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## COMPUTER SCIENCE 61A

December 4, 2015

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### 1 Rain, Rain, Go Away

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1. For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
```

```
>>> dogs[1] = list(dogs)
>>> dogs[1]
```

```
>>> dogs[0].append(2)
>>> cats
```

```
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
```

```
>>> dogs
```

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## 2 Gift in a Box

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1. (Fall 2012) Draw the environment diagram.

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```
def box(a):
    def box(b):
        def box(c):
            nonlocal a
            a = a + c
            return (a, b)
        return box
    gift = box(1)
    return (gift(2), gift(3))
box(4)
```

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### 3 The Gift & The Recurse

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1. The **quicksort** sorting algorithm is an efficient and commonly used algorithm to order the elements of a list. We choose one element of the list to be the **pivot** element and partition the remaining elements into two lists: one of elements less than the pivot and one of elements greater than the pivot. We recursively sort the two lists, which gives us a sorted list of all the elements less than the pivot and all the elements greater than the pivot, which we can then combine with the pivot for a completely sorted list.

First, implement the `quicksort_list` function. Choose the first element of the list as the pivot. You may assume that all elements are distinct.

```
def quicksort_list(lst):
    """
    >>> quicksort_list([3, 1, 4])
    [1, 3, 4]
    """

    if _____:

        _____

    pivot = lst[0]

    less = _____

    greater = _____

    return _____
```

2. We can also use quicksort to sort linked lists! Implement the `quicksort_link` function, without constructing additional `Link` instances.

You can assume that the `extend_links` function is already defined. It takes two linked lists and mutates the first so that it points to the second.

```
>>> l1, l2 = Link(1, Link(2)), Link(3, Link(4))
>>> l3 = extend_links(l1, l2)
>>> l3
Link(1, Link(2, Link(3, Link(4))))
>>> l1 is l3
True
```

```
def quicksort_link(link):
```

```
    """
```

```
    >>> s = Link(3, Link(1, Link(4)))
```

```
    >>> quicksort_link(s)
```

```
    Link(1, Link(3, Link(4)))
```

```
    """
```

```
    if _____:
```

```
        return link
```

```
    pivot, _____ = _____
```

```
    less, greater = _____
```

```
    while link is not Link.empty:
```

```
        curr, rest = link, link.rest
```

```
        if _____:
```

```
            _____
```

```
        else:
```

```
            _____
```

```
        link = _____
```

```
    less = _____
```

```
    greater = _____
```

```
    _____
```

```
    return _____
```

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## 4 Can You Take Me Higher?

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1. (Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers `x` and `y`. It returns the number of ways of reaching `y` from `x` by repeatedly incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9.

```
def inc(x):  
    return x + 1
```

```
def double(x):  
    return x * 2
```

```
def paths(x, y):  
    """Return the number of ways to reach y from x by repeated  
    incrementing or doubling.  
    >>> paths(3, 5) # inc(inc(3))  
    1  
    >>> paths(3, 6) # double(3), inc(inc(inc(3)))  
    2  
    >>> paths(3, 9) # E.g. inc(double(inc(3)))  
    3  
    >>> paths(3, 3) # No calls is a valid path  
    1  
    """  
    if x > y:  
        return _____  
  
    elif x == y:  
        return _____  
  
    else:  
        return _____
```

2. (Fall 2013) Fill in the blanks in the implementation of `pathfinder`, a higher-order function that takes an increasing function `f` and a positive integer `y`. It returns a function that takes a positive integer `x` and returns whether it is possible to reach `y` by applying `f` to `x` zero or more times. For example, 8 can be reached from 2 by applying `double` twice. A function `f` is *increasing* if  $f(x) > x$  for all positive integers `x`.

```
def pathfinder(f, y):
    """
    >>> f = pathfinder(double, 8)
    >>> {k: f(k) for k in (1, 2, 3, 4, 5)}
    {1: True, 2: True, 3: False, 4: True, 5: False}
    >>> g = pathfinder(inc, 3)
    >>> {k: g(k) for k in (1, 2, 3, 4, 5)}
    {1: True, 2: True, 3: True, 4: False, 5: False}
    """
    def find_from(x):
        while _____:
            _____
        return _____
```

3. Write a generator function that yields functions that are repeated applications of a one-argument function `f`. The first function yielded should apply `f` 0 times (the identity function), the second function yielded should apply `f` once, etc.

```
def repeated(f):
    """
    >>> [g(1) for _, g in
    ... zip(range(5), repeated(double))]
    [1, 2, 4, 8, 16]
    """
    g = _____
    while True:
        _____
        _____
```

4. Ben Bitdiddle proposes the following alternate solution. Does it work?

```
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = lambda x: f(g(x))
```

## 5 Slim Shady

1. Implement `widest_level`, which takes a `Tree` instance and returns the elements at the depth with the most elements.

```
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]),
    ...           Tree(4, [Tree(9, [Tree(2)])])]
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
    x = [t]

    while _____:
        _____

        _____ = sum(_____, [])

    return max(levels, key=_____)
```

## 6 Scheming With a Broken Heart

1. Consider the following Scheme tree data abstraction.

```
(define (make-tree entry children) (cons entry children))
(define (entry tree) (car tree))
(define (children tree) (cdr tree))
(define tree 'below-example)
;
;           5
;   +-----+-----+
;   |         |         |
;   6         7         2
;   +---+---+   |   +---+---+
;   |     |     |   |   |   |
;   9     8     1   6   4
;
;               |
;               |
;               3
```

Write a procedure `tree-sums` that takes a tree of numbers (like the one above) and outputs a list of sums from following each possible path from root to leaf.

*Hint:* You may find the `flatten` procedure helpful.

```
(define (flatten lst)
  (cond ((null? lst) nil)
        ((list? (car lst)) (append (flatten (car lst)) (
                                     flatten (cdr lst))))
        (else (cons (car lst) (flatten (cdr lst))))))
```

```
(define (tree-sums tree)
```

```
  (if _____
      _____
      (map (lambda (x) _____)
           _____)))
```

```
scm> (flatten '(0 (1) ((2)) (3 ((4)))))
(0 1 2 3 4)
scm> (tree-sums tree)
(20 19 13 16 11)
```

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## 7 Stream On

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1. Implement the `append-stream` procedure, which takes in two streams and returns a stream with the two streams concatenated. (Note that if the first stream is infinite, the result will not contain any elements from the second stream.)

```
(define (append-stream s1 s2)
```

2. Now implement `subset-stream`, which takes in a normal Scheme list and returns a stream with every possible subset of that Scheme list.

```
(define (subset-stream lst)
```

