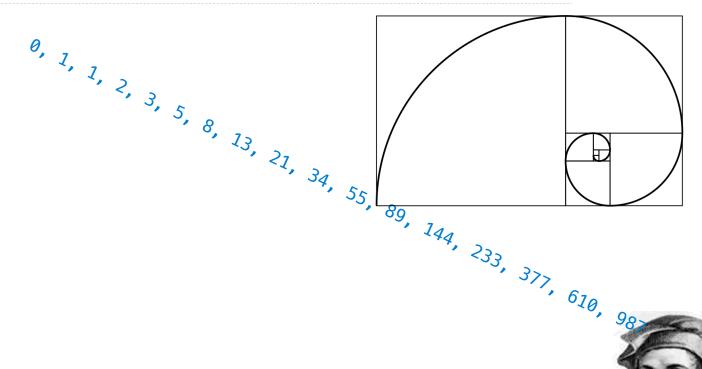






0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 98

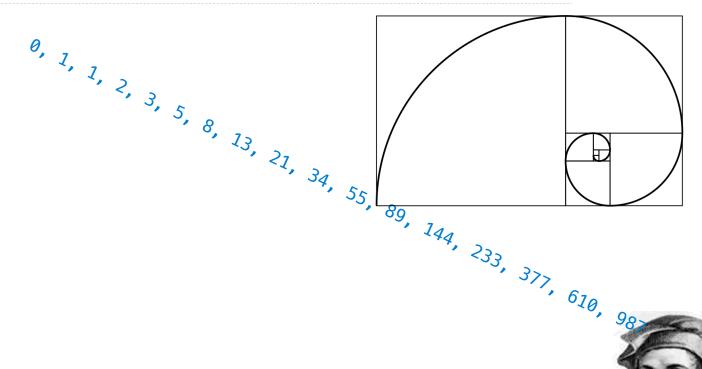




0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 5

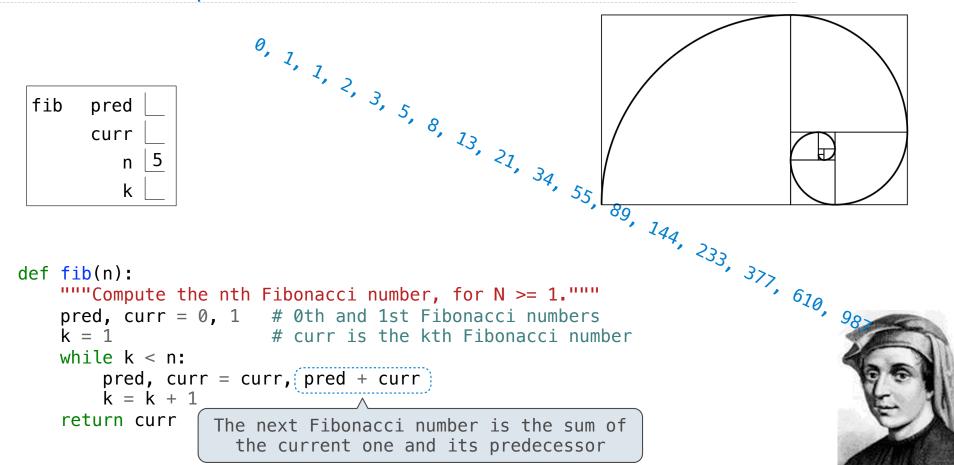


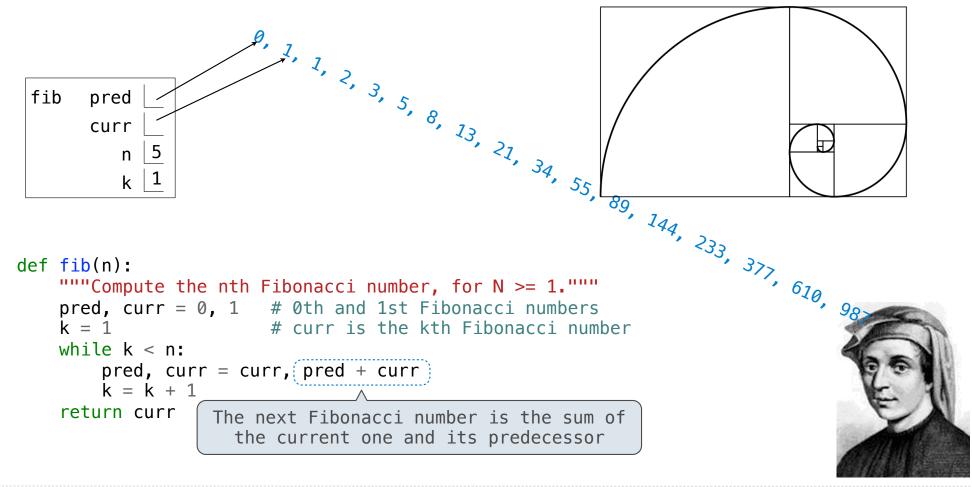


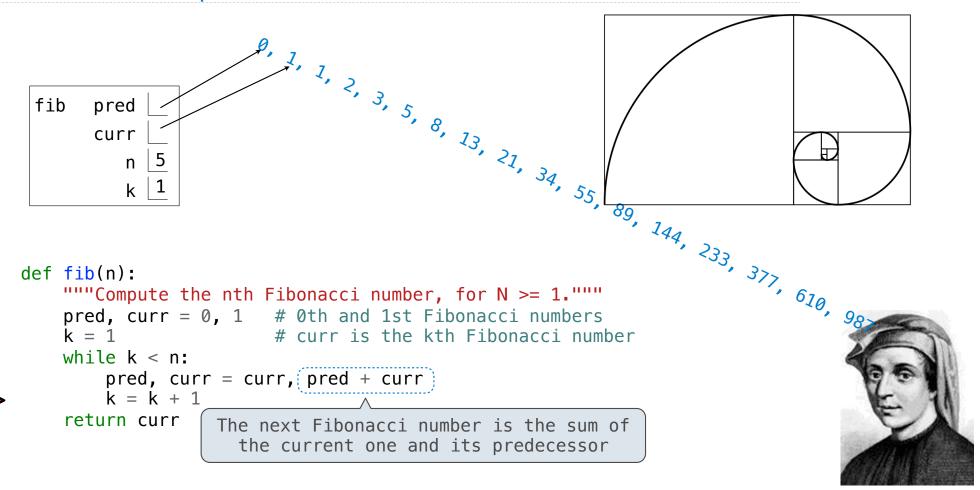


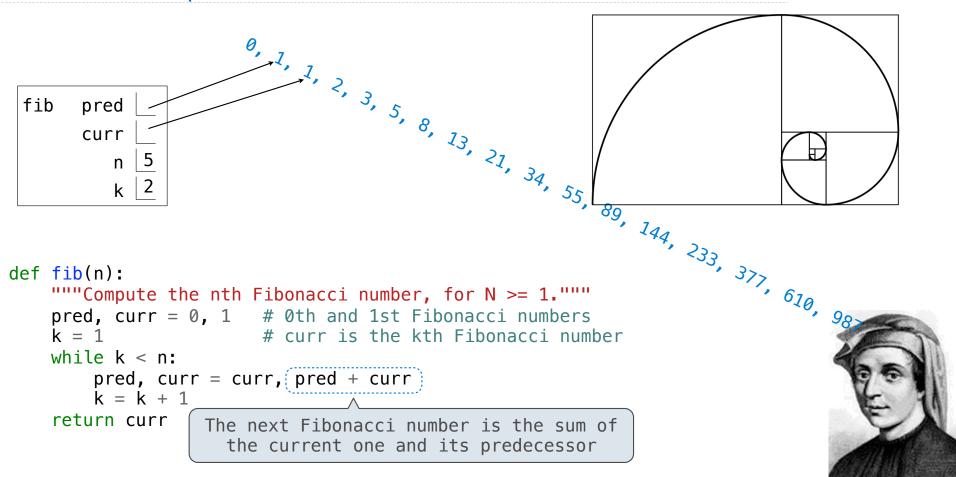
```
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 98
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1  # 0th and 1st Fibonacci numbers
    k = 1
   while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

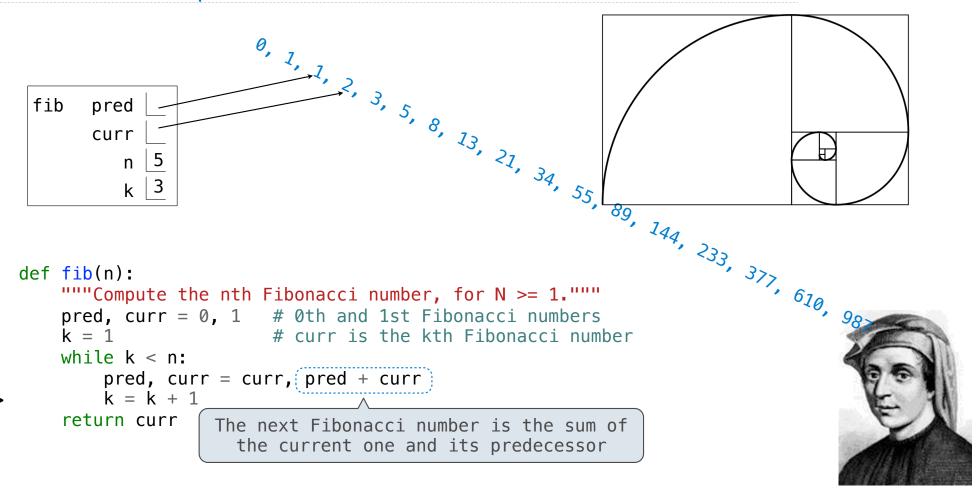
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0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 98×
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    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1  # 0th and 1st Fibonacci numbers
    k = 1
   while k < n:
        pred, curr = curr, (pred + curr)
        k = k + 1
    return curr
                  The next Fibonacci number is the sum of
                     the current one and its predecessor
```

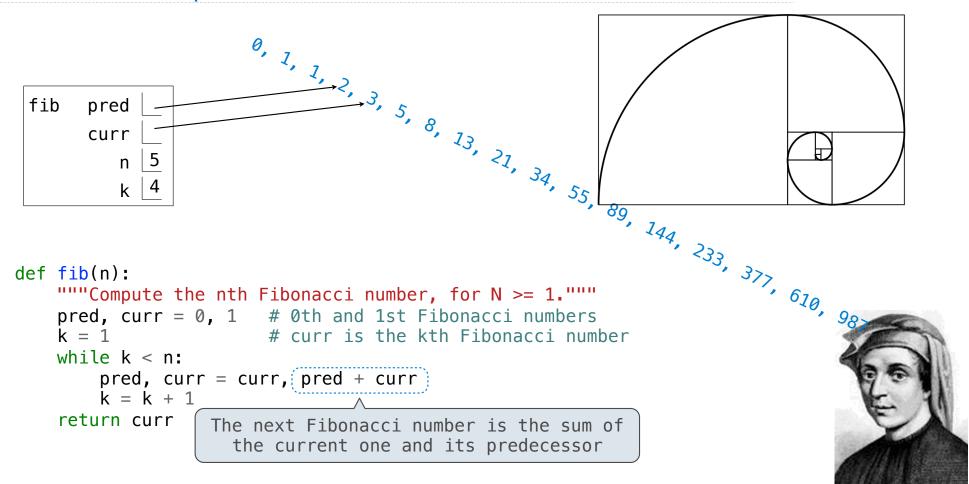


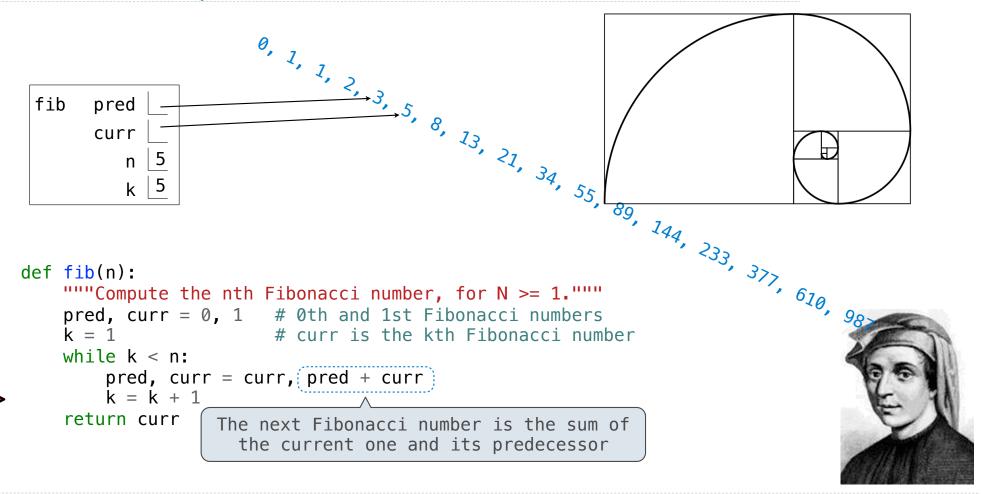












```
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 98
def fib(n):
    """Compute the nth Fibonacci number?"""
    pred, curr = 0, 1
```

k = 1while k < n: pred, curr = curr, pred + curr k = k + 1return curr



```
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 98
```



return curr

pred, curr = curr, pred + curr

k = k + 1

return curr

```
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377
Is this alternative definition of fib the
same or different from the original fib?
def fib(n):
    """Compute the nth Fibonacci number?"""
                                                                      I'm still here
    pred, curr = 0, 1 pred, curr = 1, 0
    k = 1
                       k = 0
   while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
                                           (Demo)
```

Designing Functions

Describing Functions					

A function's domain is the set of all inputs it might possibly take as arguments.

7

A function's domain is the set of all inputs it might possibly take as arguments.

A function's  $\mathit{range}$  is the set of output values it might possibly return.

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A function's range is the set of output values it might possibly return.

A pure function's behavior is the relationship it creates between input and output.

```
def square(x):
    """Return X * X."""
```

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def square(x):
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    x is a real number
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- /

```
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                                   return value is the nth Fibonacci number
     square of the input
```

7

A Guide to Designing Function	
	8

Give each function exactly one job, but make it apply to many related situations

```
Give each function exactly one job, but make it apply to many related situations >>> round(1.23)
```

Give each function exactly one job, but make it apply to many related situations

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Don't repeat yourself (DRY). Implement a process just once, but execute it many times.

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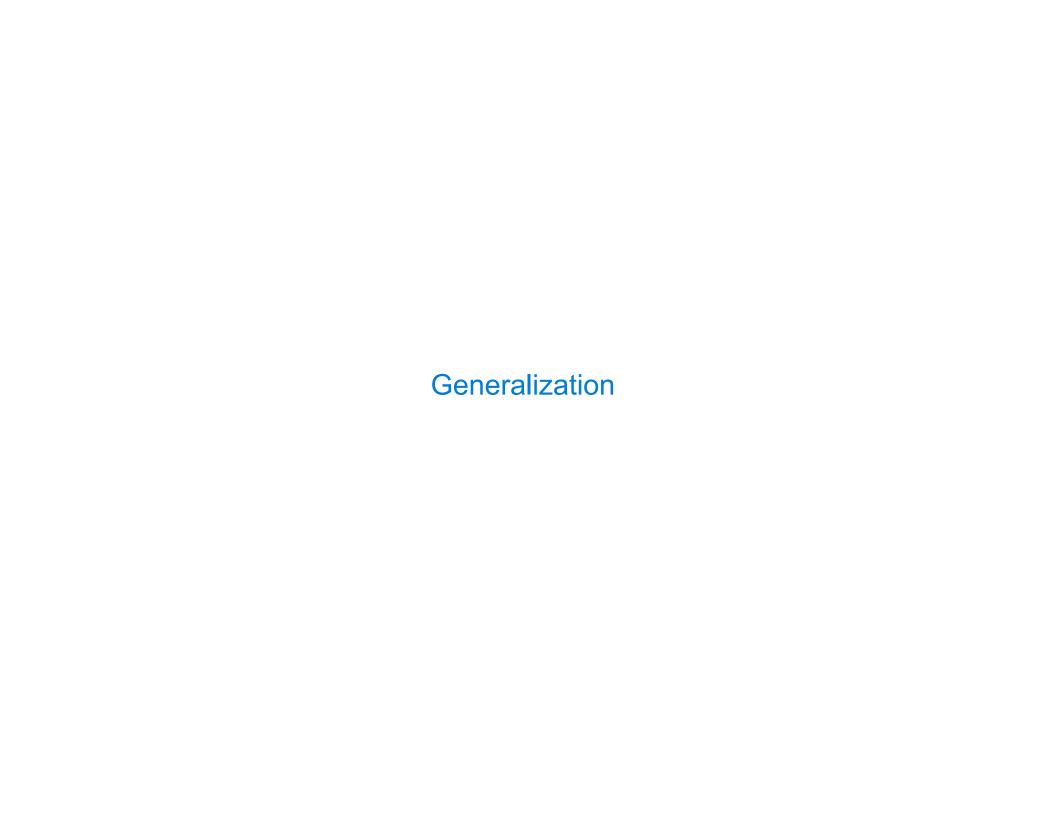
(Demo)

Give each function exactly one job, but make it apply to many related situations

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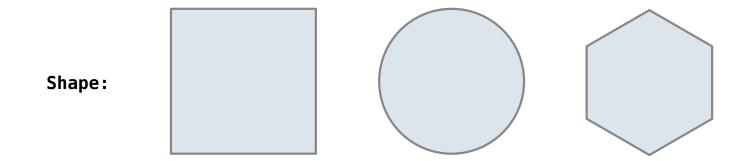
(Demo)

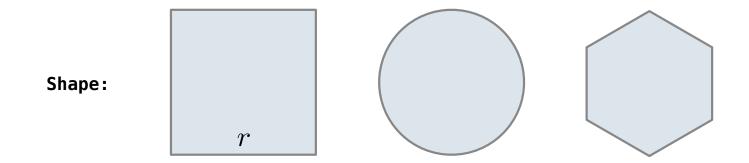


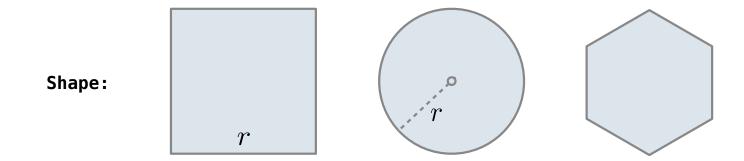


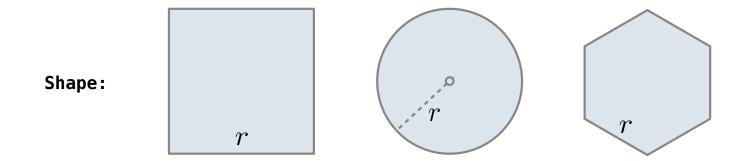
Generalizing Patterns with Arguments	

Genera	lizing	<b>Patterns</b>	with	<b>Argument</b>	S

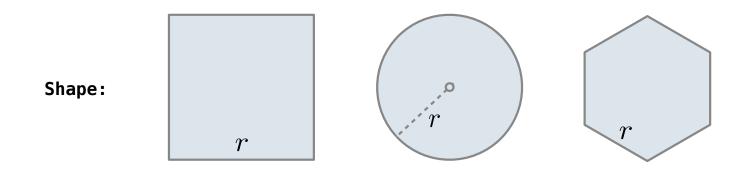






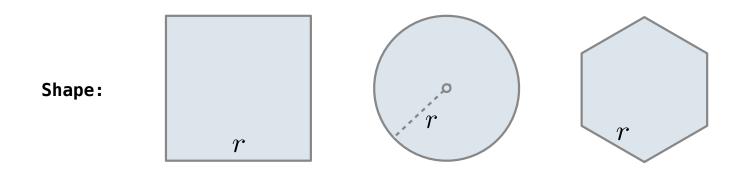


Regular geometric shapes relate length and area.



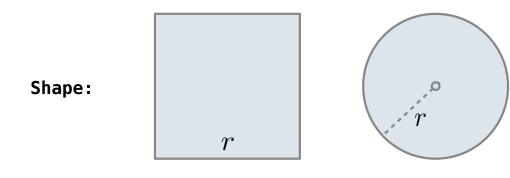
Area:

Regular geometric shapes relate length and area.



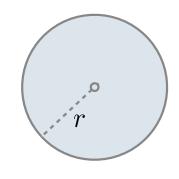
Area:  $r^2$ 

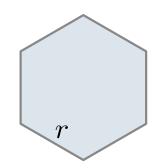
Regular geometric shapes relate length and area.



Area:  $r^2$   $\pi \cdot r^2$ 







$$r^2$$

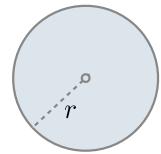
$$\pi \cdot r^2$$

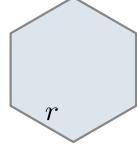
$$\frac{3\sqrt{3}}{2} \cdot r^2$$

Regular geometric shapes relate length and area.



r

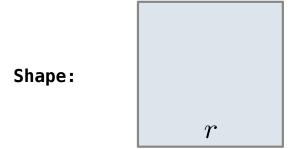


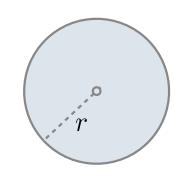


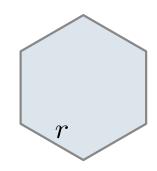
Area:

 $1 \cdot r^2$ 

Regular geometric shapes relate length and area.







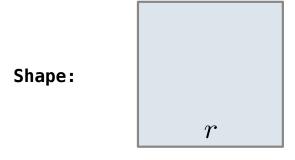
Area:

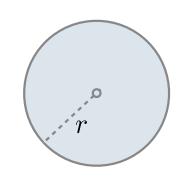
$$(1) r^2$$

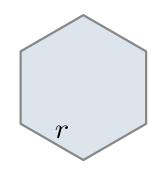
$$\pi \cdot r^2$$

$$\frac{3\sqrt{3}}{2} \cdot r^2$$

Regular geometric shapes relate length and area.







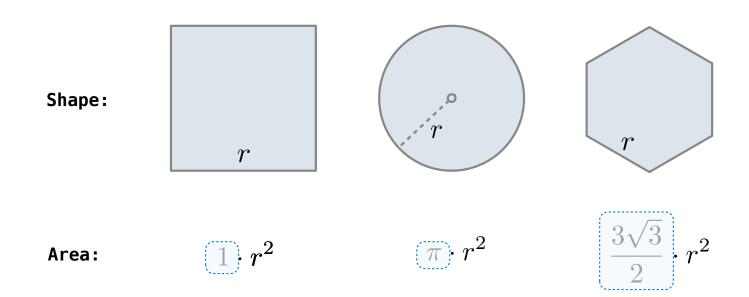
Area:

$$1 \cdot r^2$$

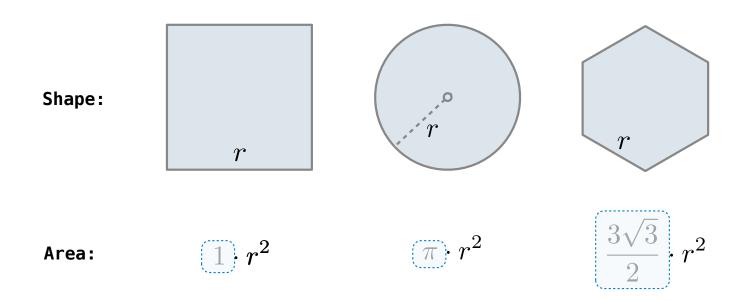
$$(\pi)$$
  $r^2$ 

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Regular geometric shapes relate length and area.

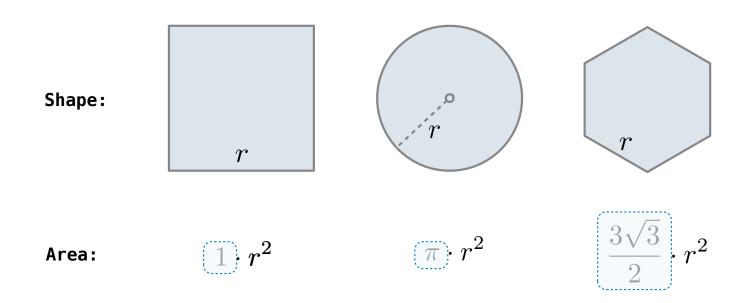


Regular geometric shapes relate length and area.



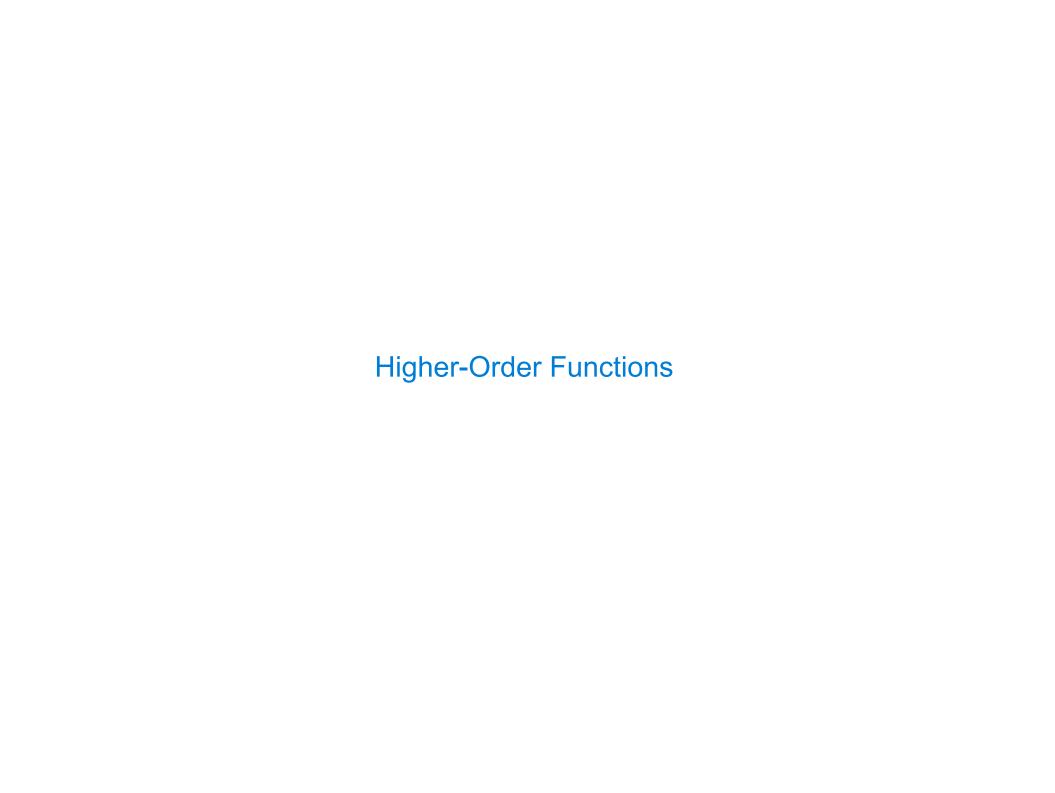
Finding common structure allows for shared implementation

Regular geometric shapes relate length and area.



Finding common structure allows for shared implementation

(Demo)



Generalizing Over Computational Processes	

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

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(Demo)

#### **Summation Example**

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

>>> summation(5, cube)
    225
    """

    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total</pre>
```

#### **Summation Example**

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return total</pre>
```

```
Function of a single argument
def cube(k):
                                (not called "term")
    return pow(k, 3)
                           A formal parameter that will
def summation(n, term)
                              be bound to a function
     """Sum the first n terms of a sequence.
    >>> summation(5, cube)
    225
     11 11 11
    total, k = 0, 1
    while k <= n:
         total, k = total + term(k), k + 1
    return total
                            The function bound to term
                                gets called here
```

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Function of a single argument
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     225
                           The cube function is passed
     11 11 11
                              as an argument value
     total, k = 0, 1
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                              as an argument value
    total, k = 0, 1
    while k <= n:
          total, k = total + term(k), k + 1
    return total
                             The function bound to term
  0 + 1 + 8 + 27 + 64 + 125
                                 gets called here
```

# Functions as Return Values

(Demo)

Locally Defined Functions	

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

>>> add_three = make_adder(3)
>>> add_three(4)
7
    """

def adder(k):
    return k + n
return adder
```

```
A function that
returns a function

def make adder(n):
    """Return a function that takes one argument k and returns k + n.

>>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """

def adder(k):
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return adder
```

```
A function that
returns a function

def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

>>> add_three = make_adder(3)
    The name add_three is bound
    to a function

7
    """

def adder(k):
    return k + n
    another def statement
return adder
```

```
A function that returns a function

def make adder(n):

"""Return a function that takes one argument k and returns k + n.

>>> add three = make adder(3)

>>> add_three(4)

The name add_three is bound to a function

7

"""

def adder(k):
    return k + n

return adder

Can refer to names in the enclosing function
```

Call Expressions as Operator Expressions	
	16

make\_adder(1) ( 2

make\_adder(1) ( 2 )

