# 61A Lecture 28

Friday, November 7

# Announcements

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- Midterm survey due Monday 11/10 @ 11:59pm (Thanks!)

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- Homework 8 due Wednesday 11/12 @ 11:59pm (Scheme!)

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- Homework 8 due Wednesday 11/12 @ 11:59pm (Scheme!)
- Project 4 due Thursday 11/20 @ 11:59pm (Big!)

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Special form to create dynamically scoped procedures (**mu** special form only exists in Project 4)

*mu*

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# Tail Recursion

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But... no for/while statements!  Can we make basic iteration efficient?  Yes!

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(Demo)

Interactive Diagram

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Recursive call is a tail call

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# Tail Recursion Examples

## Which Procedures are Tail Recursive?

Which of the following procedures run in constant space? $\Theta(1)$

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;; Compute the length of s.
(define (length s)
  (+ 1 (if (null? s)
           -1
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```scheme
;; Return the nth Fibonacci number.
(define (fib n)
  (define (fib-iter current k)
    (if (= k n)
        current
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                  (+ k 1))          ) )
  (if (= 1 n) 0 (fib-iter 1 2)))
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```scheme
;; Return whether s contains v.
(define (contains s v)
  (if (null? s)
      false
      (if (= v (car s))
          true
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```scheme
;; Return whether s has any repeated elements.
(define (has-repeat s)
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# Map and Reduce

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```
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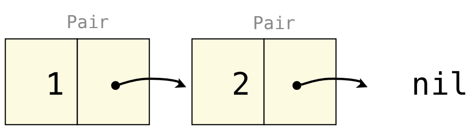

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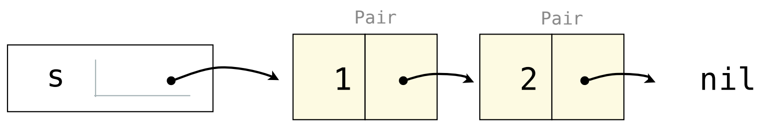

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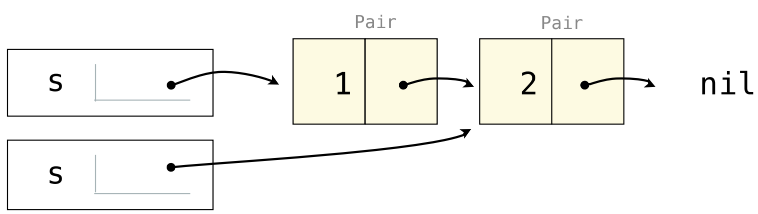

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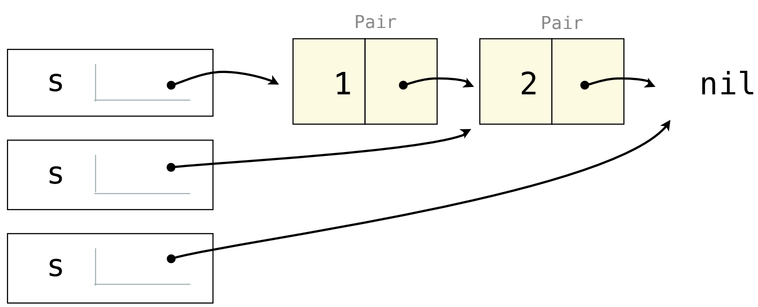

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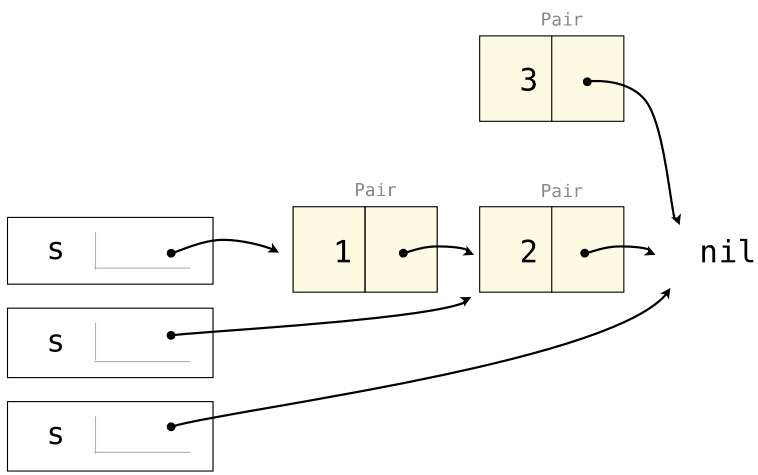

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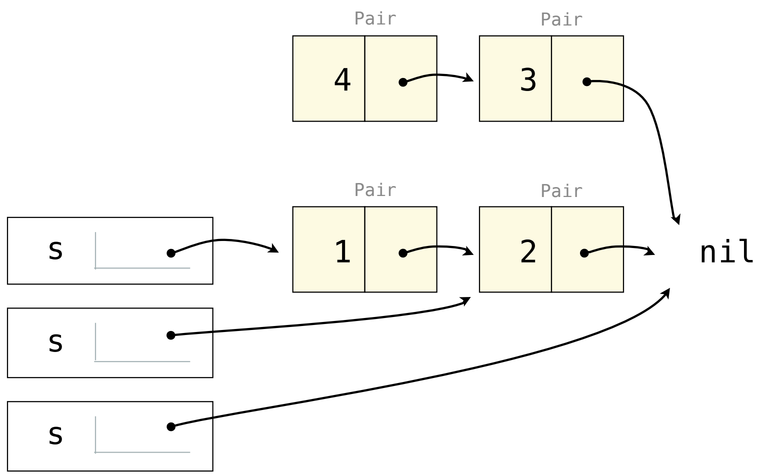

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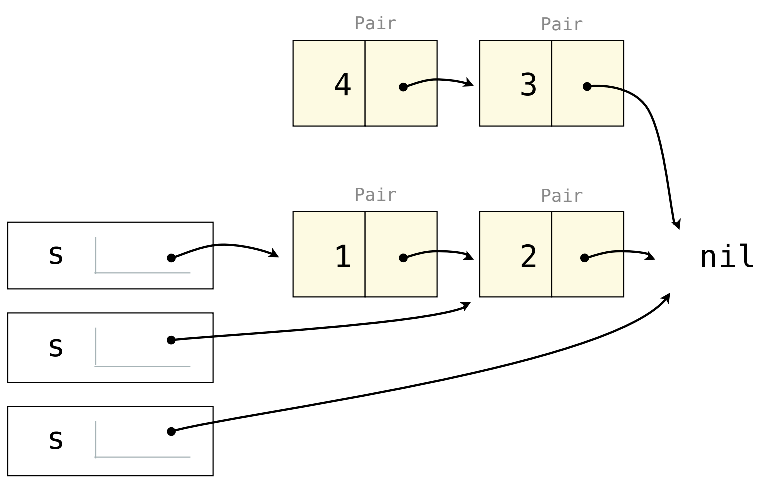

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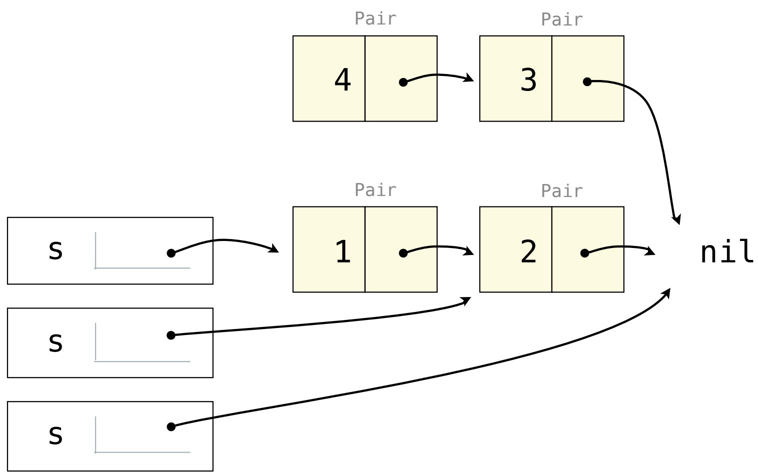

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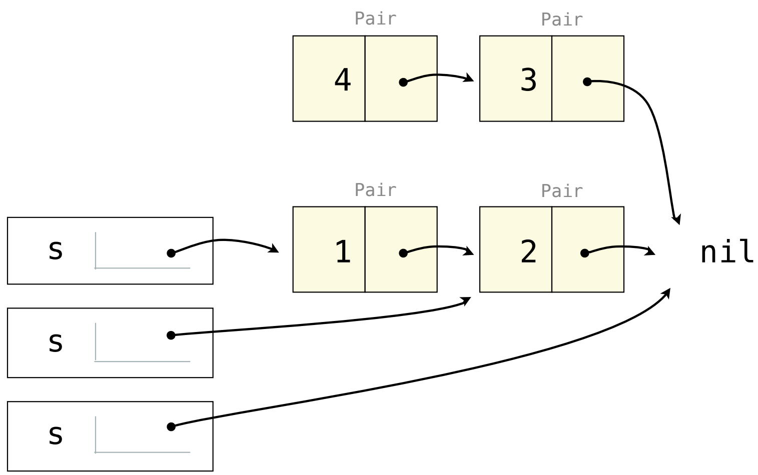


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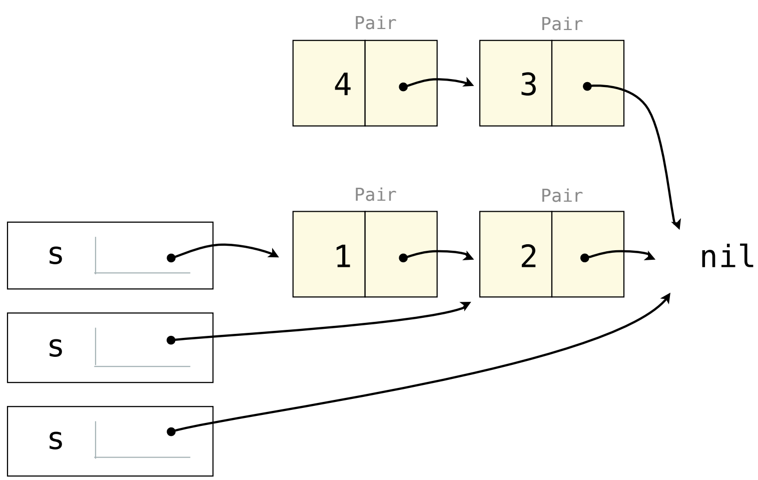

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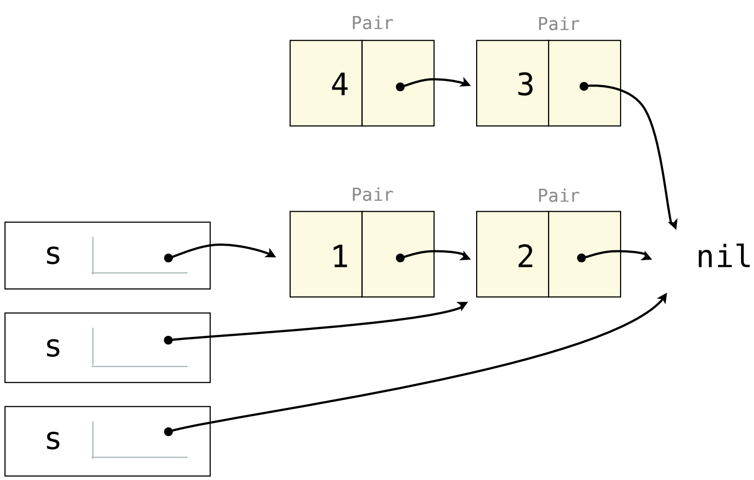

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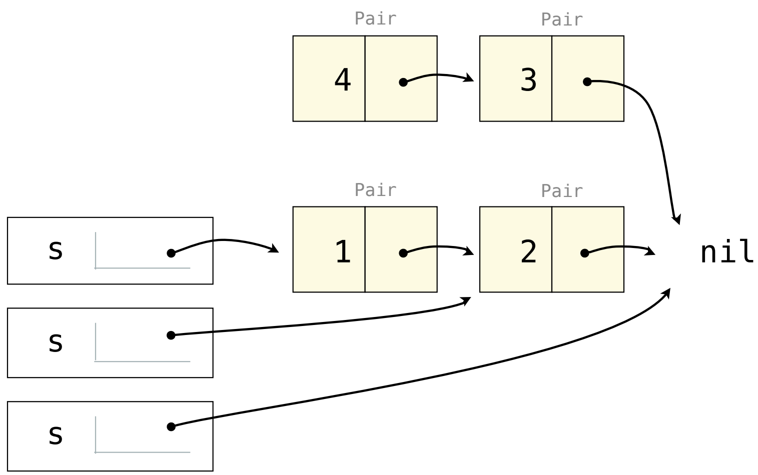

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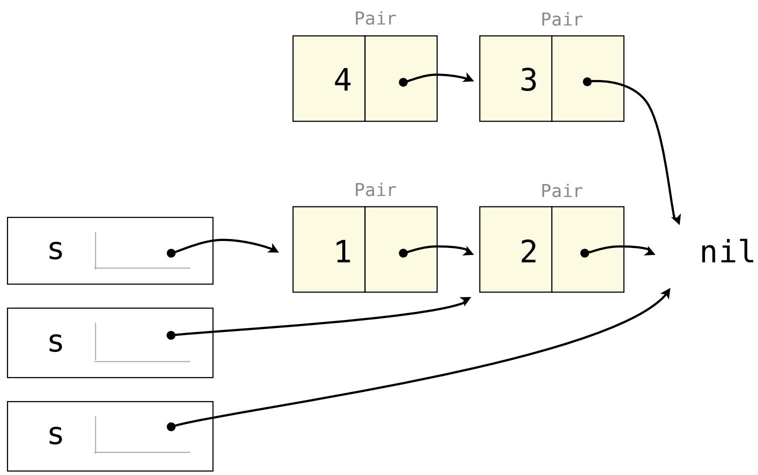

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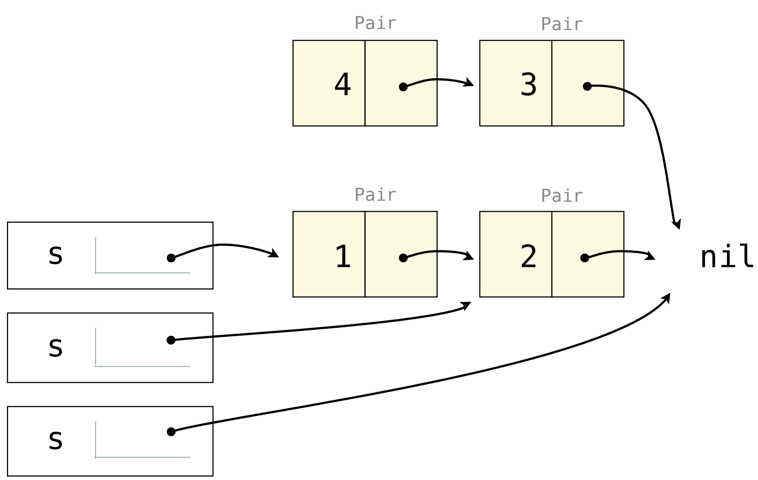


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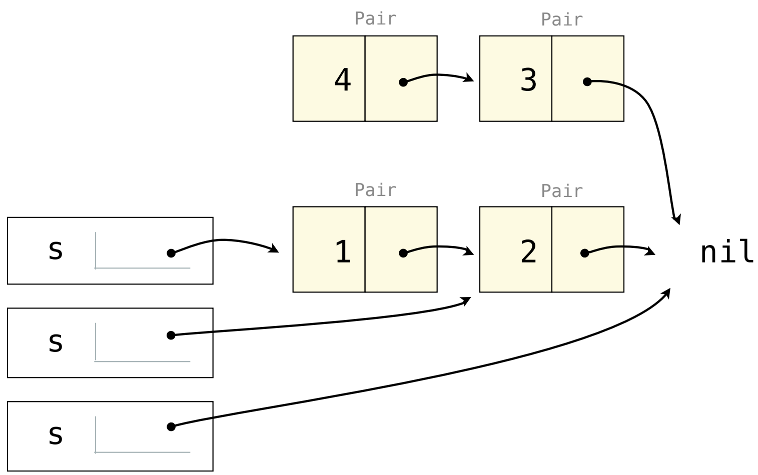


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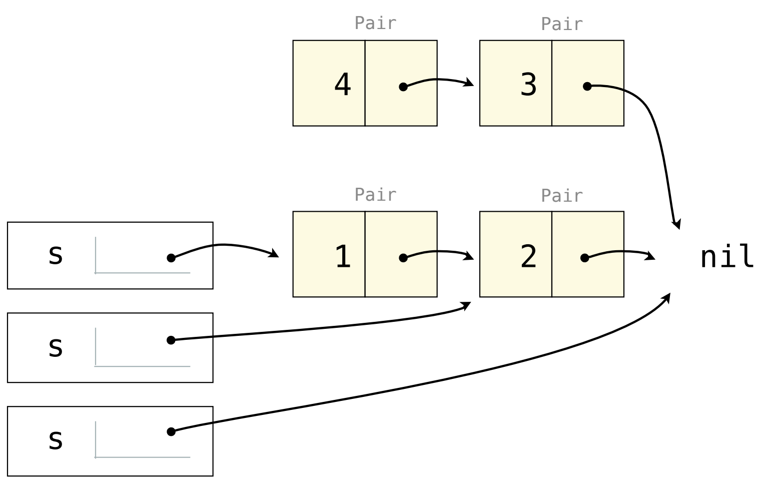


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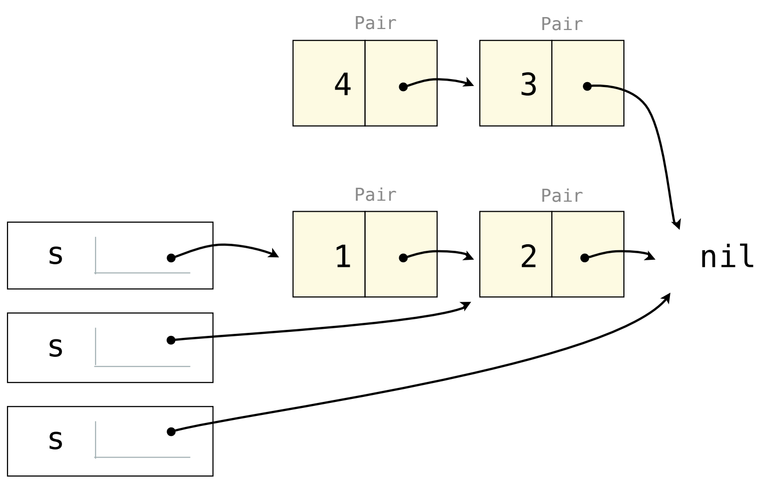


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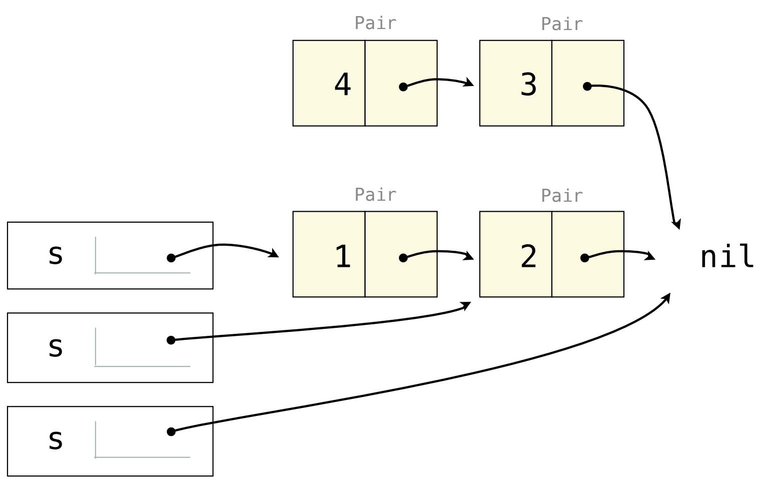


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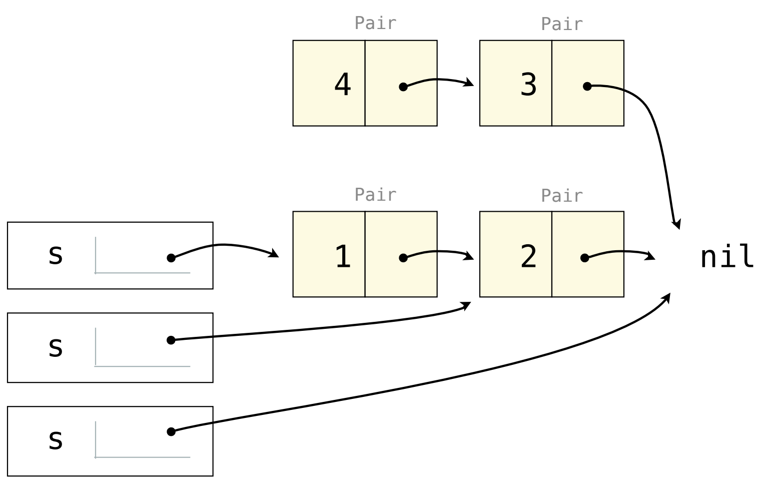


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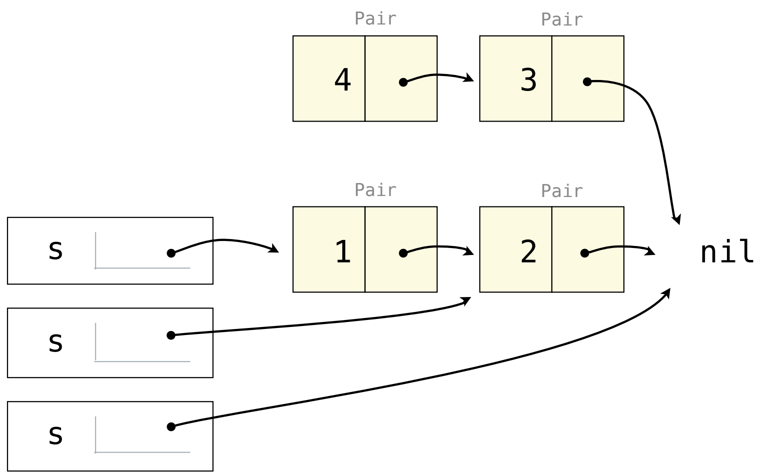


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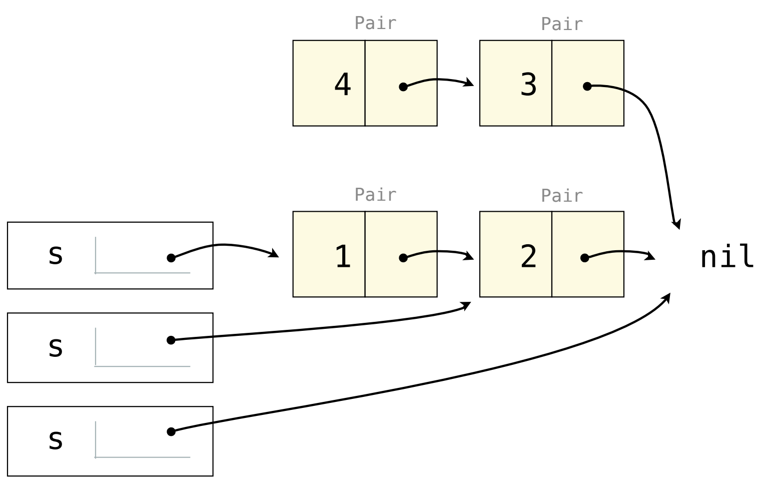


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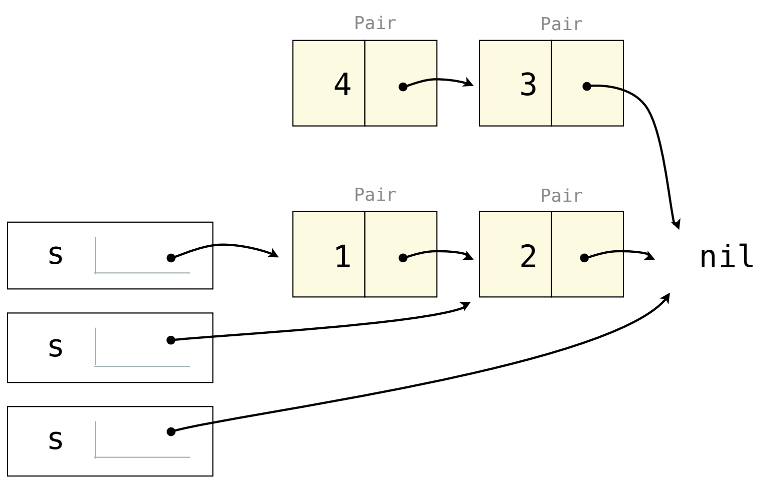


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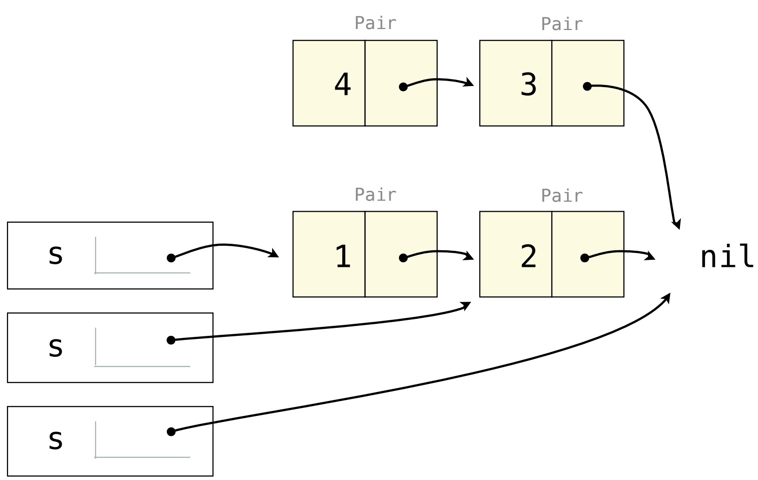


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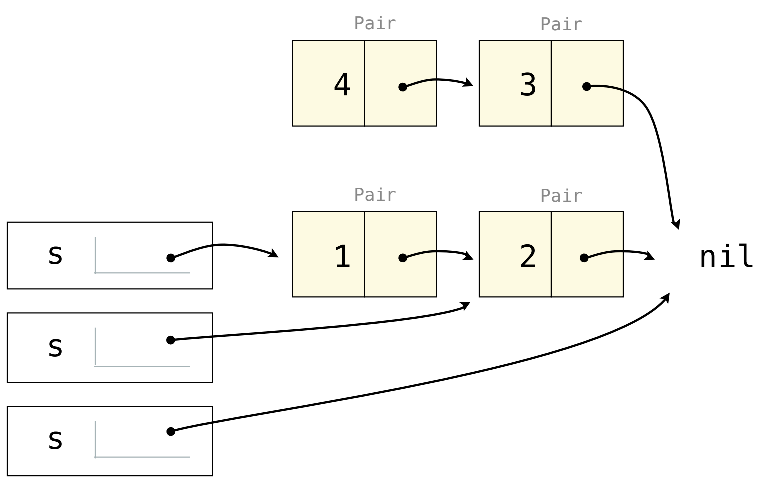


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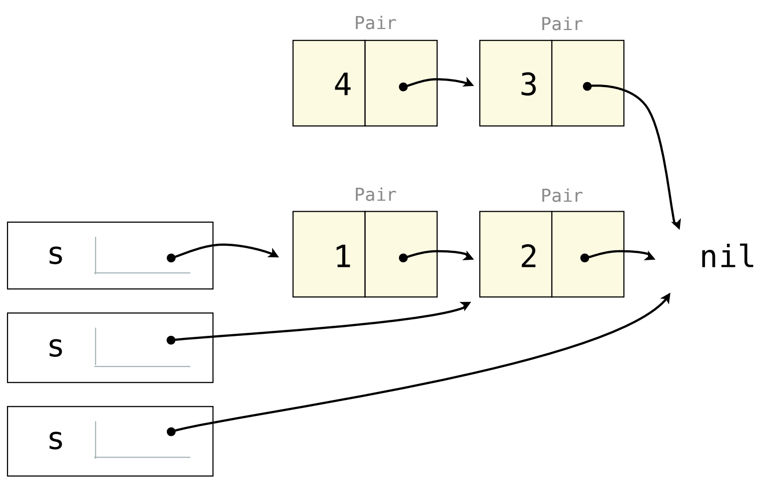


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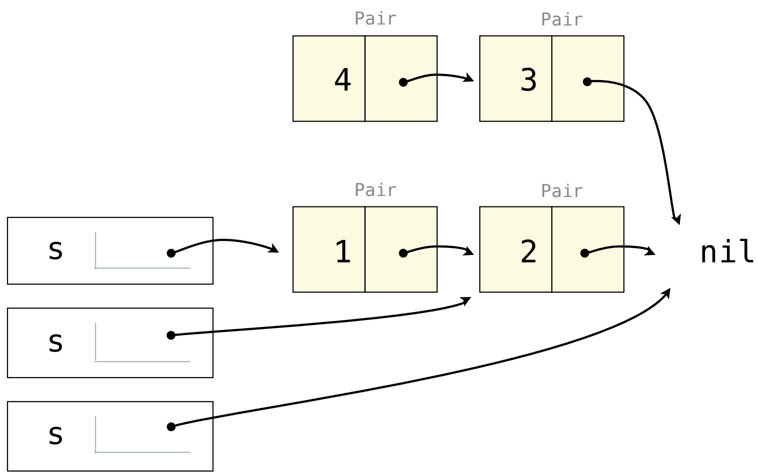


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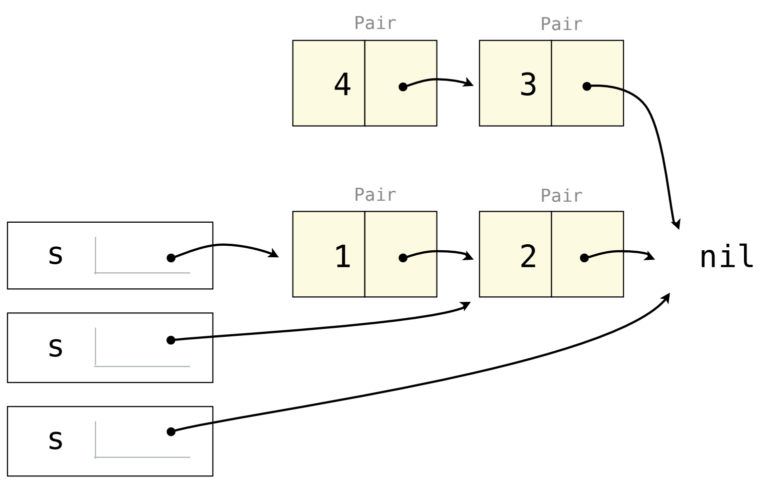


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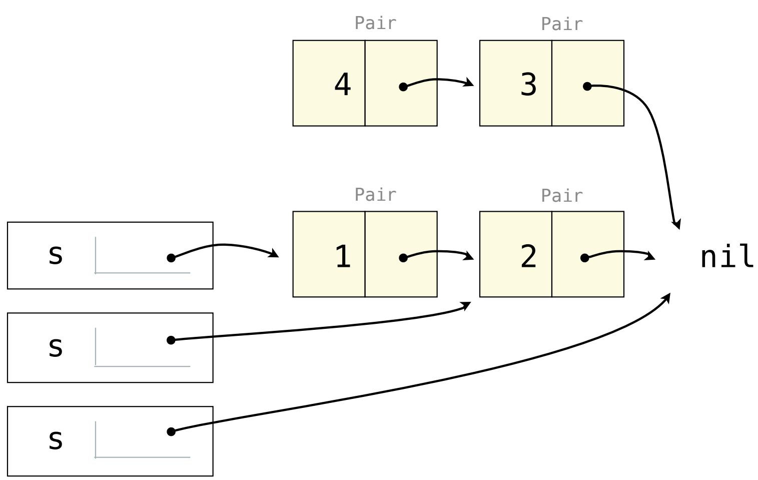


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# General Computing Machines

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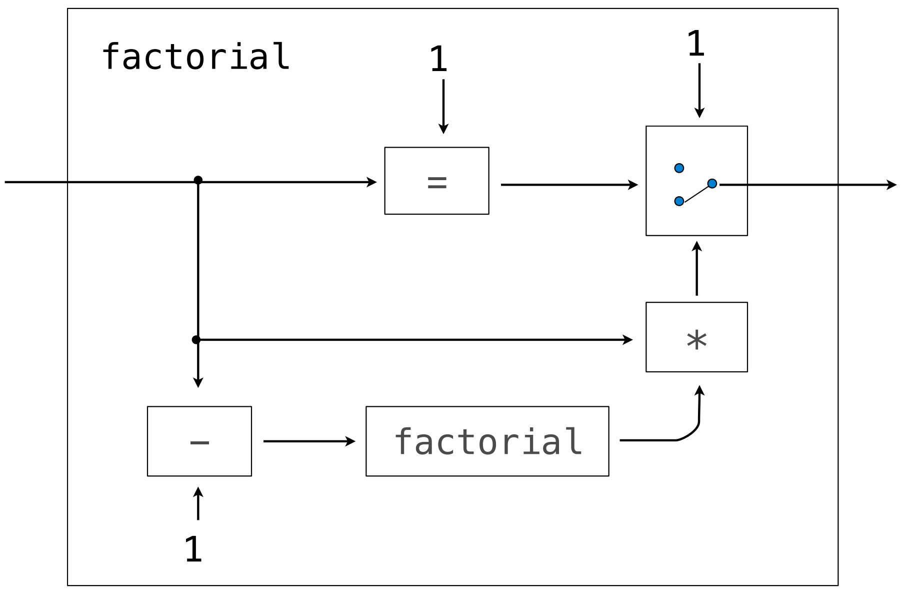

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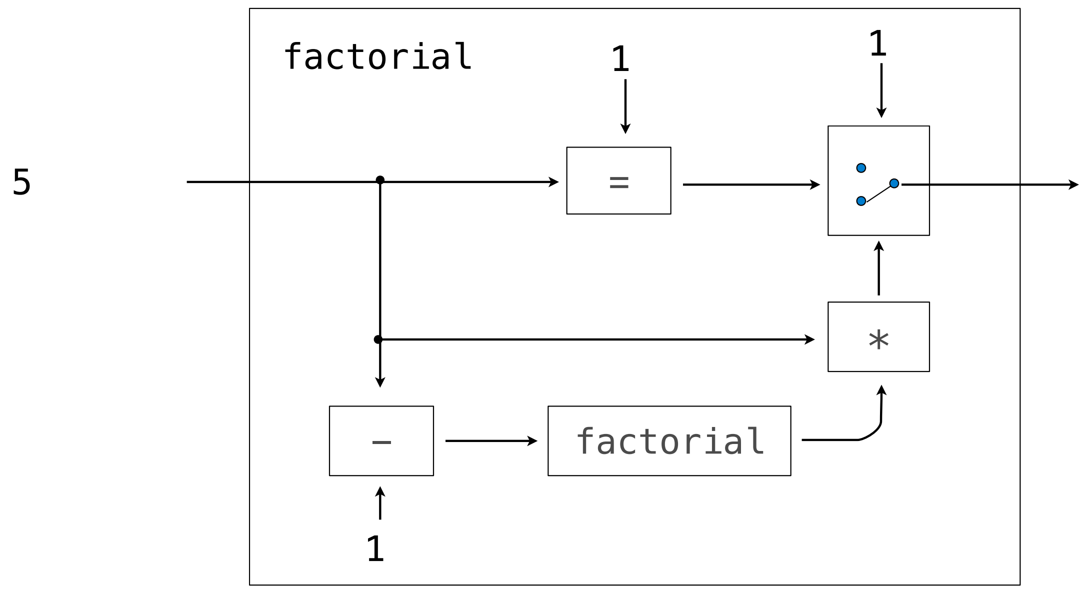

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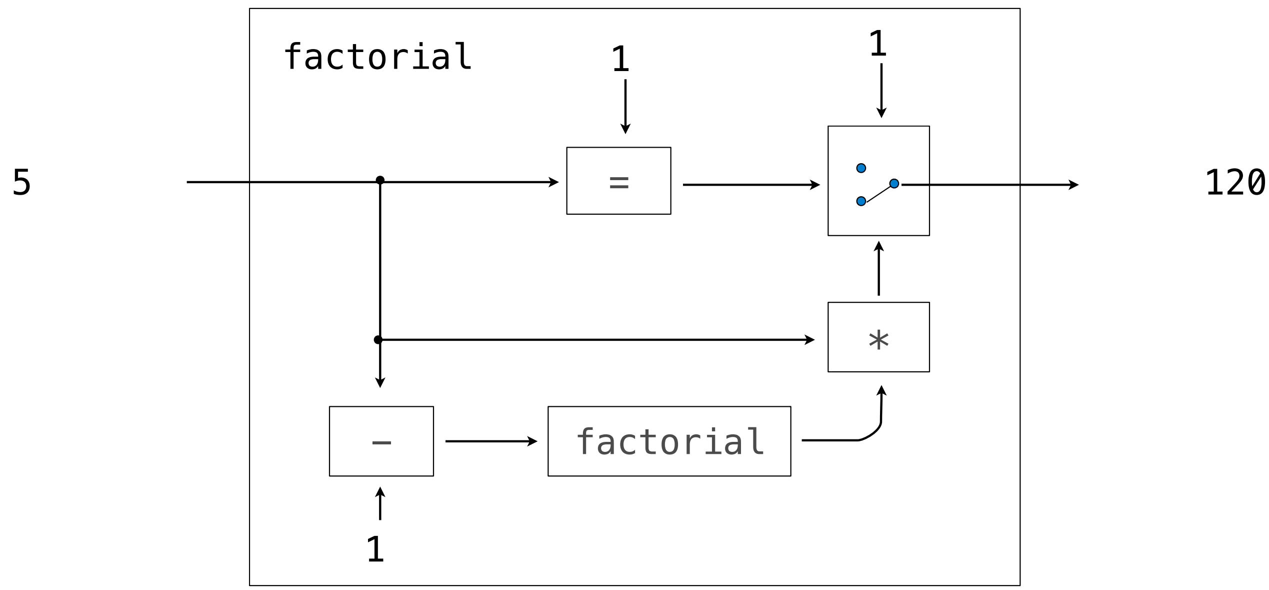

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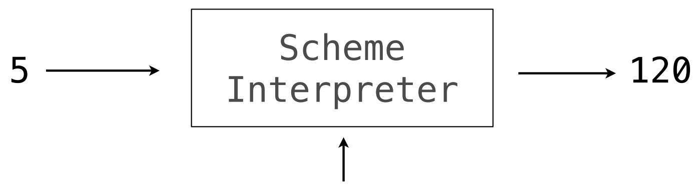


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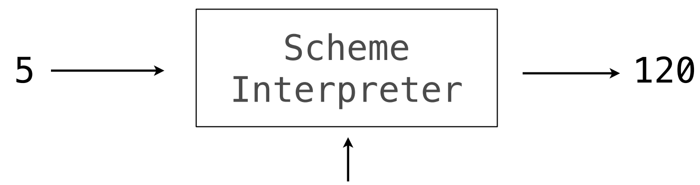


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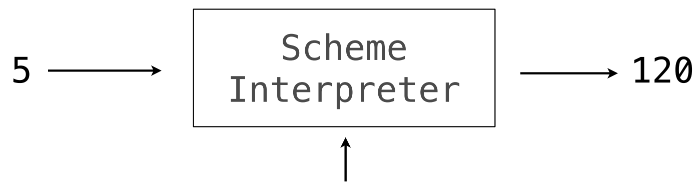


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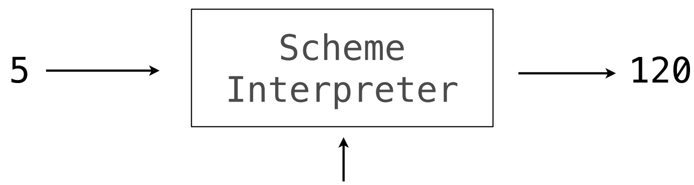


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Internally, it is just a set of evaluation rules