

## 61A Extra Lecture 13

## Announcements

## Prediction

## Regression

Given a set of  $(x, y)$  pairs, find a function  $f(x)$  that returns good  $y$  values

pairs =  $[(64.75, 163.5), (63.75, 147.5), (72.5, 224), \dots]$

Height in inches:  
5 feet 4 3/4 inches

Weight in pounds

Data from a health  
survey of 6342 adults

Measuring error:  $|y - f(x)|$  or  $(f(x) - y)^2$  are both typical

Over the whole set of  $(x, y)$  pairs, we can average this "squared error"

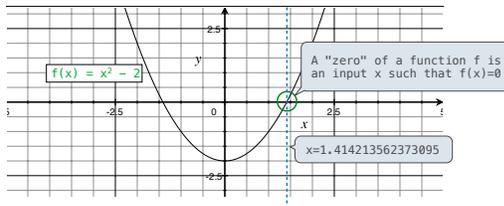
Squared error has the wrong units, so it's common to take the square root

The result is the "root mean squared error" of a predictor  $f$  on a set of  $(x, y)$  pairs

(Demo)

## Purpose of Newton's Method

Quickly finds accurate approximations to zeroes of differentiable functions!



Application: Find the minimum of a function by finding the zero of its derivative

## Approximate Differentiation

Differentiation can be performed symbolically or numerically

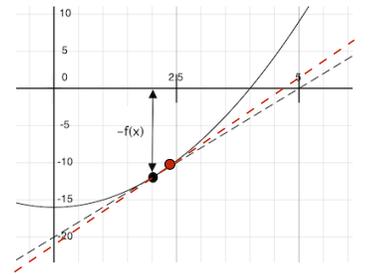
$$f(x) = x^2 - 16$$

$$f'(x) = 2x$$

$$f'(2) = 4$$

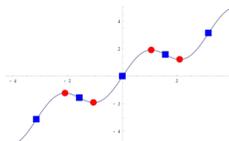
$$f'(x) = \lim_{a \rightarrow 0} \frac{f(x+a) - f(x)}{a}$$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a} \quad (\text{if } a \text{ is small})$$



## Critical Points and Inverses

Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0



The global minimum of convex functions that are (mostly) twice-differentiable can be computed numerically using techniques that are similar to Newton's method

(Demo)

## Multiple Linear Regression

Given a set of  $(x, y)$  pairs, find a linear function  $f(x)$  that returns good  $y$  values

A linear function has the form  $w \cdot x + b$  for vectors  $w$  and  $x$  and scalar  $b$

(Demo)

Note: Root mean squared error can be optimized through linear algebra alone, but numerical optimization works for a much larger class of related error measures

## Classification

## Classification

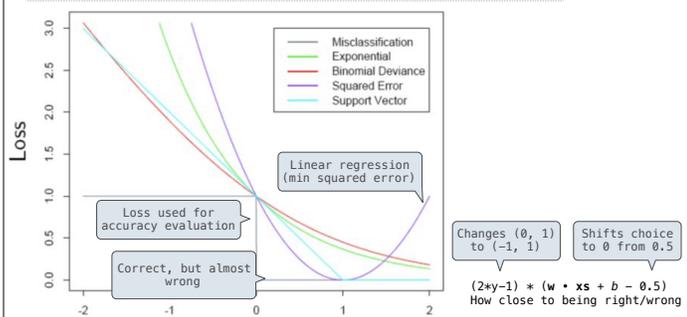
When the output of prediction is a category instead of a quantity, prediction is called *classification* instead of *regression*

The approach is basically the same, except that the output is coerced into a category

Error is measured by *accuracy*: the proportion of categorical predictions that were correct

(Demo)

## Loss Functions



## Language Models

## Natural Language Can Be Predicted

Programs should be written for people ~~to~~ read.

X                      Y

"should be written for": 1  
"be written for": 1  
"written for": 1  
"for": 1  
"nice computer": 0

(Demo)