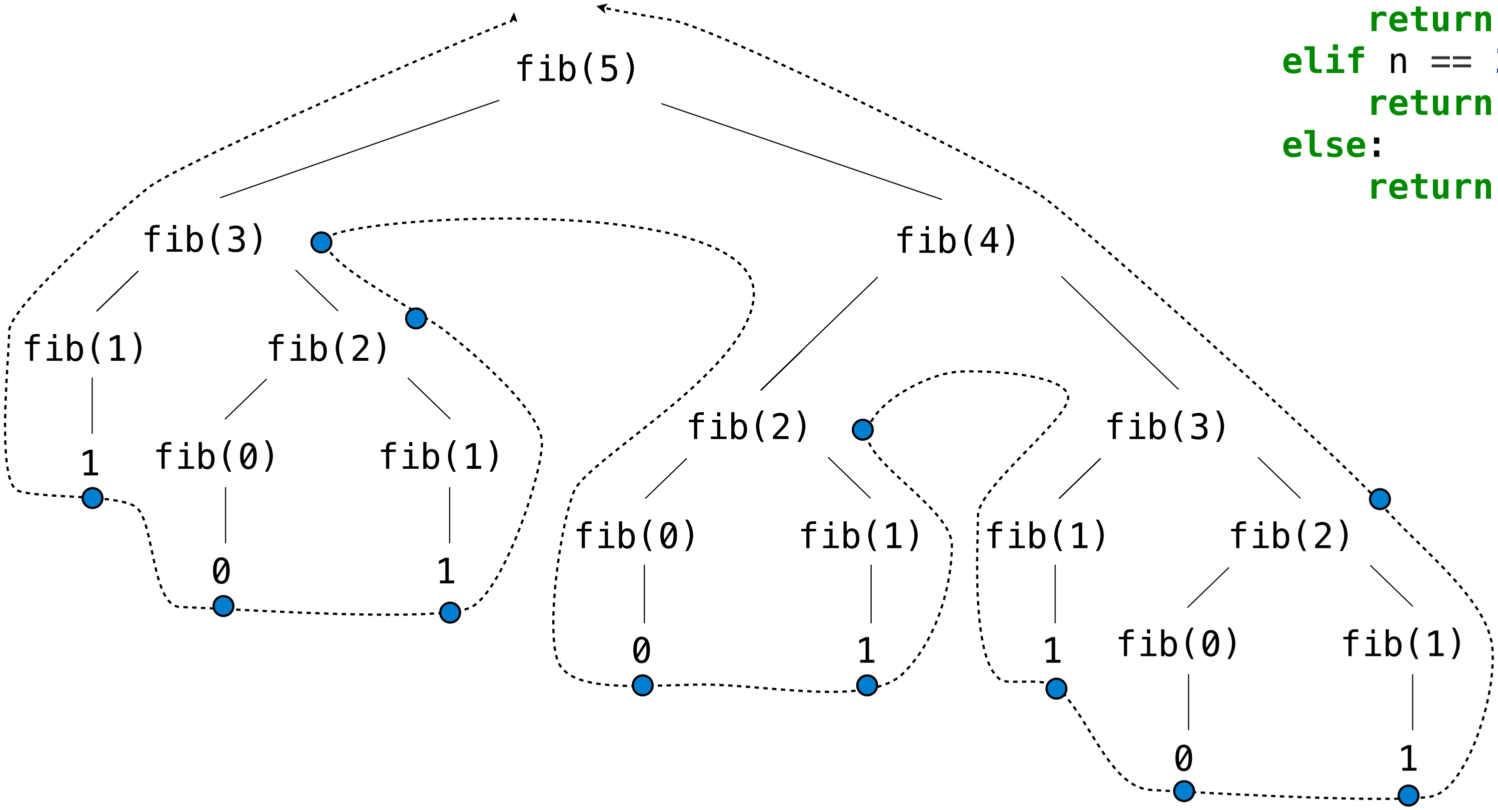

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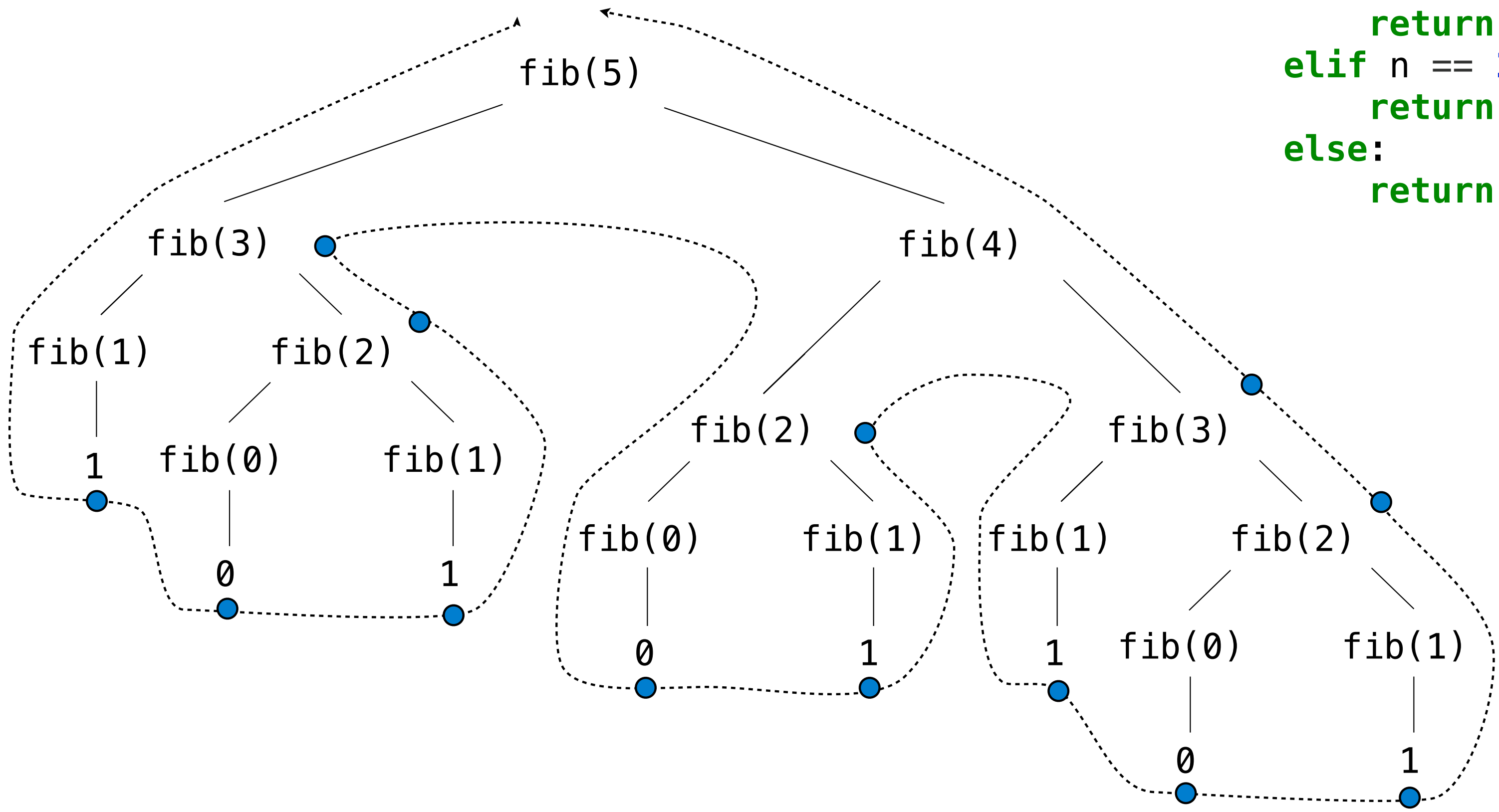


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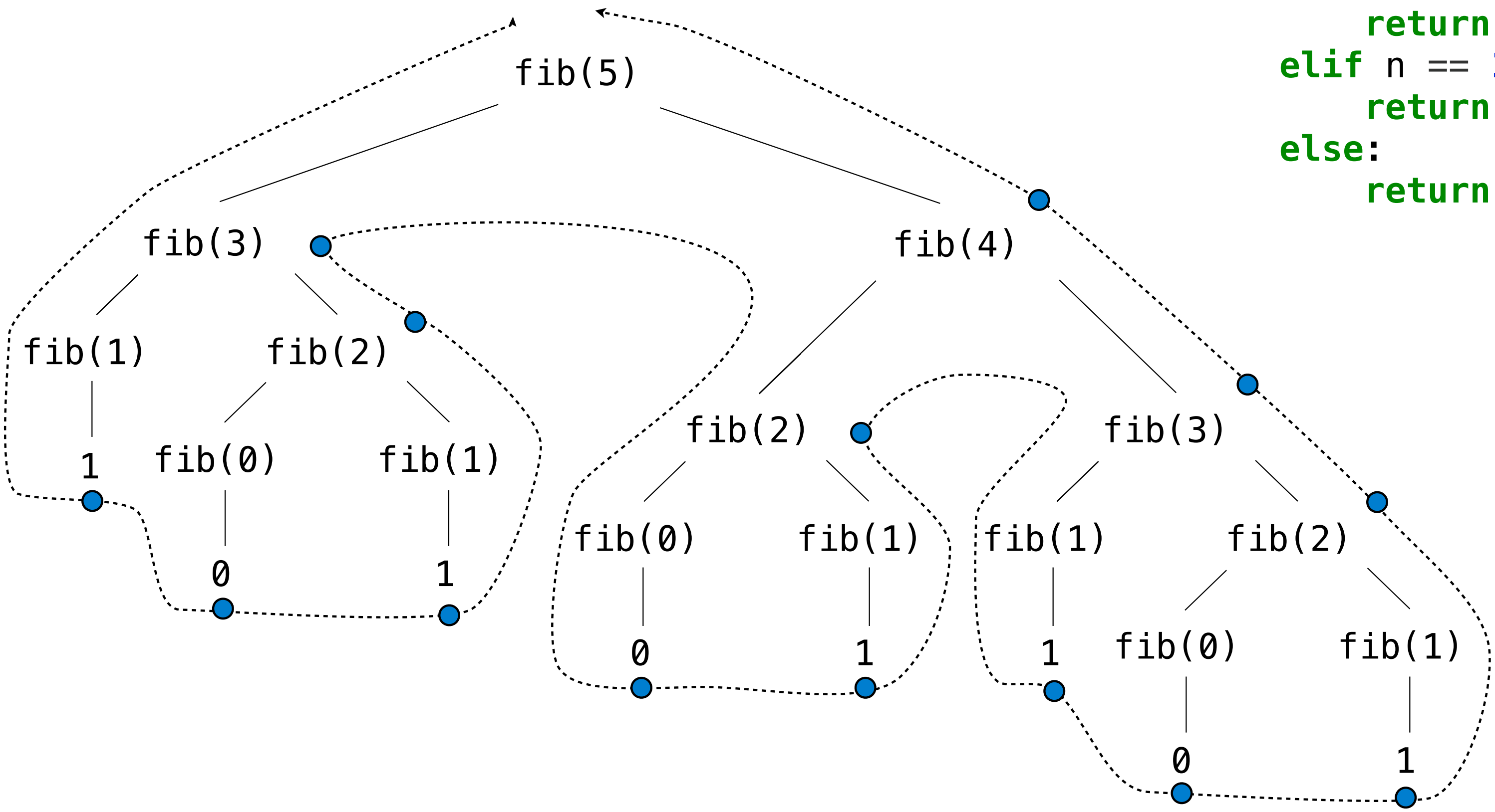

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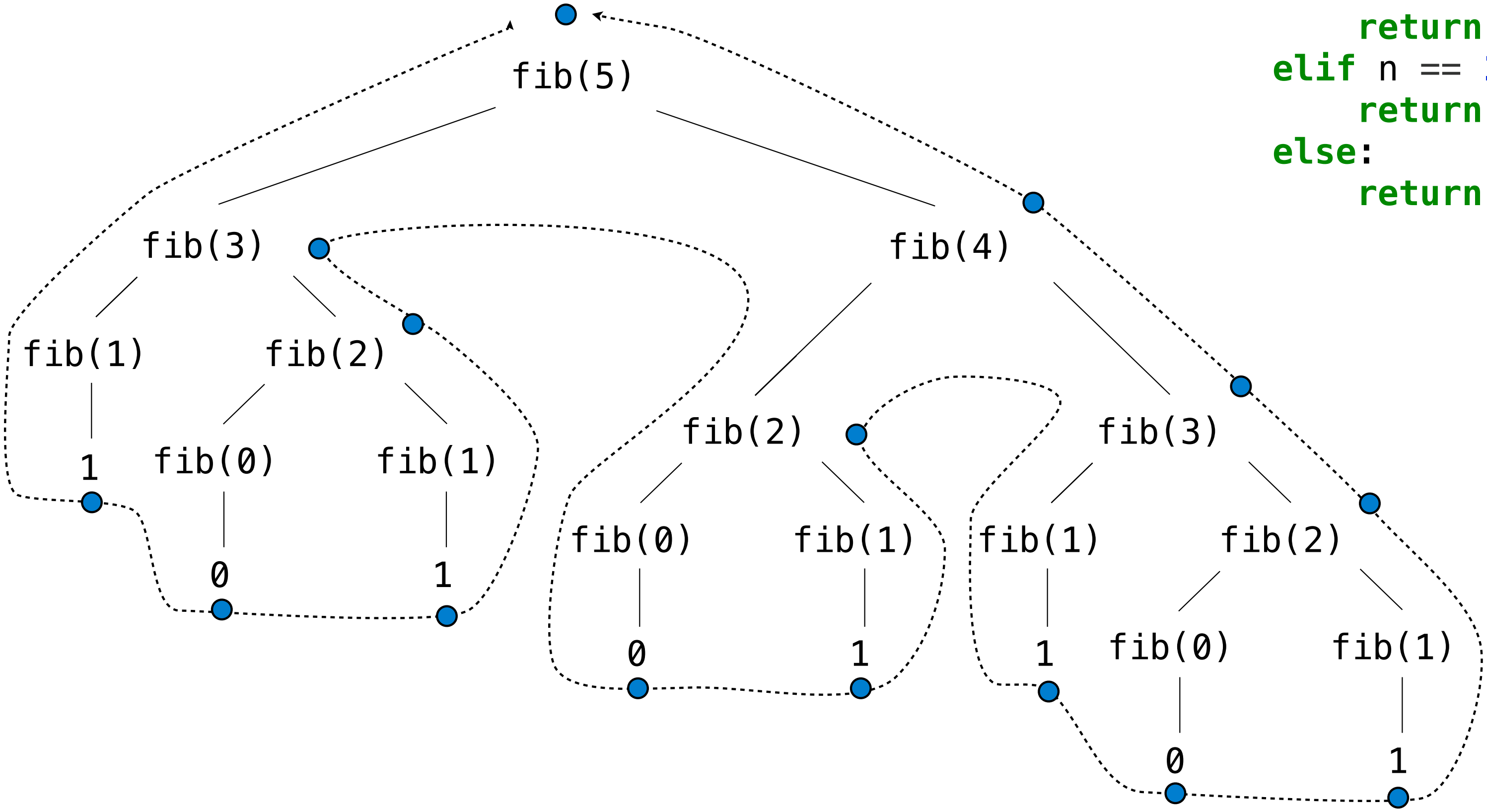

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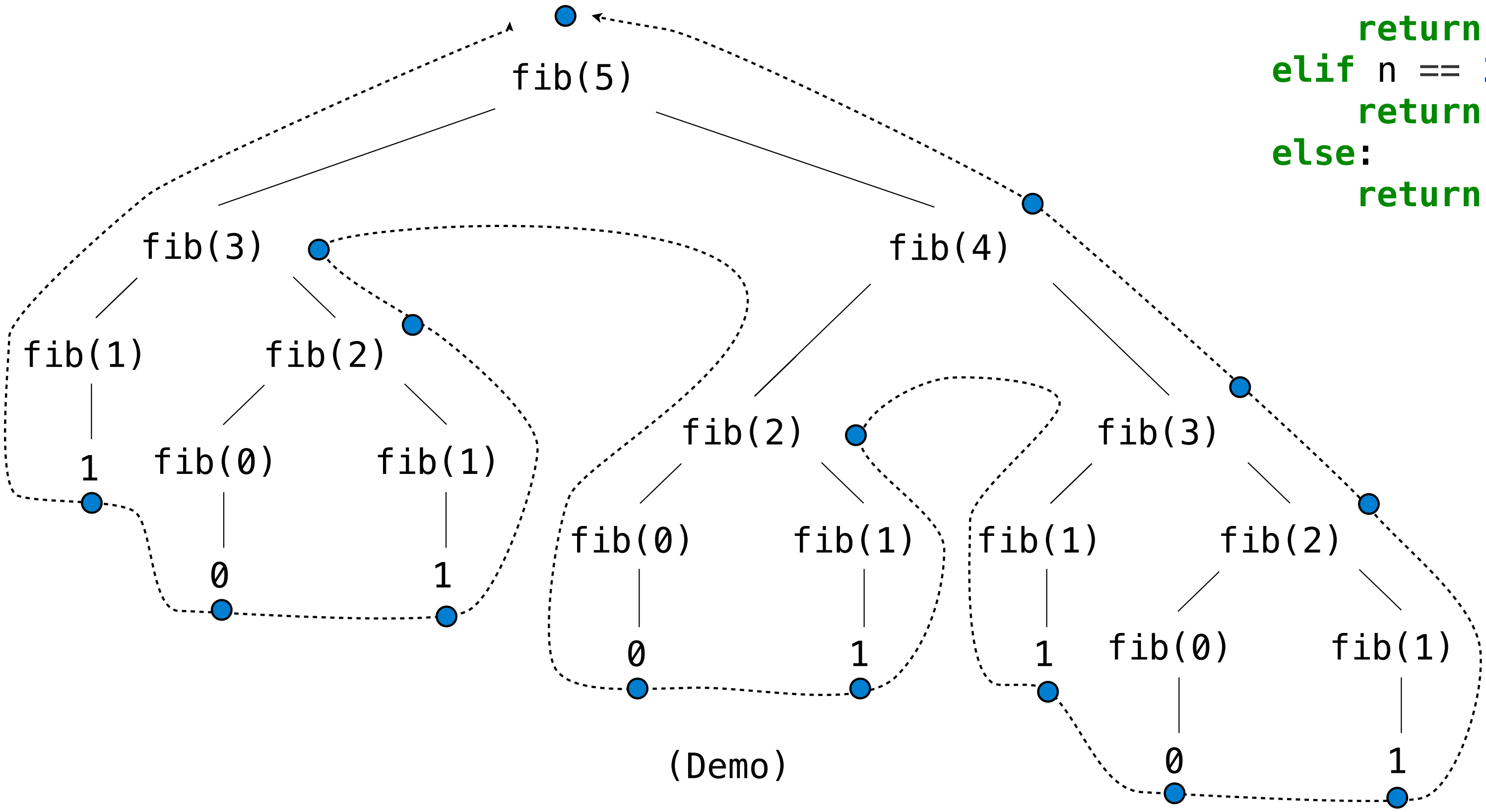

(Demo)

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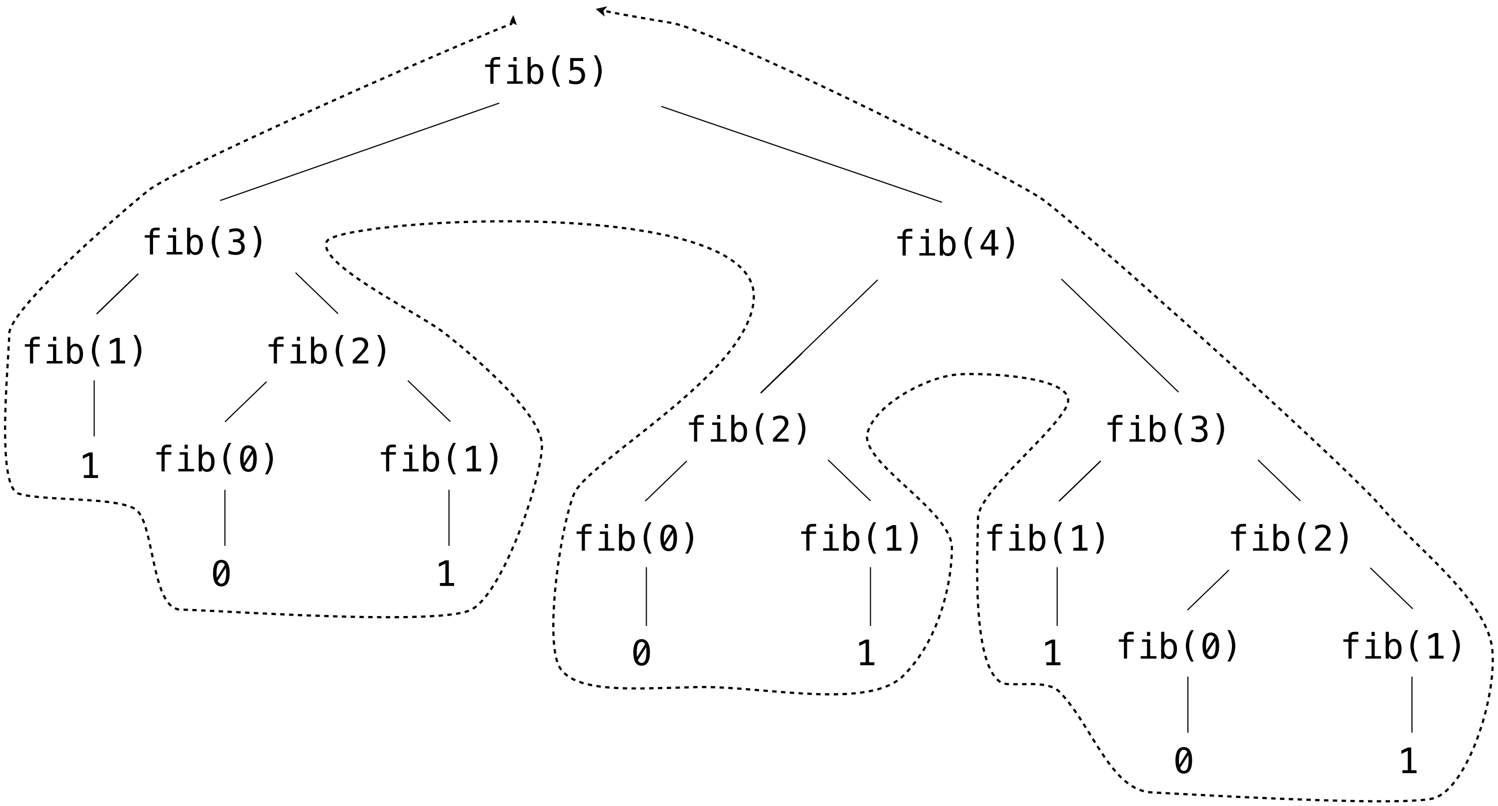
Memoized Tree Recursion



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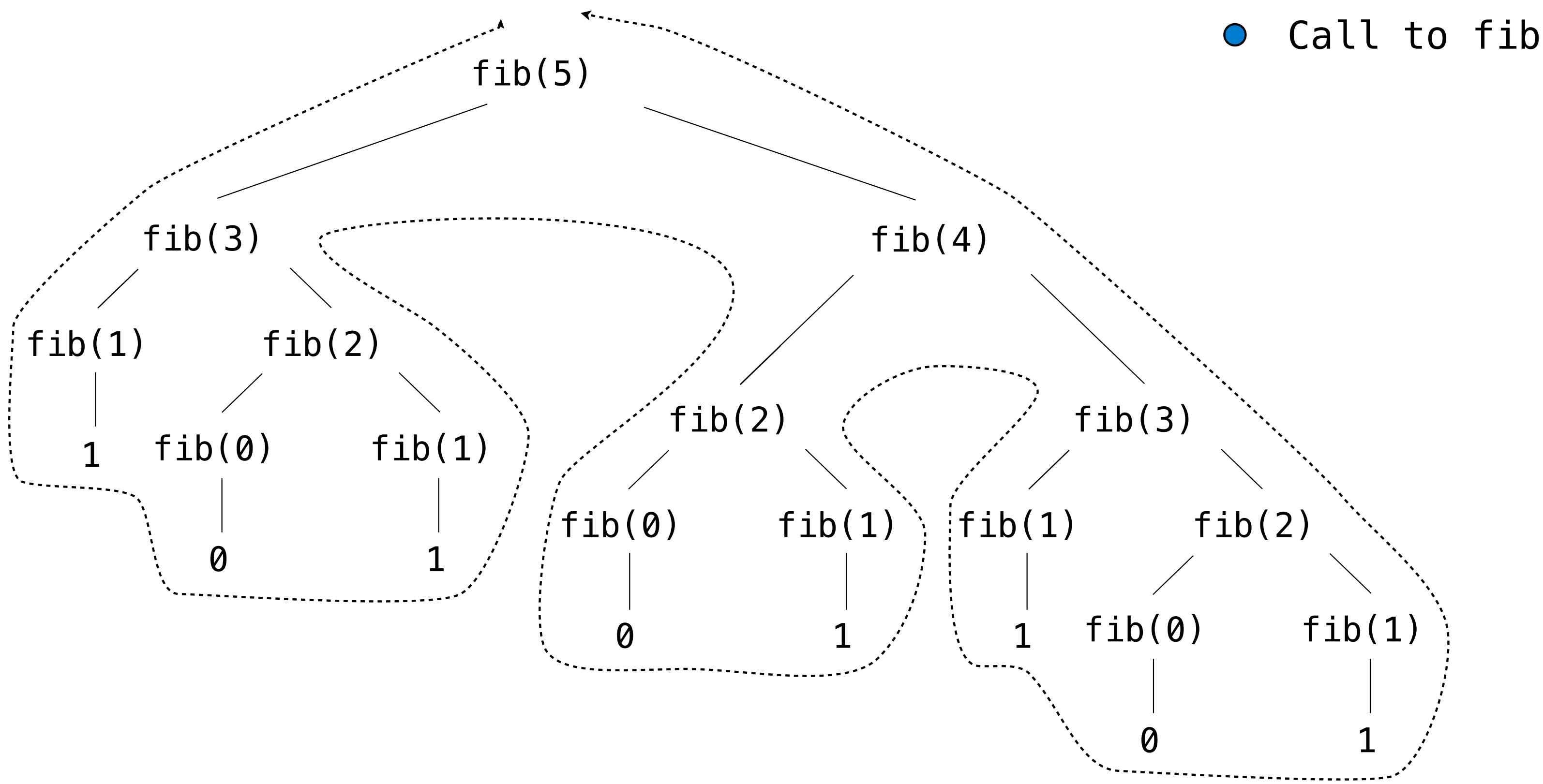

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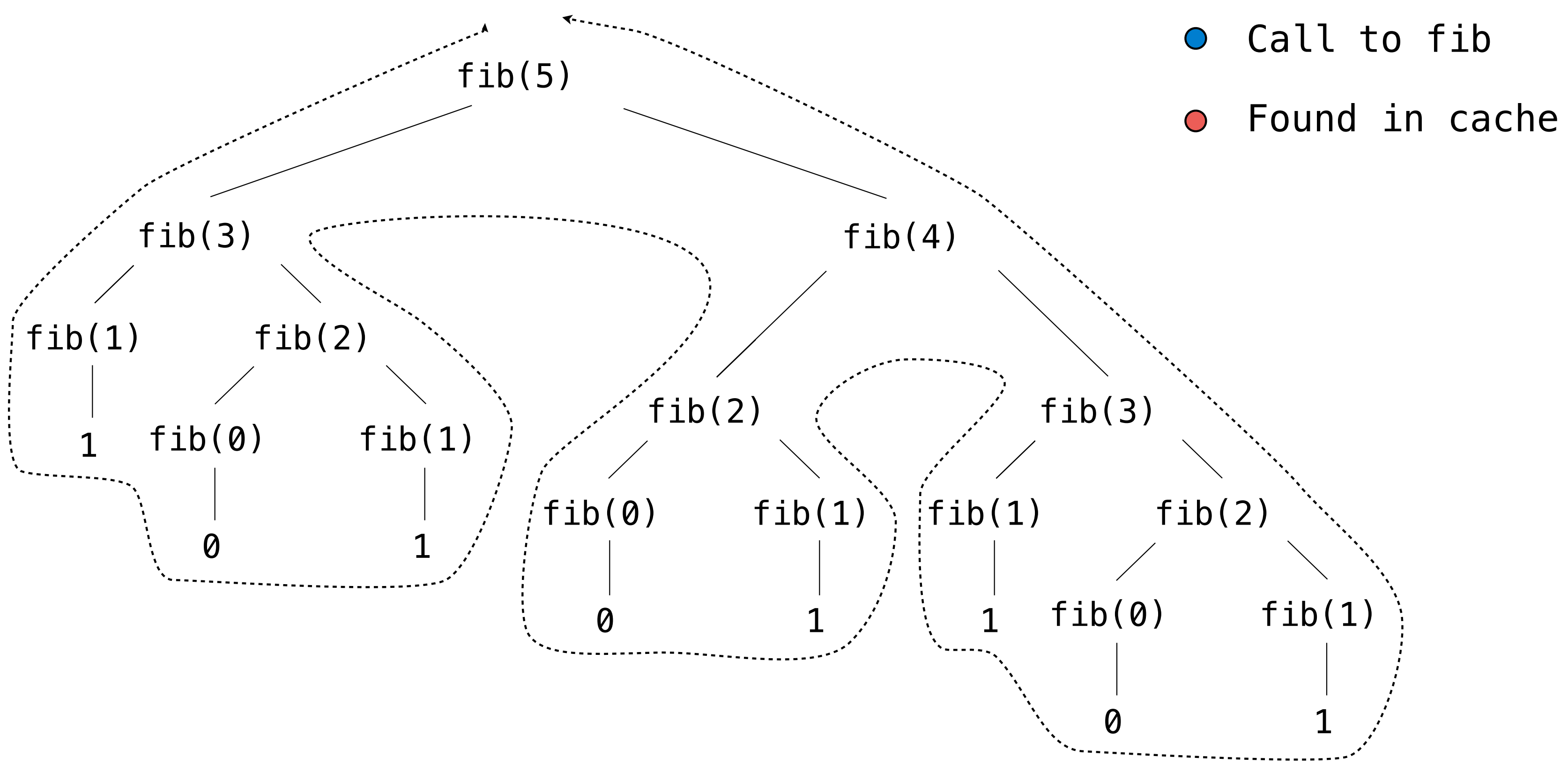

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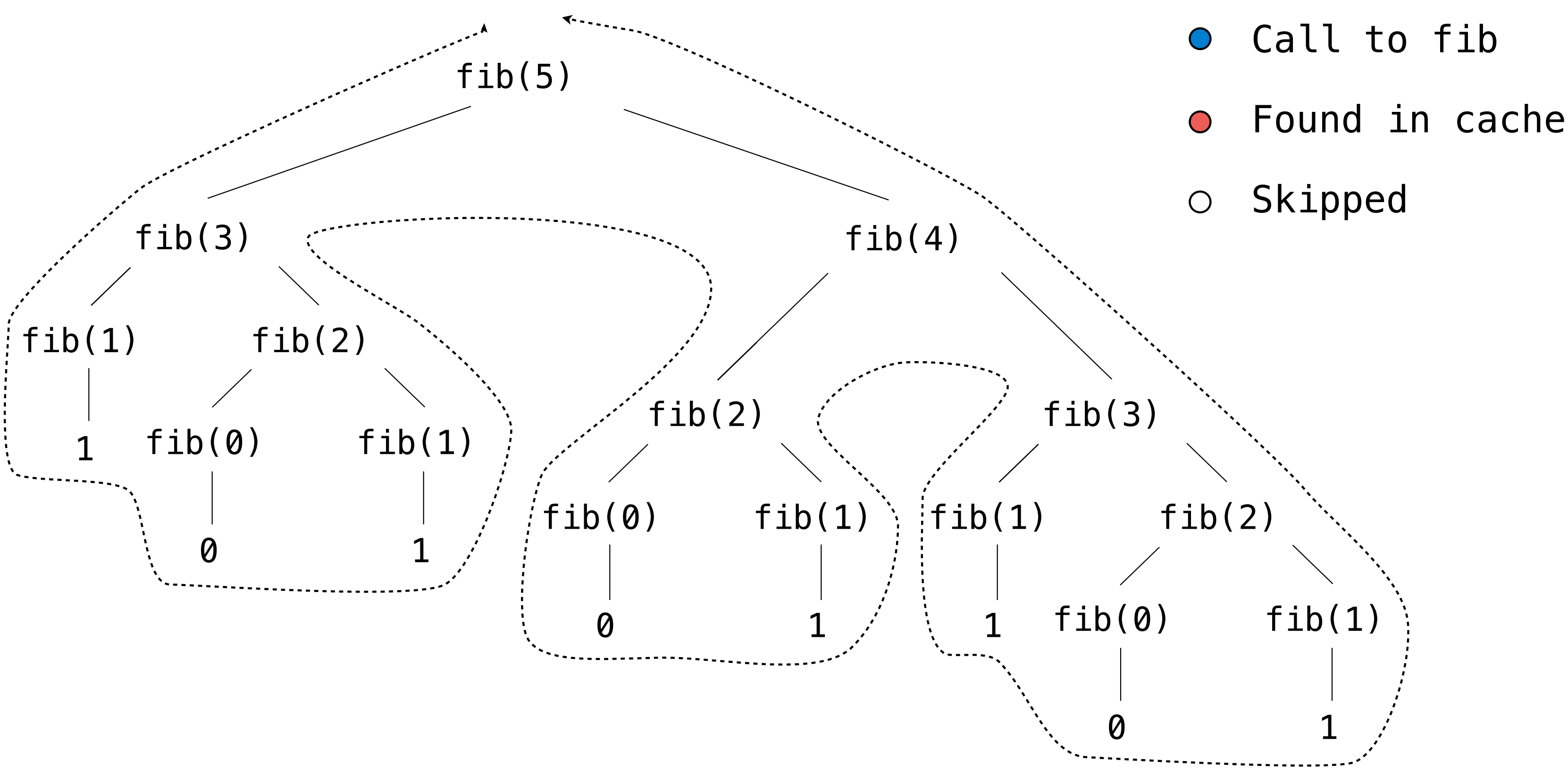

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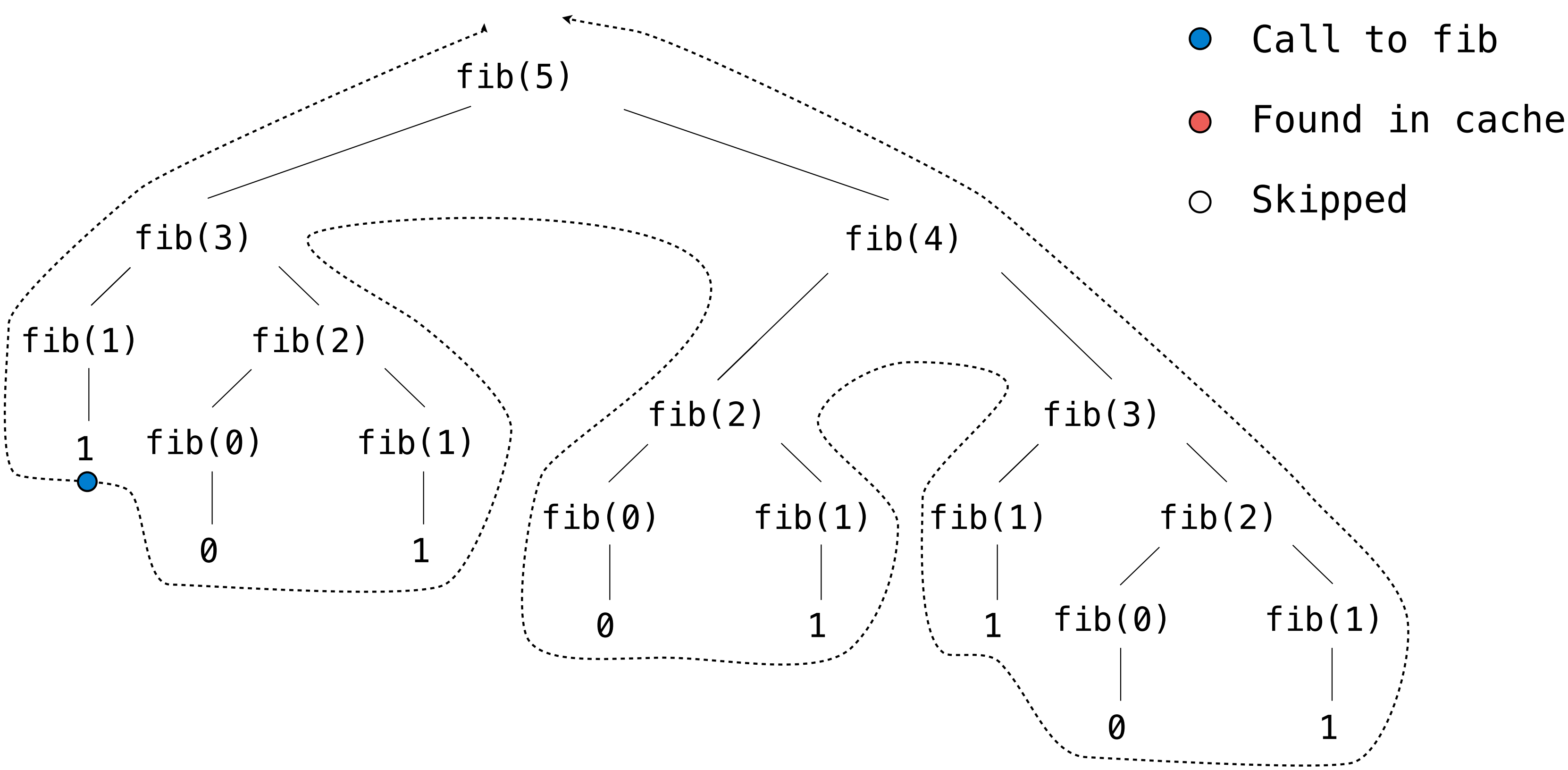

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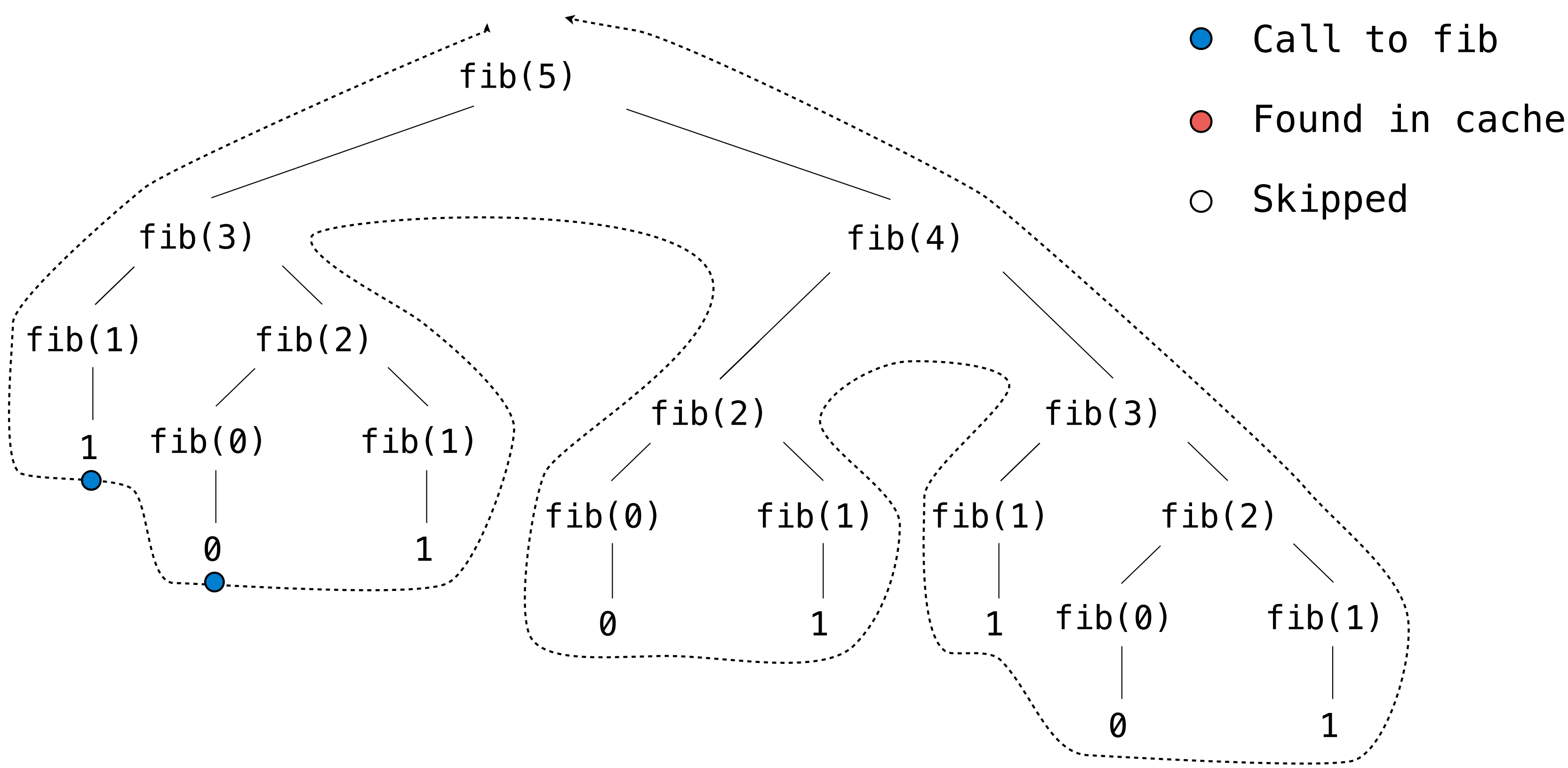

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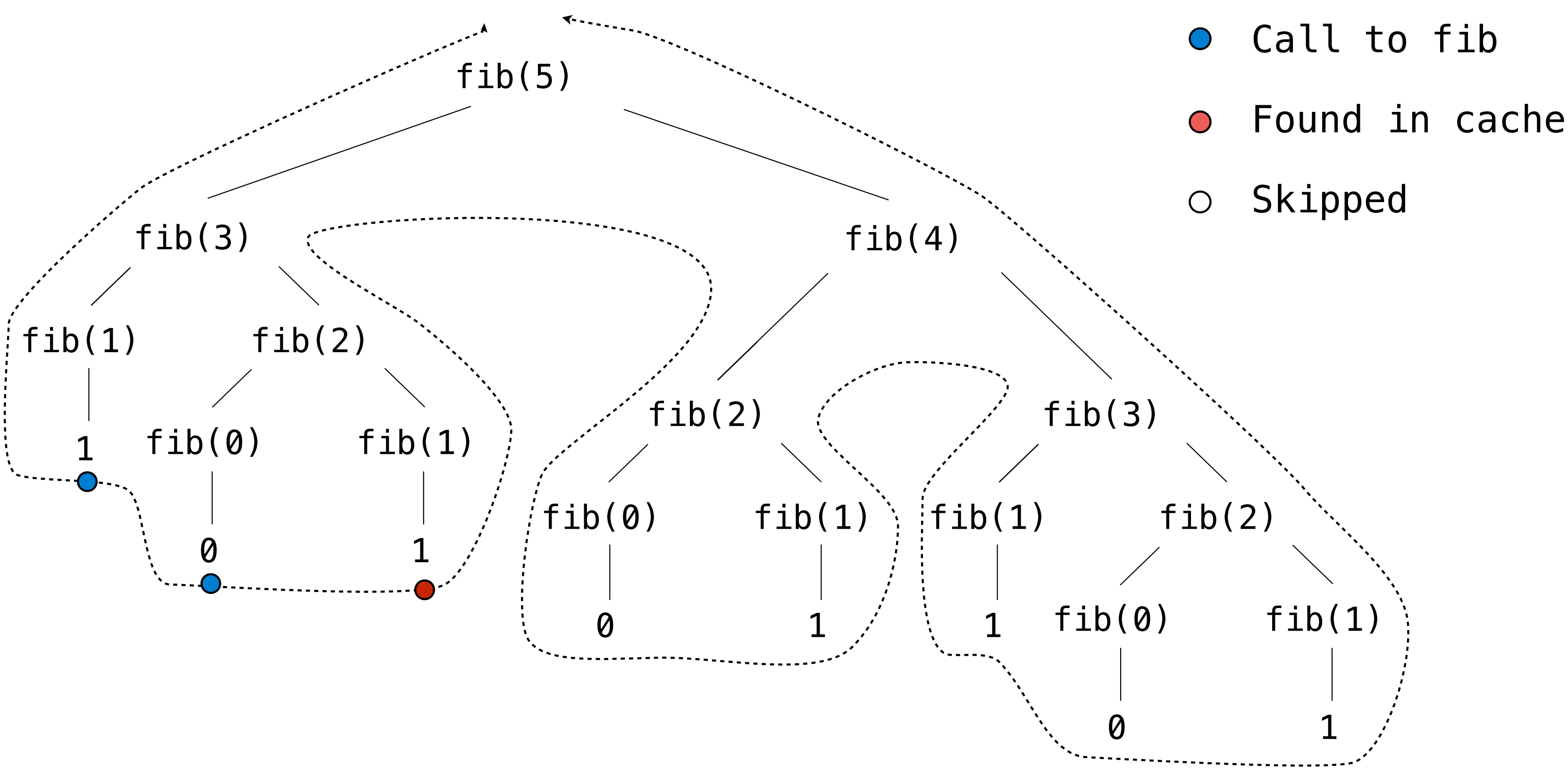

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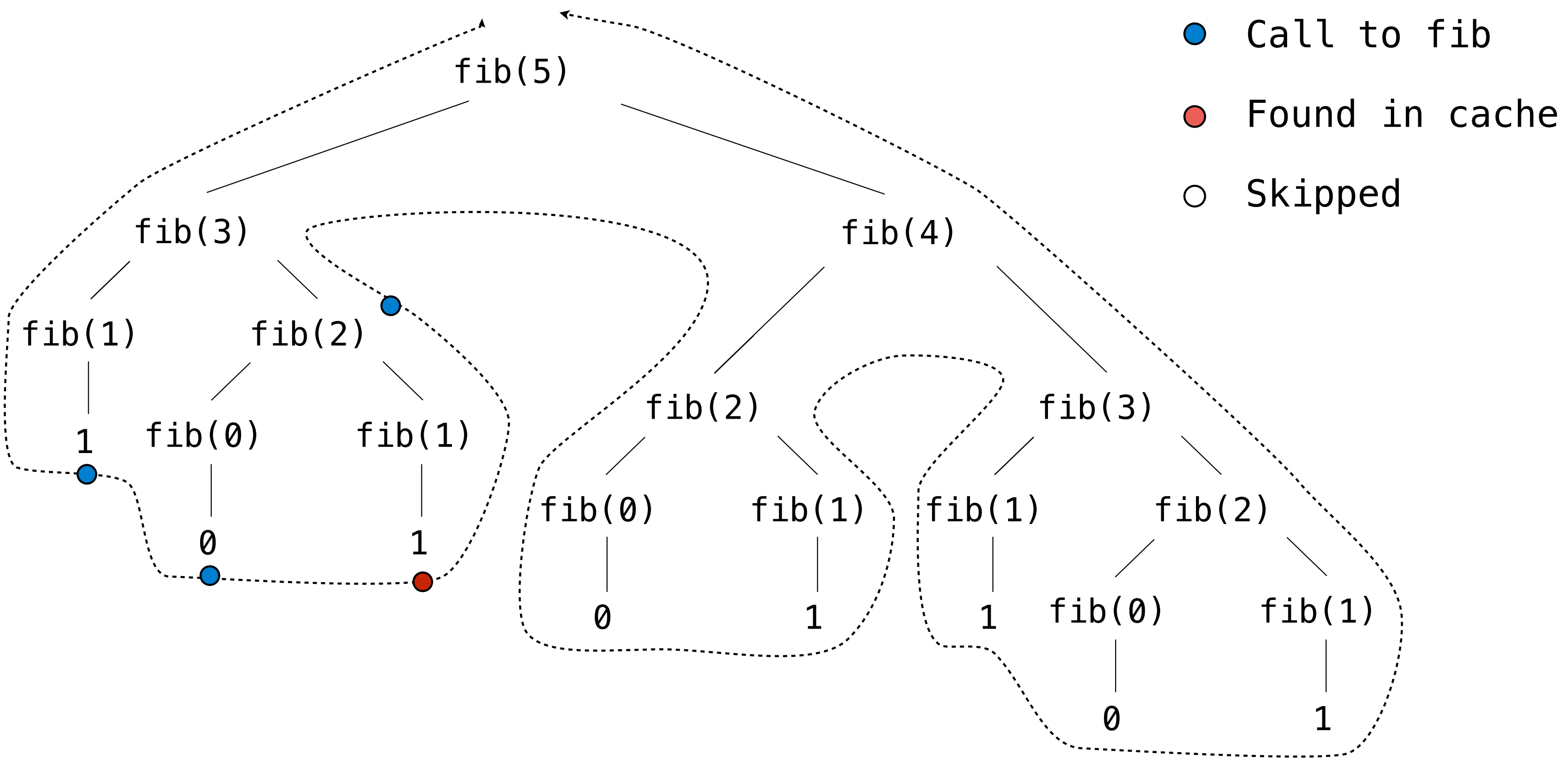

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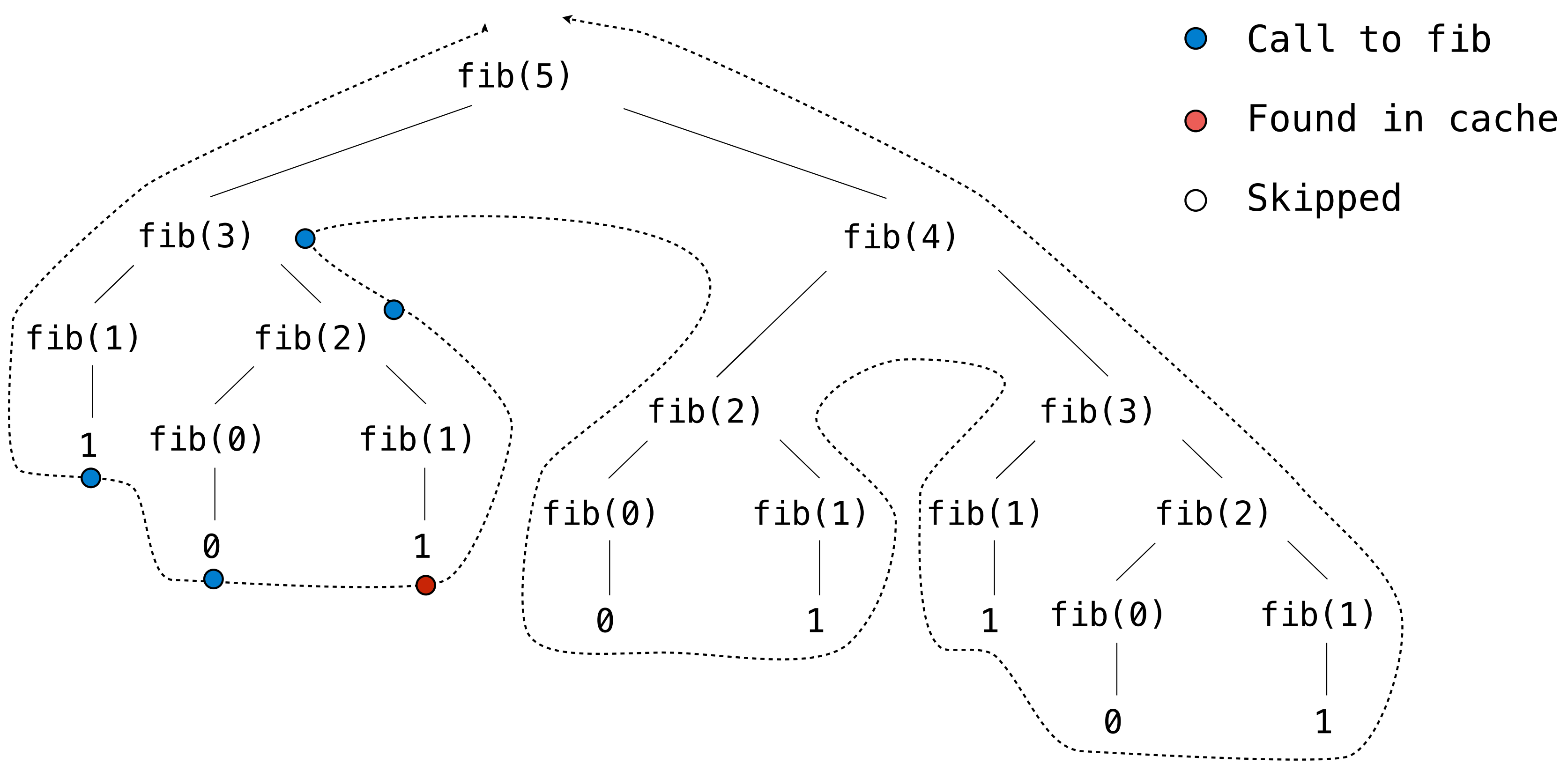

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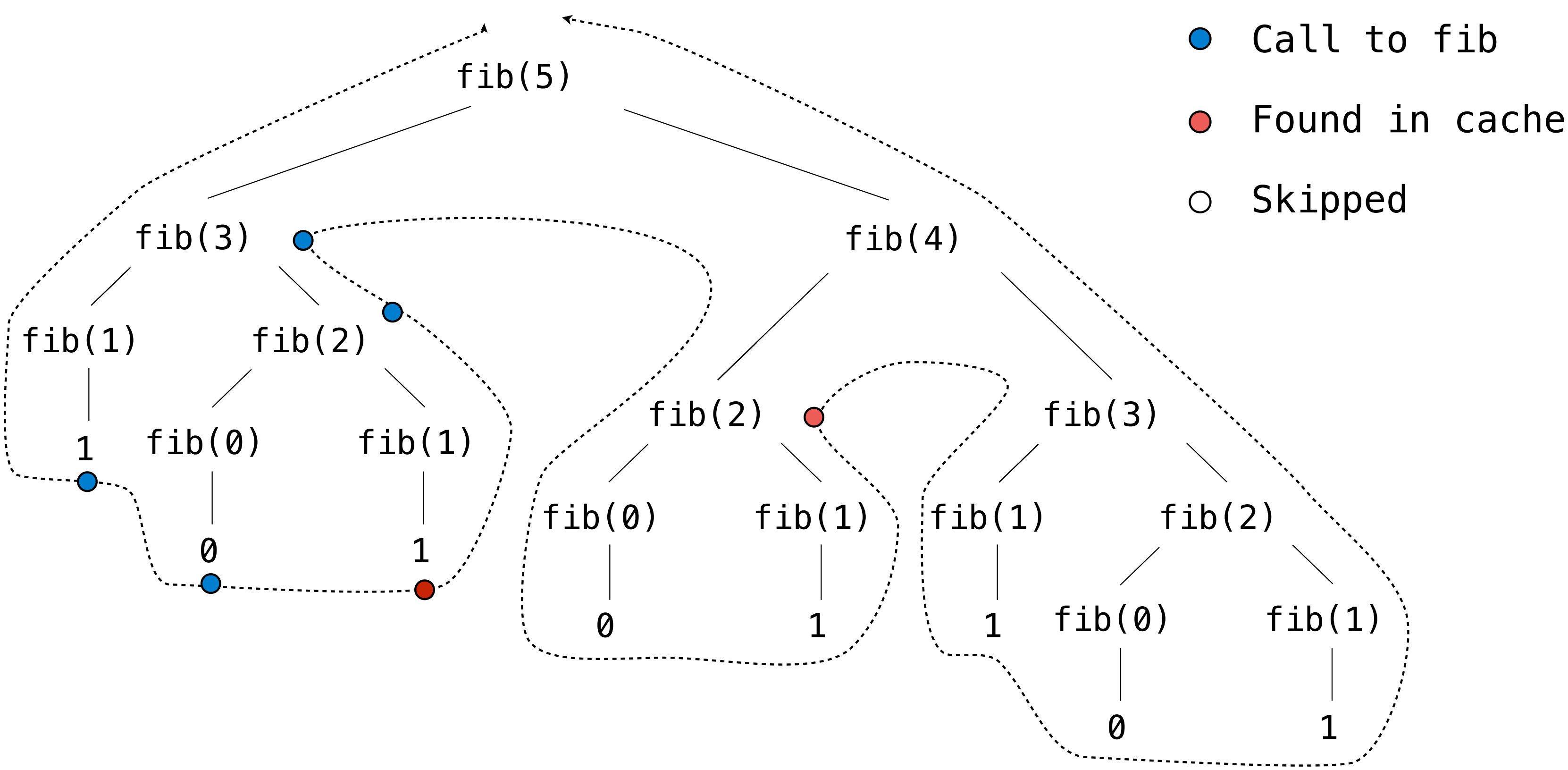

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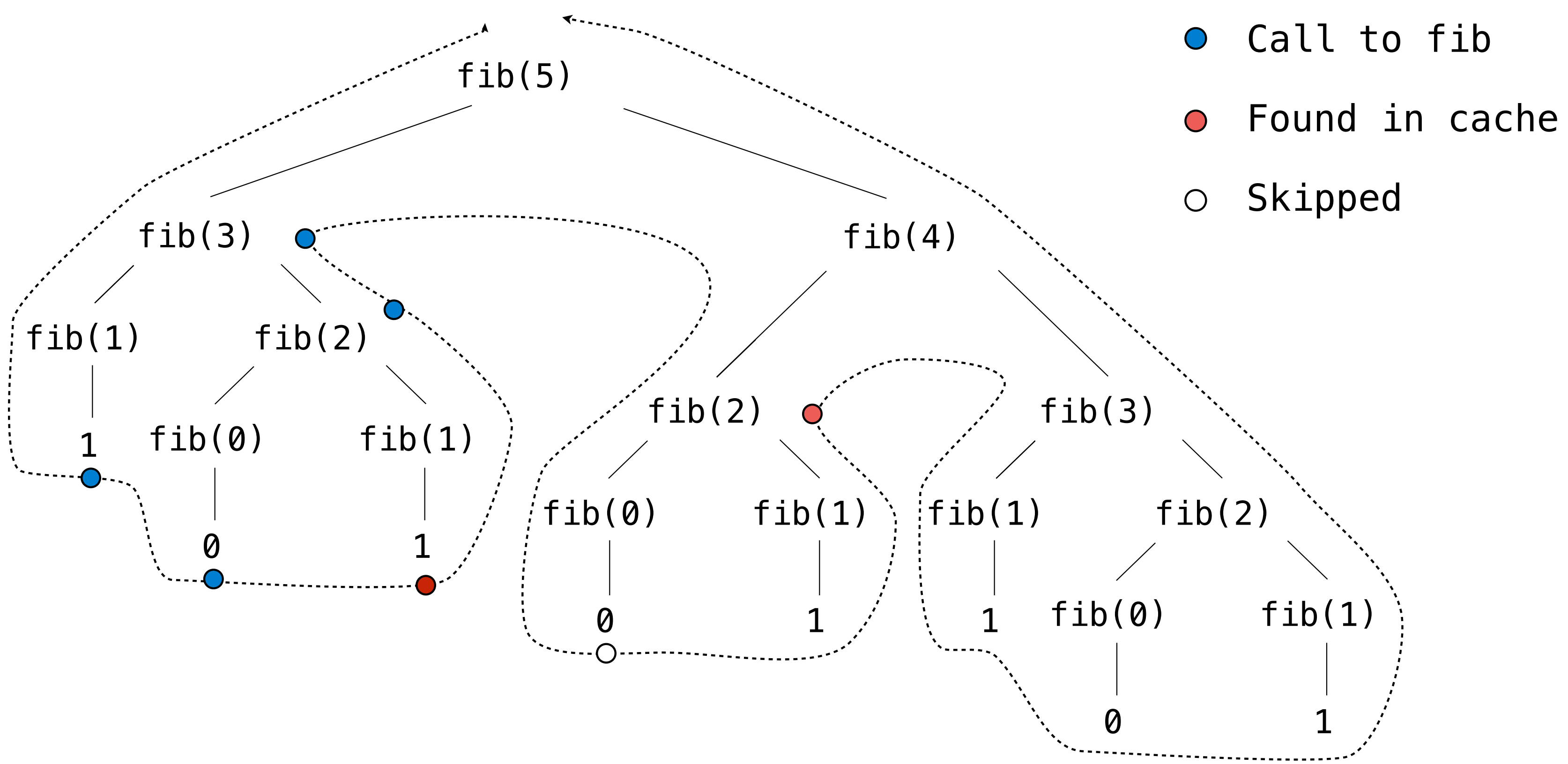

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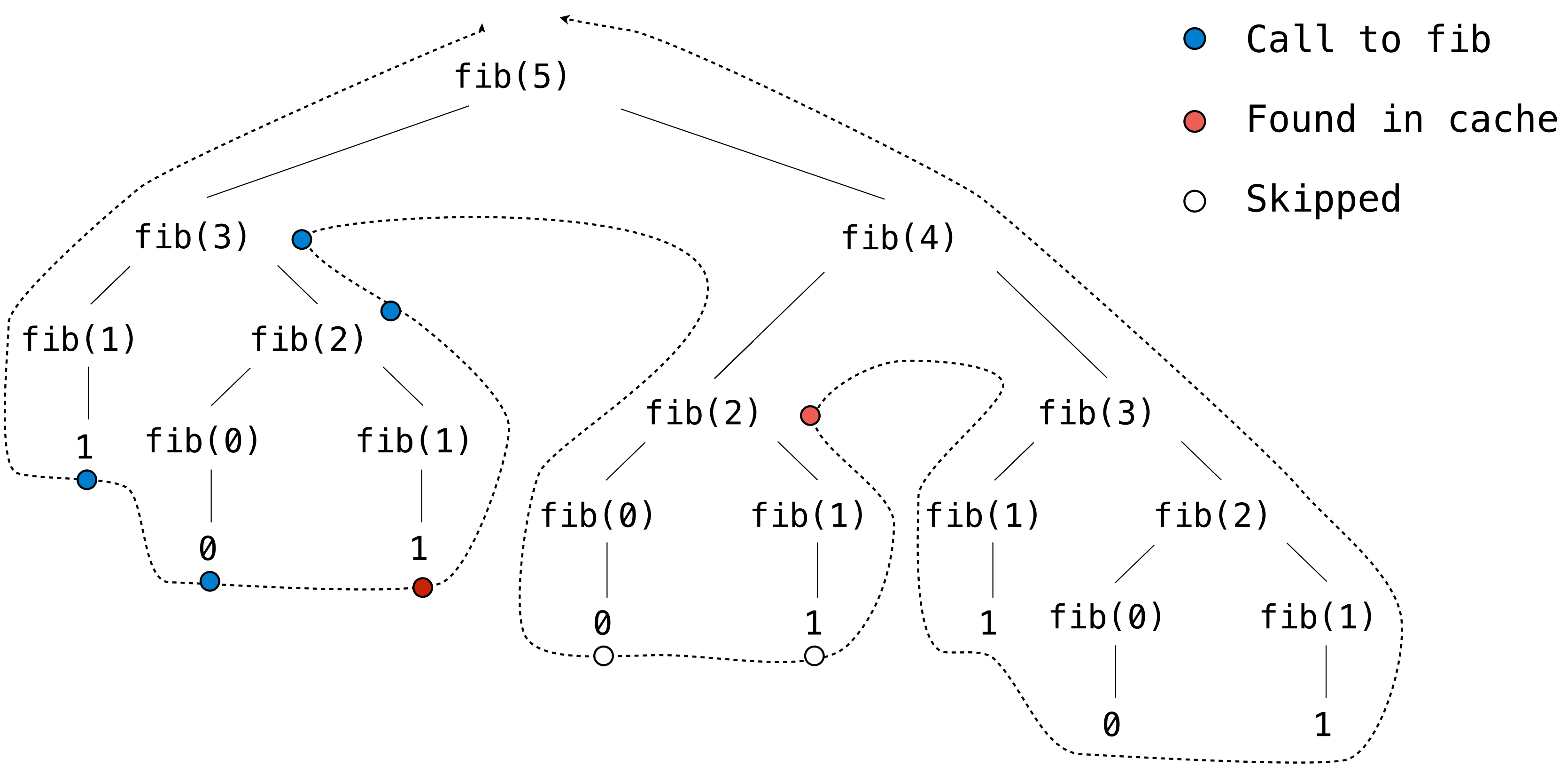

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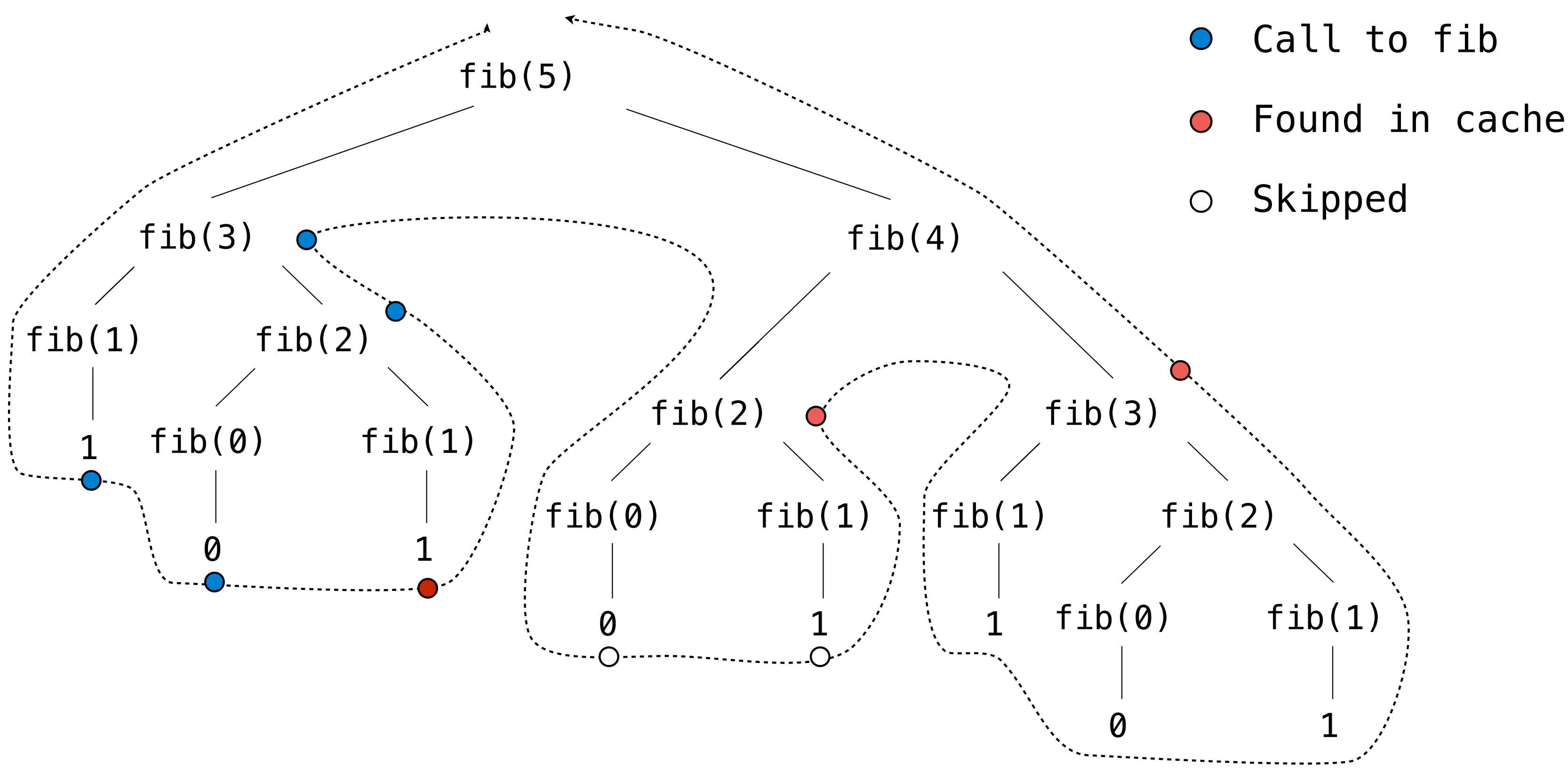

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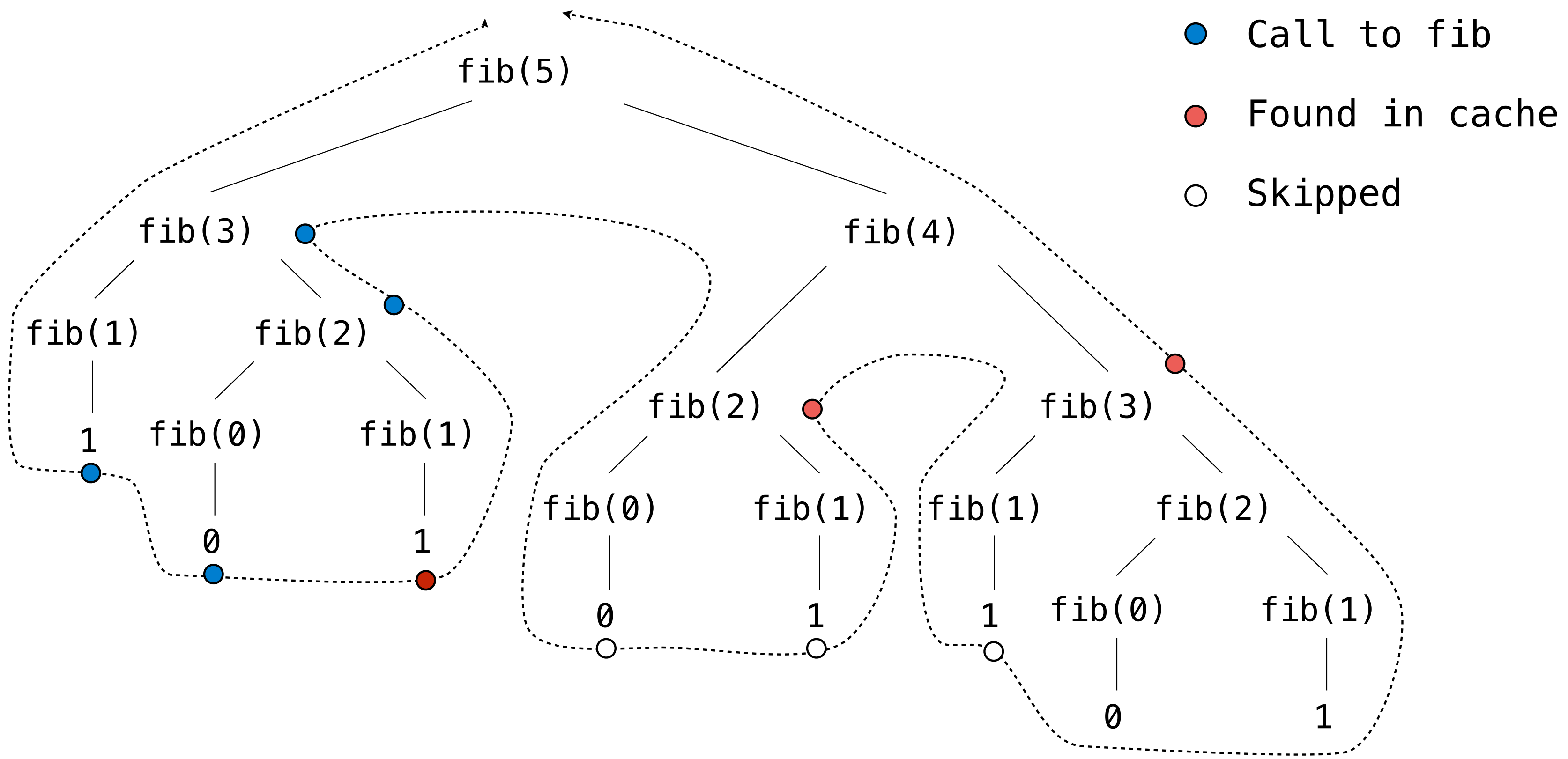

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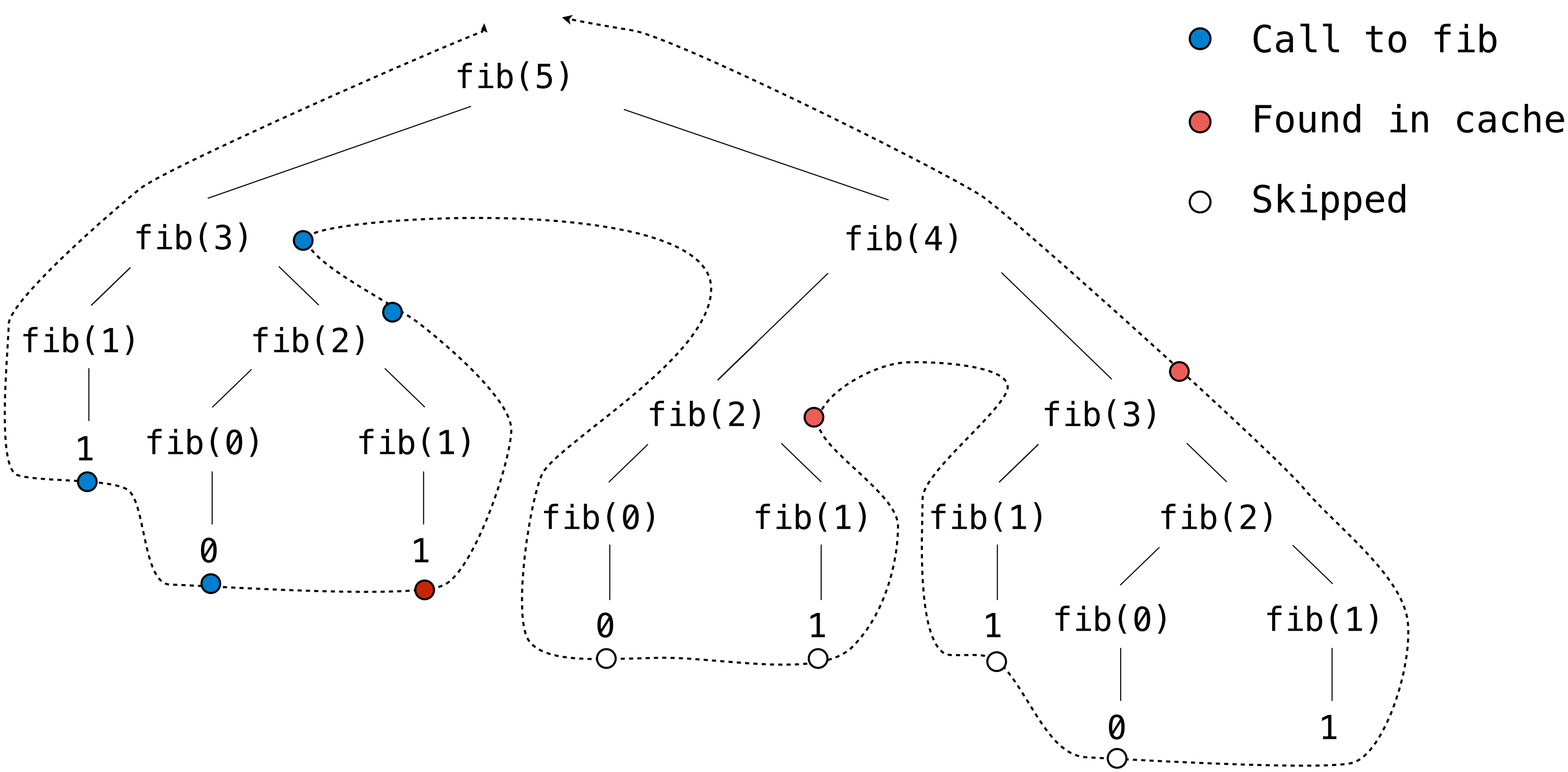

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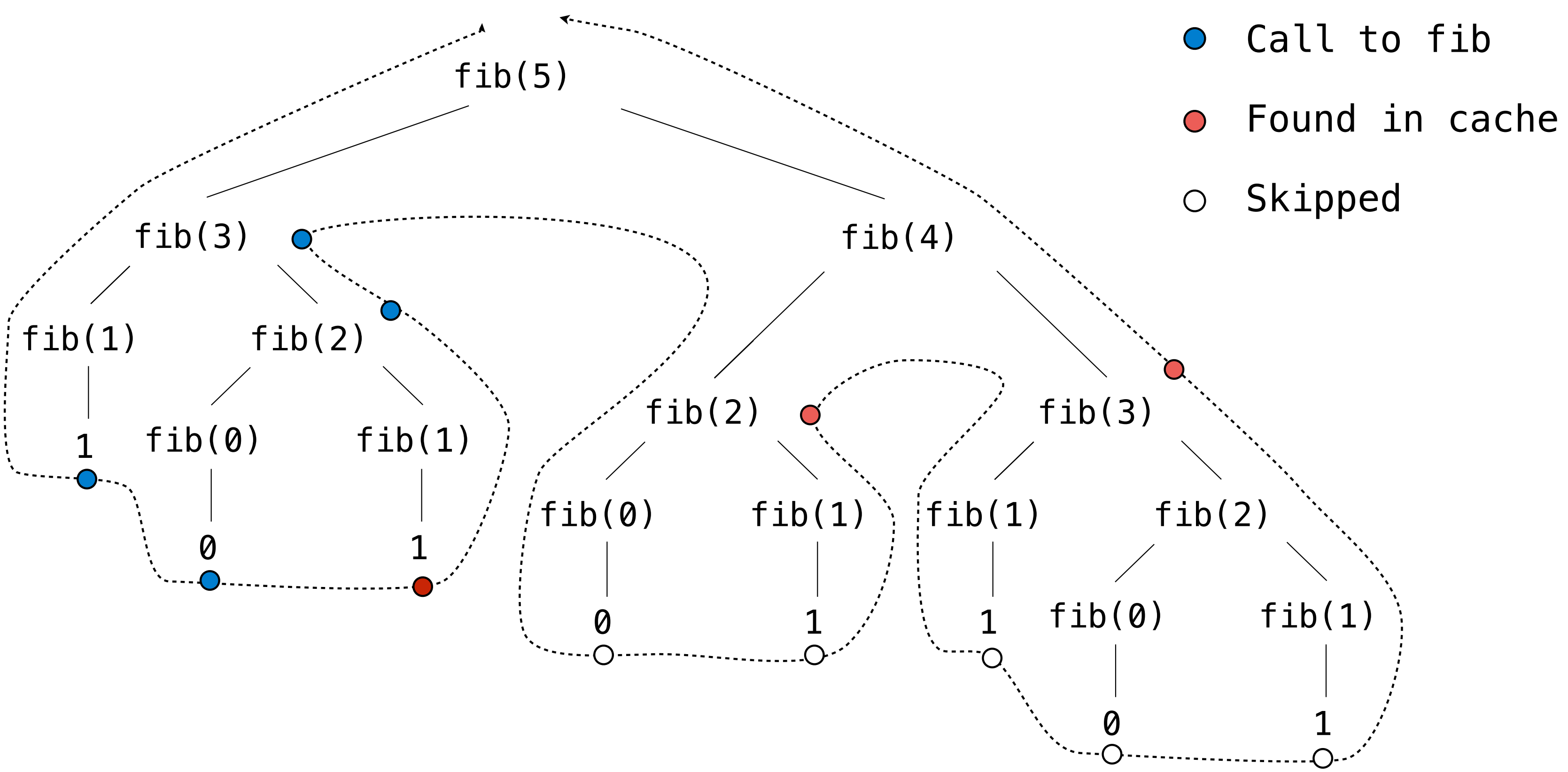

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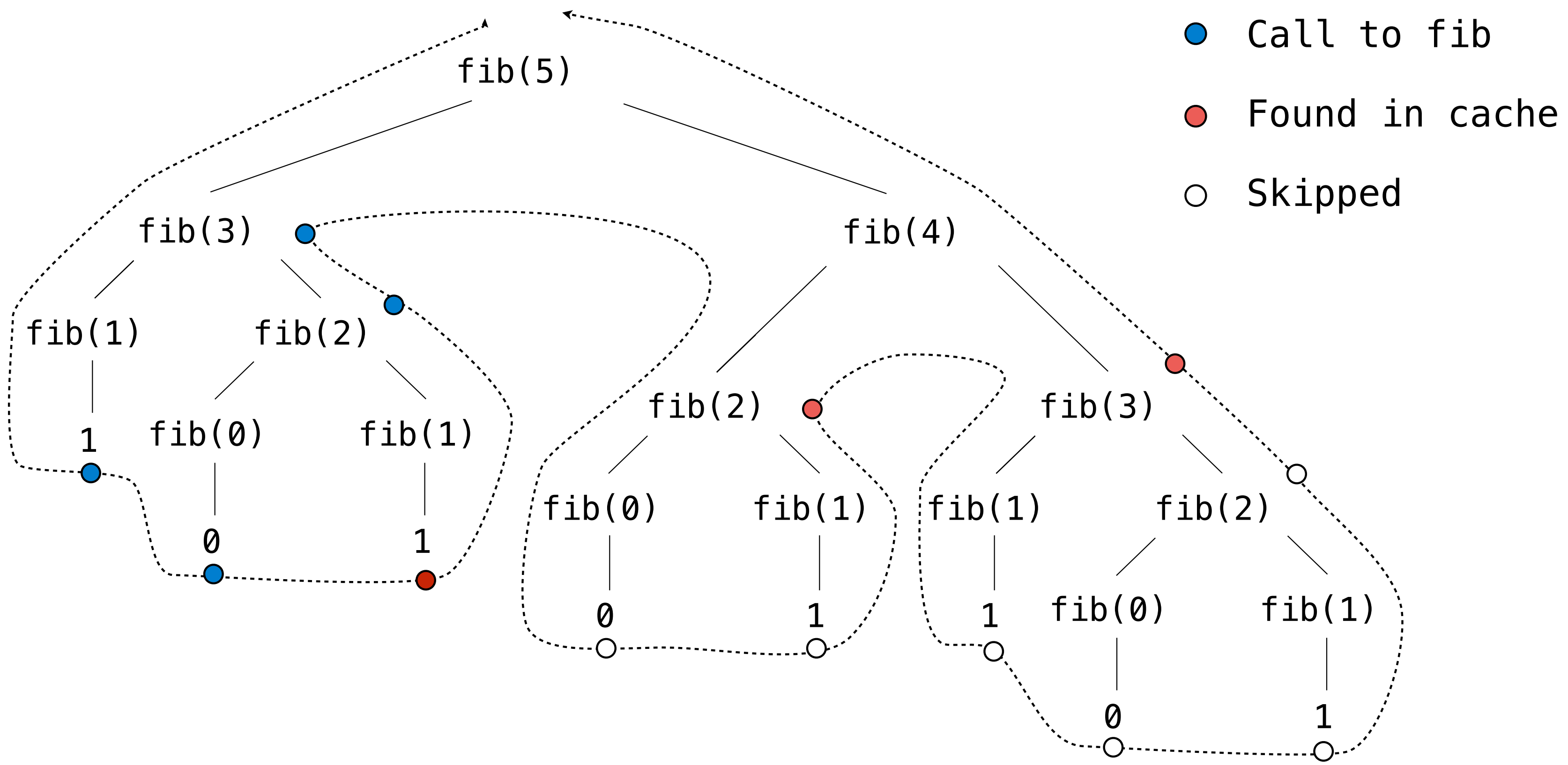

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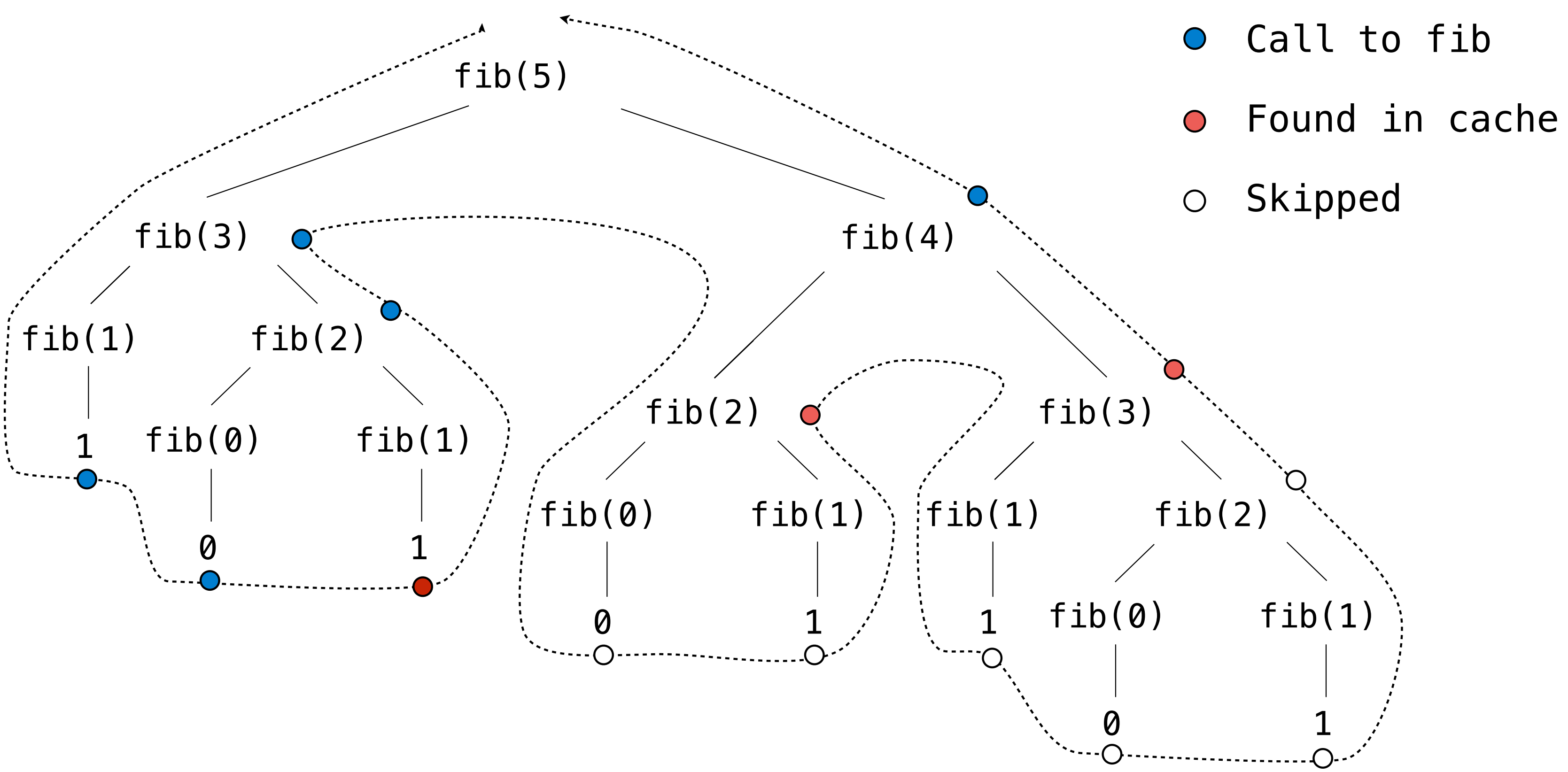

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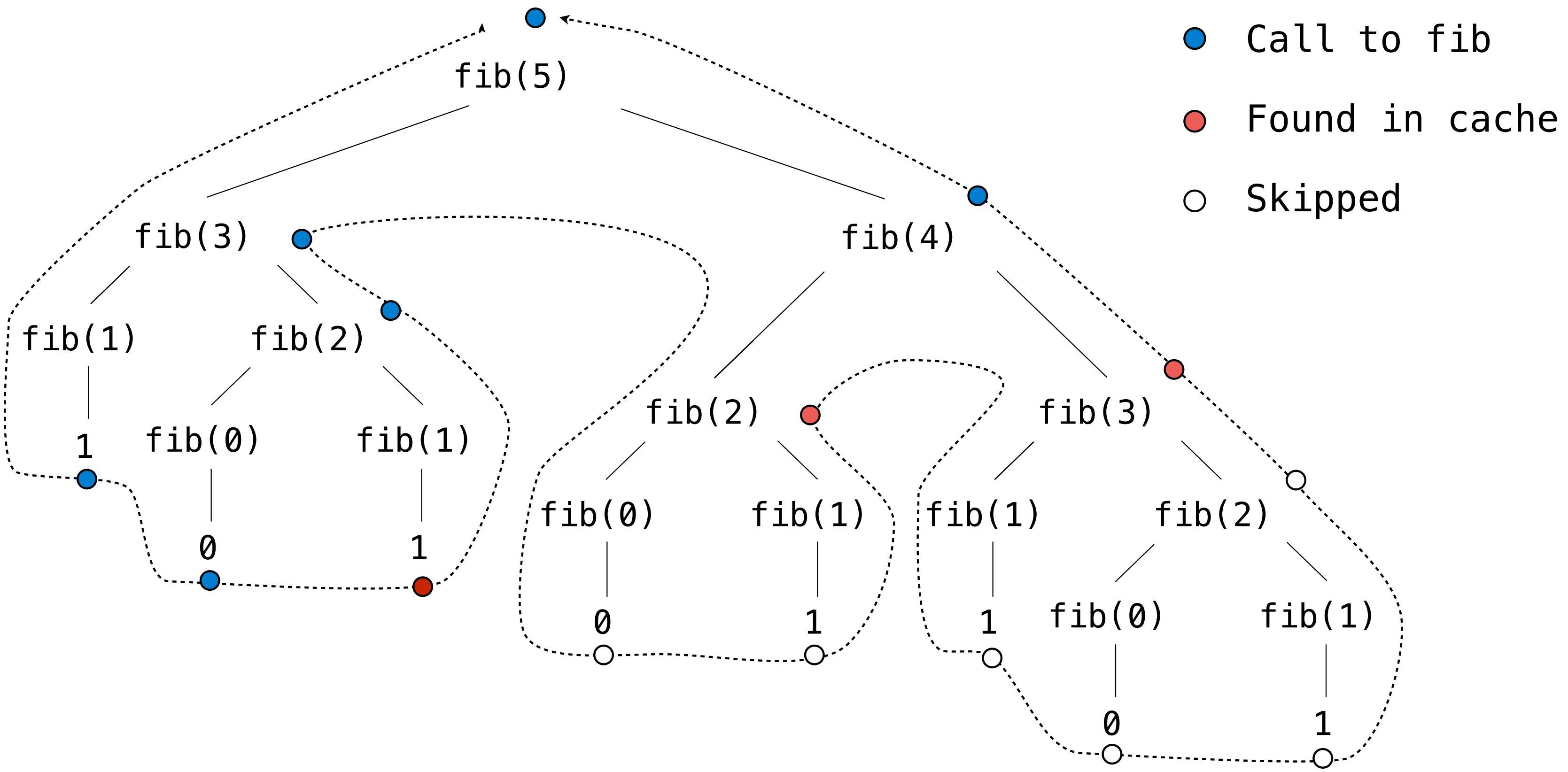

Linked List Class

Linked Lists as Objects

Linked Lists as Objects

Linked list idea: Pairs are sufficient to represent sequences of arbitrary length

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1 2 3 4

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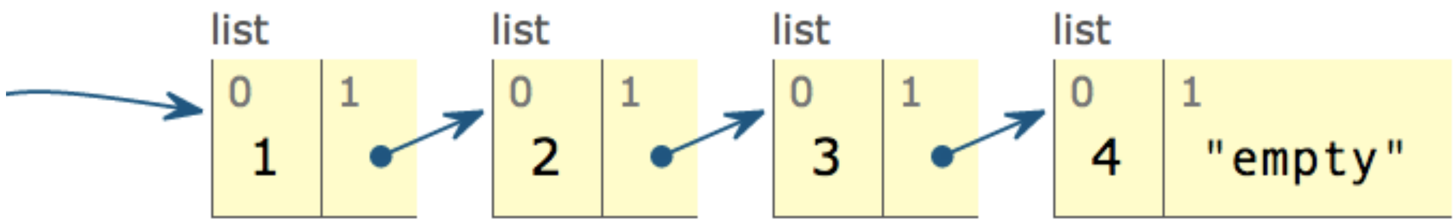

Linked Lists as Objects

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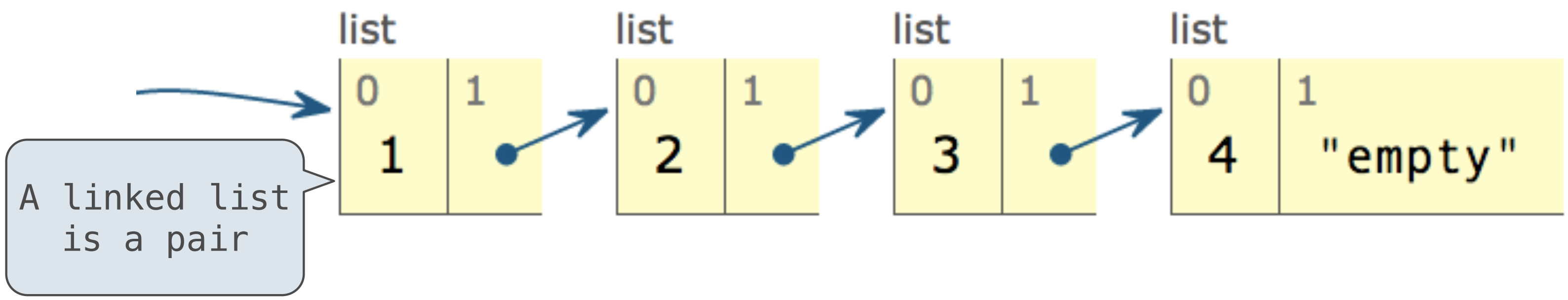

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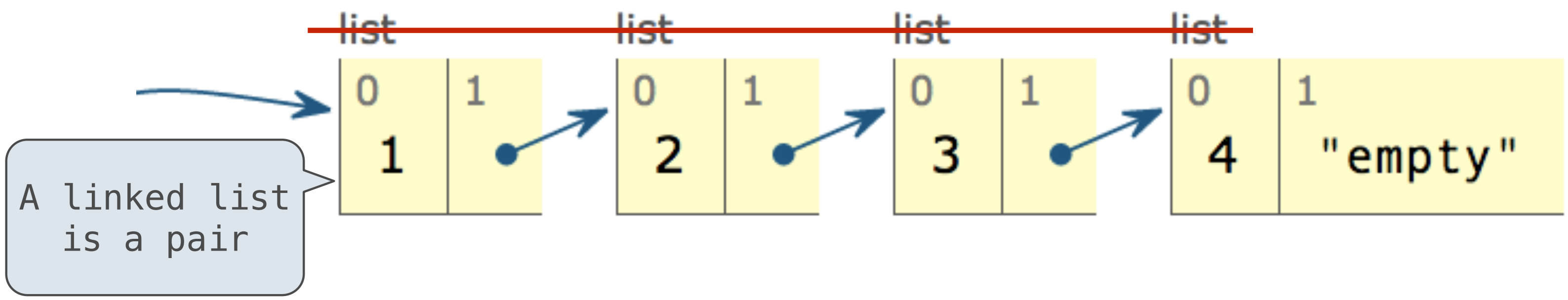

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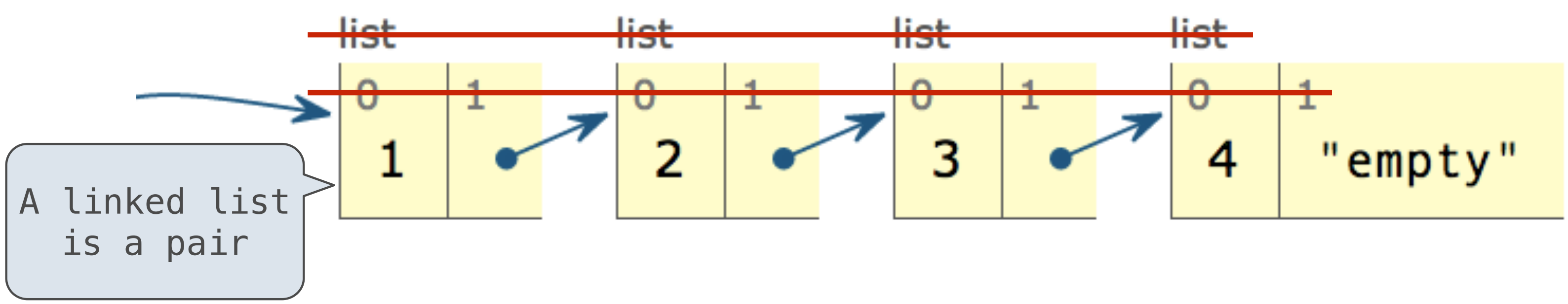

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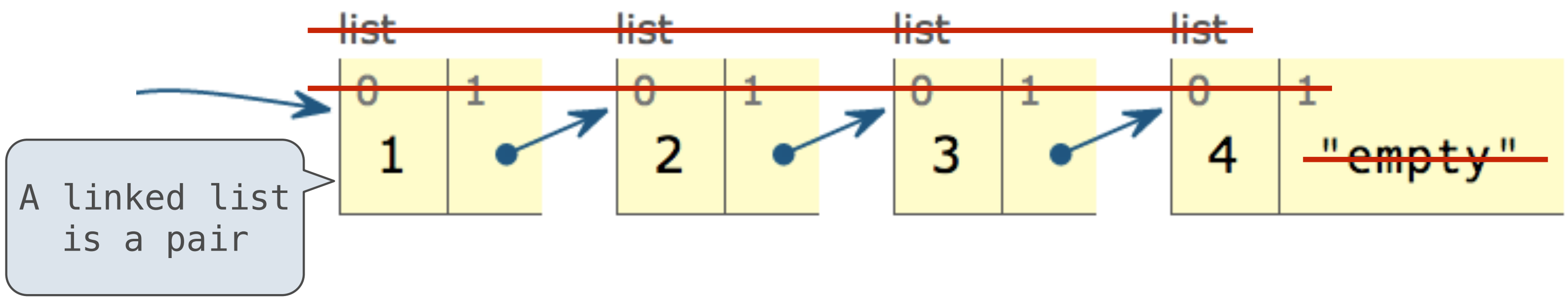

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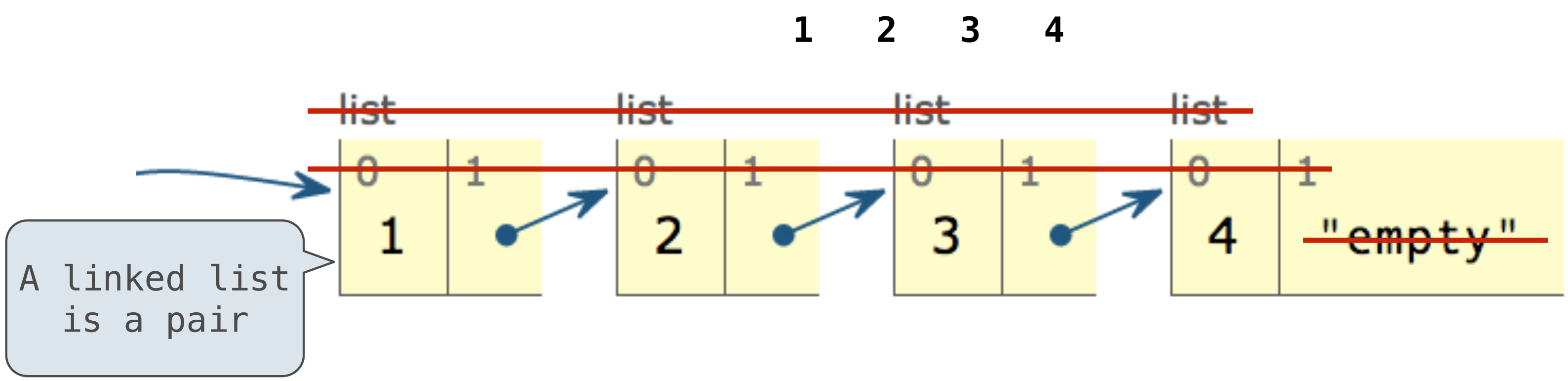

Data abstraction (old way):

Linked Lists as Objects

Linked list idea: Pairs are sufficient to represent sequences of arbitrary length

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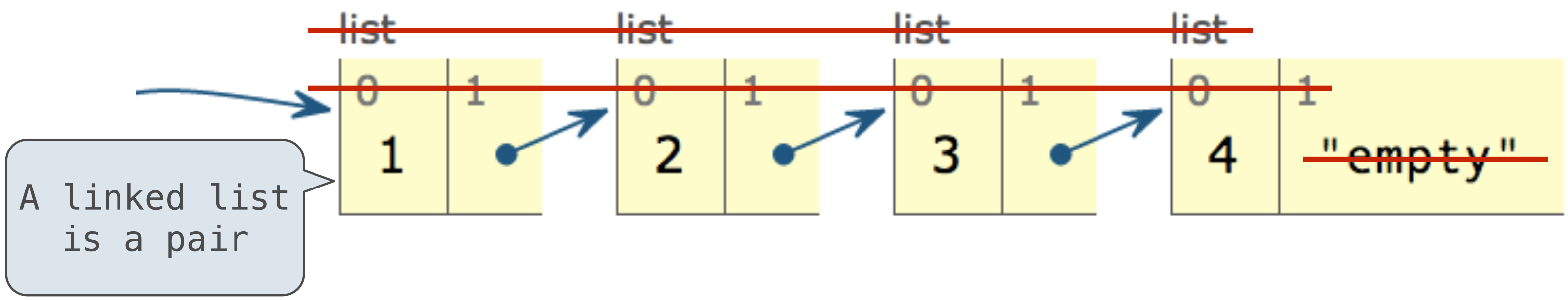

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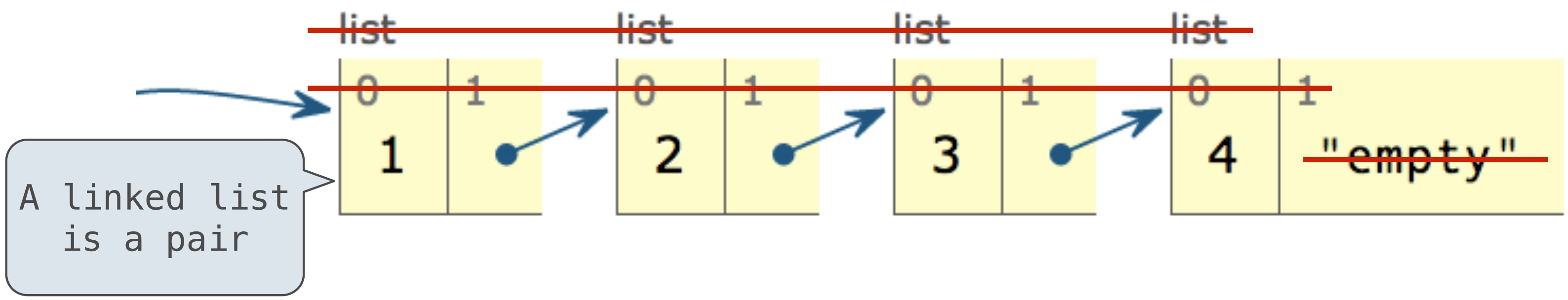

Data abstraction (old way):

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>>> s = link(1, link(2, link(3, link(4, empty))))
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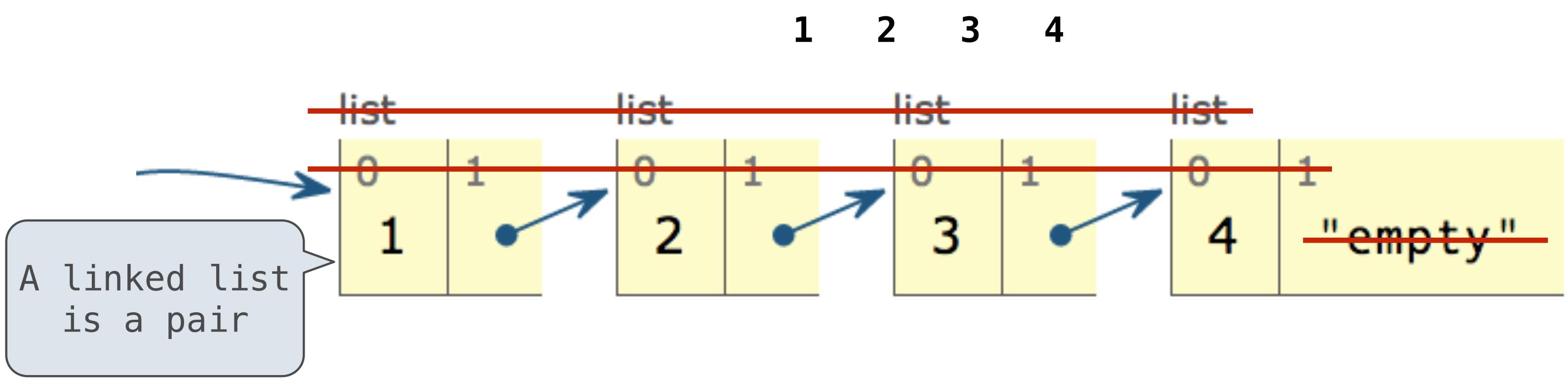

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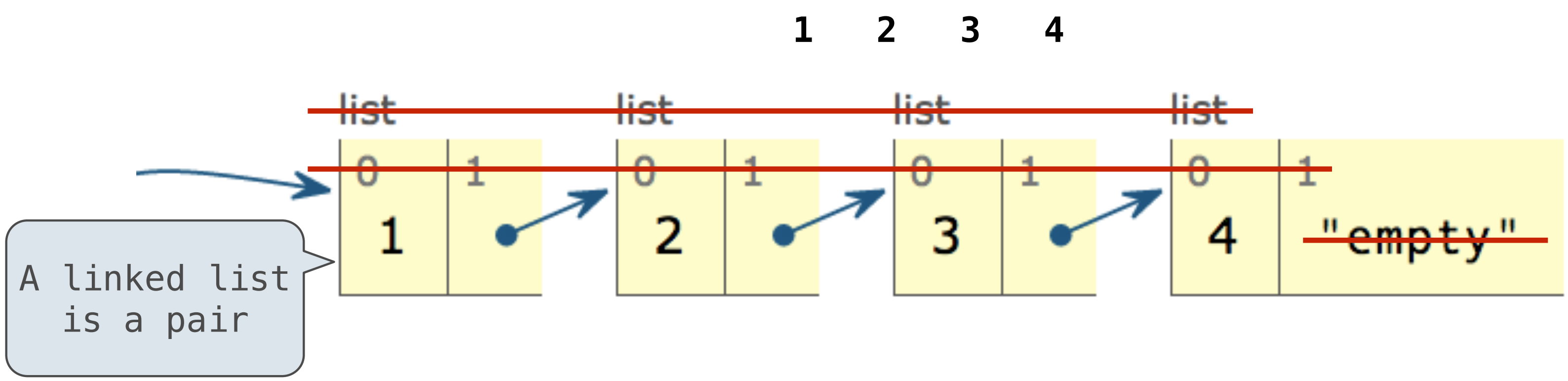

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>>> s = link(1, link(2, link(3, link(4, empty))))
>>> len_link(s)
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```

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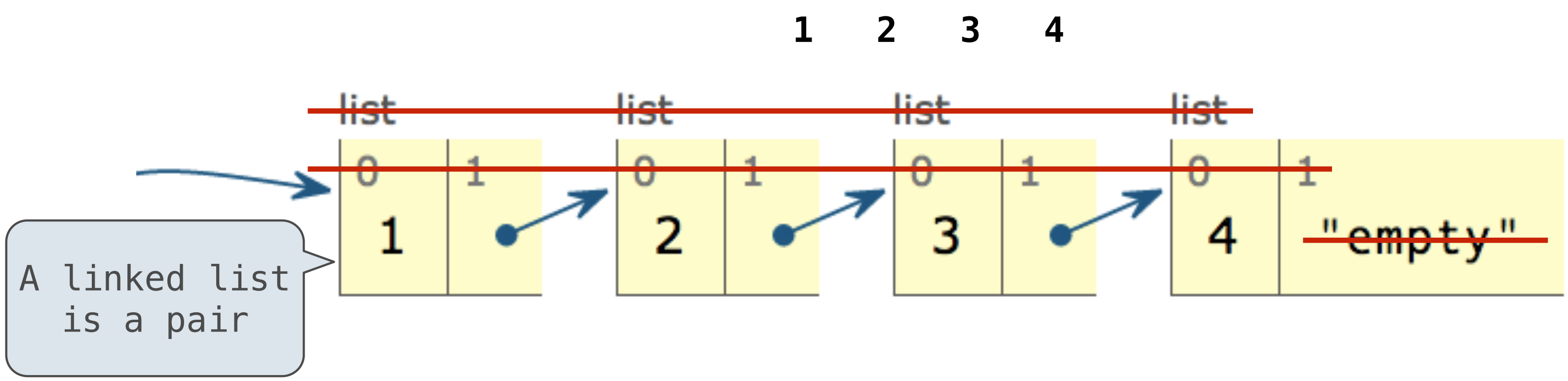

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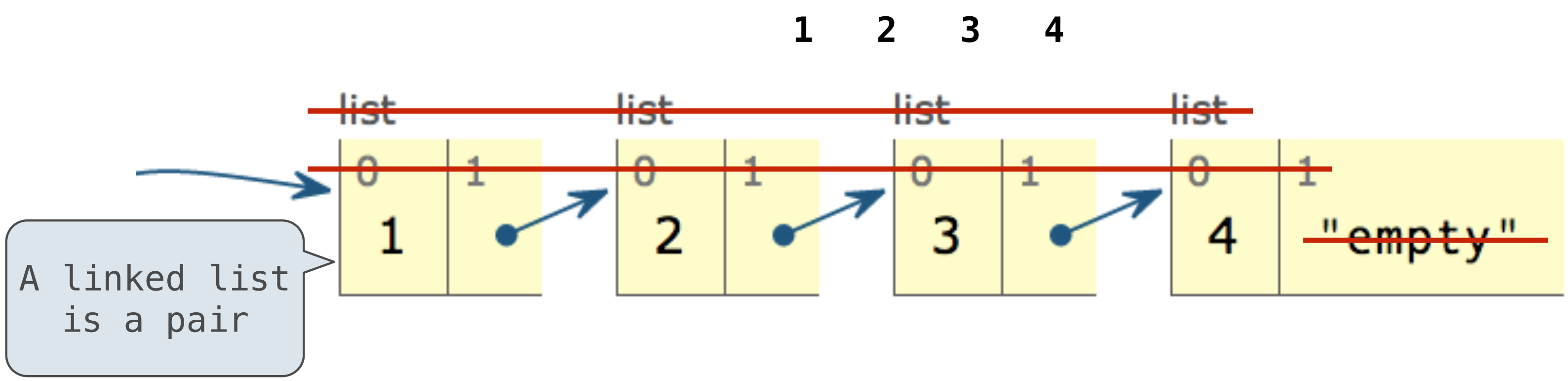

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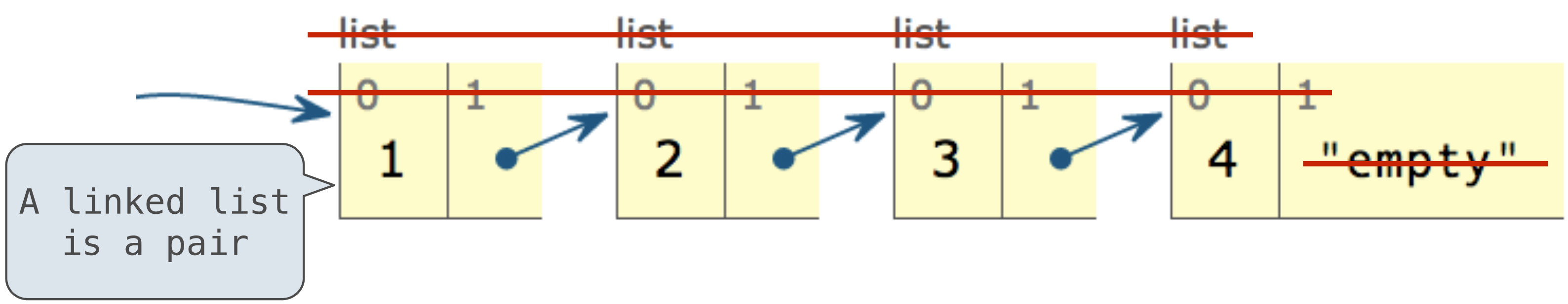

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>>> s[2]
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```

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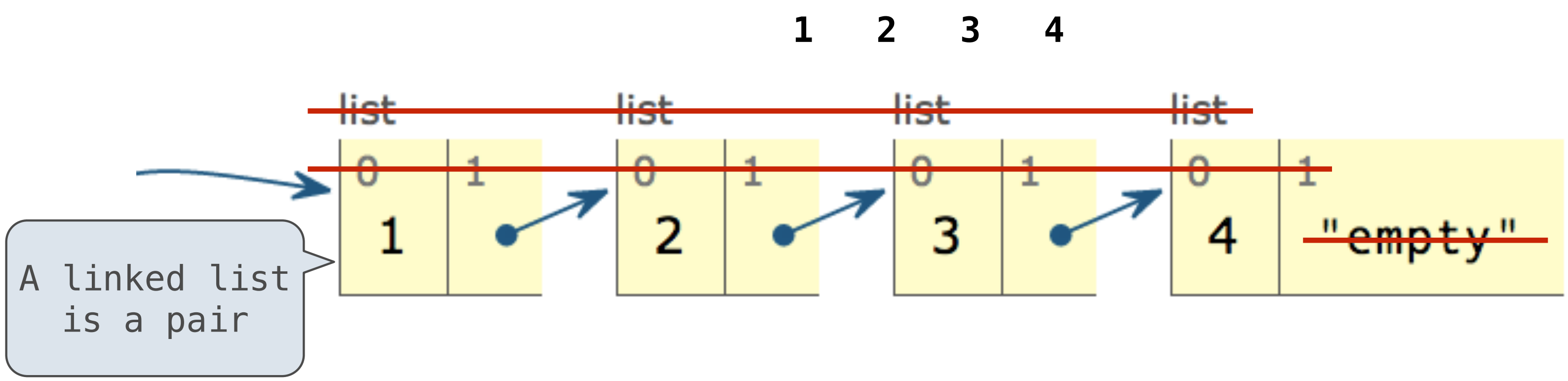

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[1, [2, [3, [4, 'empty']]]]
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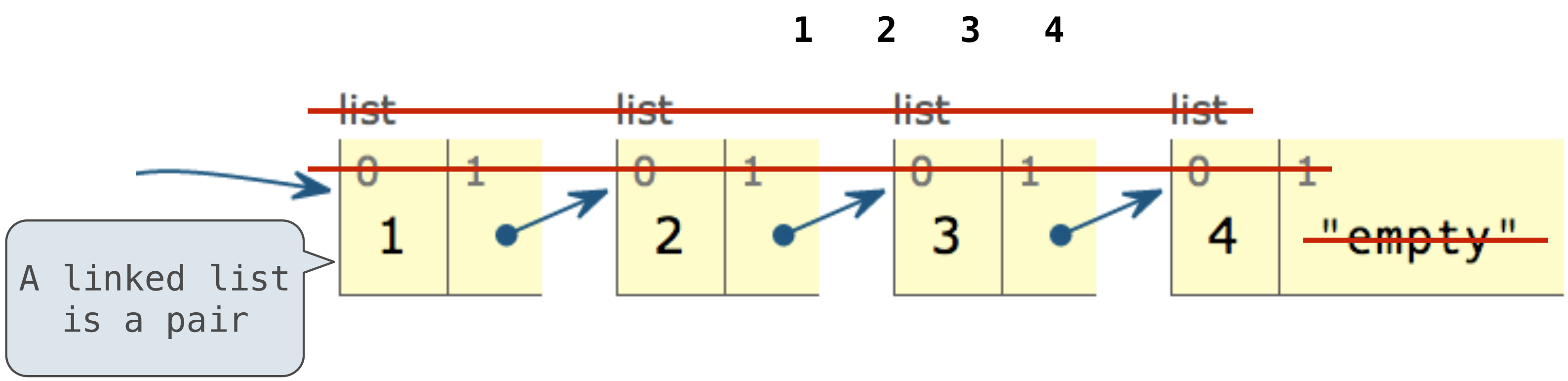

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Linked List Class

More special method names:

`__getitem__`	Element selection []
`__len__`	Built-in len function

Linked List Class

Linked list class: pairs are two-attribute objects

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class Link:

    def __init__(self, first, rest=empty):
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Linked list class: pairs are two-attribute objects

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class Link:


    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest

    def __getitem__(self, i):
        if i == 0:
            return self.first
        else:
            return self.rest[i-1]
```

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Yes, this call is recursive too

More special method names:

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Linked List Class

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class Link:

    empty = ()

    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest

    def __getitem__(self, i):
        if i == 0:
            return self.first
        else:
            return self.rest[i-1]

    def __len__(self):
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Some zero length sequence

Calls this method

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Methods can be recursive too!

(Demo)

Tree Class

Tree Class

A Tree has an entry (any value) at its root and a list of branches

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class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
```

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```
class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
        for branch in branches:
            assert isinstance(branch, Tree)
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Built-in isinstance function:
returns True if branch has a class
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def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
```

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def fib_tree(n):
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    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        return Tree(left.entry + right.entry, (left, right))
```

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(Demo)

Example: Hailstone Trees

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```
Pick a positive integer n as the start
```

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`Pick a positive integer n as the start`

`If n is even, divide it by 2`

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```
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If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1
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Continue this process until n is 1
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4

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4

8

Example: Hailstone Trees

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4

8

16

Example: Hailstone Trees

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4

8

16

32

Example: Hailstone Trees

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```

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2
4
8
16
32
64
```

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2

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128

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If n is even, divide it by 2

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```

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Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

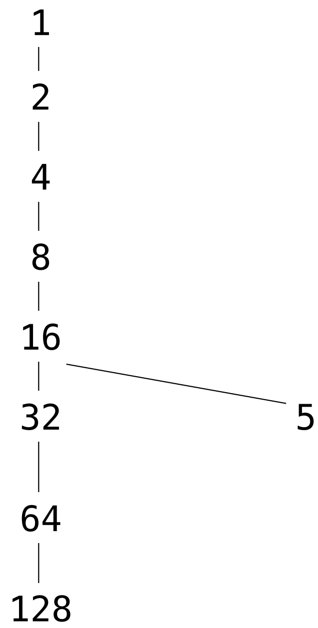

Example: Hailstone Trees

```
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Continue this process until n is 1
```

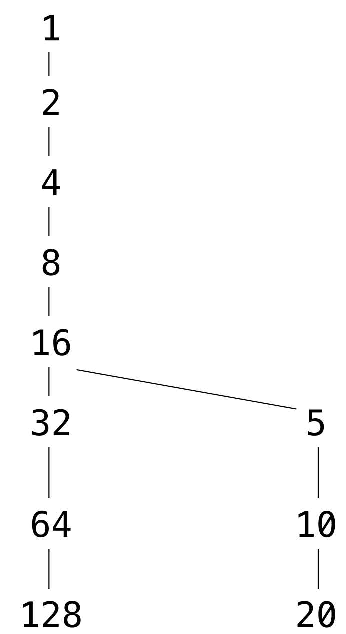

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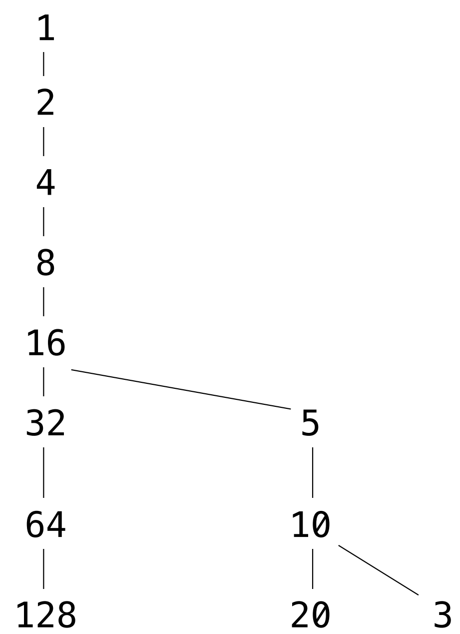

Example: Hailstone Trees

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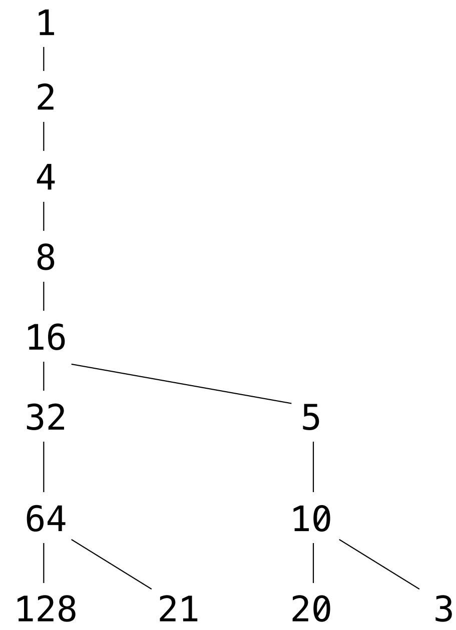
`If n is odd, multiply it by 3 and add 1`

`Continue this process until n is 1`



Example: Hailstone Trees

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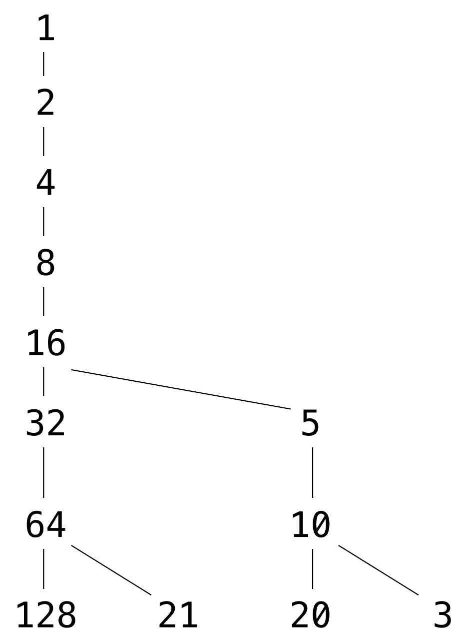



```
All starting n that give an
8-number-long hailstone sequence
```

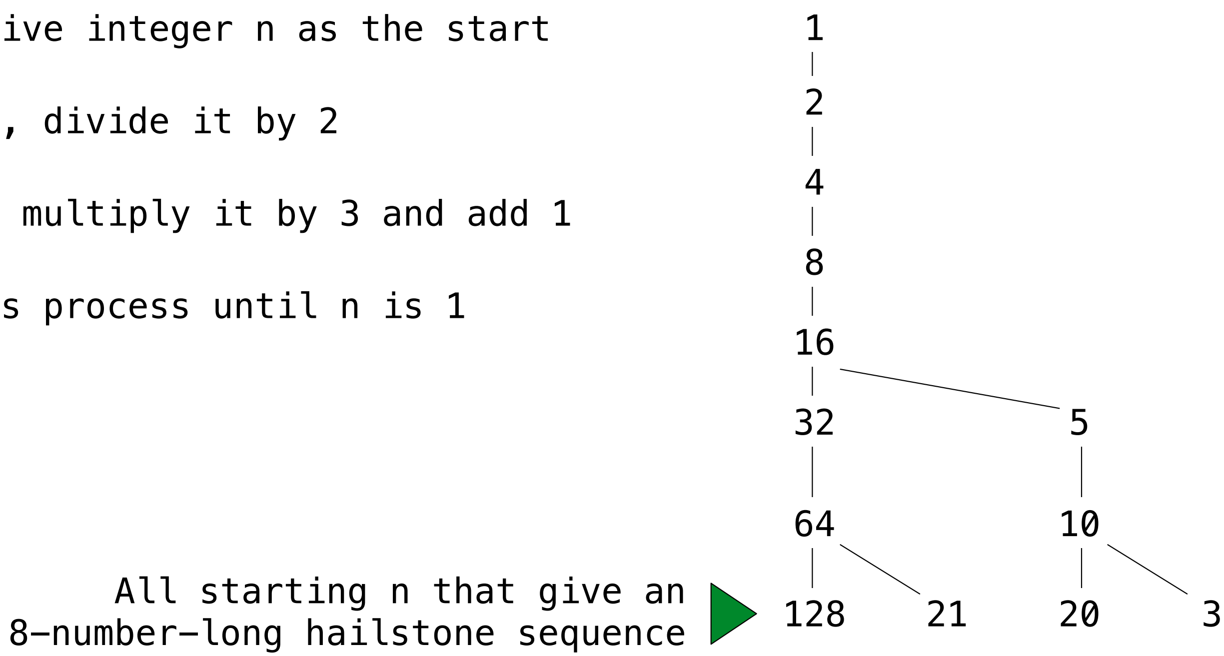
Example: Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```



(Demo)