# 61A Lecture 7

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# Announcements

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## Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person
- Your score is the number of entries against which you win more than 50.00001% of the time
- All strategies must be deterministic, pure functions of the players' scores
- All winning entries will receive extra credit
- The real prize: honor and glory

**Fall 2011 Winners**

Kaylee Mann
Yan Duan & Ziming Li
Brian Prike & Zhenghao Qian
Parker Schuh & Robert Chatham

**Fall 2012 Winners**

Chenyang Yuan
Joseph Hui

**Fall 2013 Winners**

Paul Bramsen
Sam Kumar & Kangsik Lee
Kevin Chen

**Fall 2014 Winners**

Alan Tong & Elaine Zhao
Zhenyang Zhang
Adam Robert Villaflor & Joany Gao
Zhen Qin & Dian Chen
Zizheng Tai & Yihe Li

**Spring 2015 Winners**

Sinho Chewi & Alexander Nguyen Tran
Zhaoxi Li
Stella Tao and Yao Ge

**Fall 2015 Winners**

Micah Carroll & Vasilis Oikonomou
Matthew Wu
Anthony Yeung and Alexander Dai

Your name could be here FOREVER!

**Fall 2016 Winners...**

cs61a.org/proj/hog_contest

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# Order of Recursive Calls

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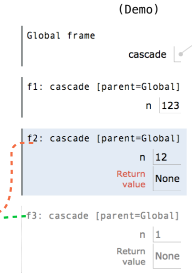
## The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```




Program output:

```
123
12
1
12
```

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram

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## Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

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# Example: Inverse Cascade

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## Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(                    )
shrink = lambda n: f_then_g(                    )
```

## Tree Recursion

## Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **n:** | 0, | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | ... , | 35 |
| **fib(n):** | 0, | 1, | 1, | 2, | 3, | 5, | 8, | 13, | 21, | ... , | 9,227,465 |

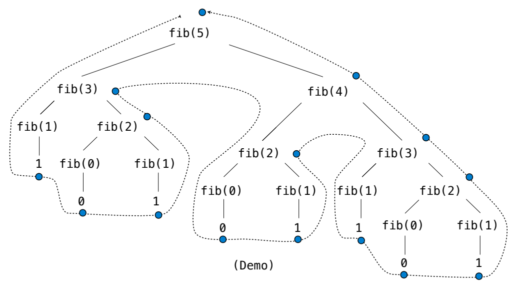
```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg



## A Tree-Recursive Process

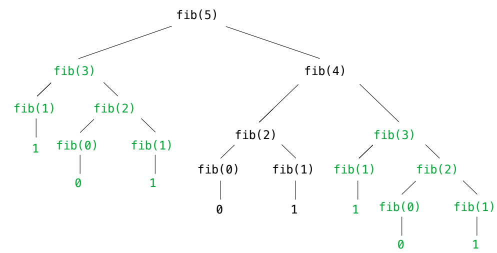
The computational process of fib evolves into a tree structure






## Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times




(We will speed up this computation dramatically in a few weeks by remembering results)



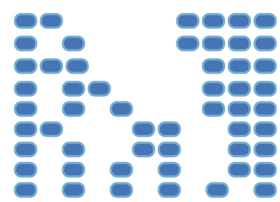
## Example: Counting Partitions

## Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
2 + 2 + 2 = 6
1 + 1 + 2 + 2 = 6
1 + 1 + 1 + 1 + 2 = 6
1 + 1 + 1 + 1 + 1 + 1 = 6





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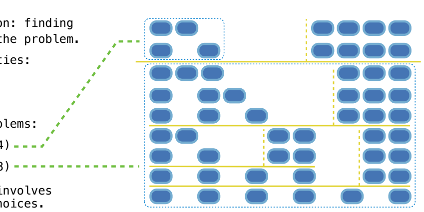
The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.





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- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
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- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

(Demo)

Interactive Diagram