CS 61A Lecture 11

Announcements

Dictionaries

```
{'Dem': 0}
```

Limitations on Dictionaries

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The second restriction is part of the dictionary abstraction

If you want to associate multiple values with a key, store them all in a sequence value

Box-and-Pointer Notation

The Closure Property of Data Types

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Lists can contain lists as elements (in addition to anything else)

Box-and-Pointer Notation in Environment Diagrams

Interactive Diagram

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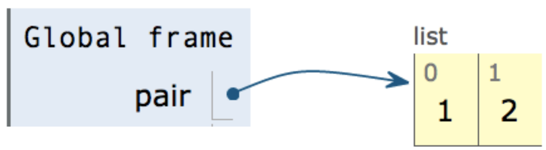


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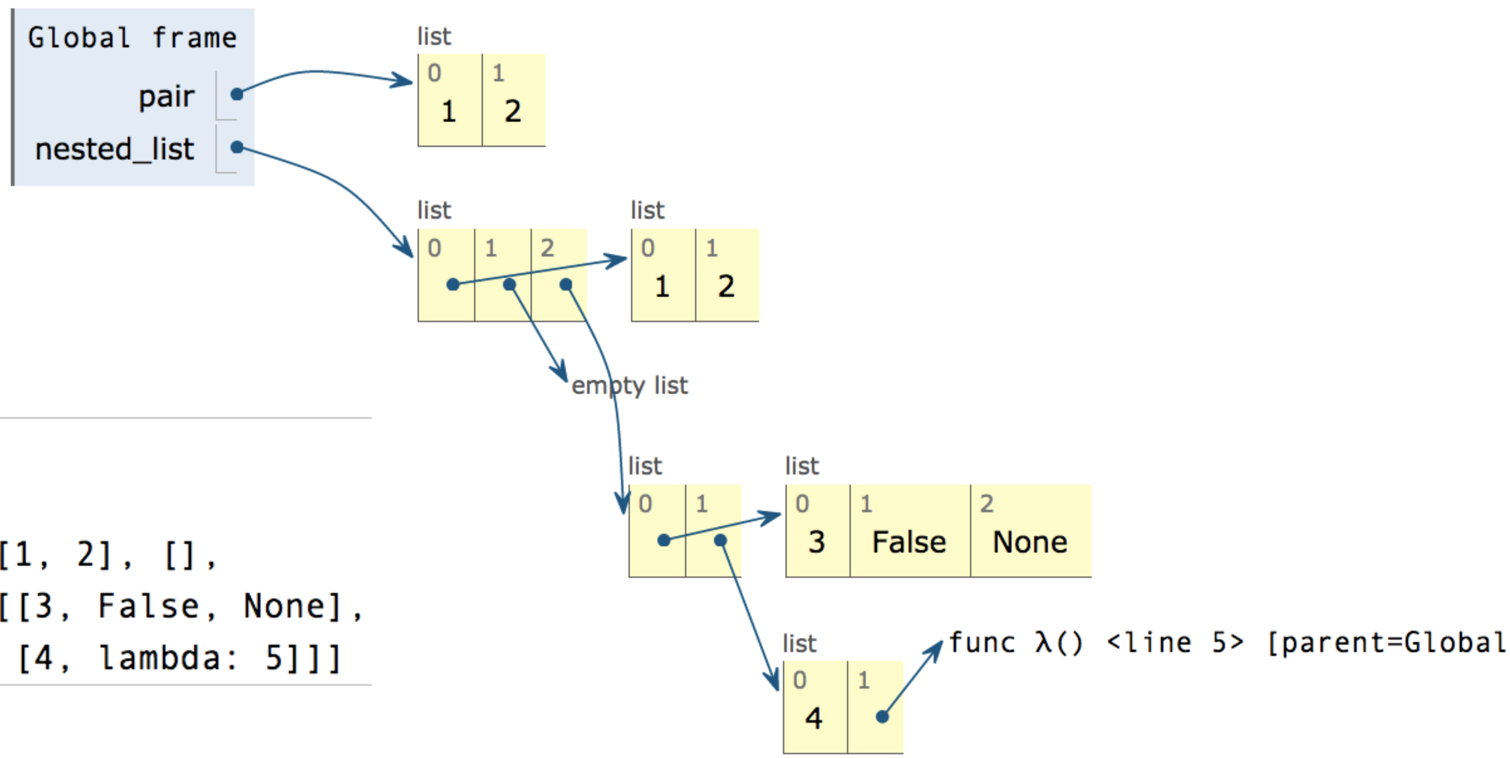


```
pair = [1, 2]

nested_list = [[1, 2], [],
               [[3, False, None],
                [4, lambda: 5]]]
```

Interactive Diagram

Slicing

(Demo)

Slicing Creates New Values

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digits = [1, 8, 2, 8]
start =  digits[:1]
middle = digits[1:3]
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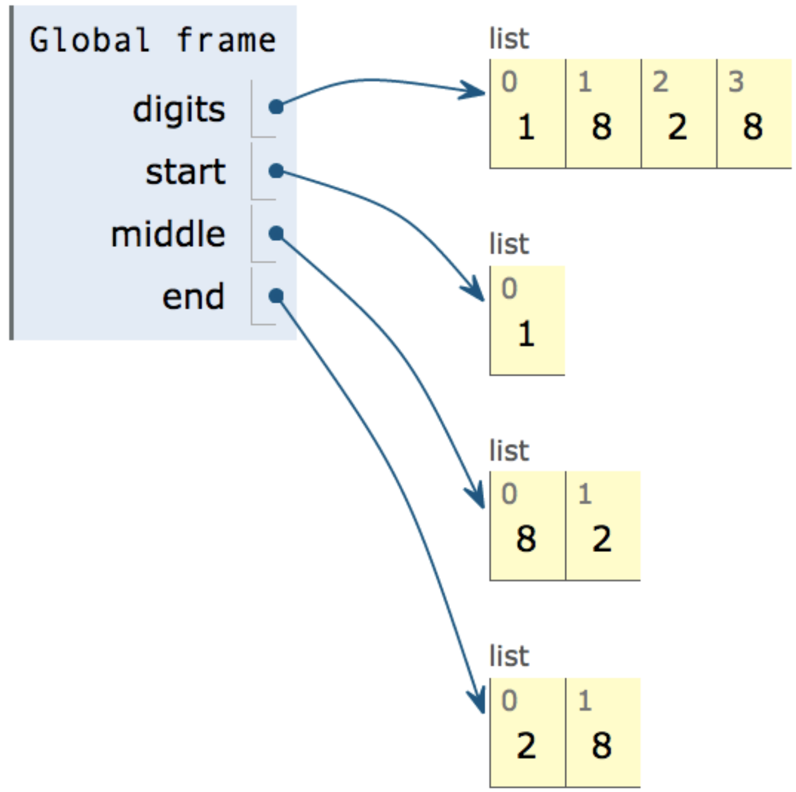

Interactive Diagram

Processing Container Values

Sequence Aggregation

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Several built-in functions take iterable arguments and aggregate them into a value

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- **sum**(iterable[, start]) -> value

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Return the sum of an iterable of numbers (NOT strings) plus the value
of parameter 'start' (which defaults to 0).  When the iterable is
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  max(a, b, c, ...[, key=func]) -> value

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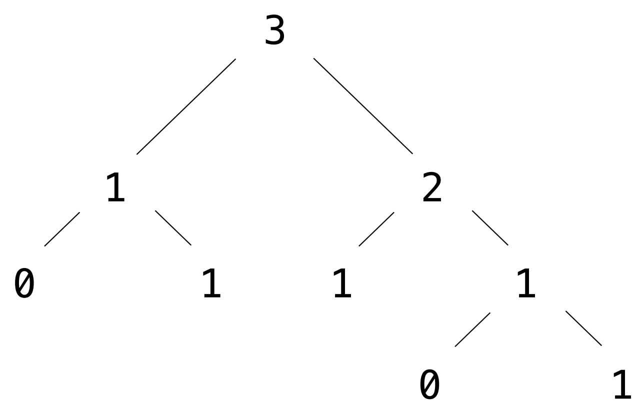
(Demo)

Trees

Tree Abstraction



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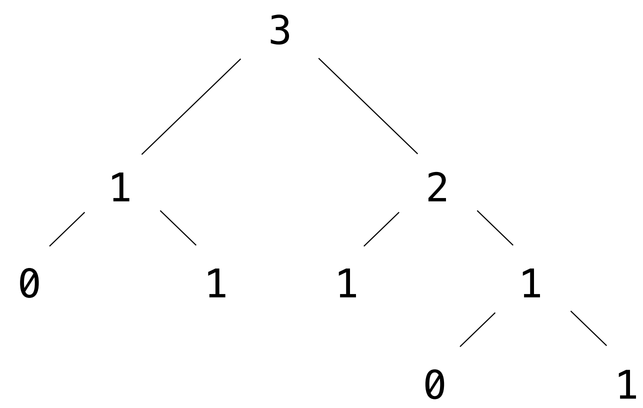

Recursive description **(wooden trees):**

Relative description **(family trees):**

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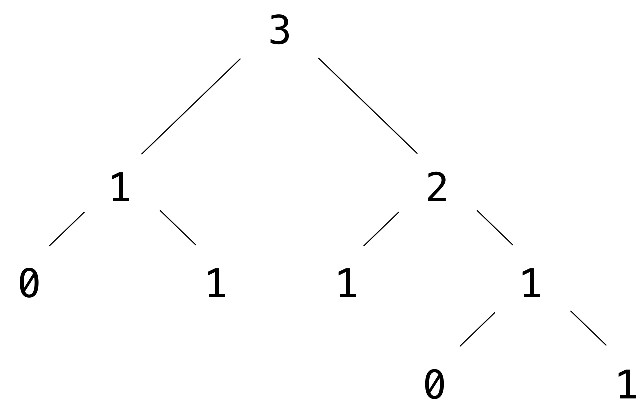

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A **tree** has a **root** value and a list of **branches**

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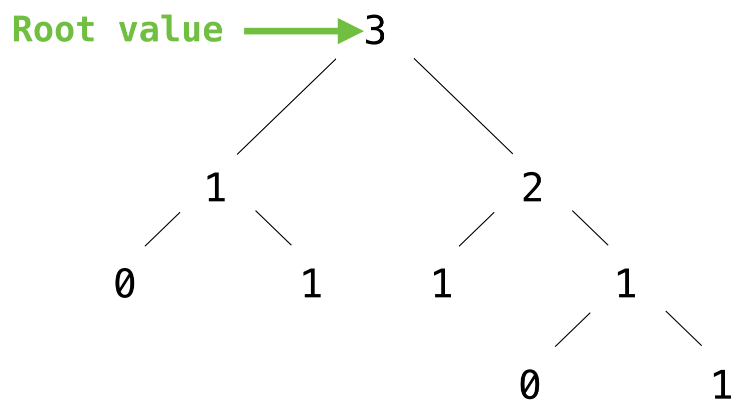

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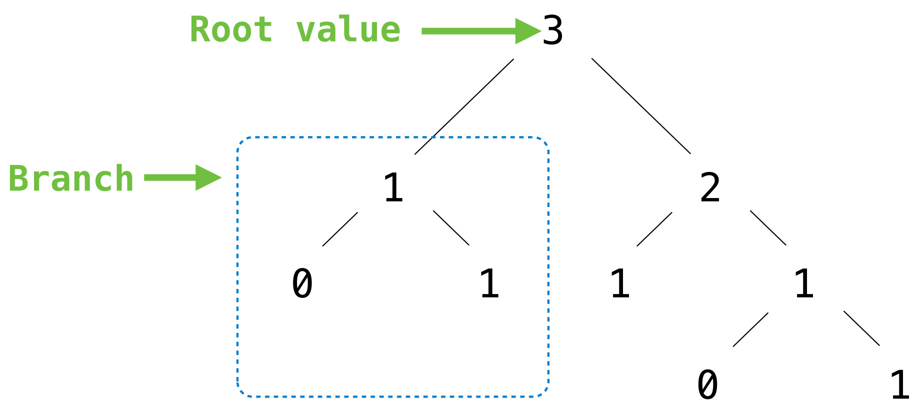

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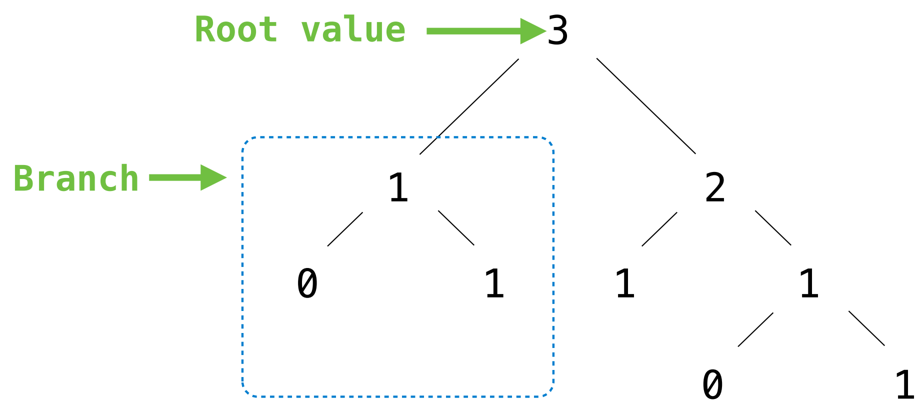

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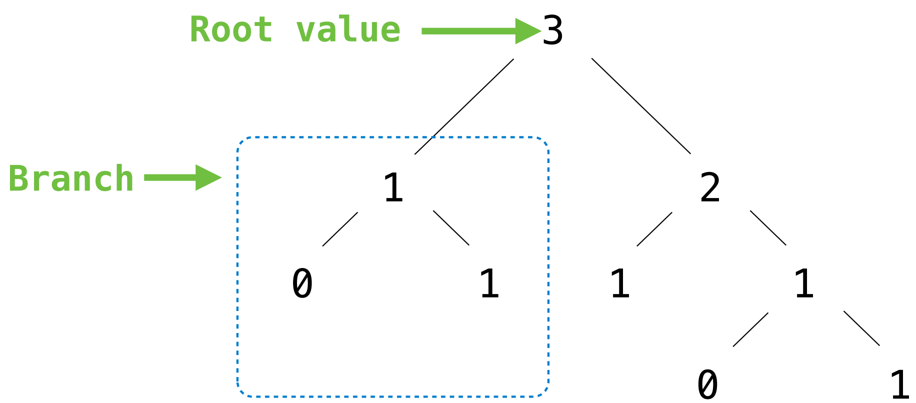

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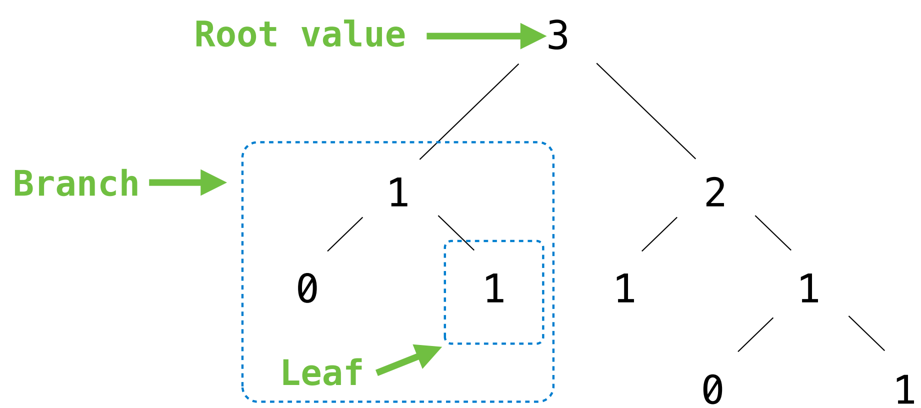

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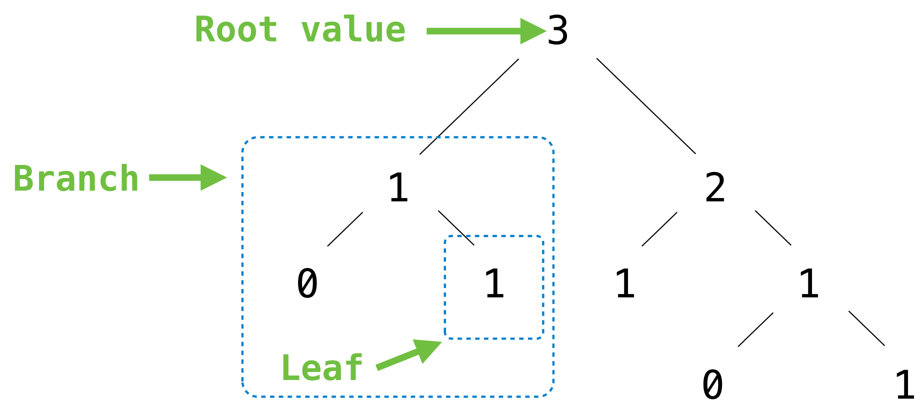

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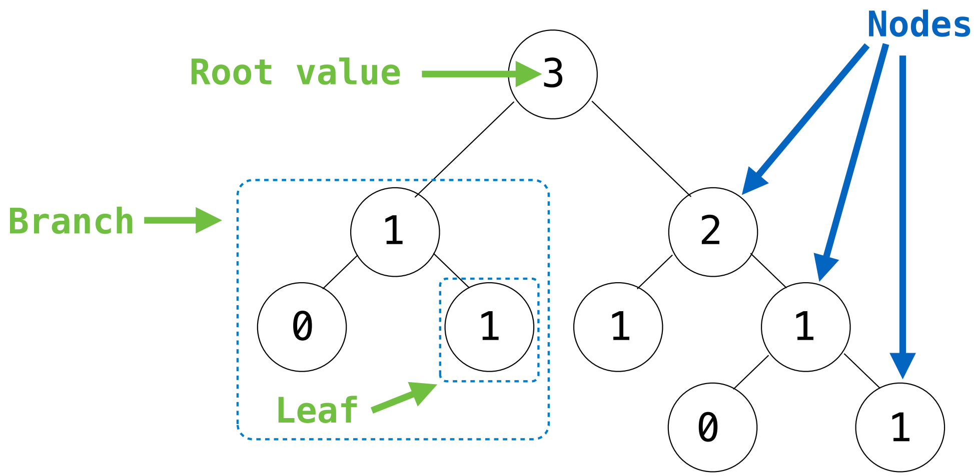

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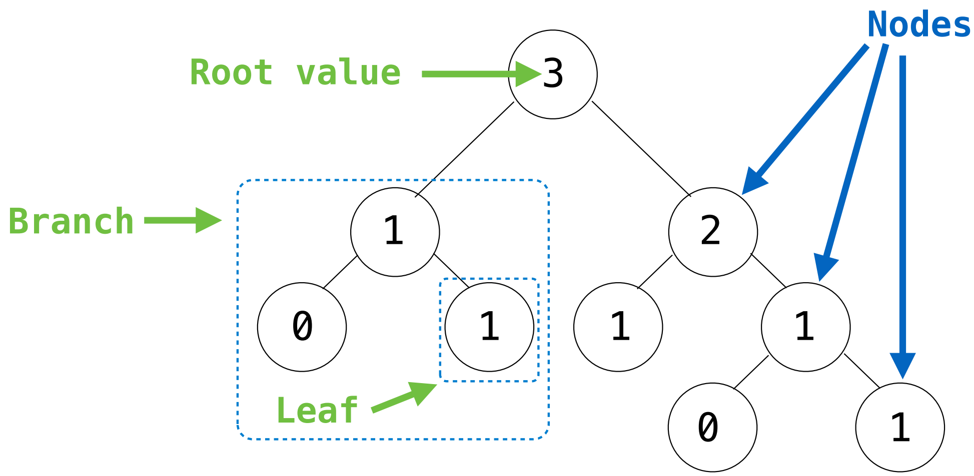

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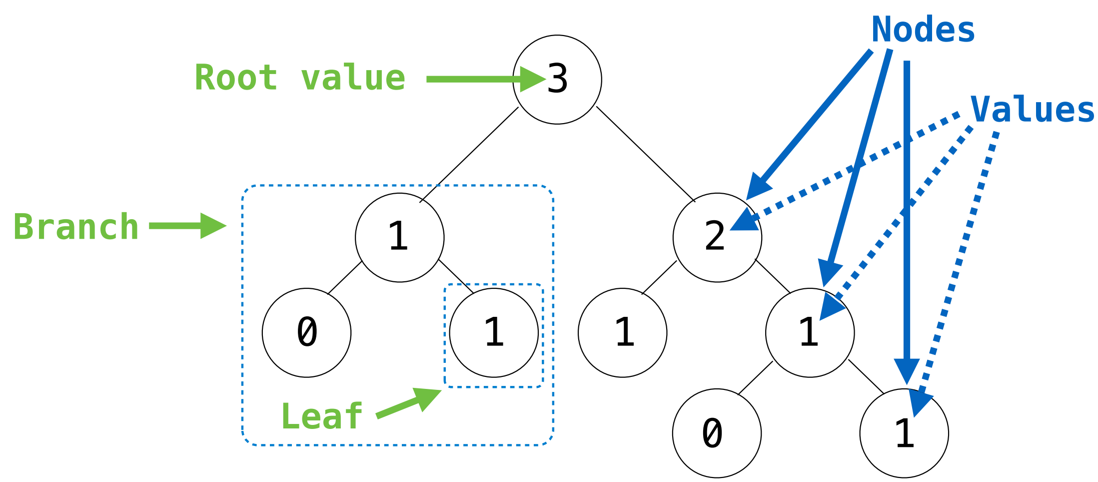

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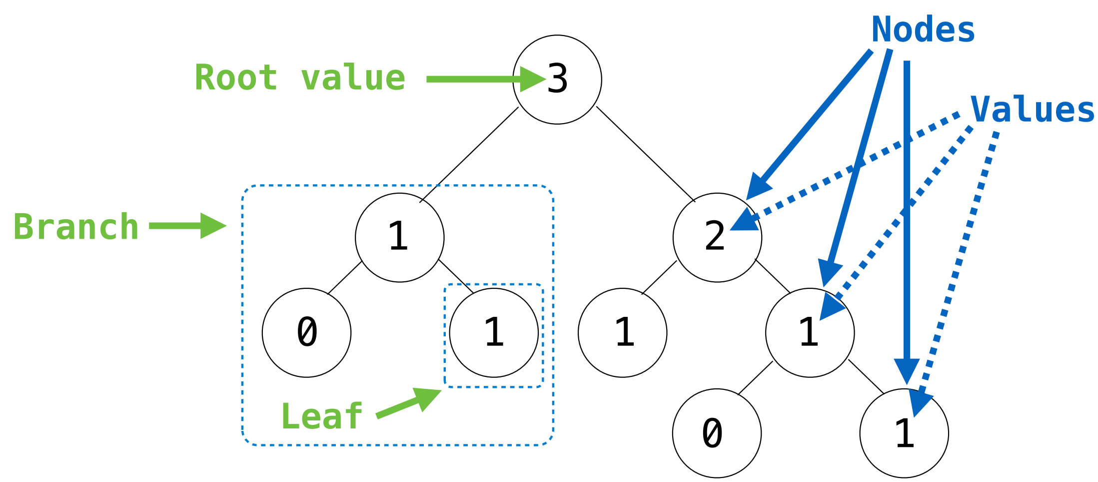

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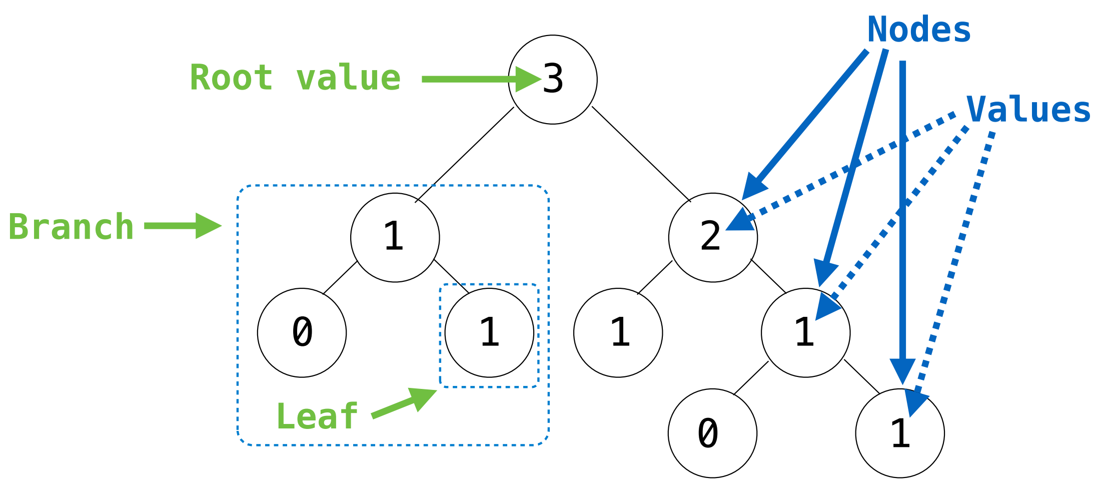

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People often refer to values by their locations: "each parent is the sum of its children"

Implementing the Tree Abstraction

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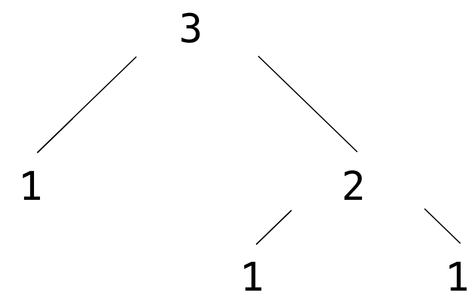

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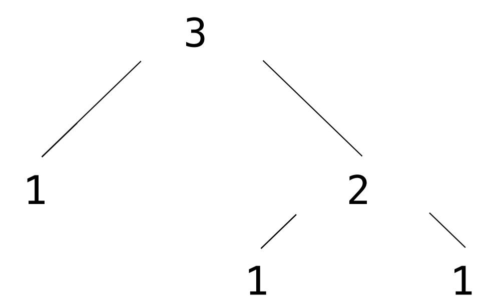


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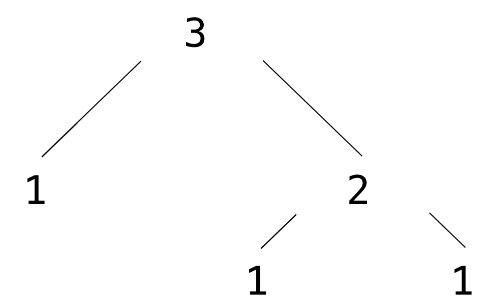


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def tree(root, branches=[]):
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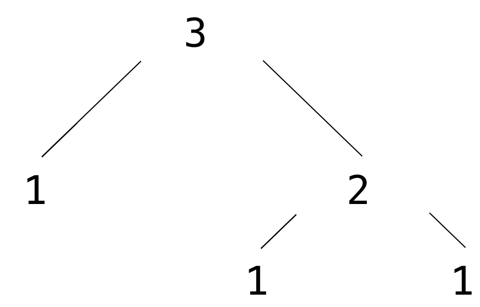


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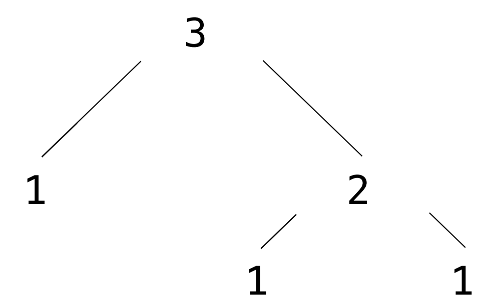

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def tree(root, branches=[]):
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def root(tree):
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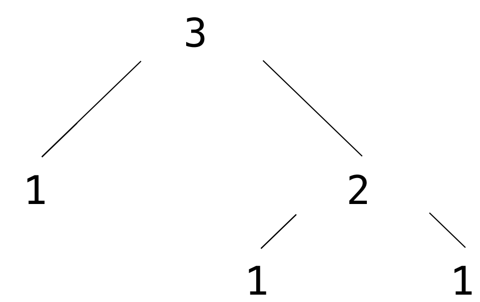


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def tree(root, branches=[]):
   return [root] + branches

def root(tree):
    return tree[0]
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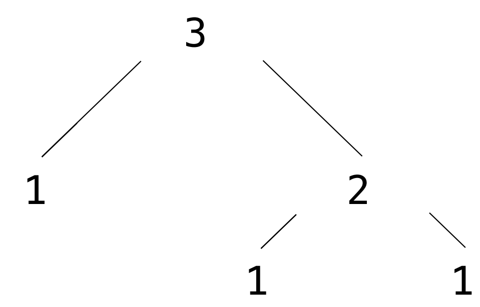


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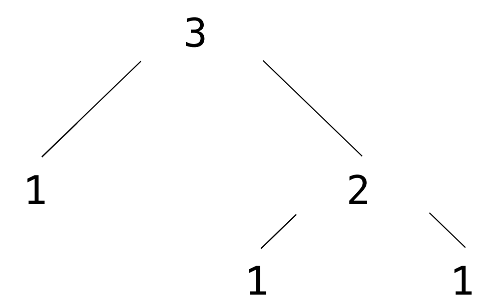


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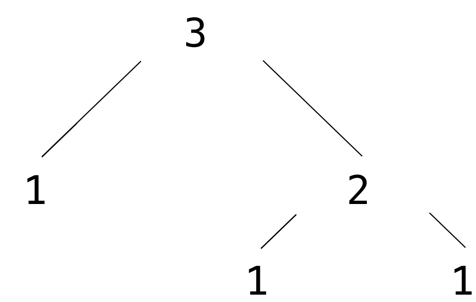


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Implementing the Tree Abstraction

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def tree(root, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [root] + list(branches)

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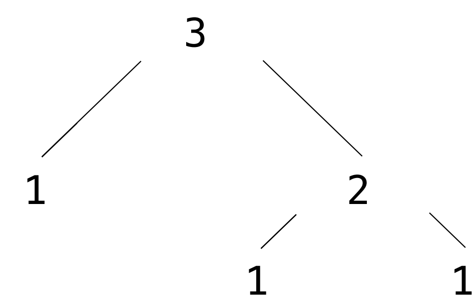


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Creates a list from a sequence of branches

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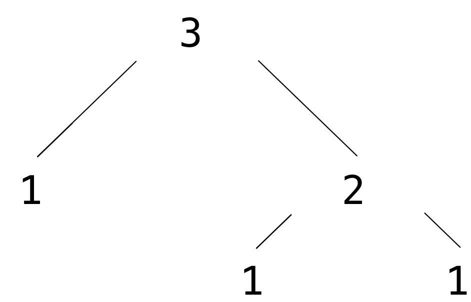


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Verifies the tree definition

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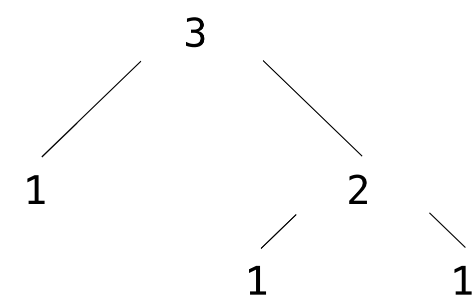


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def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
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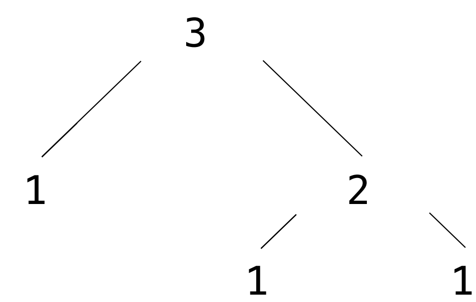


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Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

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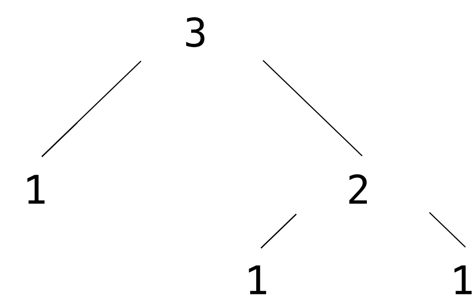


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def tree(root, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [root] + list(branches)

def root(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

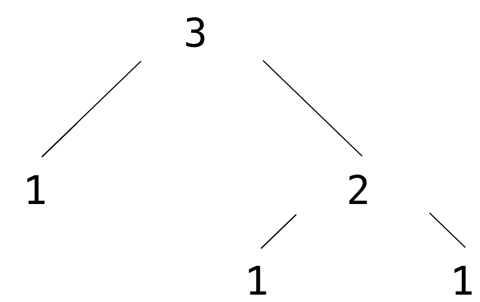
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

- A tree has a root value and a list of branches
- Each branch is a tree

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def is_leaf(tree):
    return not branches(tree)
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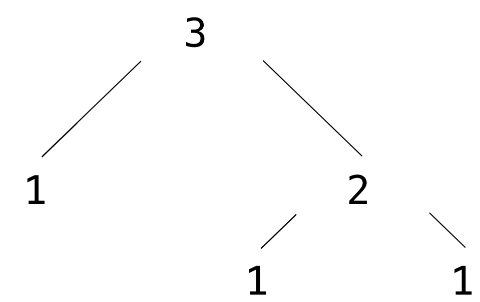


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(Demo)

Tree Processing

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def leaves(tree):
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    >>> leaves(fib_tree(5))
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Example: Printing Trees

(Demo)