61A Lecture 23

Friday, October 24

Announcements

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- No lecture next Monday, No lab next Tuesday & Wednesday, No office hours next Mon–Wed

Mutable Linked Lists

Recursive Lists Can Change

Attribute assignment statements can change first and rest attributes of a Link

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The rest of a linked list can contain the linked list as a sub-list

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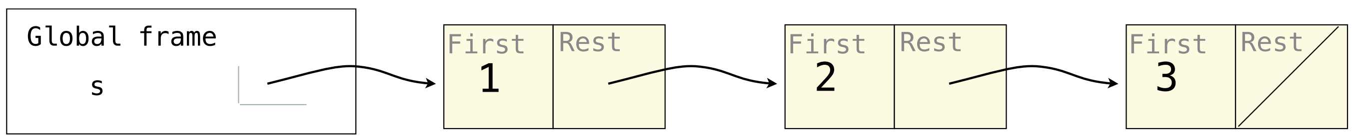

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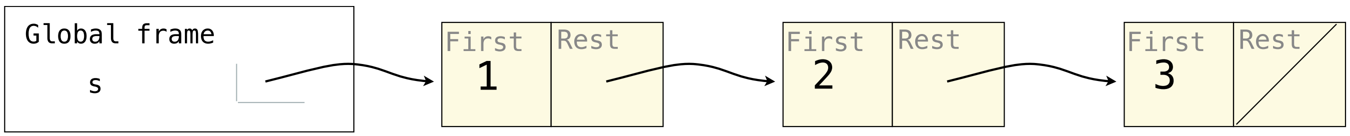

Note: The actual environment diagram is much more complicated.

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>>> s.first = 5
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>>> s = Link(1, Link(2, Link(3)))
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>>> t = s.rest
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>>> s.first
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>>> s.rest.rest.rest.rest.rest.first
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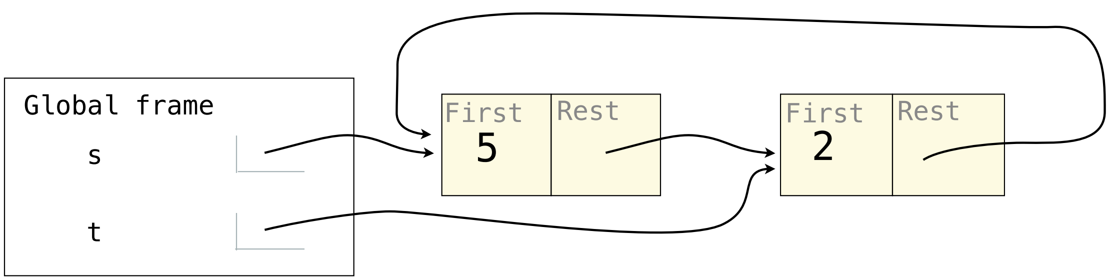

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Hailstone Trees

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Pick a positive integer n as the start
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If n is even, divide it by 2

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16

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 |
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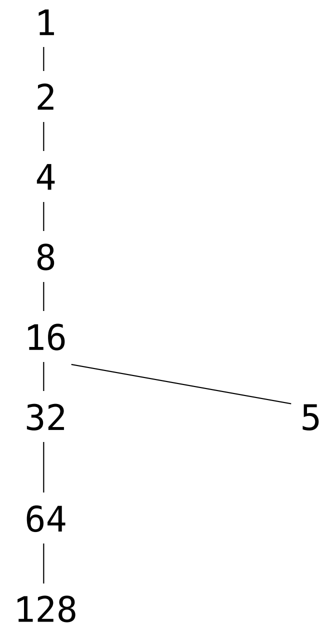

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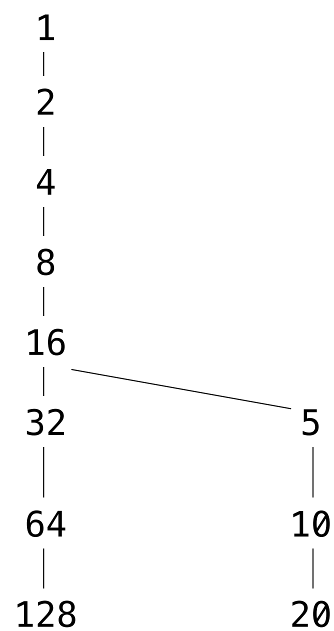

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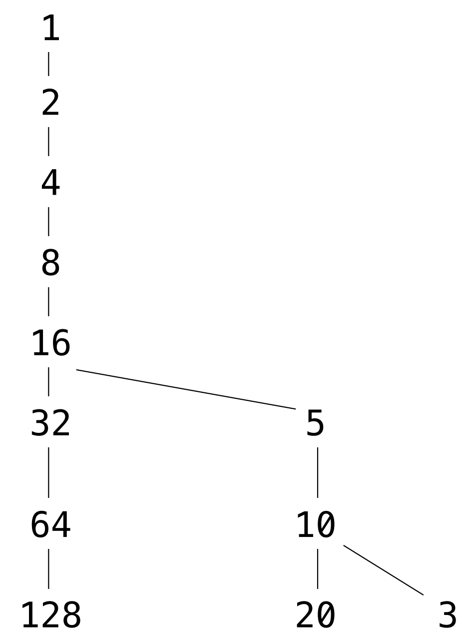

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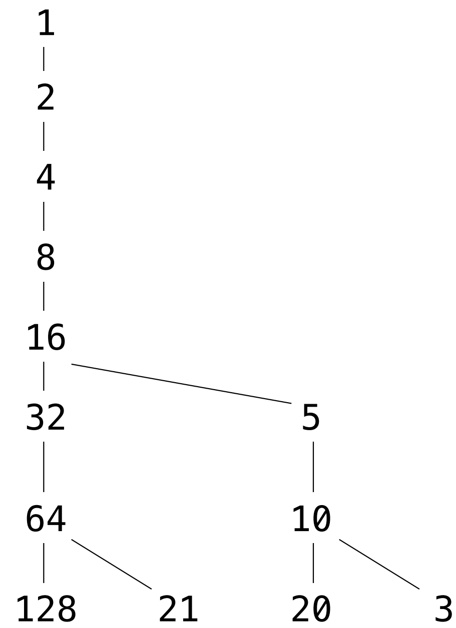

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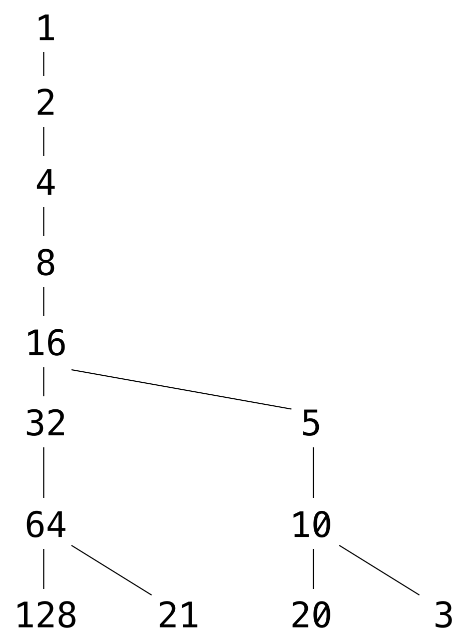
All possible n that start a
length-8 hailstone sequence



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Question: Write hailstone_tree(k, n=1), which returns a Tree in which the paths from the leaves to the root are all possible hailstone sequences of length k ending in n.

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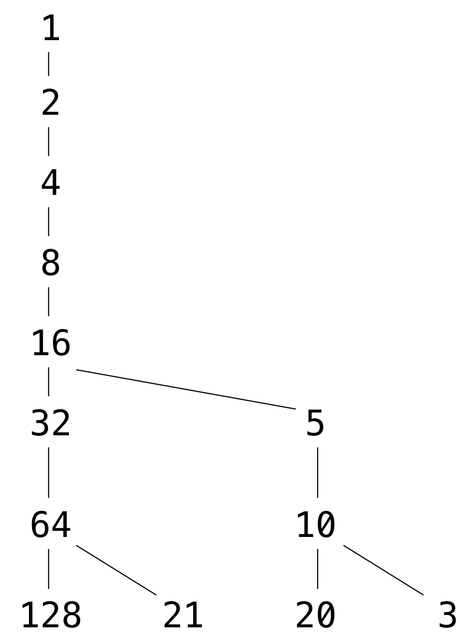

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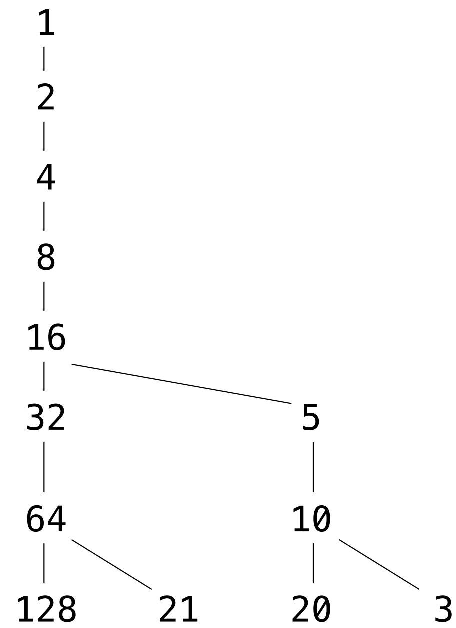

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(Demo)

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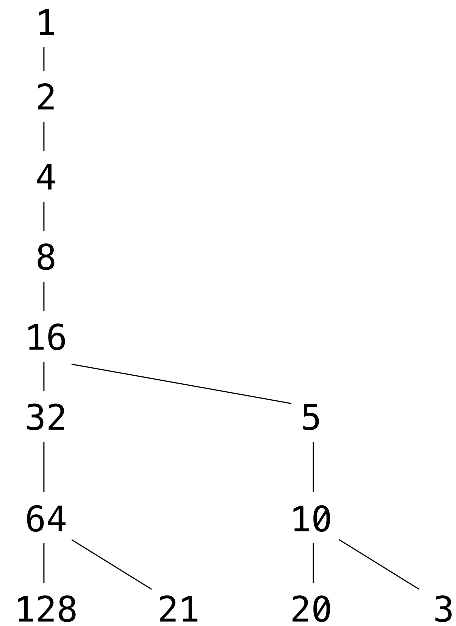


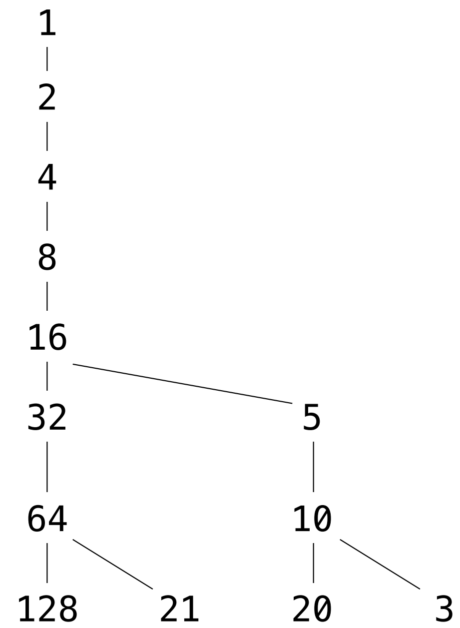
All possible n that start a length-8 hailstone sequence

(Demo)

Question: List the leaves of a Tree instance

Binary Hailstone Trees

Hailstone Trees using BinaryTree



Hailstone Trees using BinaryTree

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class BinaryTree(Tree):
    empty = Tree(None)
    empty.is_empty = True

    def __init__(self, entry, left=empty, right=empty):
        Tree.__init__(self, entry, (left, right))
        self.is_empty = False

    @property
    def left(self):
        return self.branches[0]

    @property
    def right(self):
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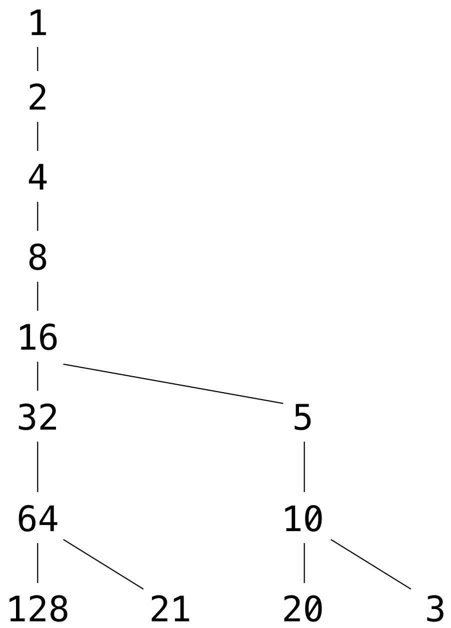

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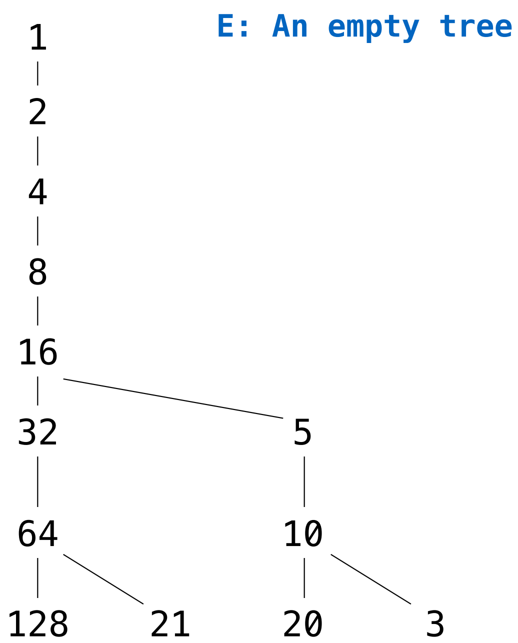

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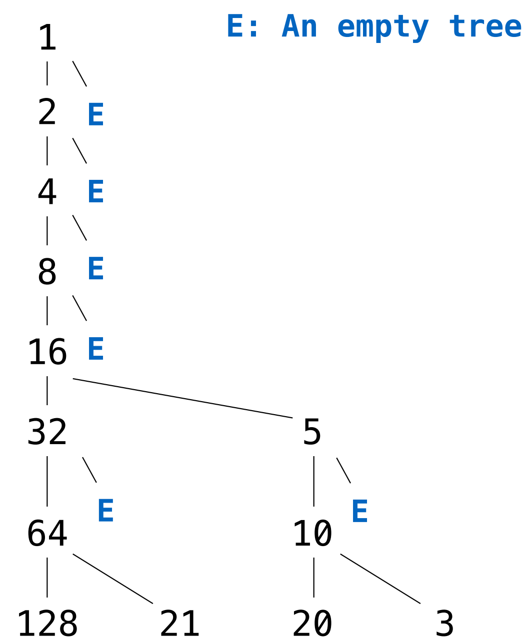

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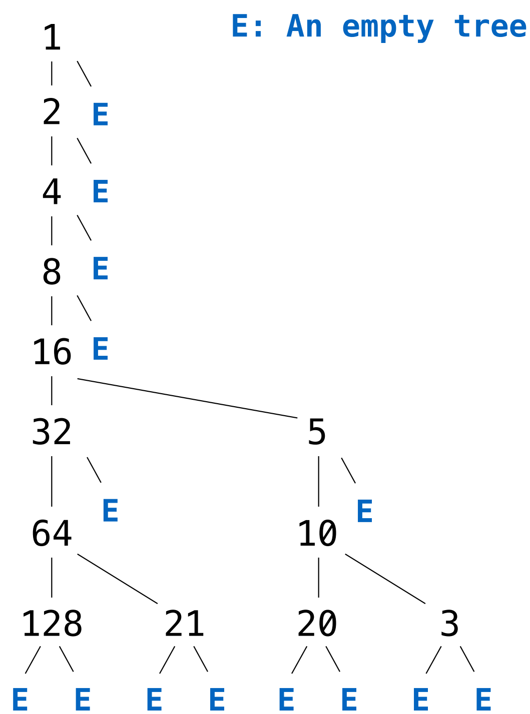

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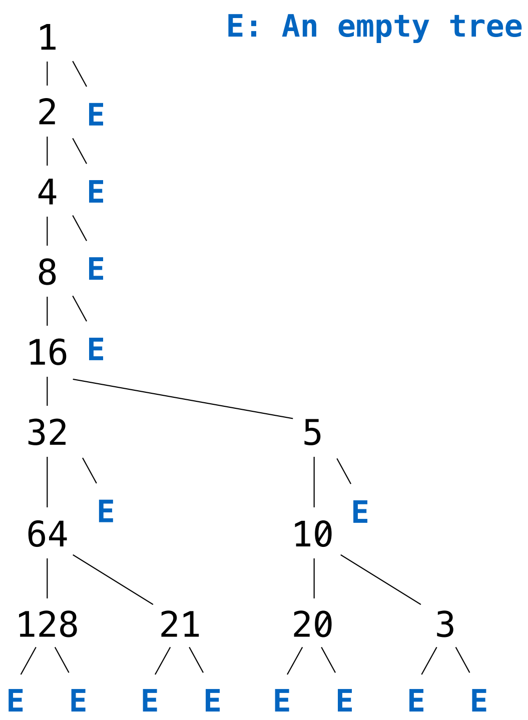

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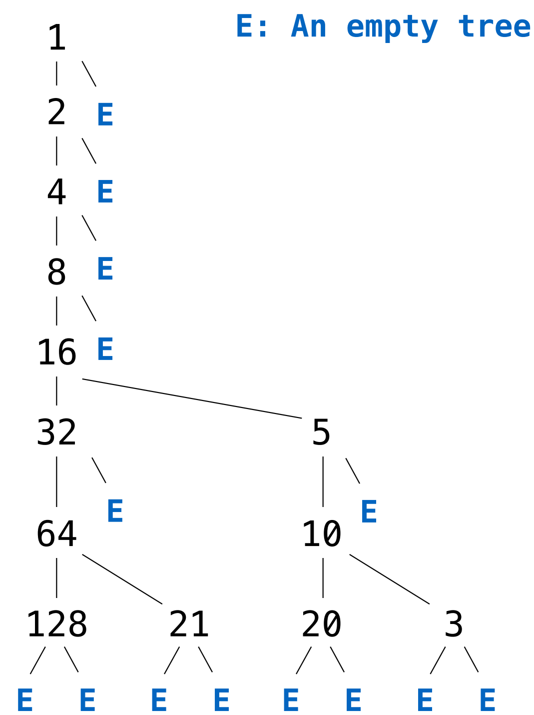


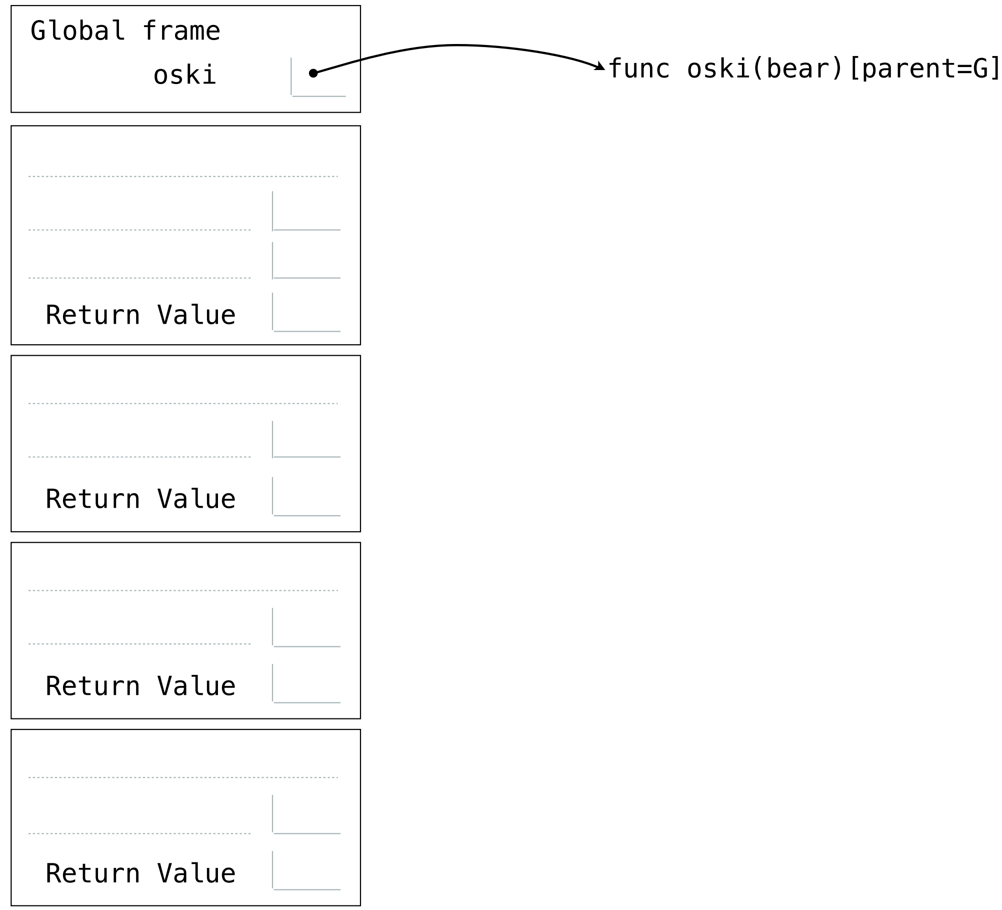
(Demo)

Question: List the entries in the longest path in a binary tree for which all elements are less than k

Environment Diagrams

Go Bears!

```
def oski(bear):
    def cal(berk):
        nonlocal bear
        if bear(berk) == 0:
            return [berk+1, berk-1]
        bear = lambda ley: berk-ley
        return [berk, cal(berk)]
    return cal(2)
oski(abs)
```



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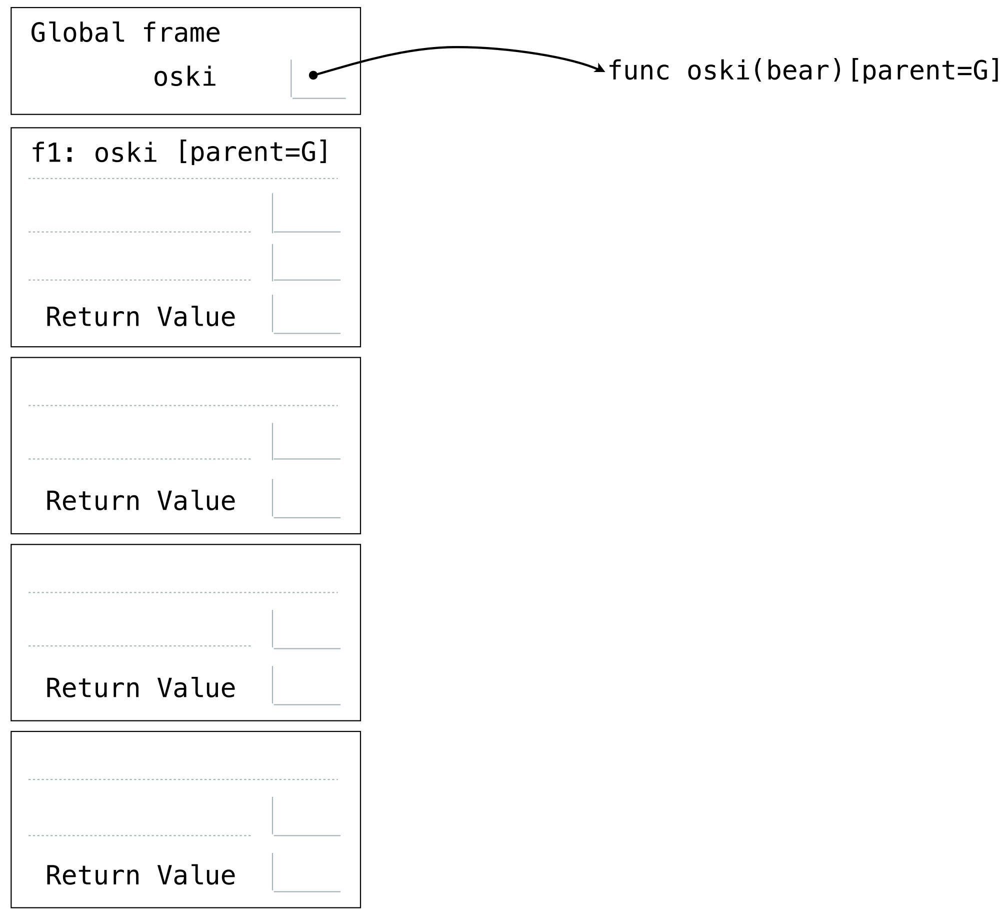

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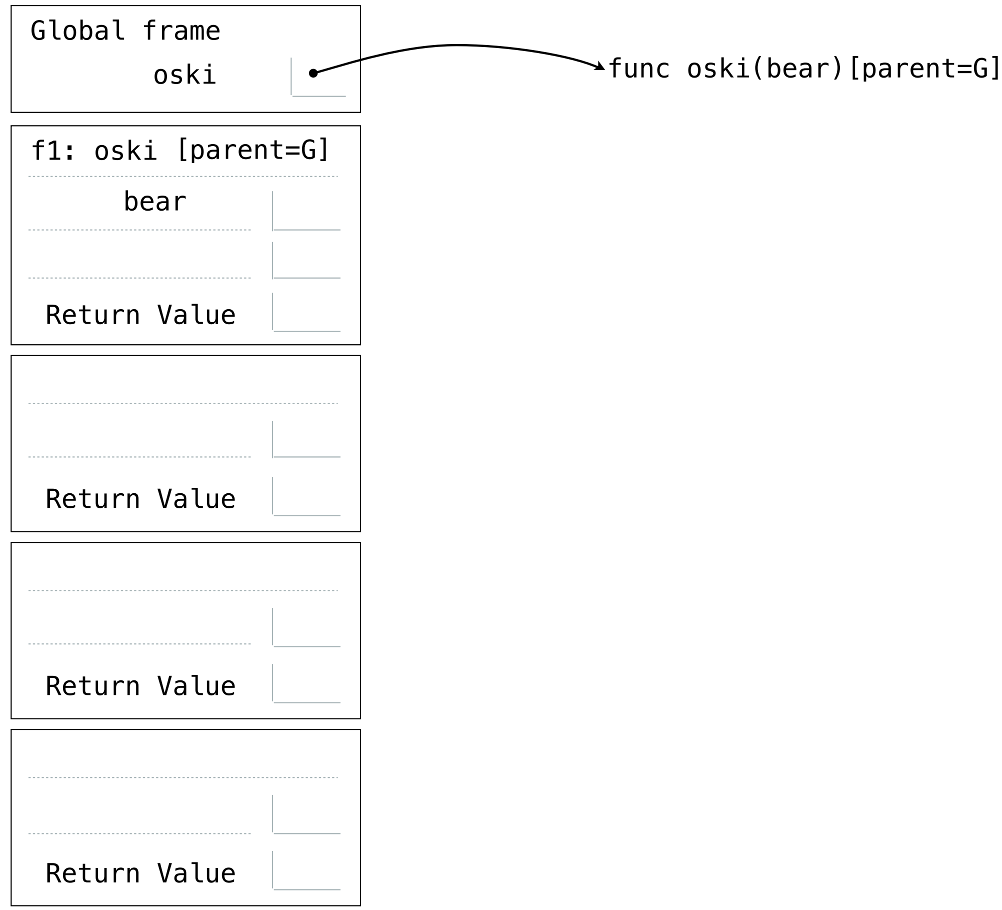

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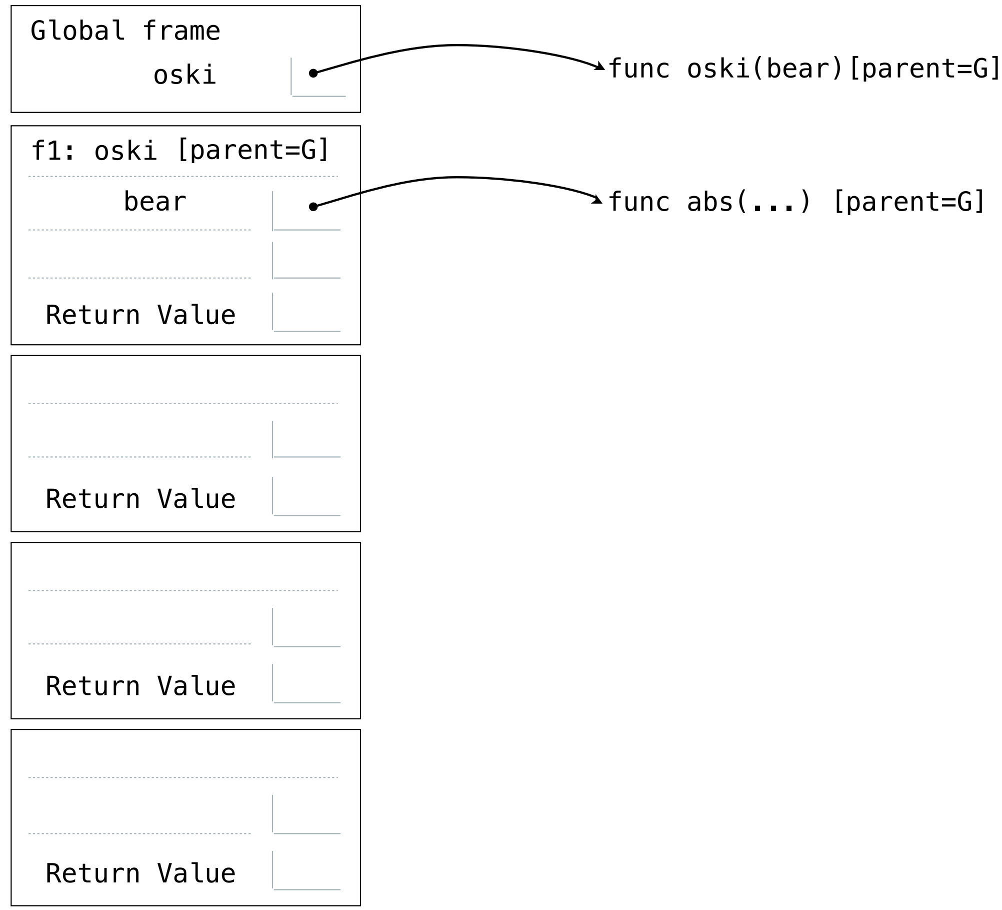

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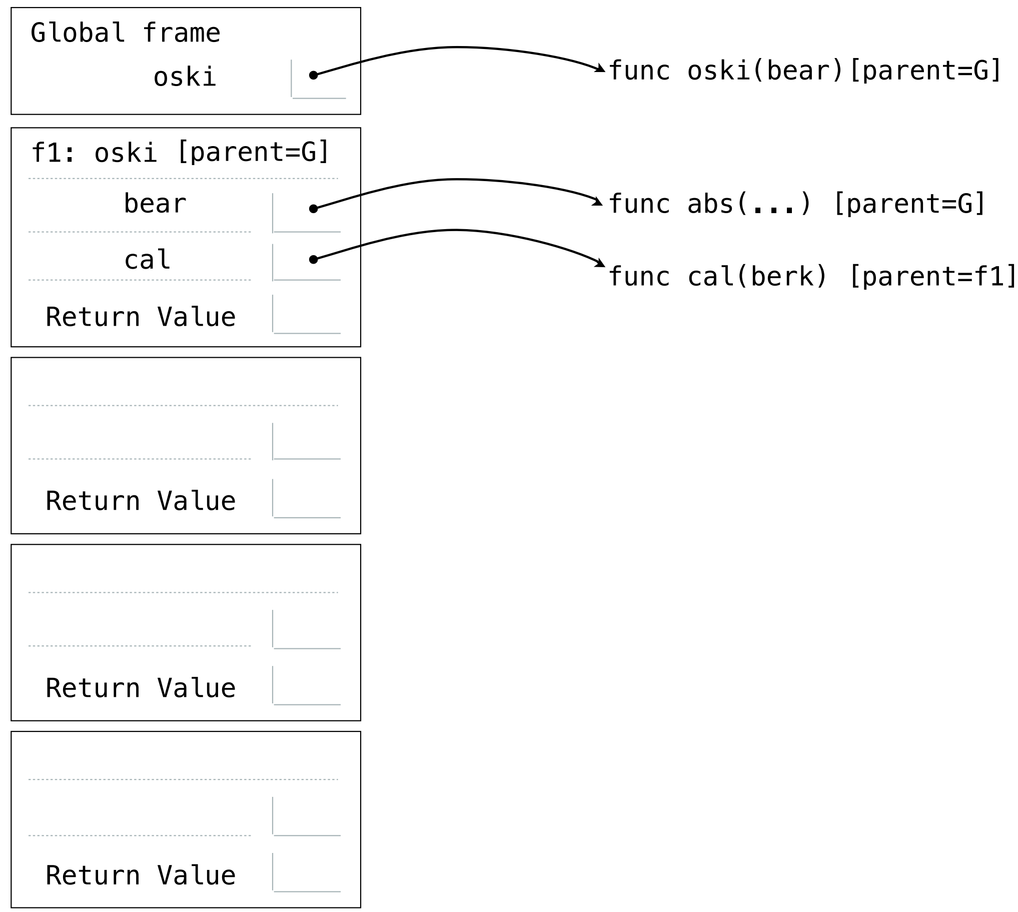

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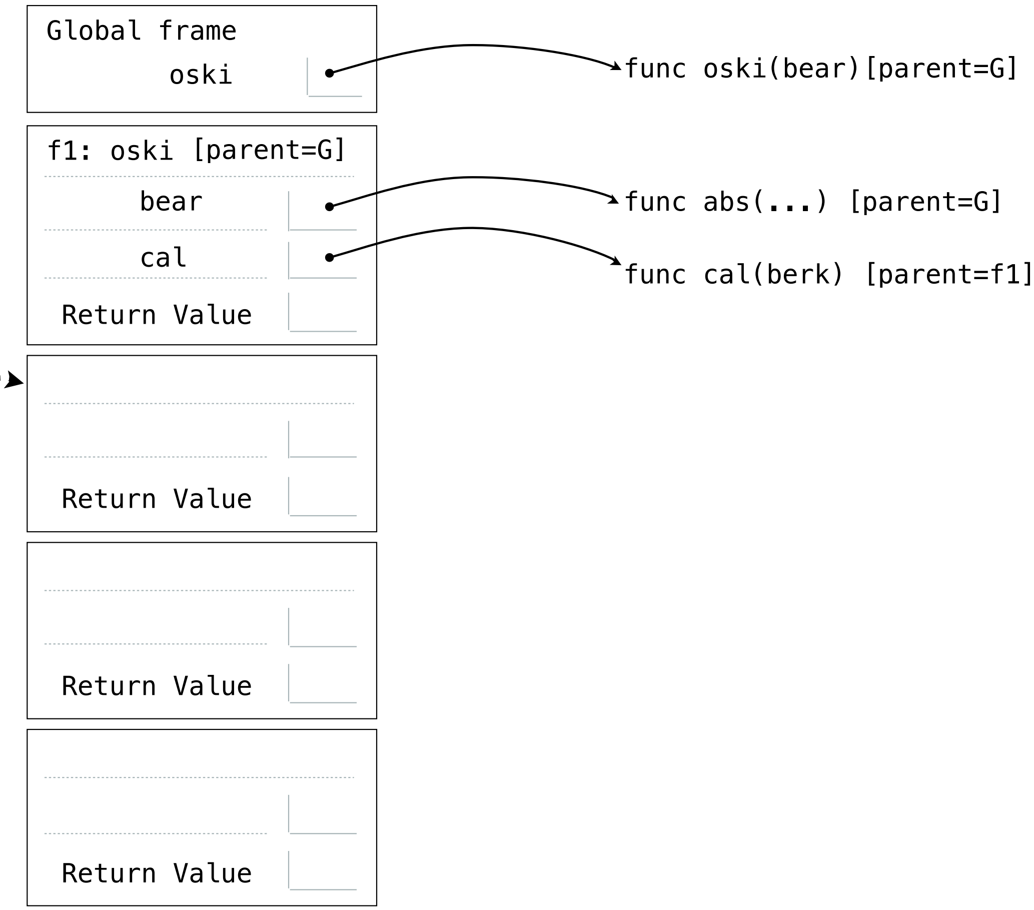

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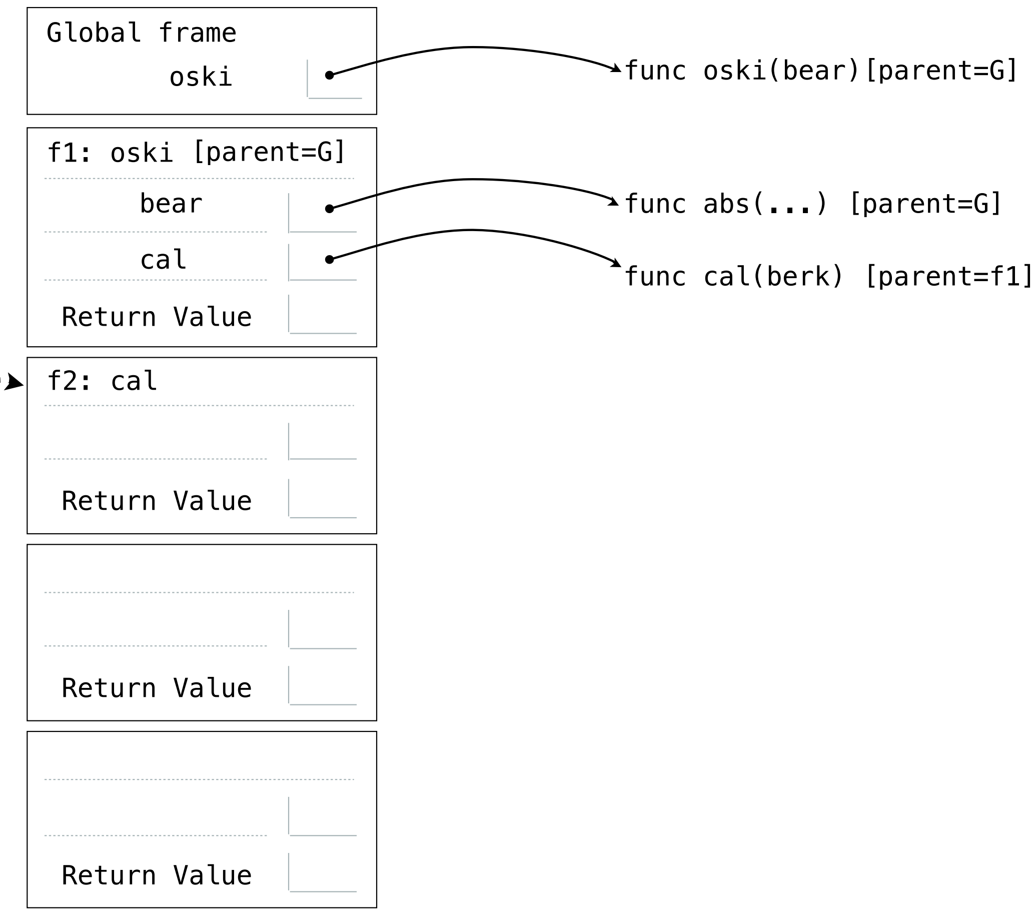

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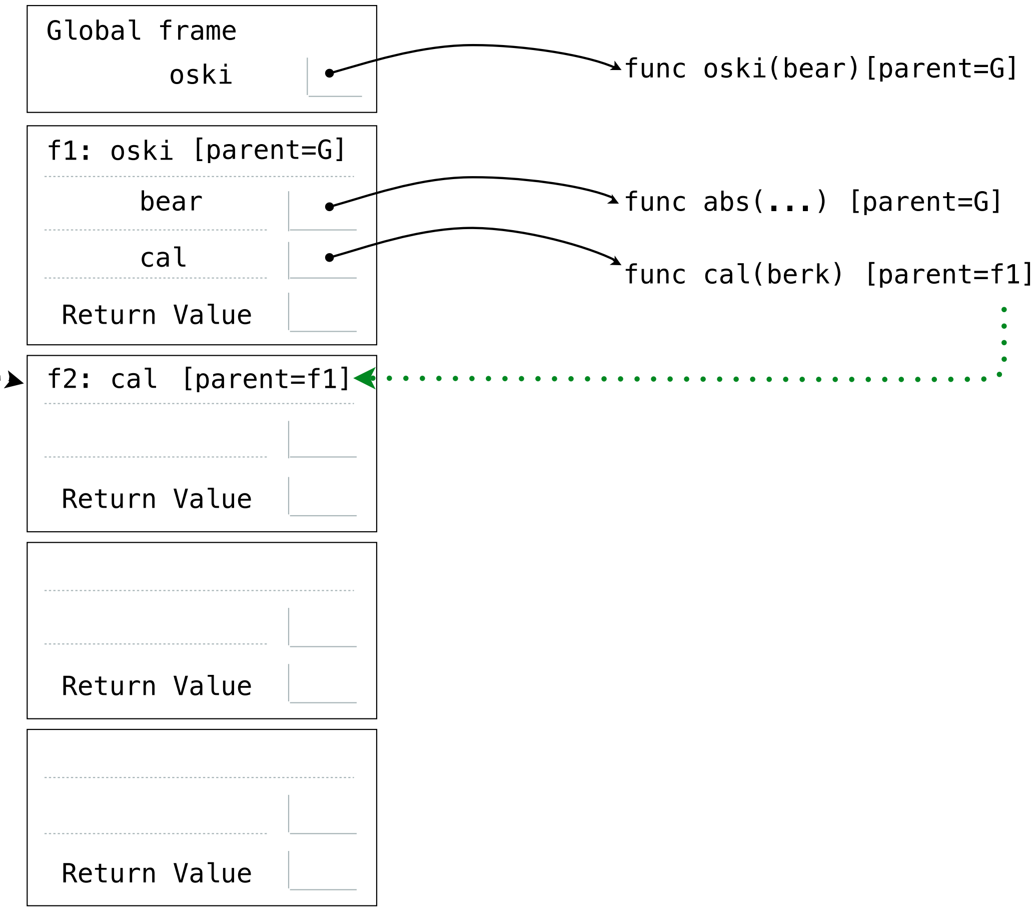

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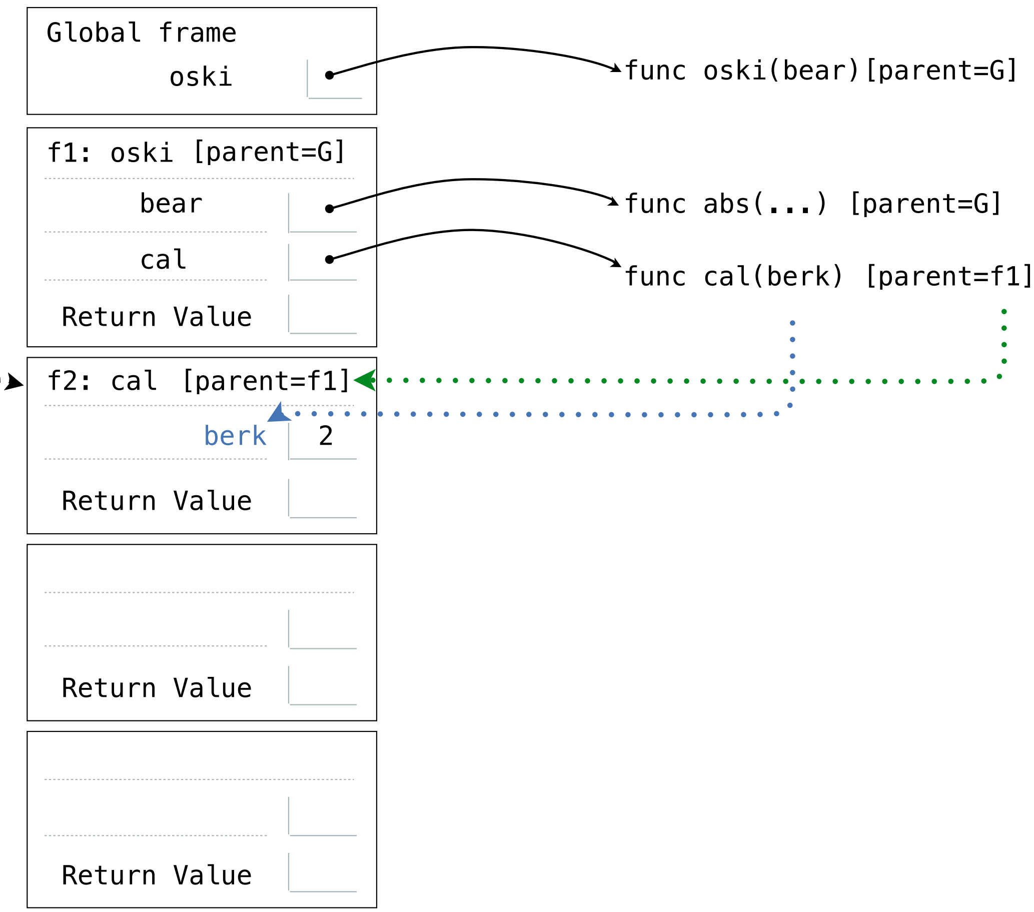

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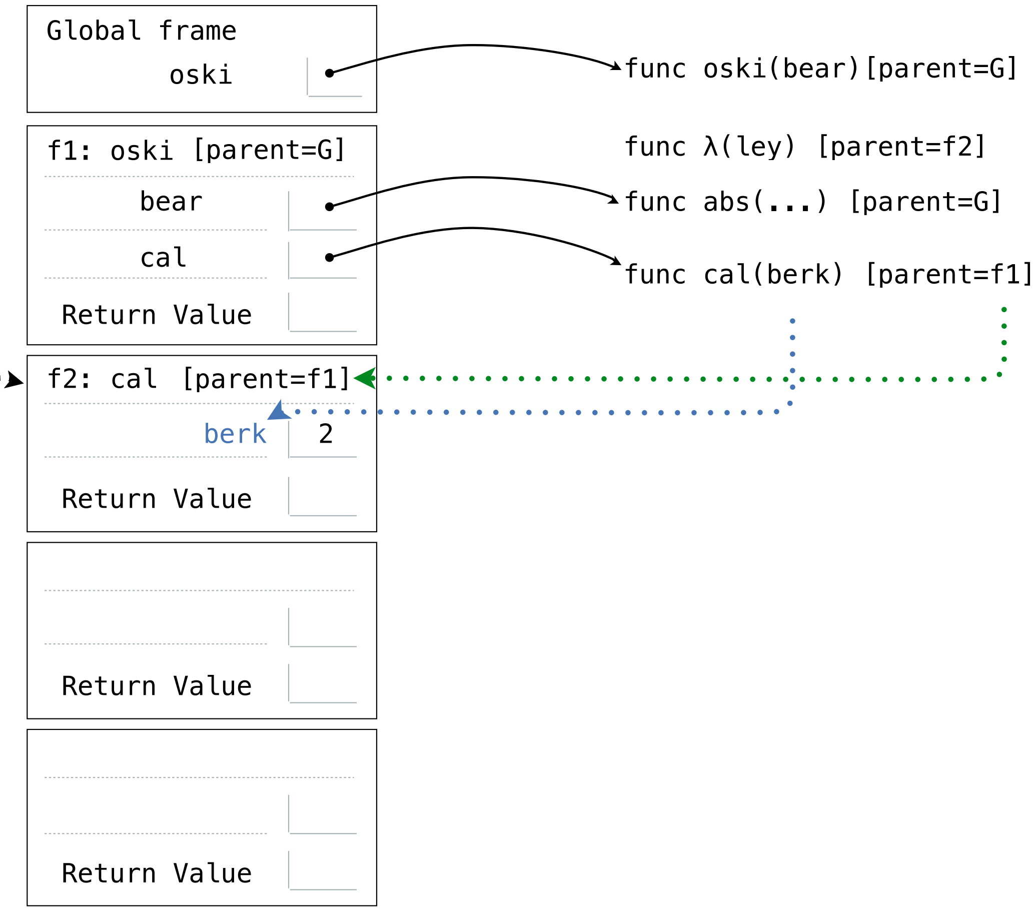

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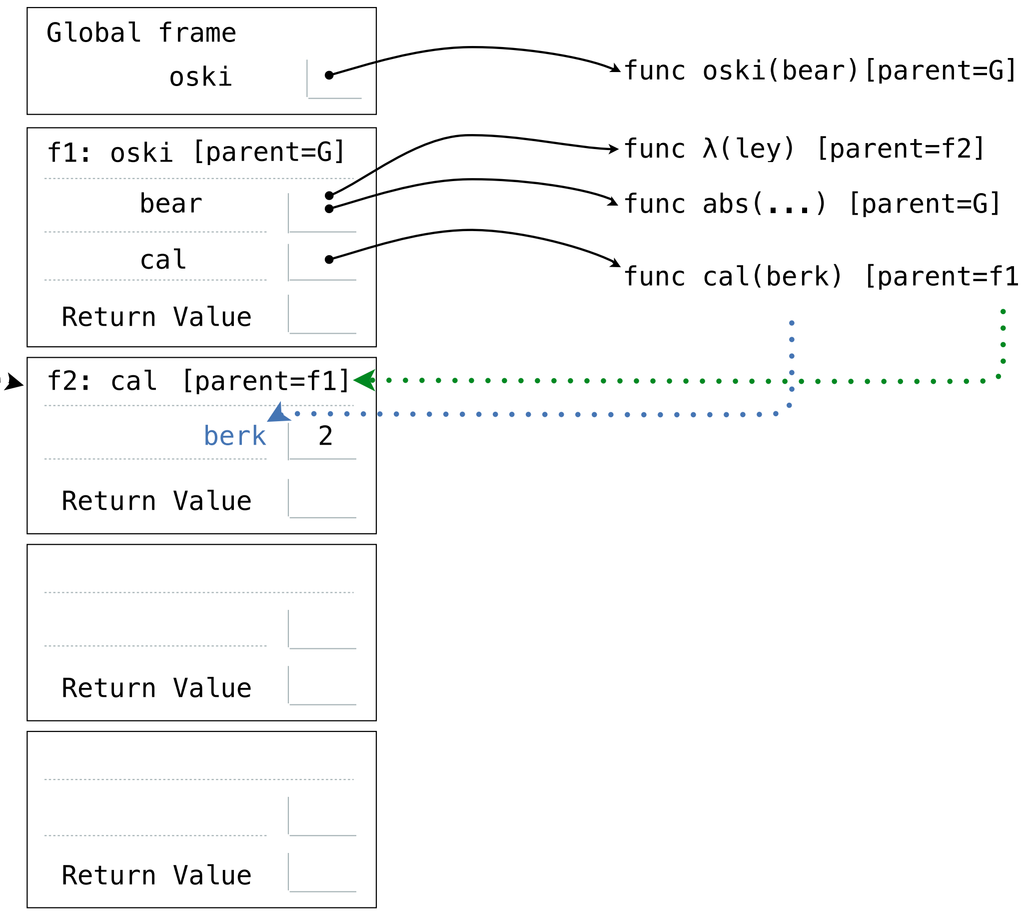

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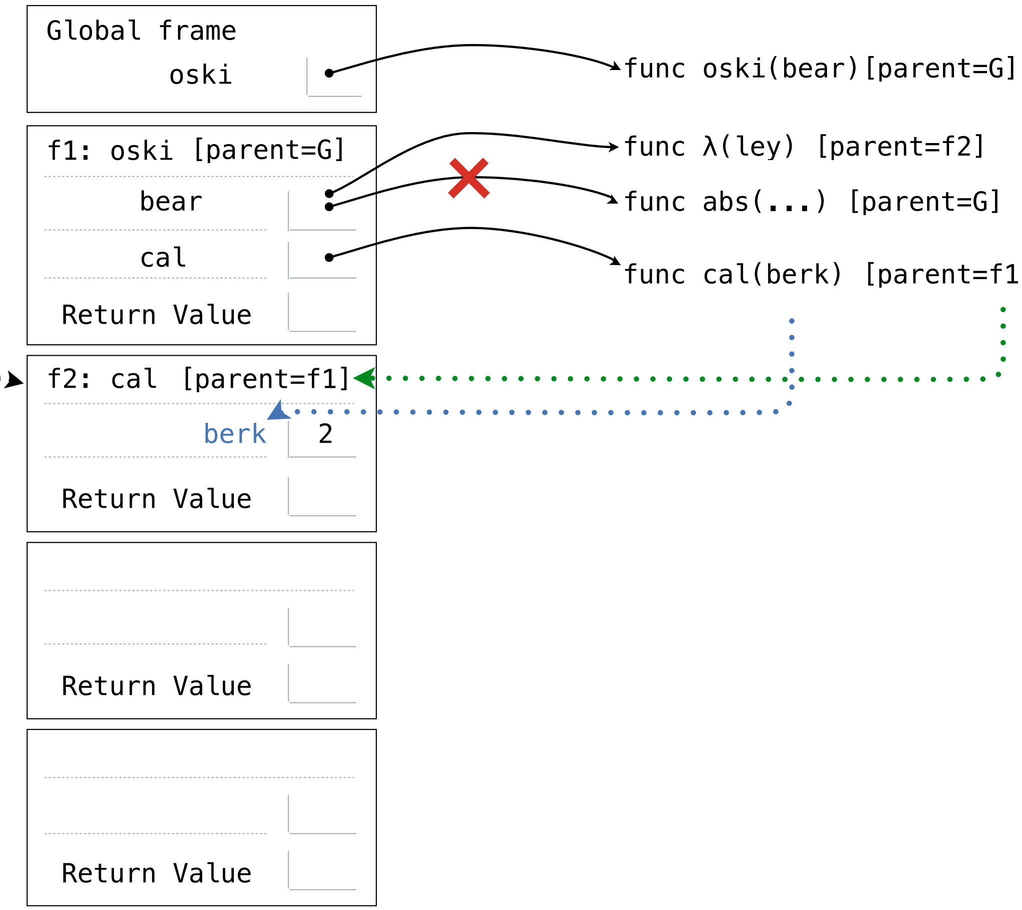

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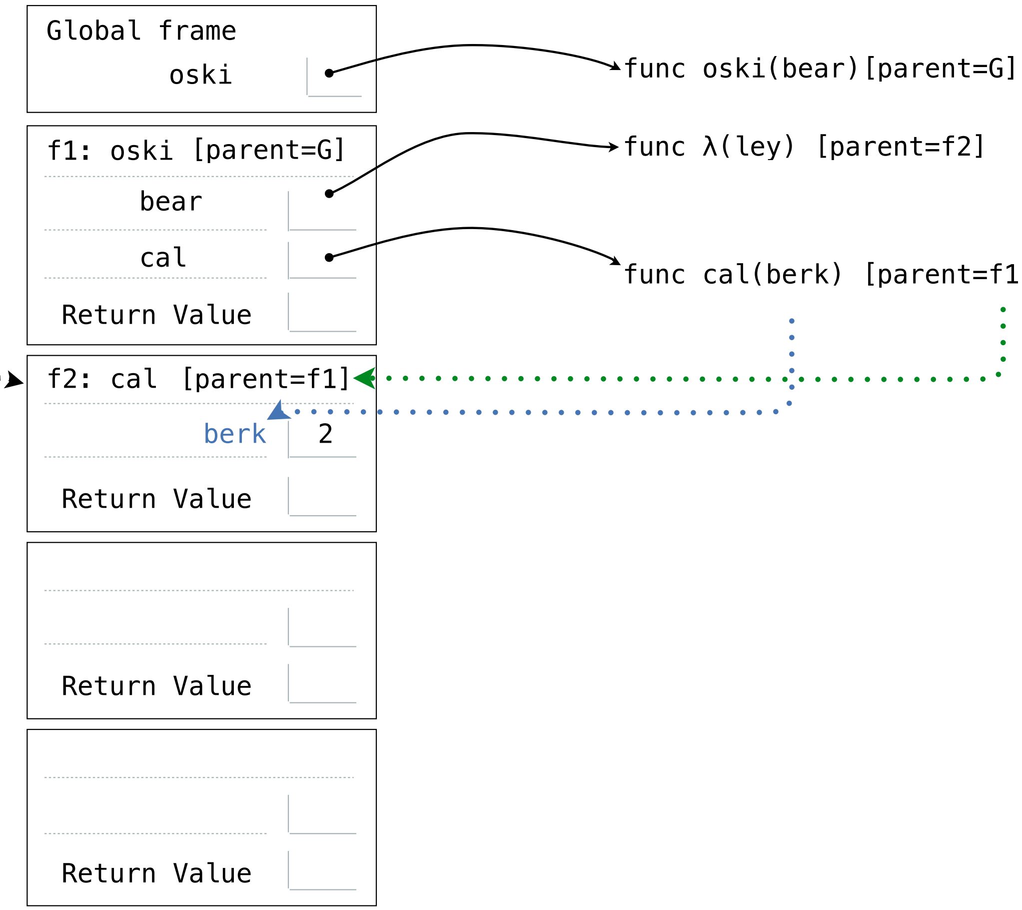

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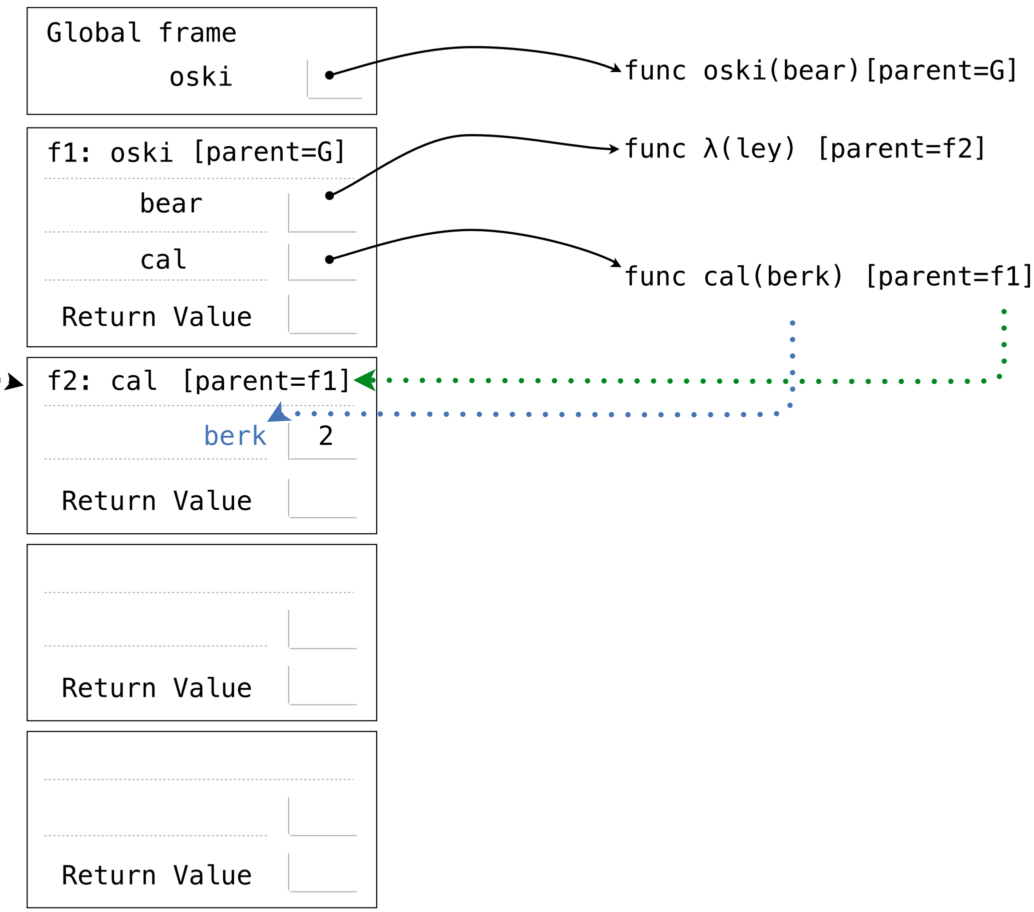

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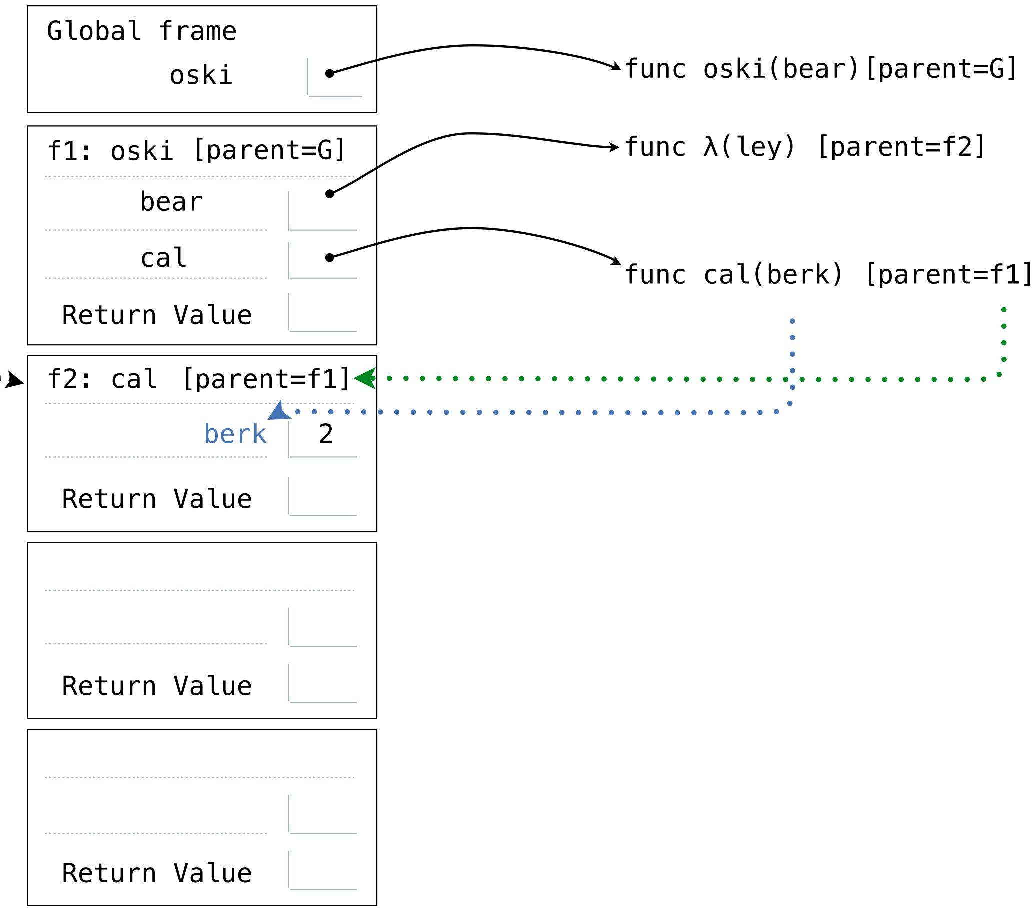

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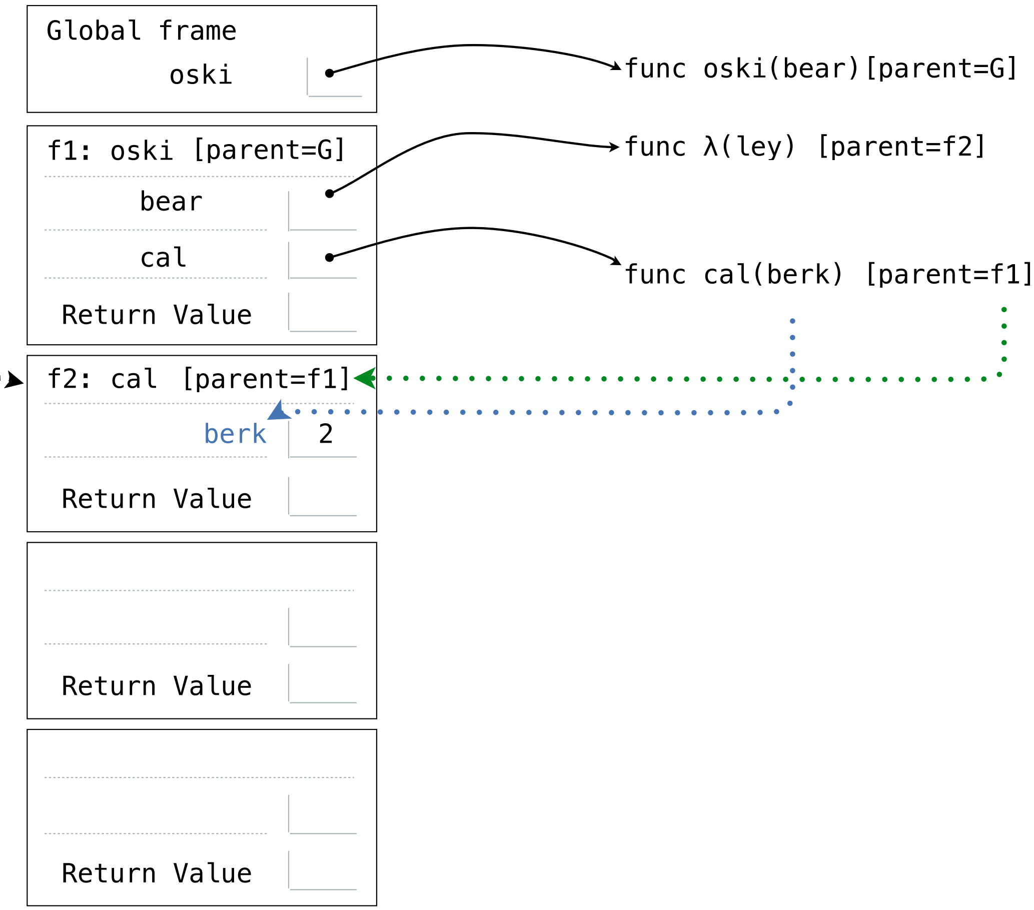

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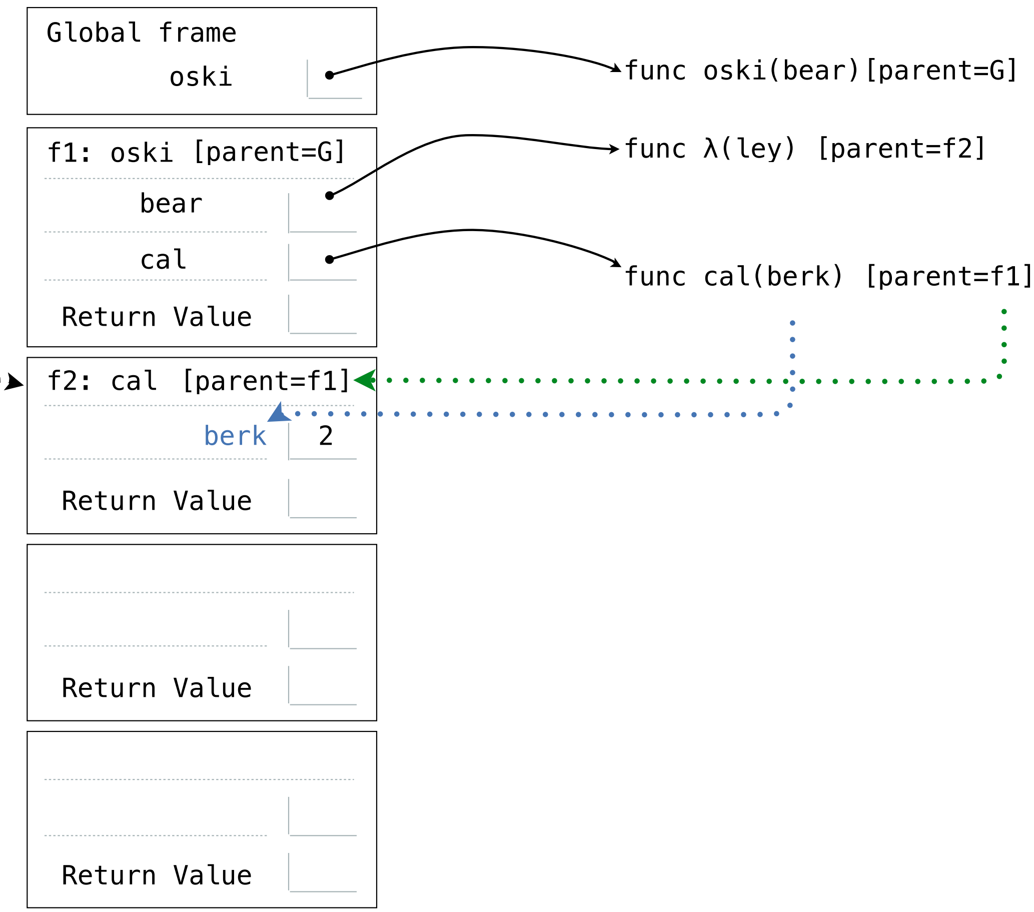

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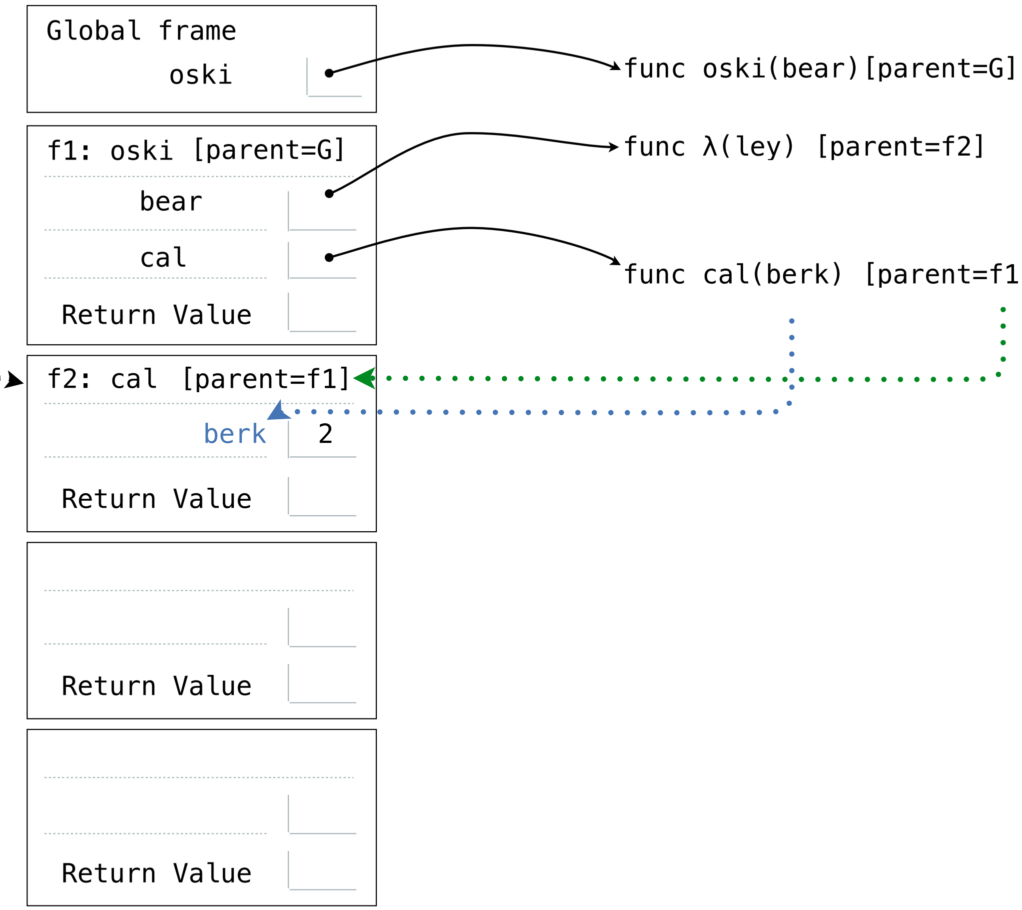

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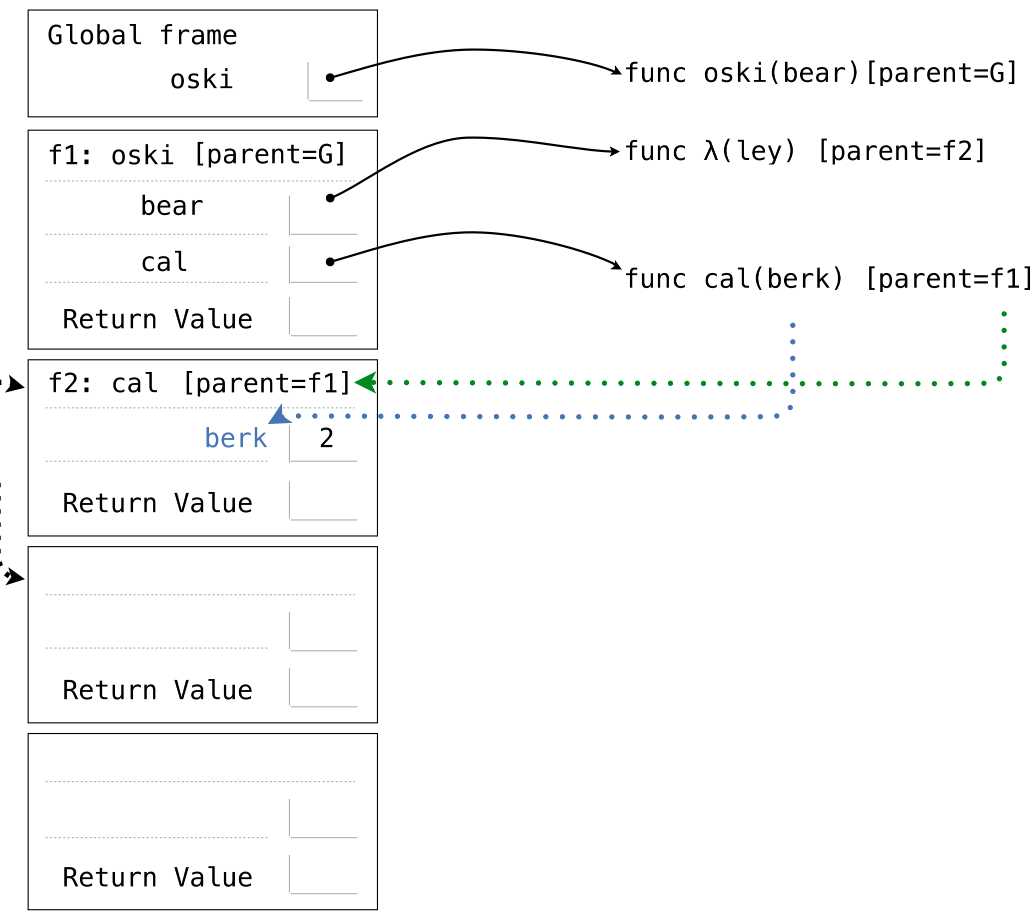

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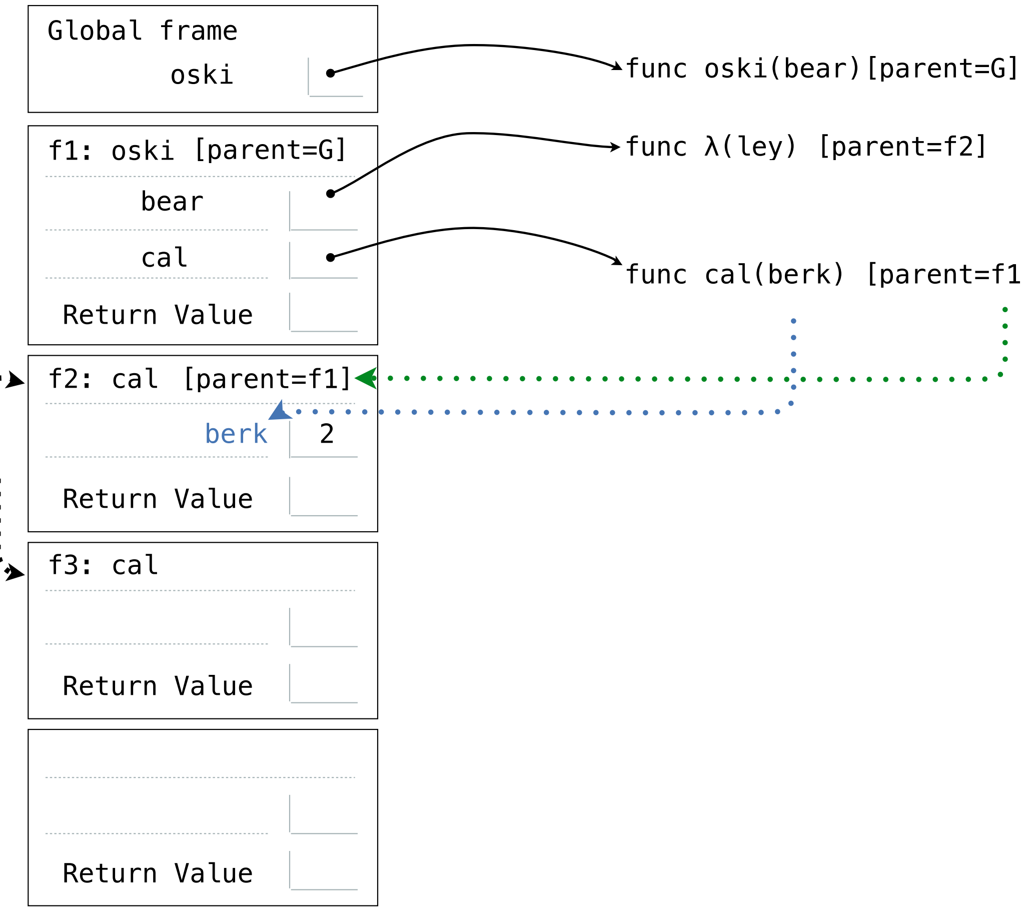

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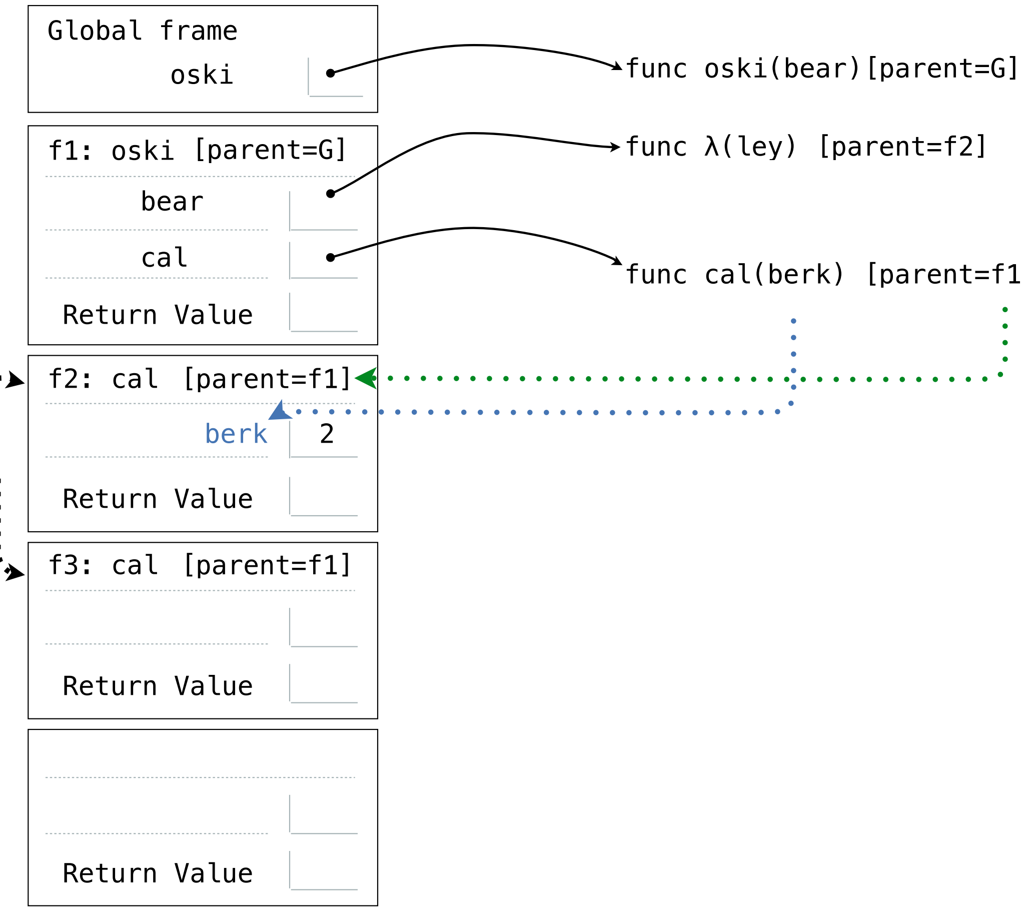

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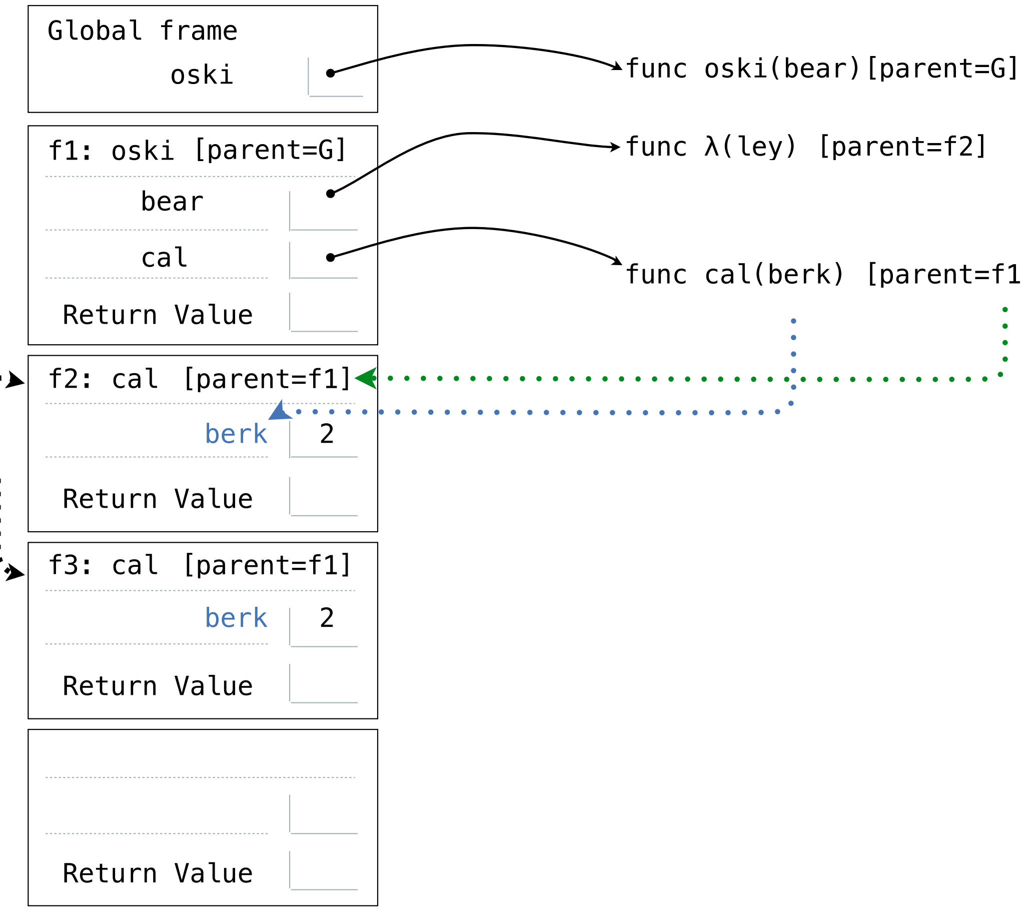

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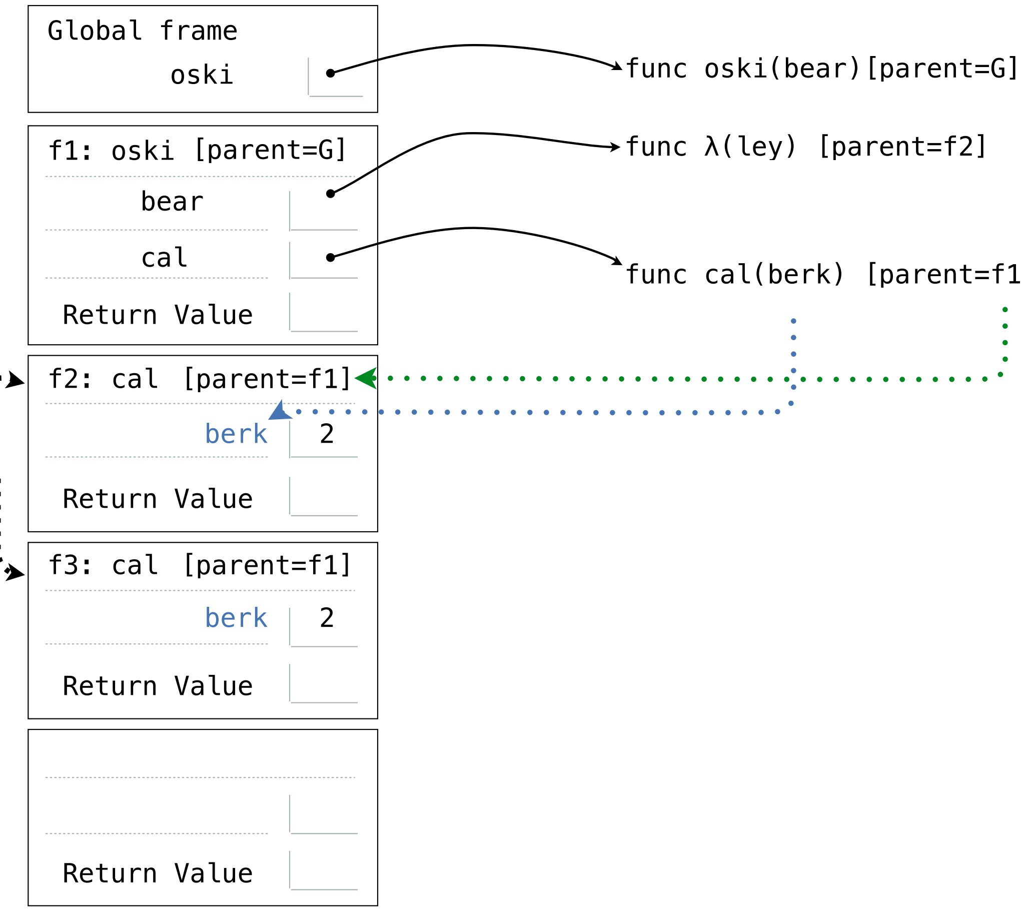

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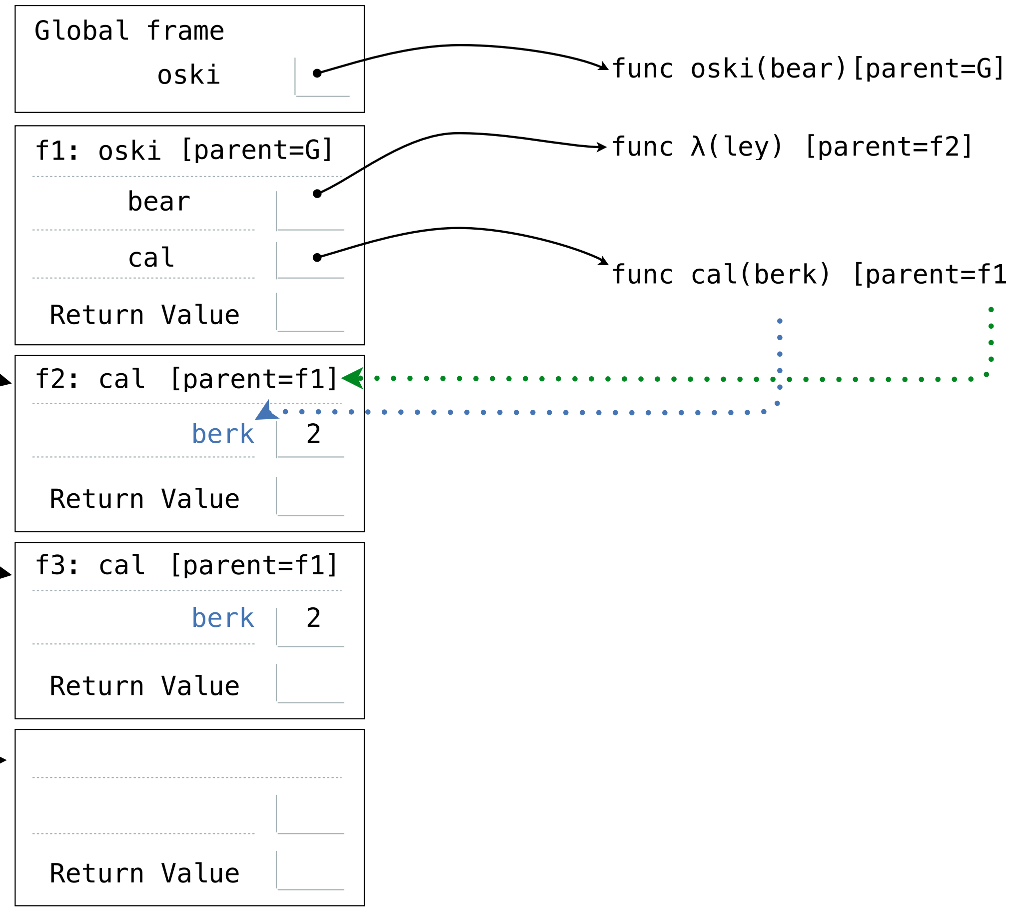

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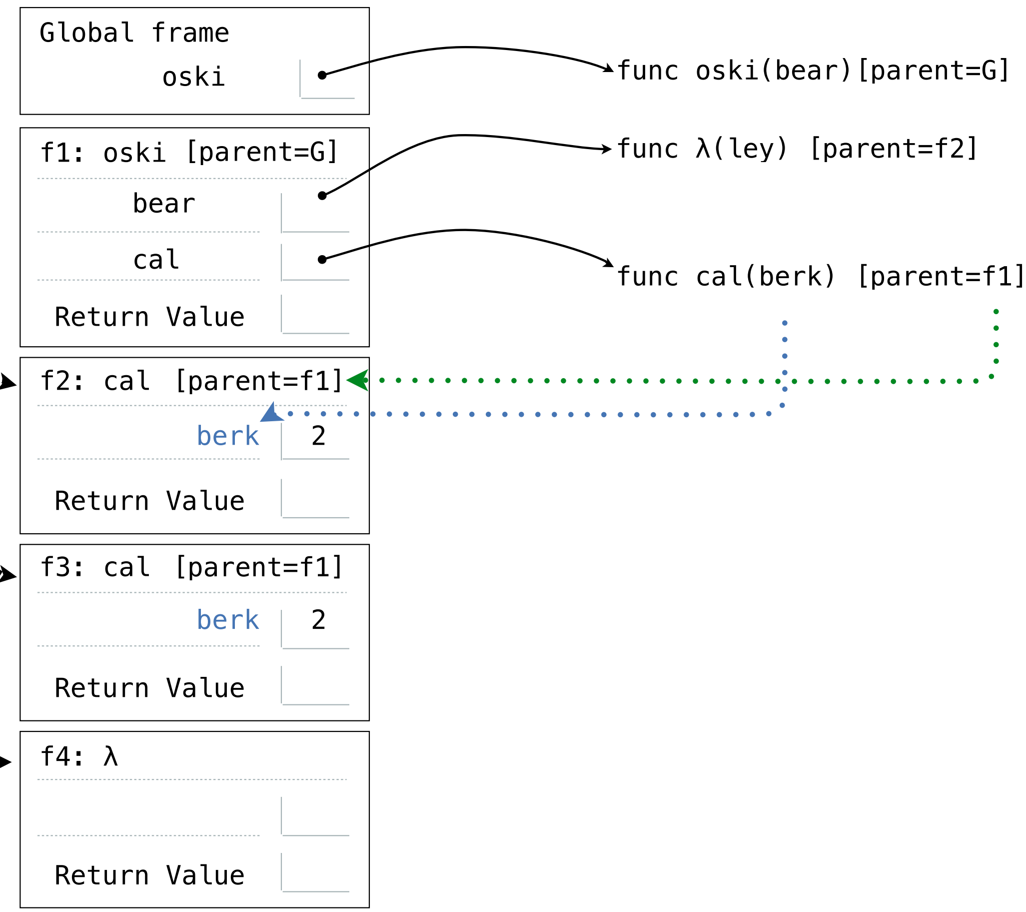

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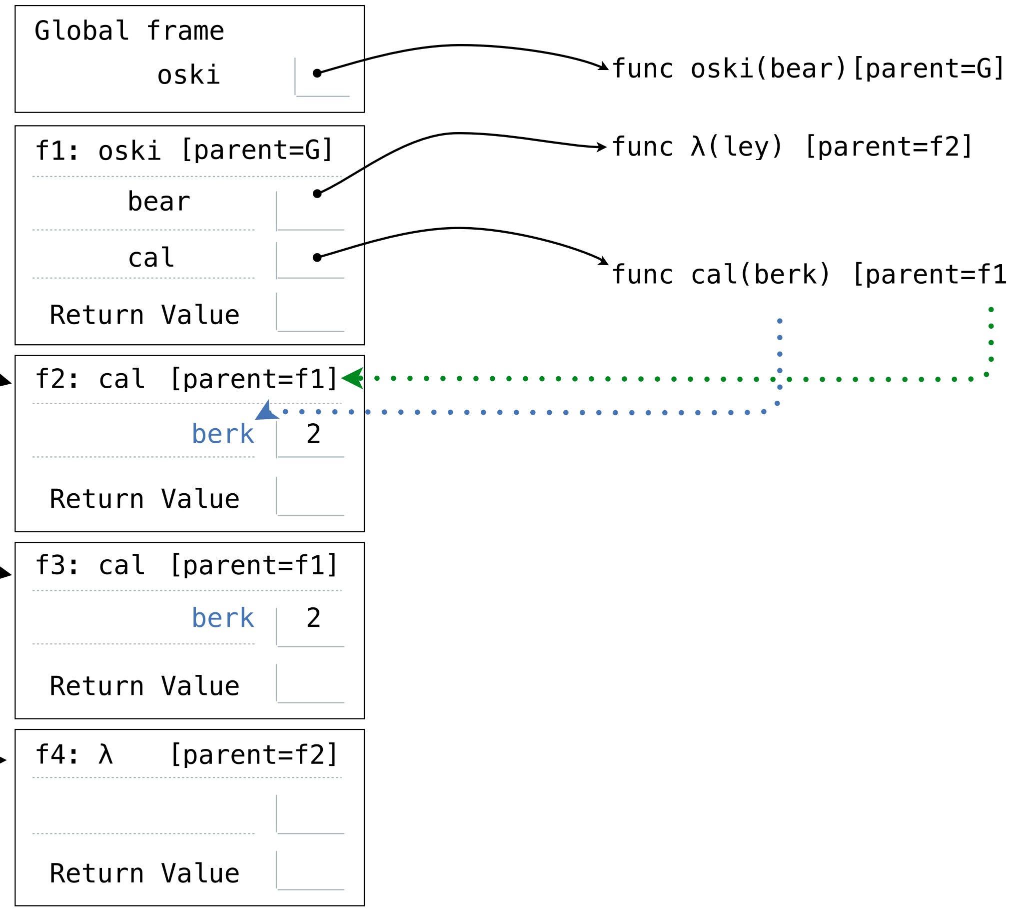

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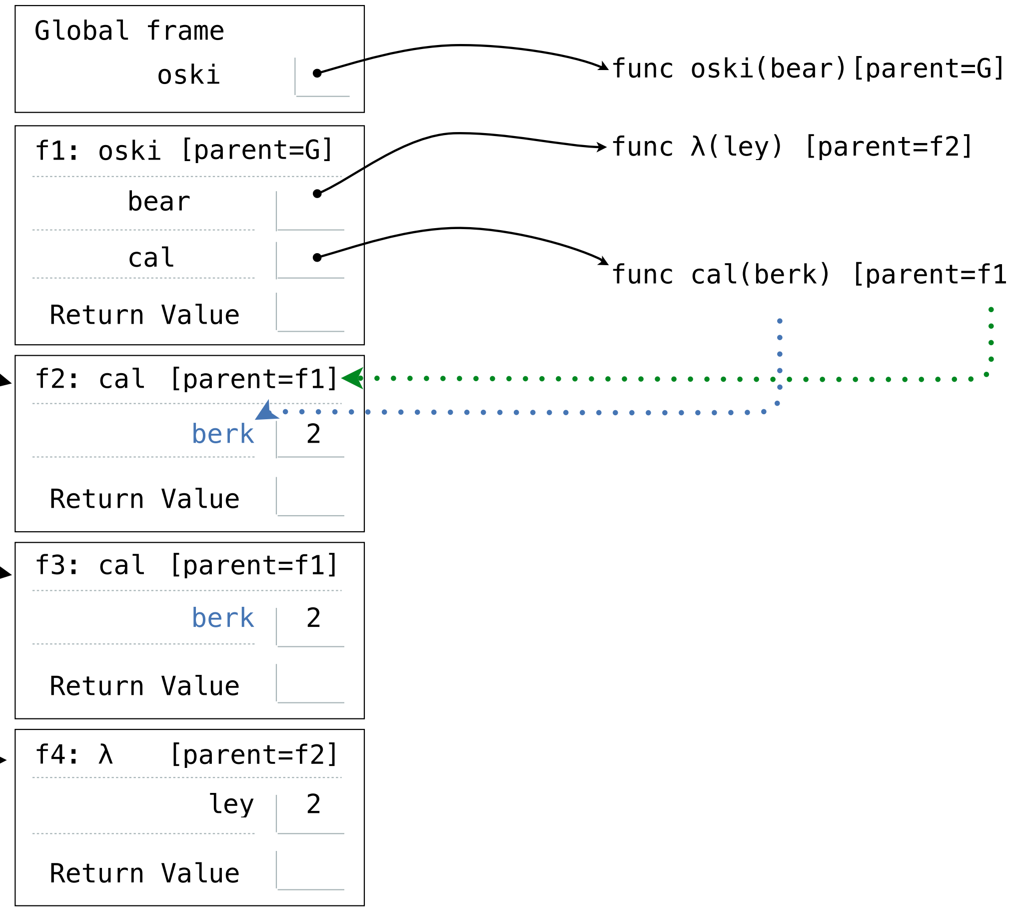

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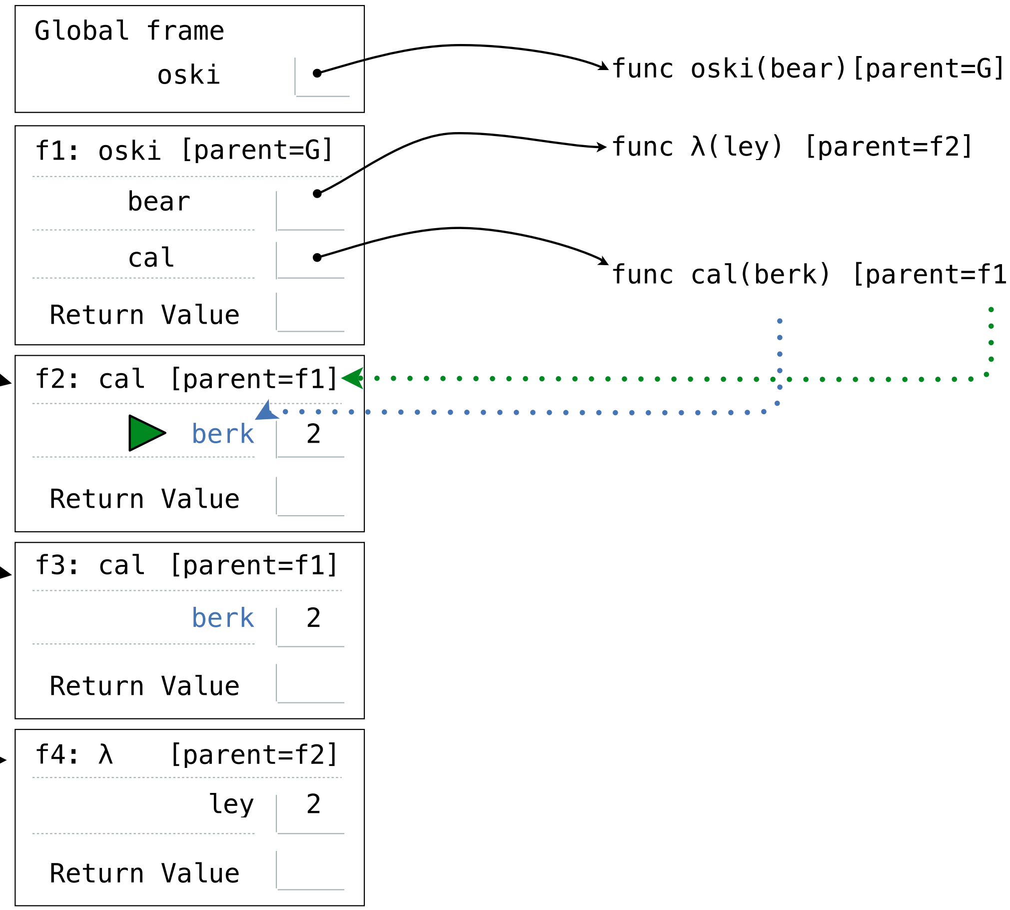

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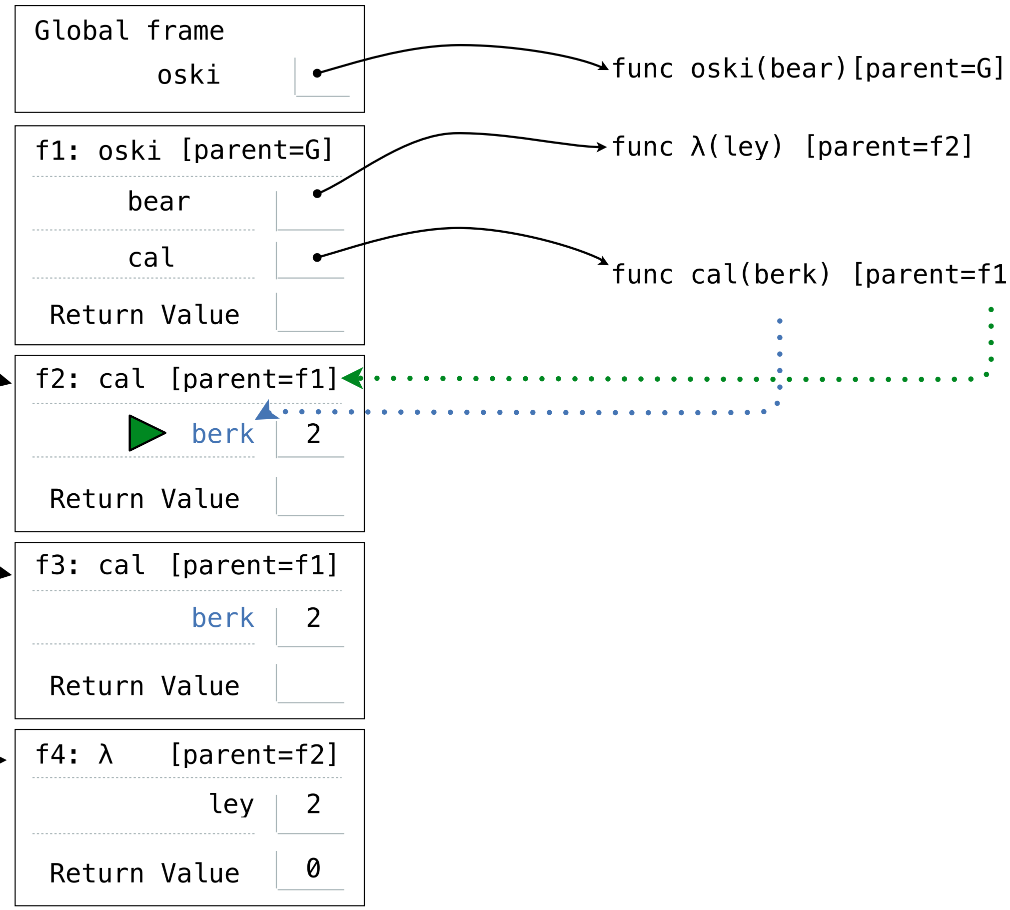

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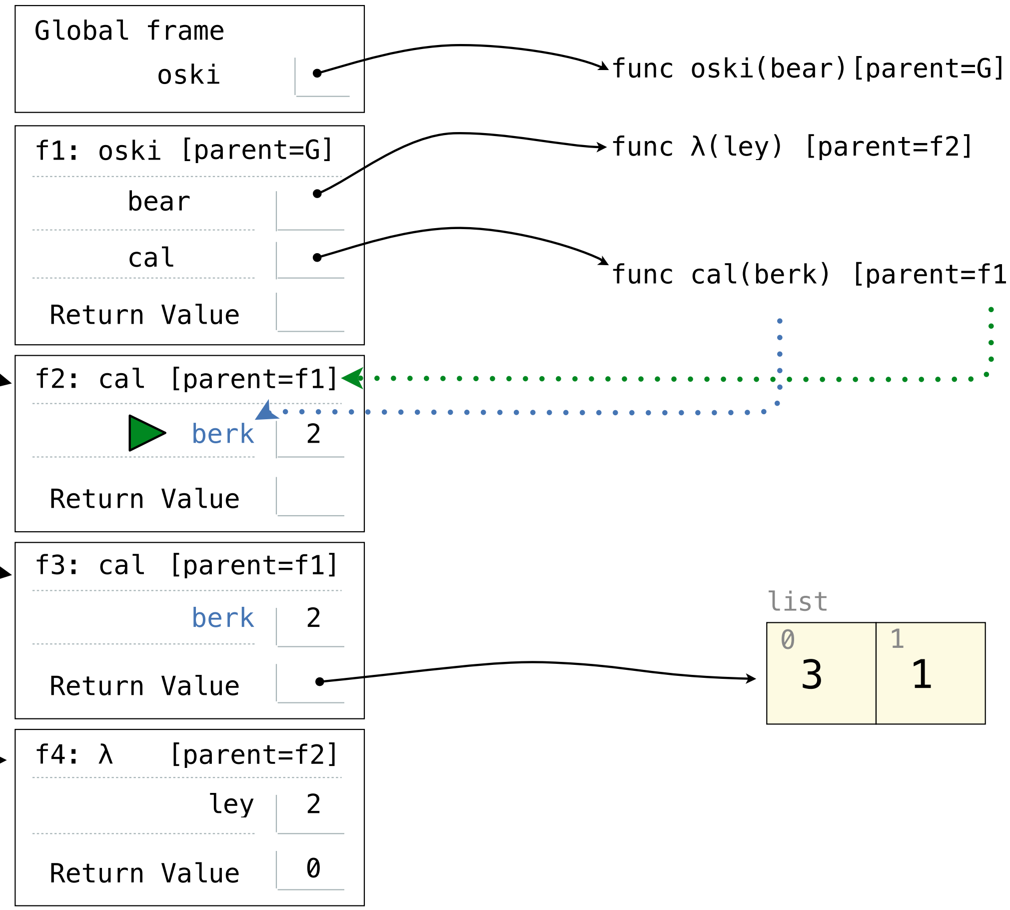

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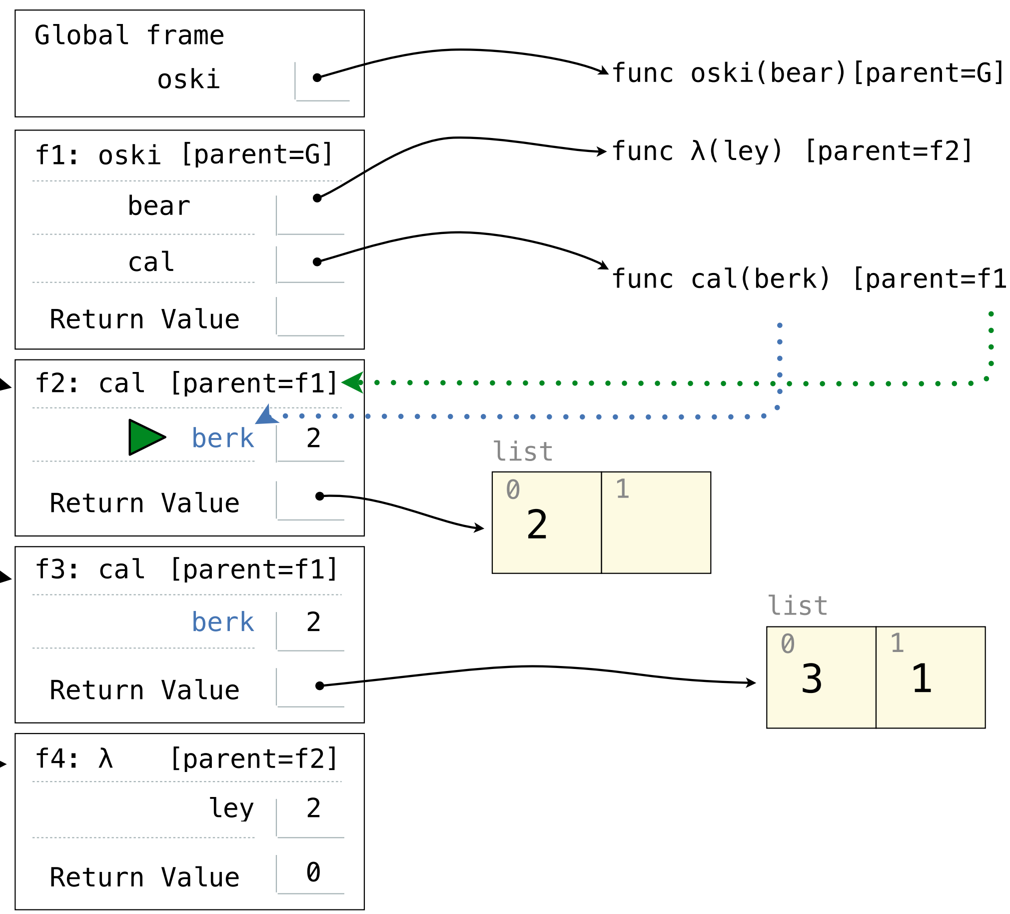

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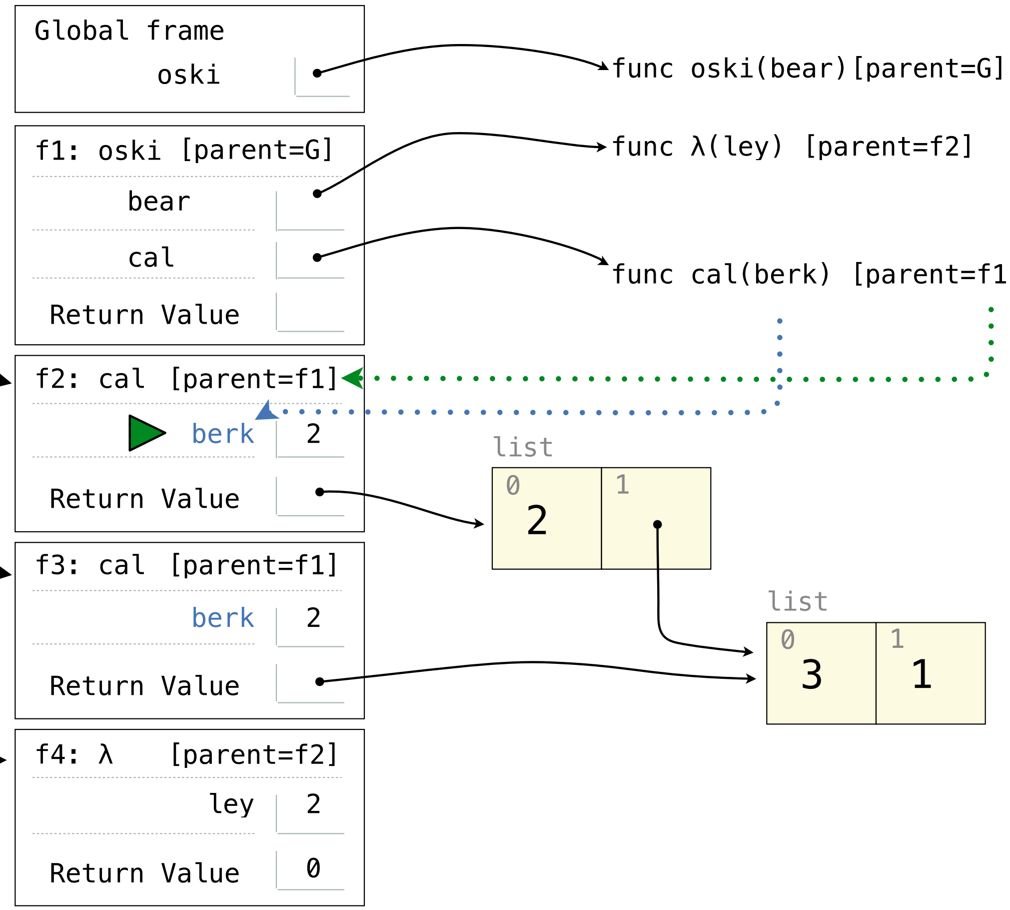

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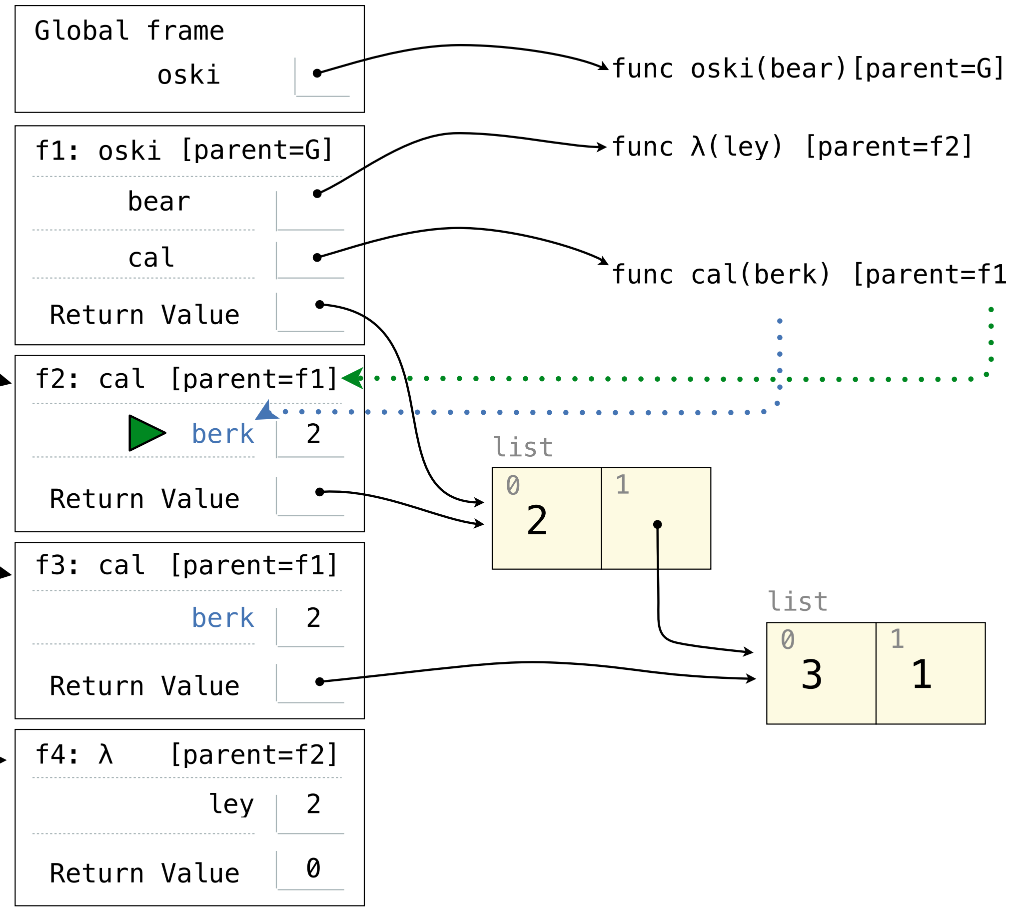

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