61A Lecture 22

Wednesday, October 22

Announcements

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  - Conflict form submissions are due Wednesday 10/22!

Sets

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One more built-in Python container type

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- Set literals are enclosed in braces

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>>> s = {3, 2, 1, 4, 4}
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>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
```

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```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
```

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>>> s
{1, 2, 3, 4}
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True
>>> len(s)
4
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- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
```

Sets

One more built-in Python container type

- Set literals are enclosed in braces
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- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Implementing Sets

Implementing Sets

```
What we should be able to do with a set:
```

Implementing Sets

What we should be able to do with a set:

- **Membership testing:** Is a value an element of a set?

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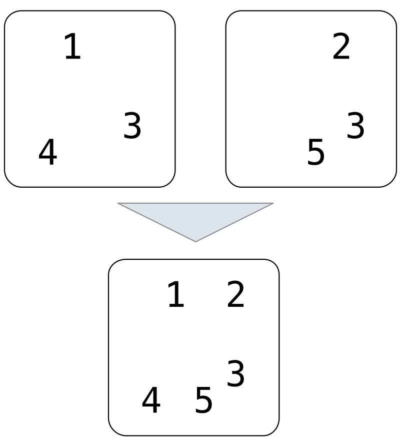
Union



Implementing Sets

What we should be able to do with a set:

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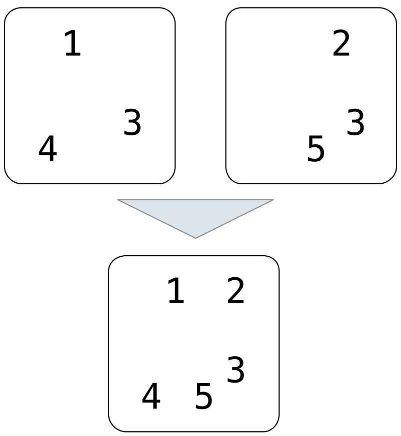
Union



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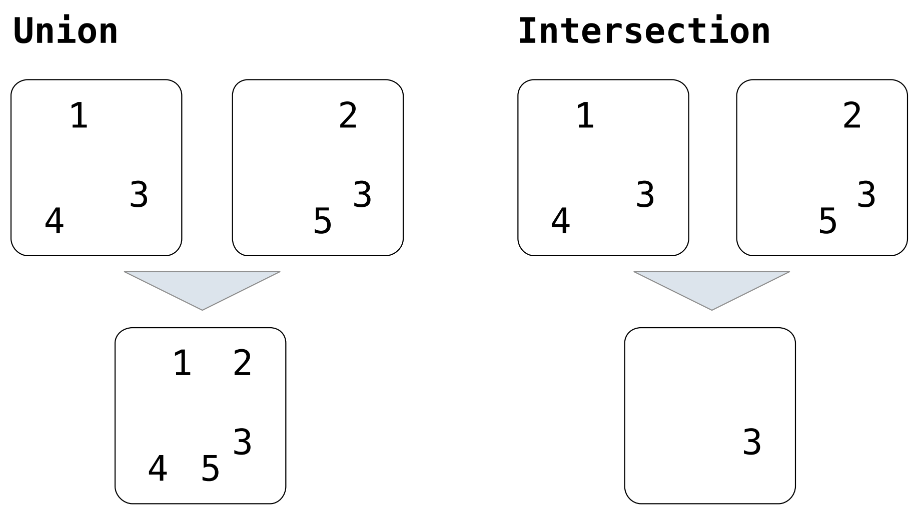

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- **Membership testing:** Is a value an element of a set?
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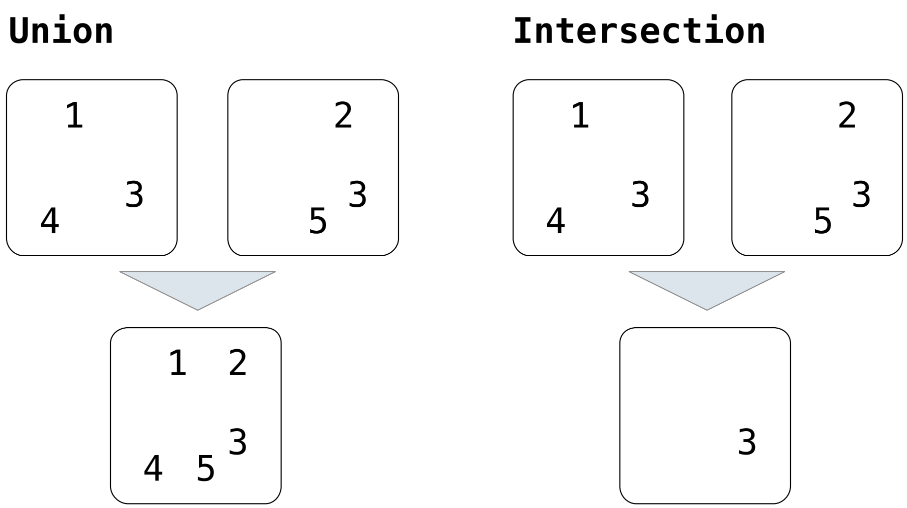
- **Adjoin:** Return a set with all elements in s and a value v



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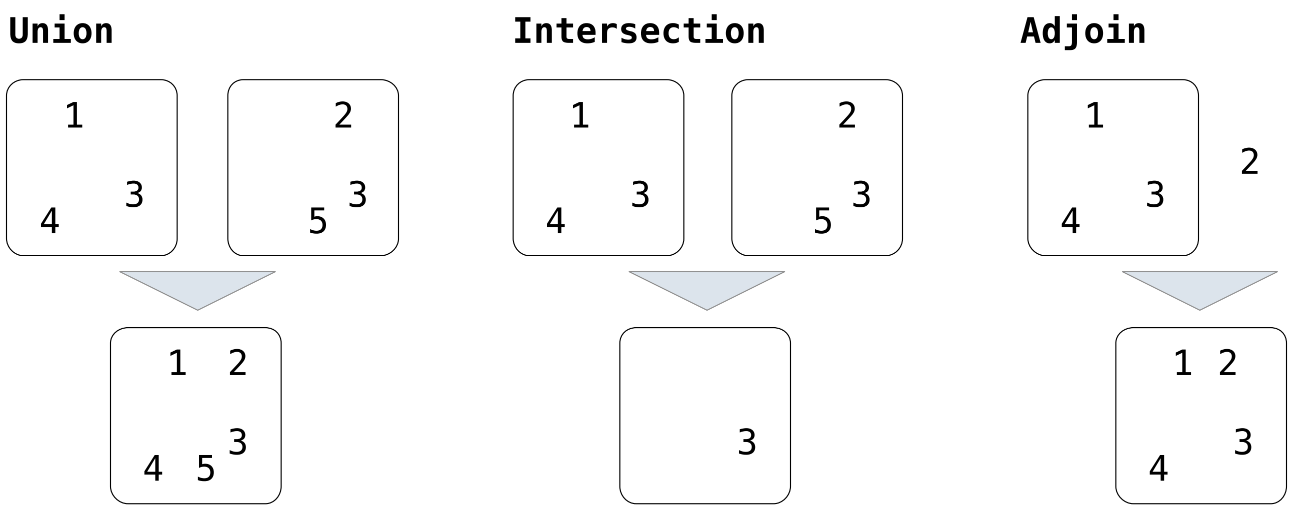

Sets as Unordered Sequences

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Proposal 1: A set is represented by a linked list that contains no duplicate items.

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def set_contains(s, v):
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    >>> s = Link(1, Link(2, Link(3)))
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Assuming v either does not appear in s ***or*** *appears in a uniformly distributed random location*

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(Demo)

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def adjoin_set(s, v):
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Sets as Unordered Sequences

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def adjoin_set(s, v):
    if set_contains(s, v):
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def intersect_set(set1, set2):
    in_set2 = lambda v: set_contains(set2, v)
    return keep_if(set1, in_set2)
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def union_set(set1, set2):
    not_in_set2 = lambda v: not set_contains(set2, v)
    set1_not_set2 = keep_if(set1, not_in_set2)
    return extend(set1_not_set2, set2)
```

Sets as Unordered Sequences

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(Demo)

Sets as Ordered Sequences

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Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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Parts of the program that...	Assume that sets are...	Using...

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values		

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set

Sets as Ordered Sequences

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set
Implement set operations		

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Implement set operations	Ordered linked lists	

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Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set
Implement set operations	Ordered linked lists	first, rest, <, >, ==

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set
Implement set operations	Ordered linked lists	first, rest, <, >, ==

Different parts of a program may make different assumptions about data

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def intersect_set(set1, set2):
    if empty(set1) or empty(set2):
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        e1, e2 = set1.first, set2.first
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        if e1 == e2:
            return Link(e1, intersect_set(set1.rest, set2.rest))
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        elif e1 < e2:
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Order of growth?

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Order of growth? $\Theta(n)$

Sets as Binary Search Trees

Binary Search Trees

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Proposal 3: A set is represented as a Tree with two branches. Each entry is:

Binary Search Trees

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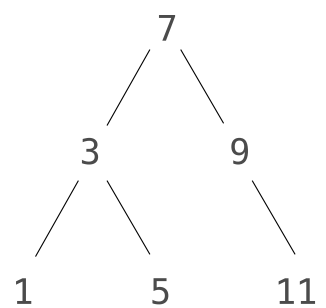

Binary Search Trees

Proposal 3: A set is represented as a Tree with two branches. Each entry is:

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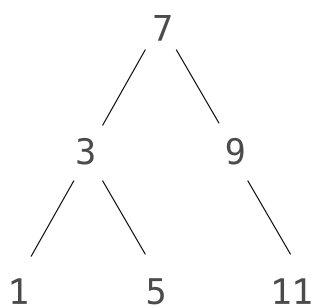

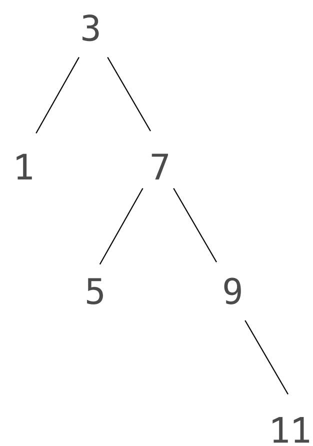

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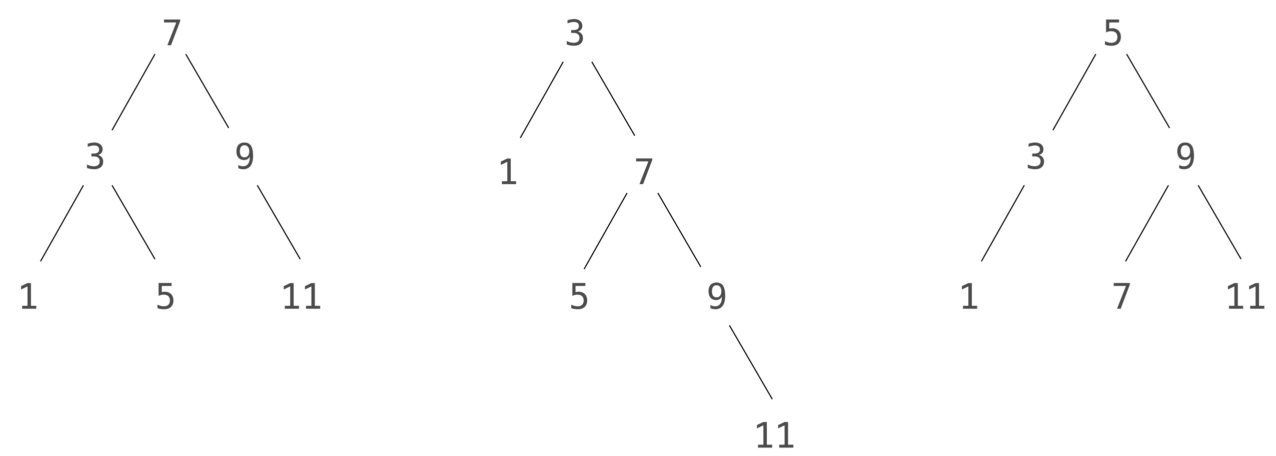

Binary Tree Class

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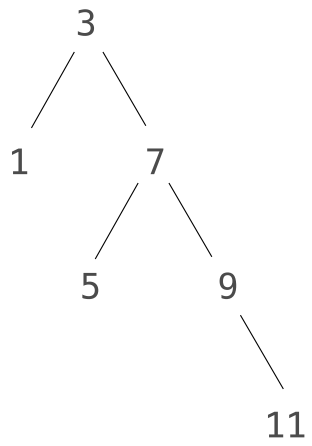

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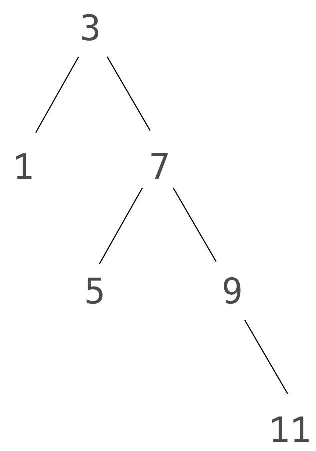

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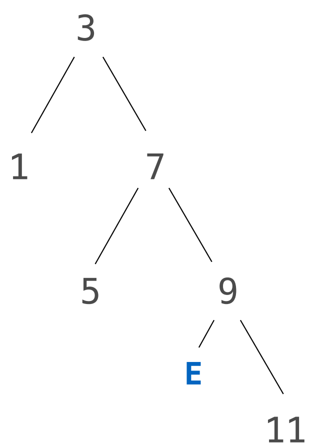

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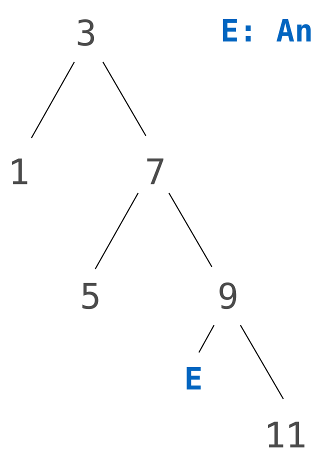

E: An empty tree

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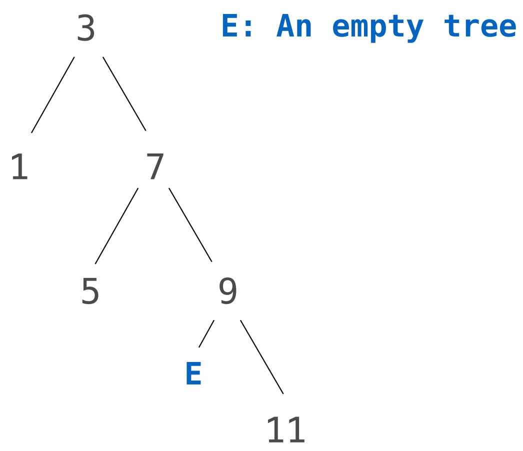

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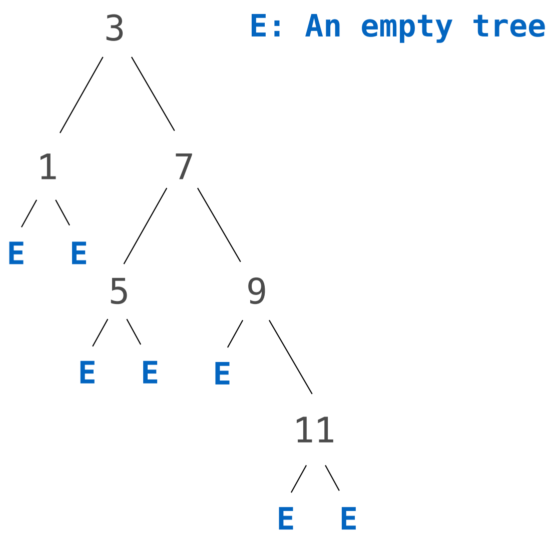

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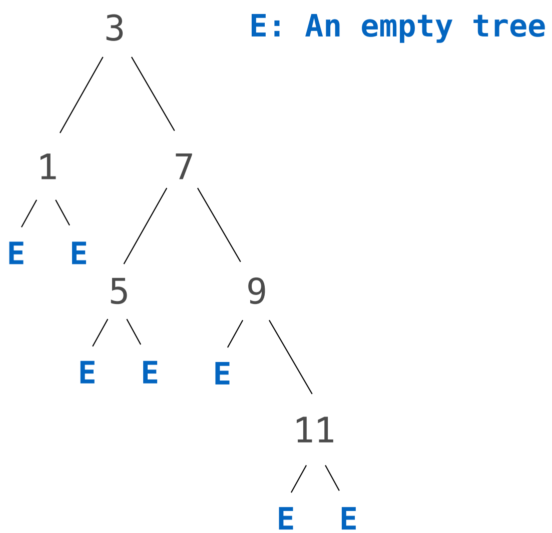


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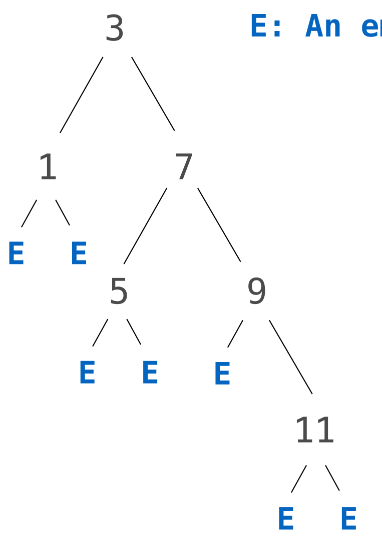

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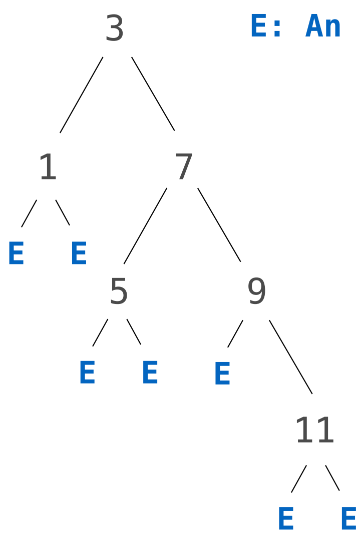

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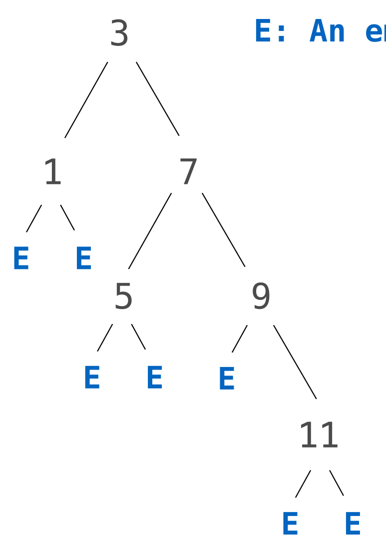

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    def left(self):
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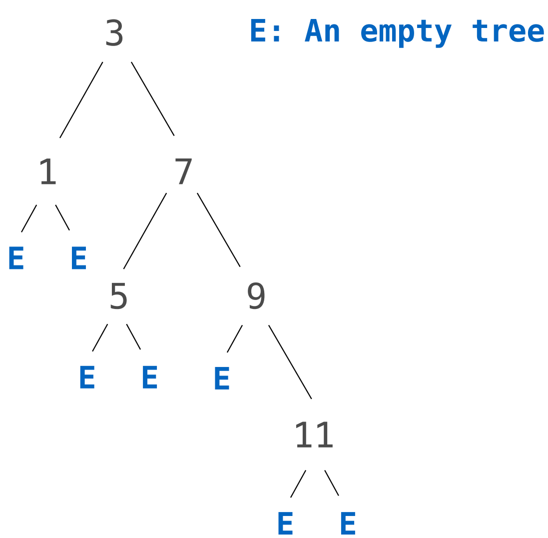


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    def right(self):
        return self.branches[1]
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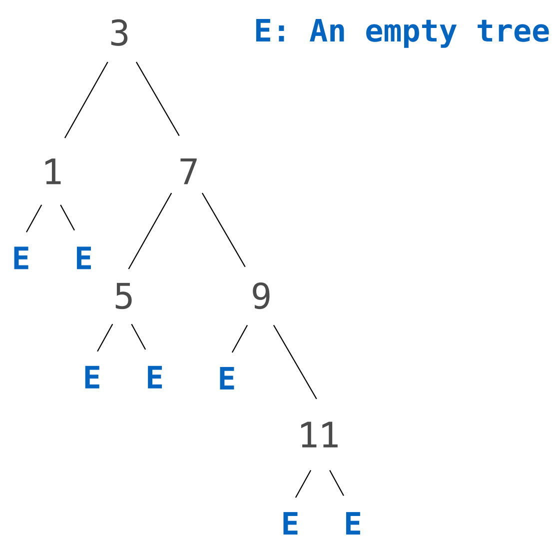


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Bin = BinaryTree
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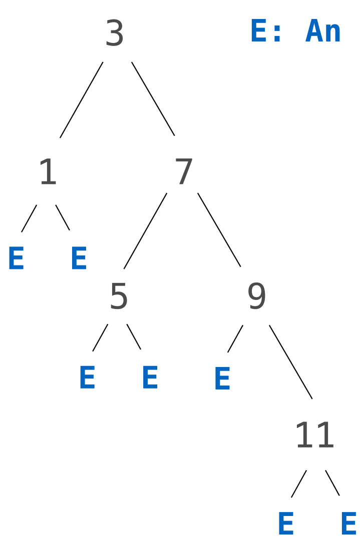

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Bin = BinaryTree
t = Bin(3, Bin(1),
           Bin(7, Bin(5),
                  Bin(9, Bin.empty,
                         Bin(11))))
```

Membership in Binary Search Trees

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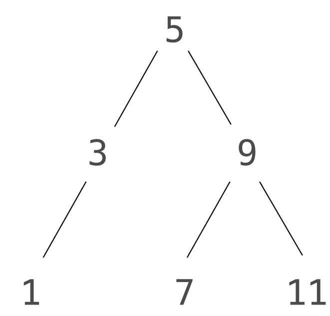

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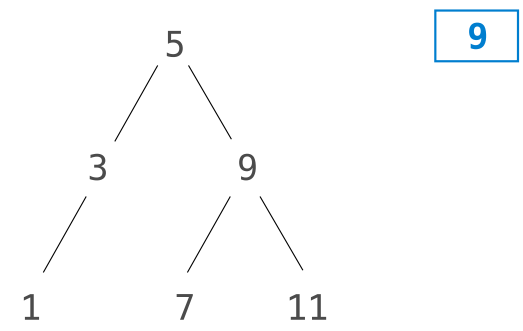

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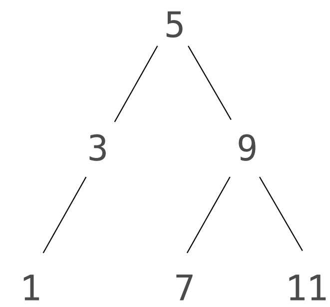

9

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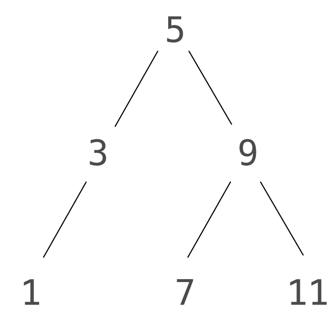

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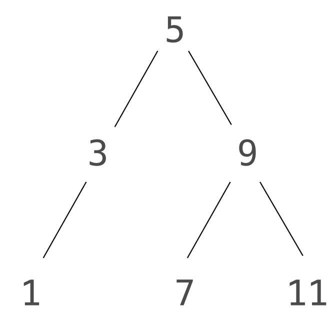

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        return set_contains(s.right, v)
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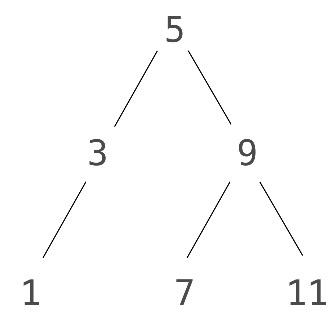

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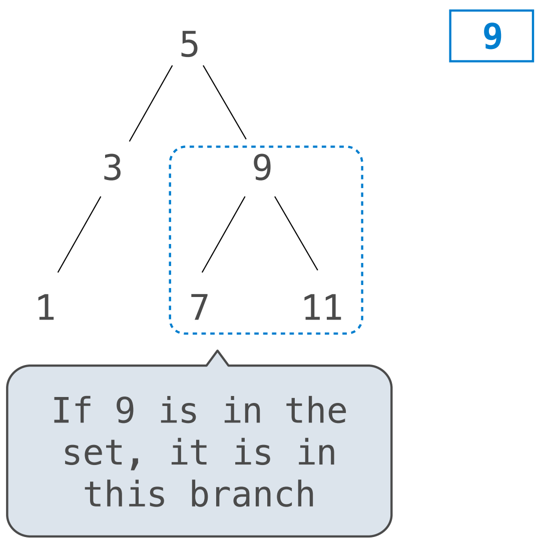

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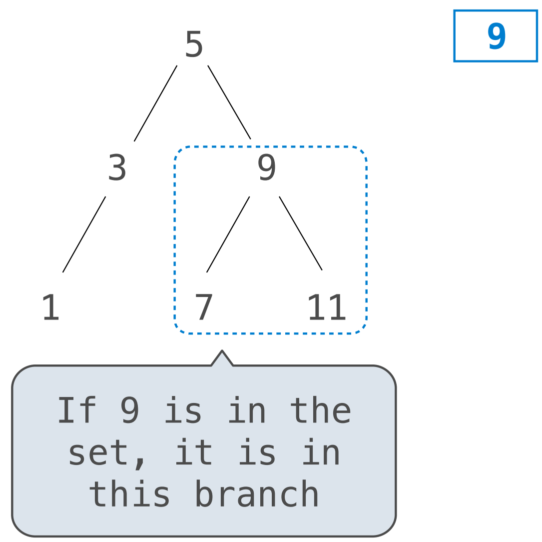

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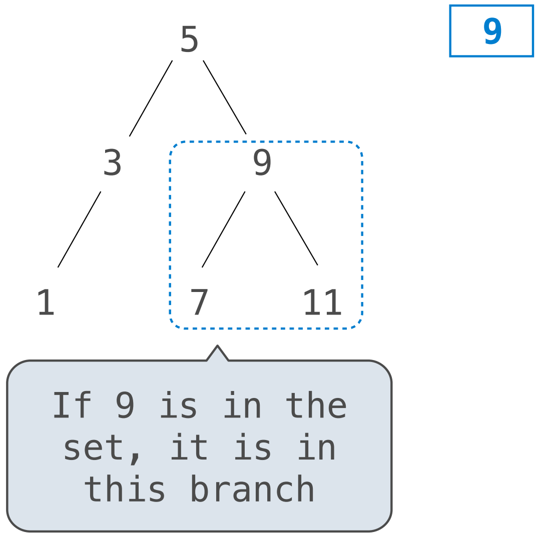

Order of growth?

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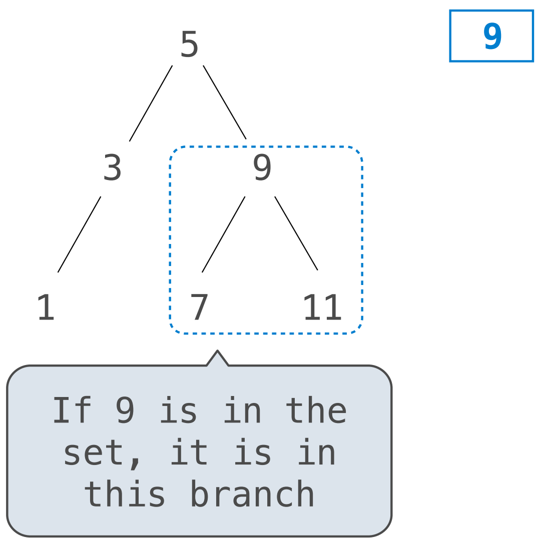

Order of growth? $\Theta(h)$ on average

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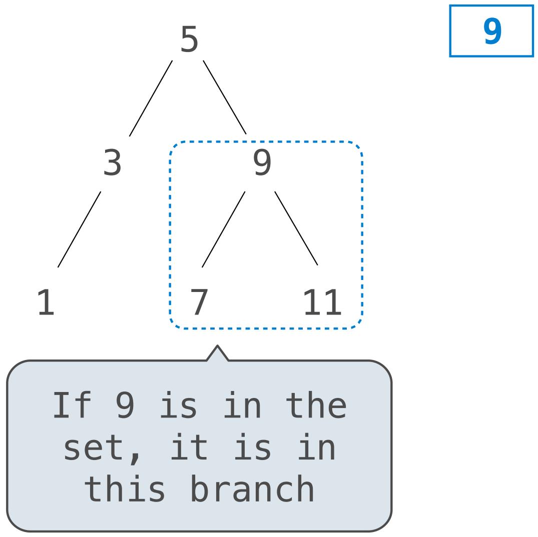

Order of growth? $\Theta(h)$ on average $\Theta(\log n)$ on average for a balanced tree

Adjoining to a Tree Set

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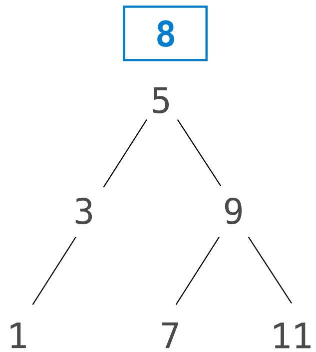

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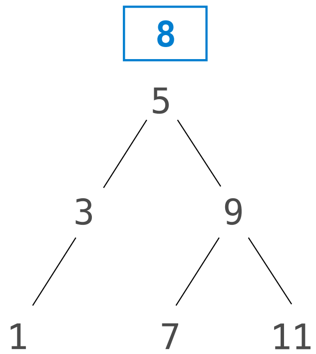

Right!

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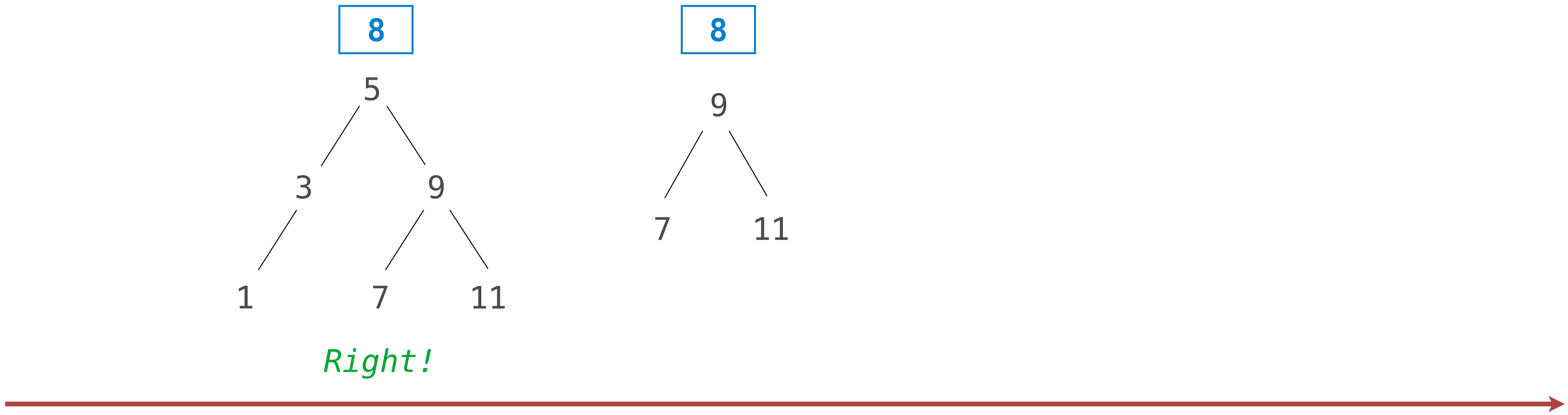

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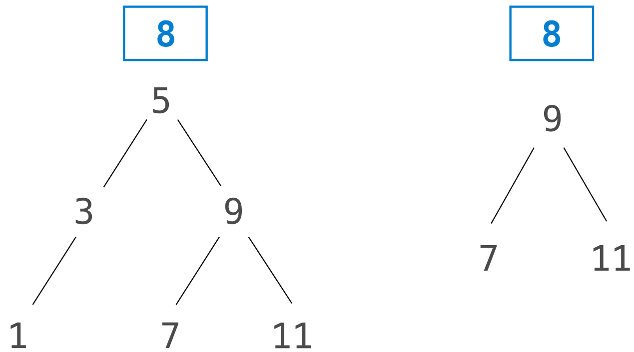

Right! *Left!*

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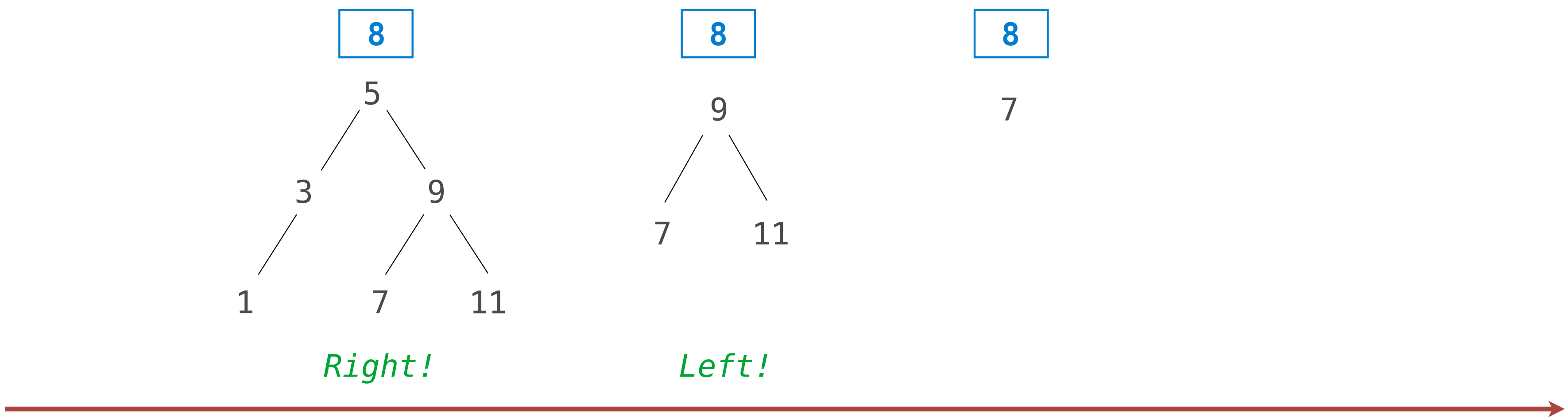

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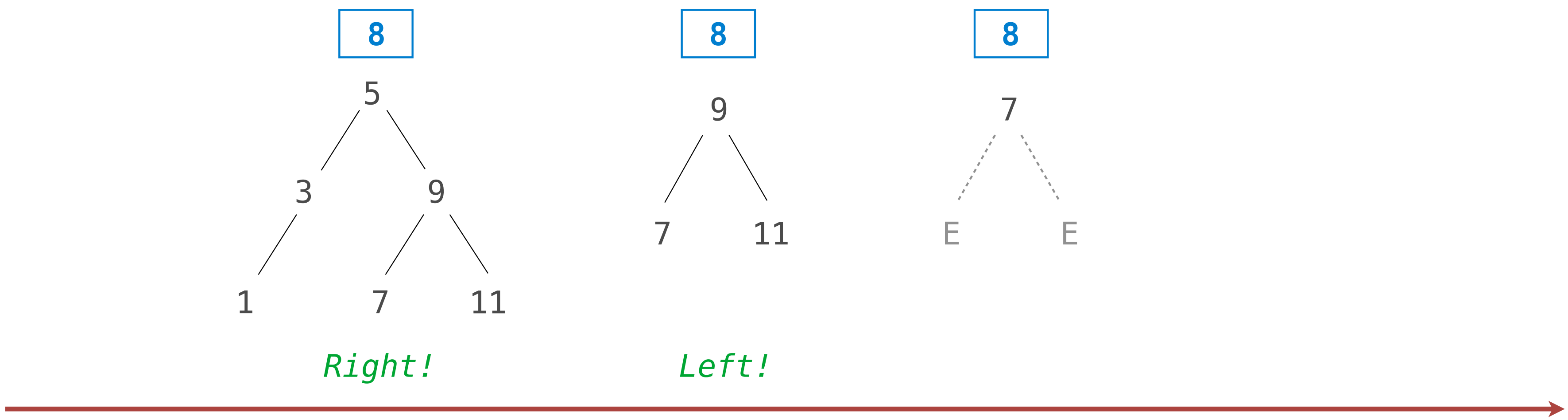

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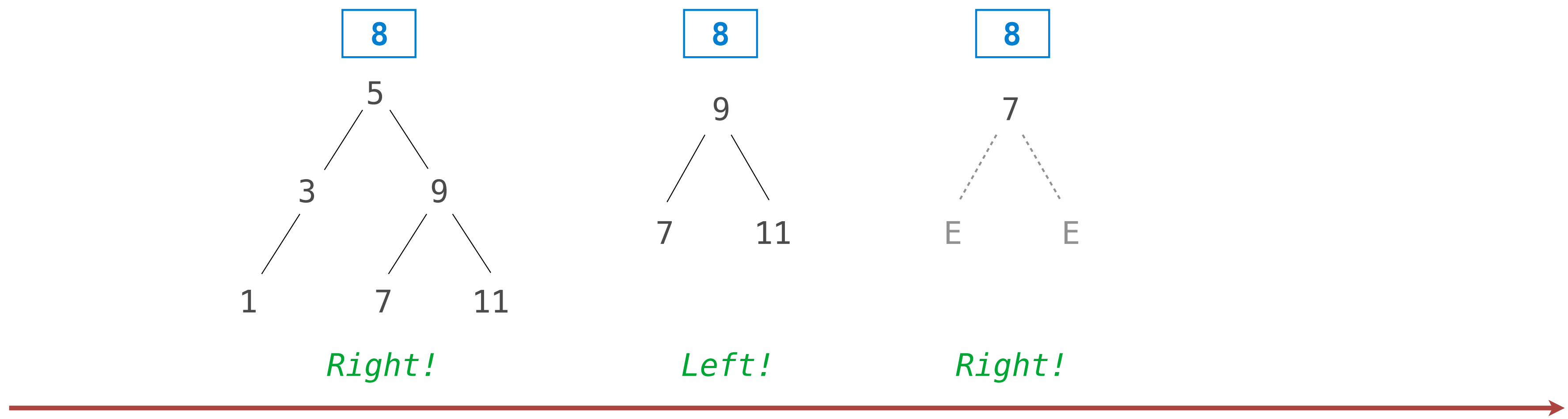

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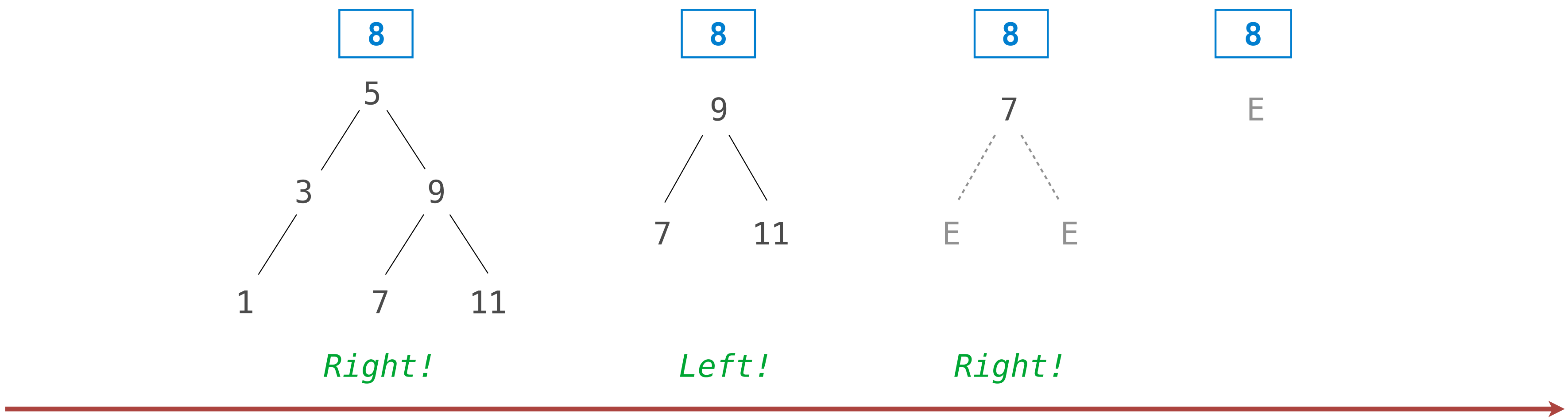

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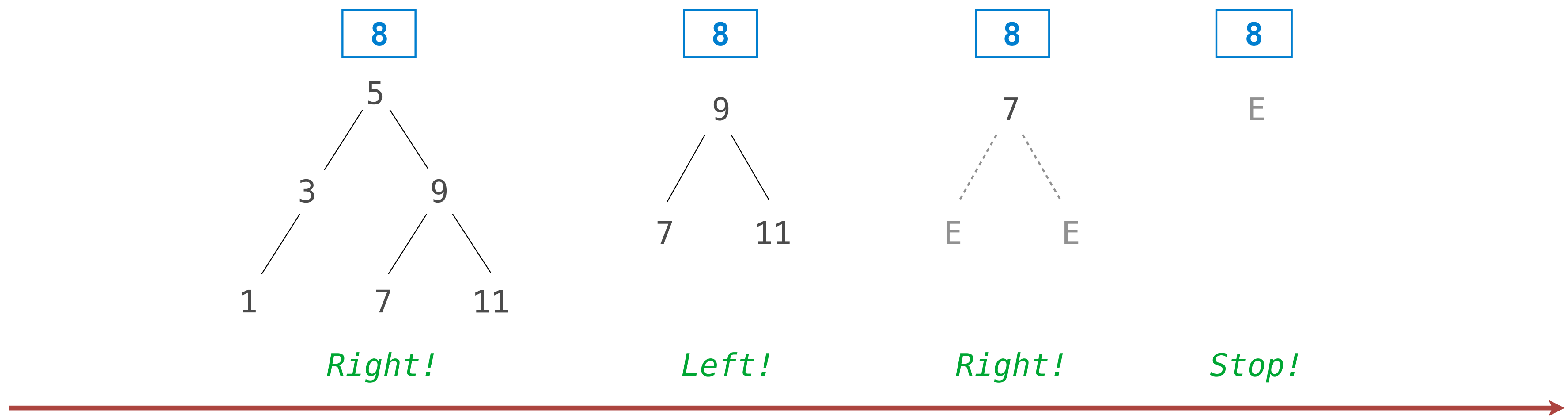

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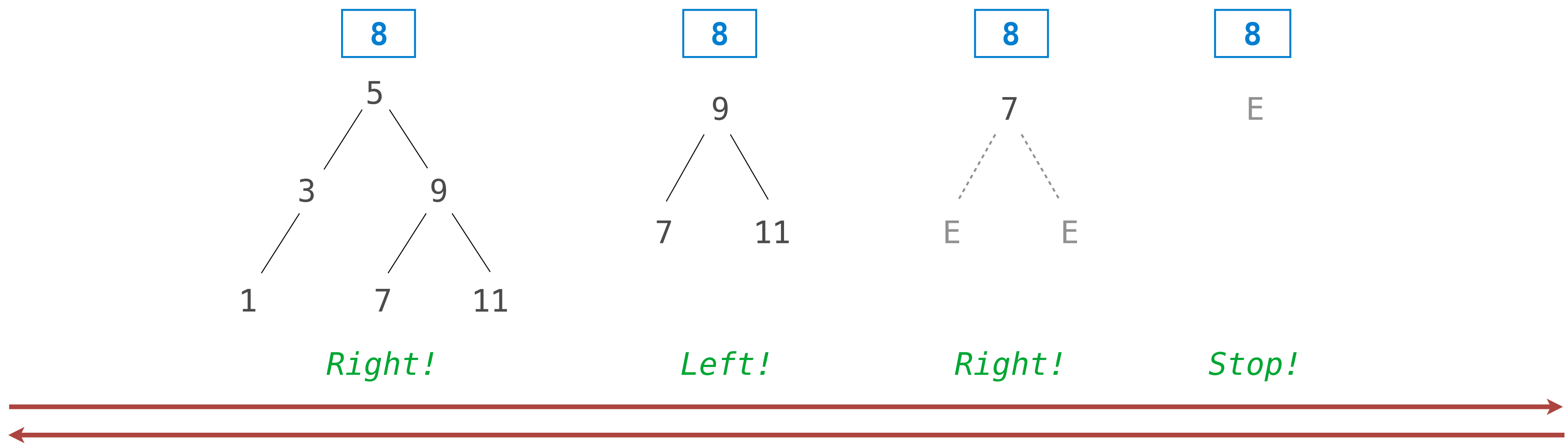

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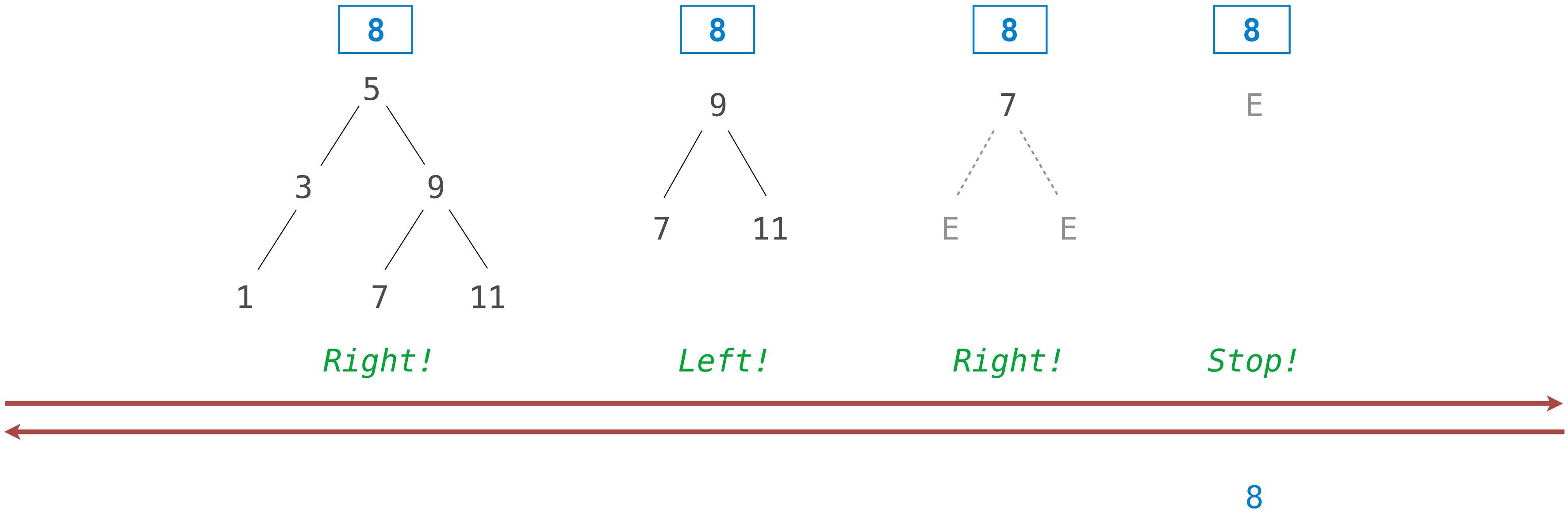

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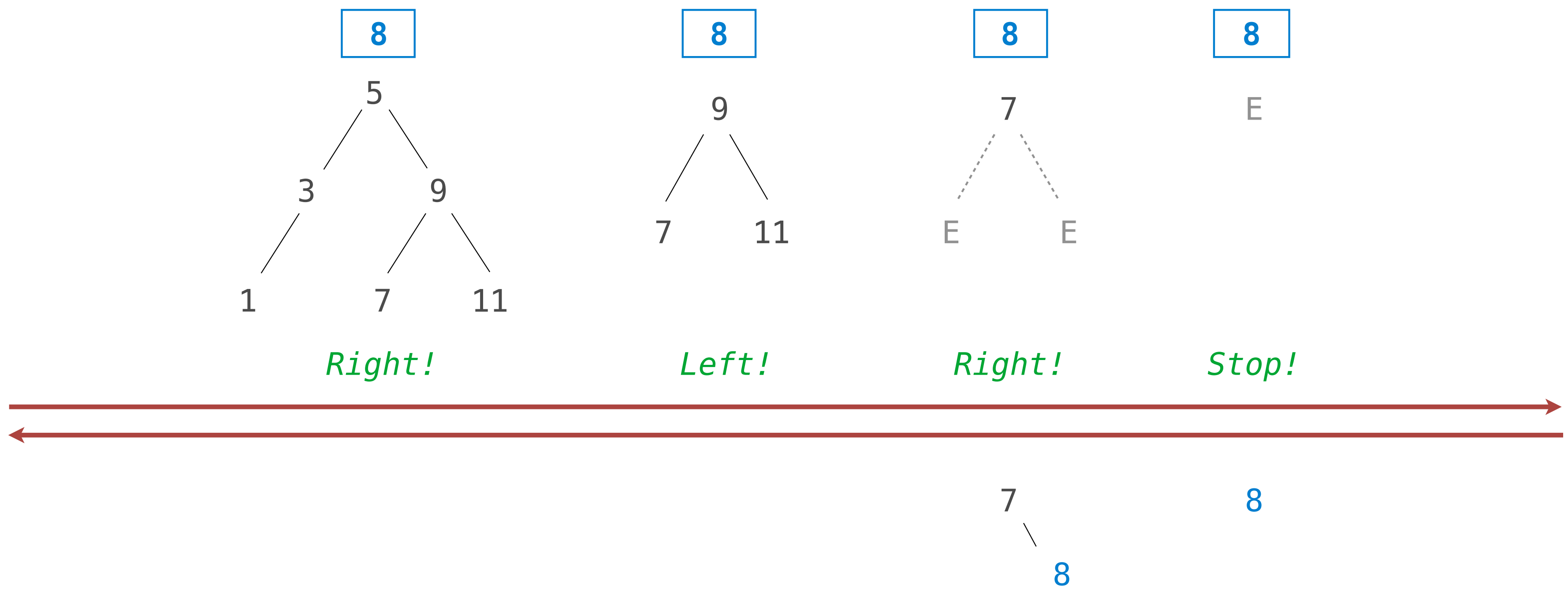

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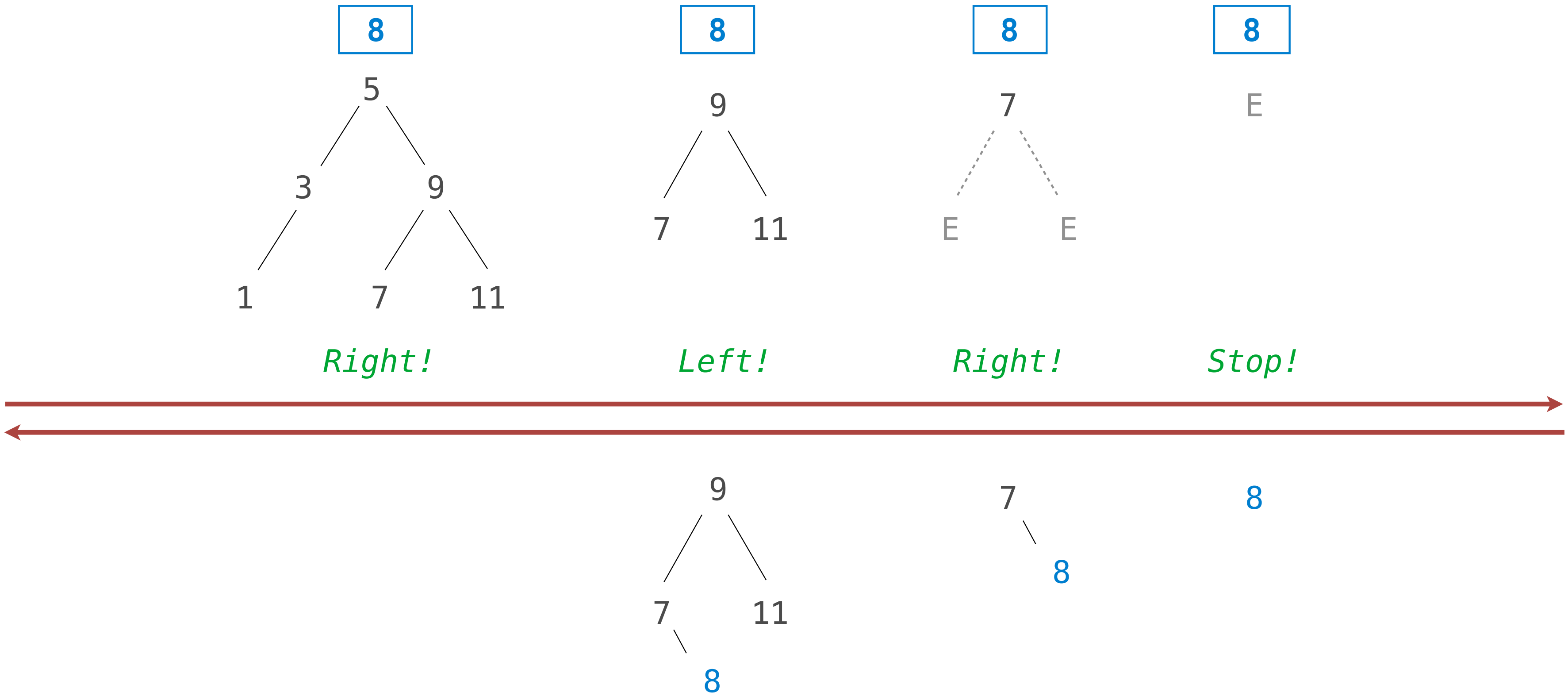

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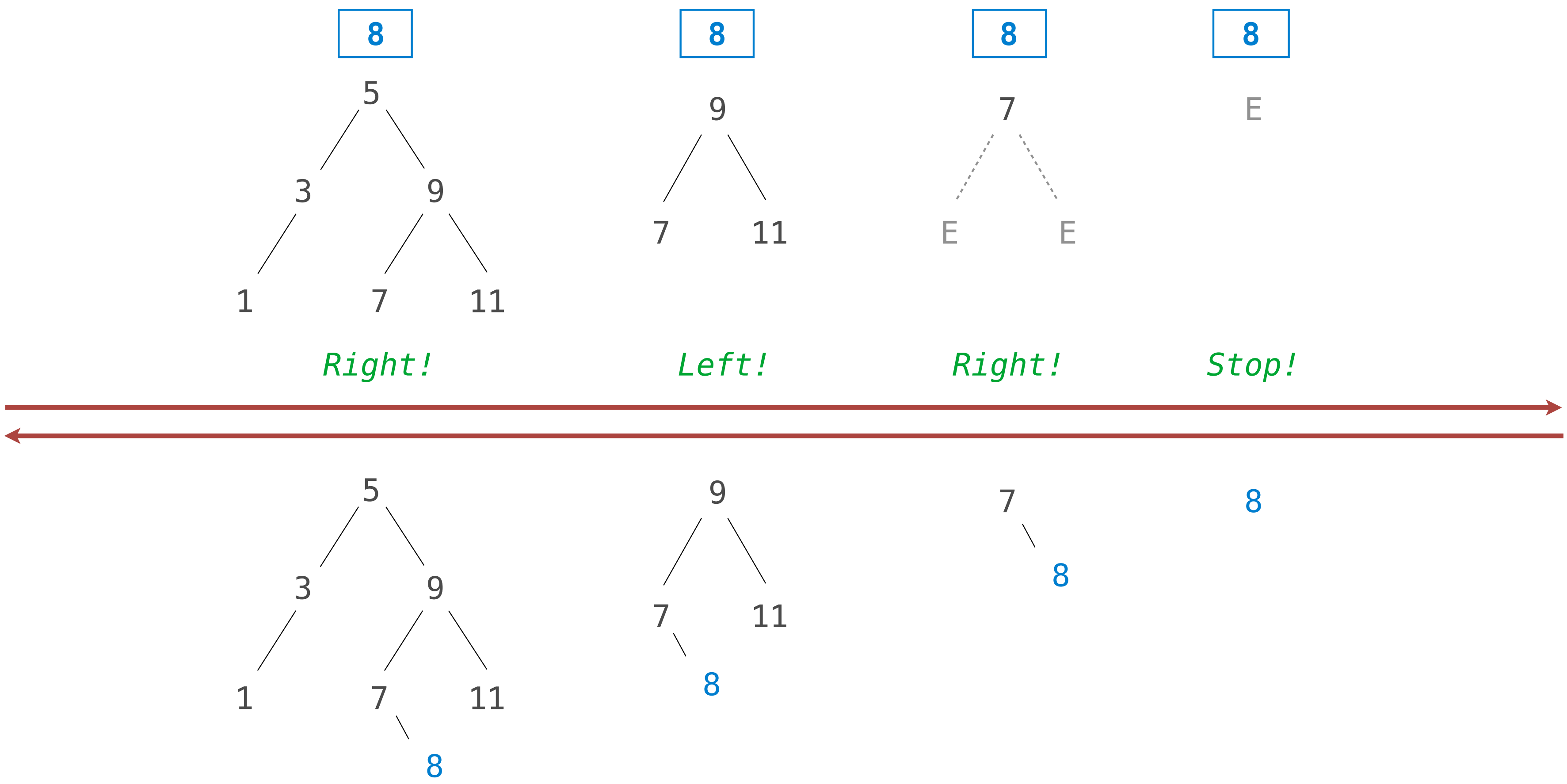

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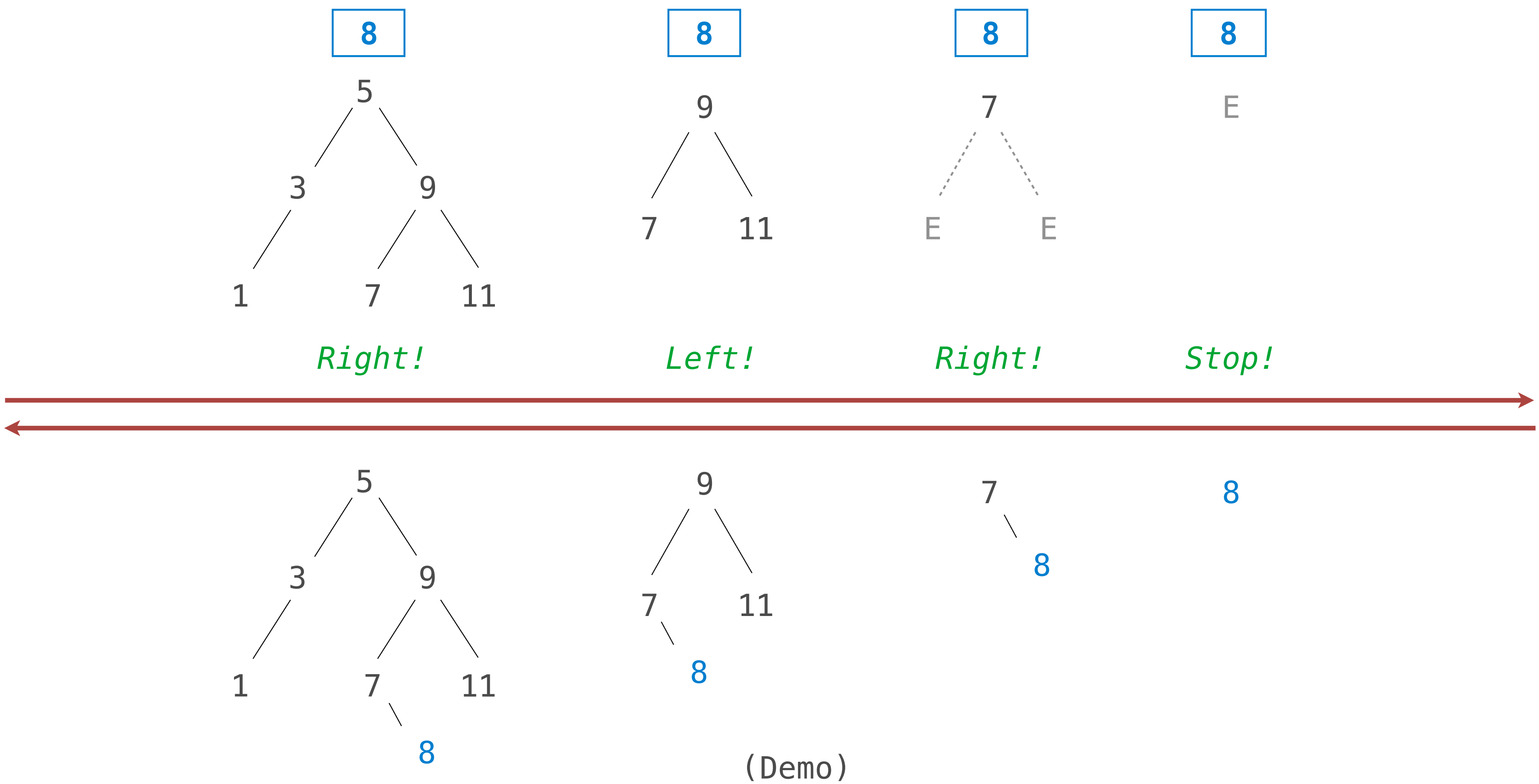

(Demo)