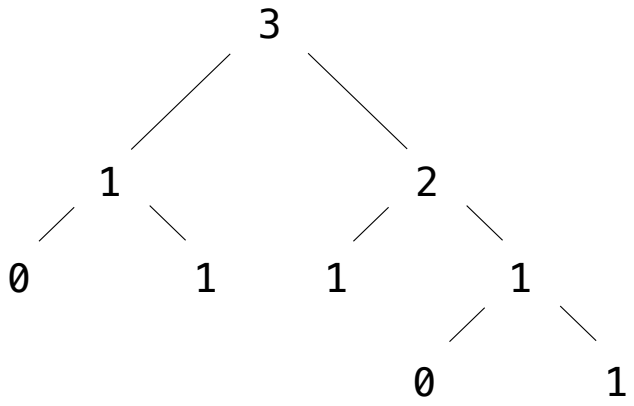
# 61A Lecture 19

# Announcements

# Tree Class

# Tree Review



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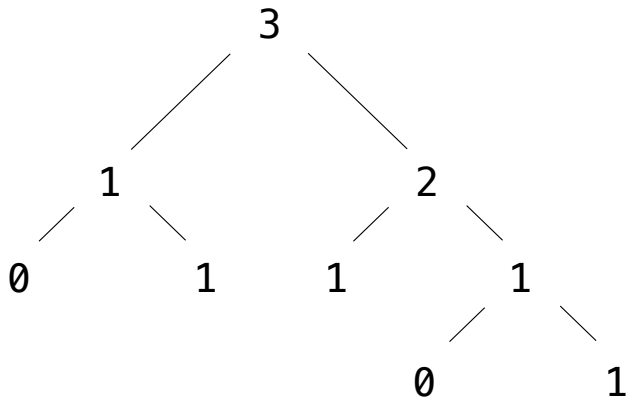


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**Relative description (family trees):**

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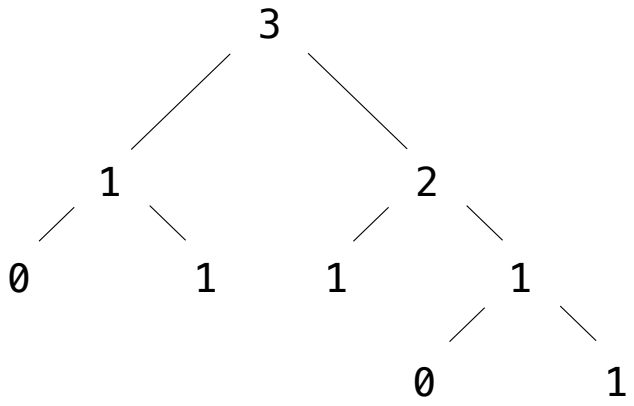


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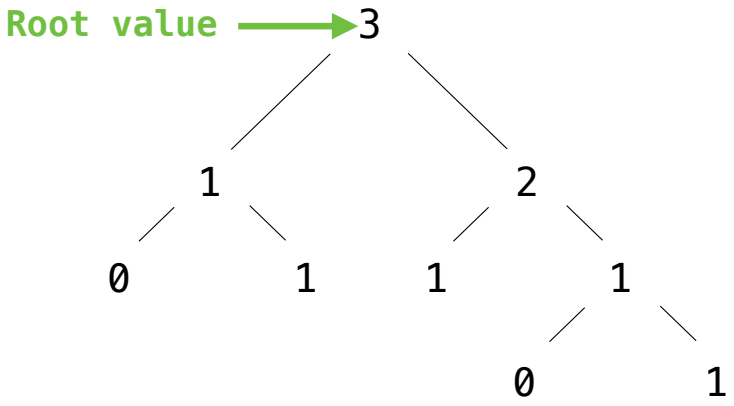


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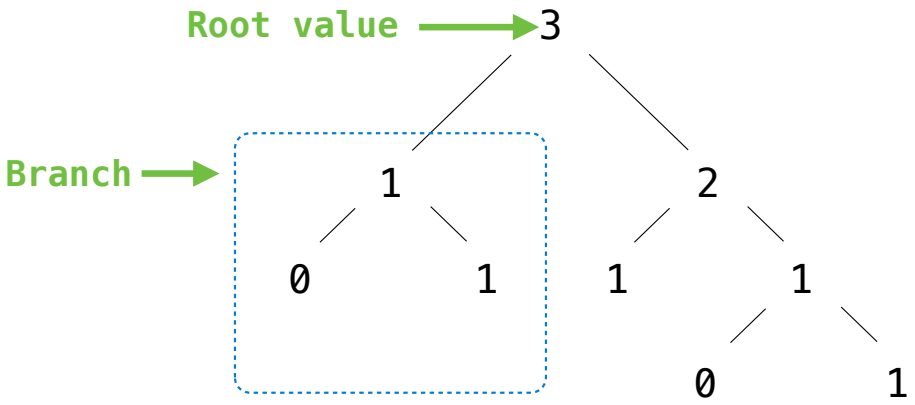


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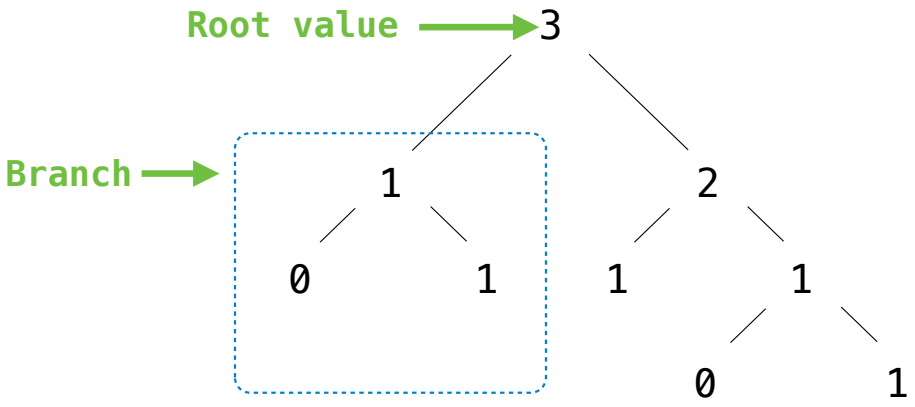


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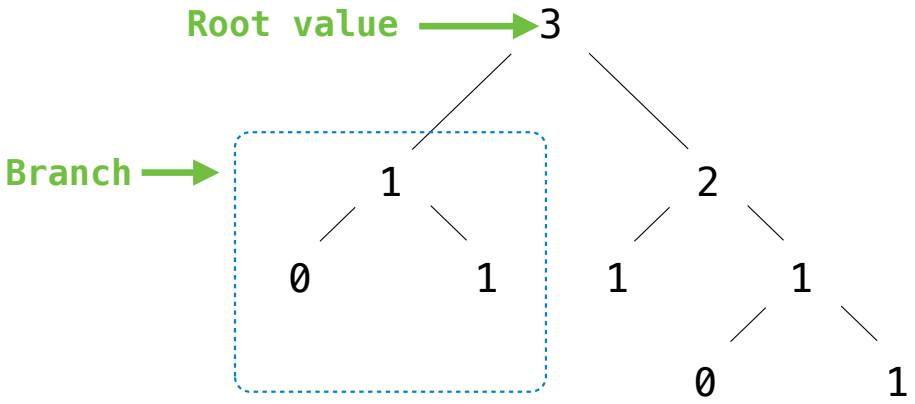


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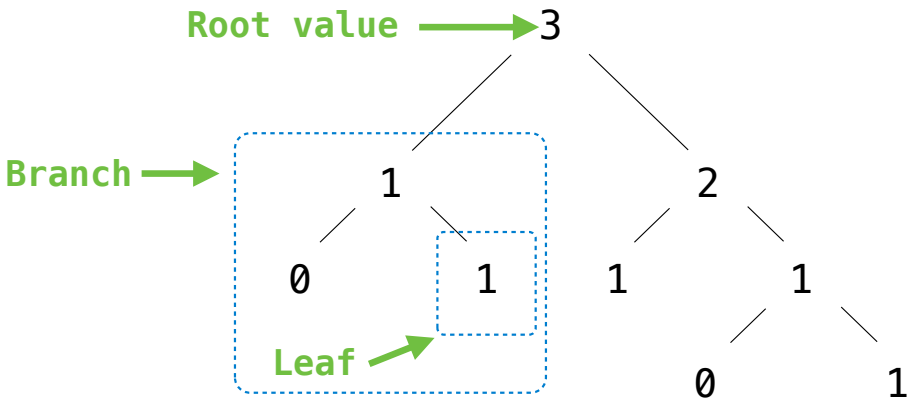


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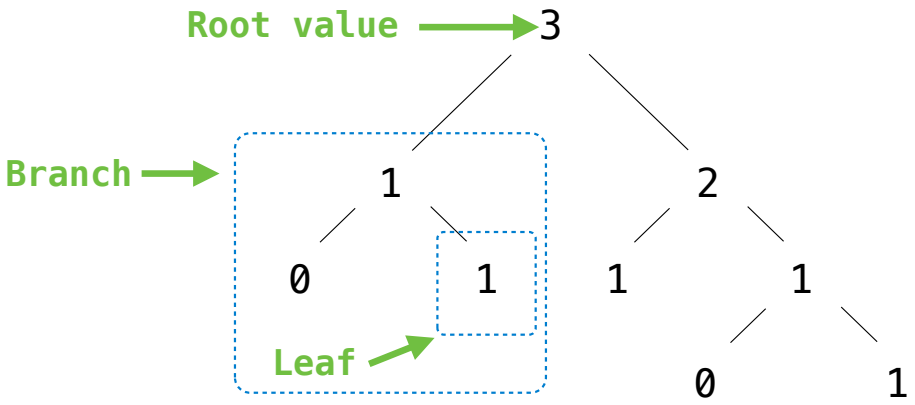


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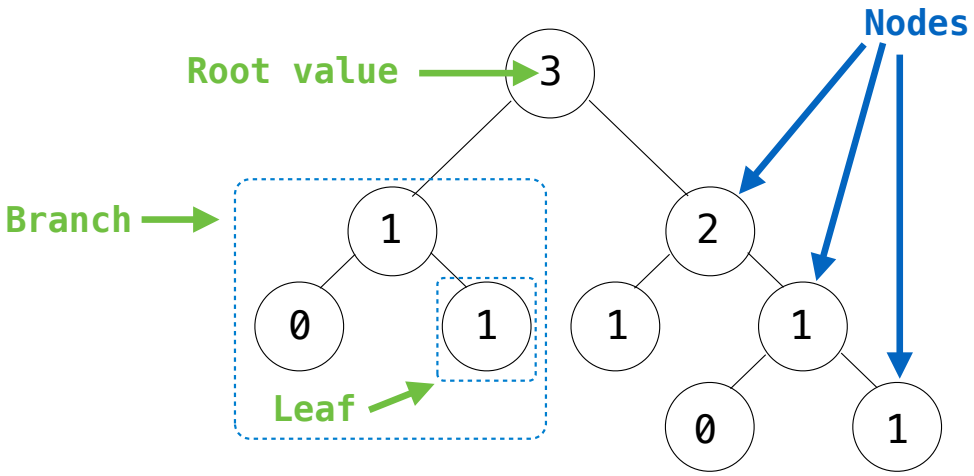


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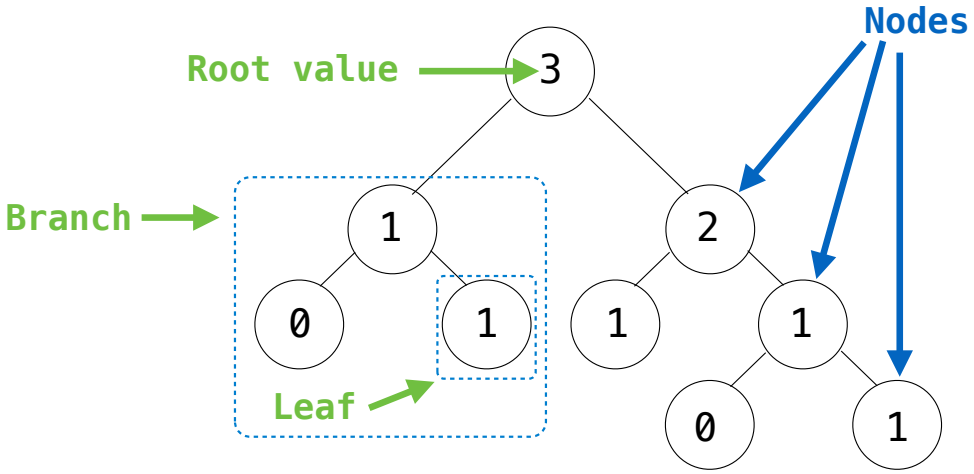


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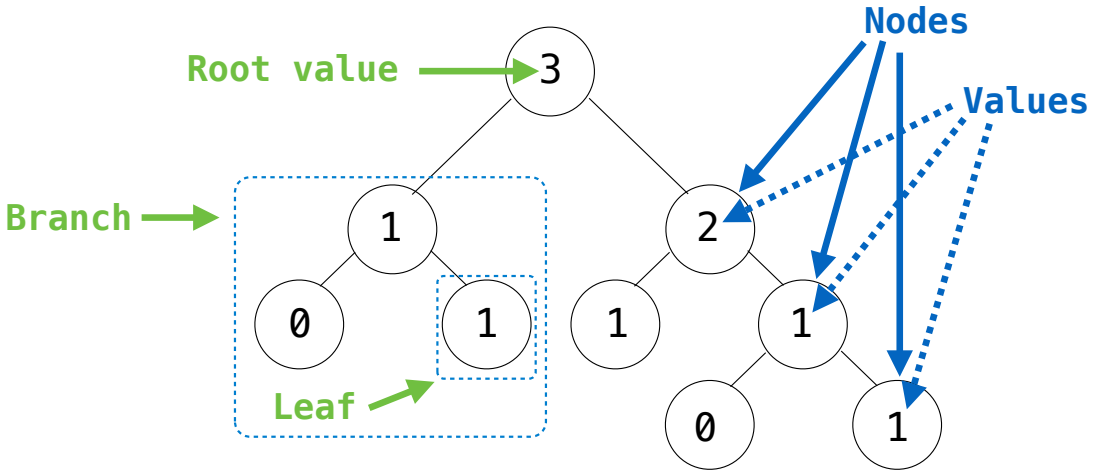


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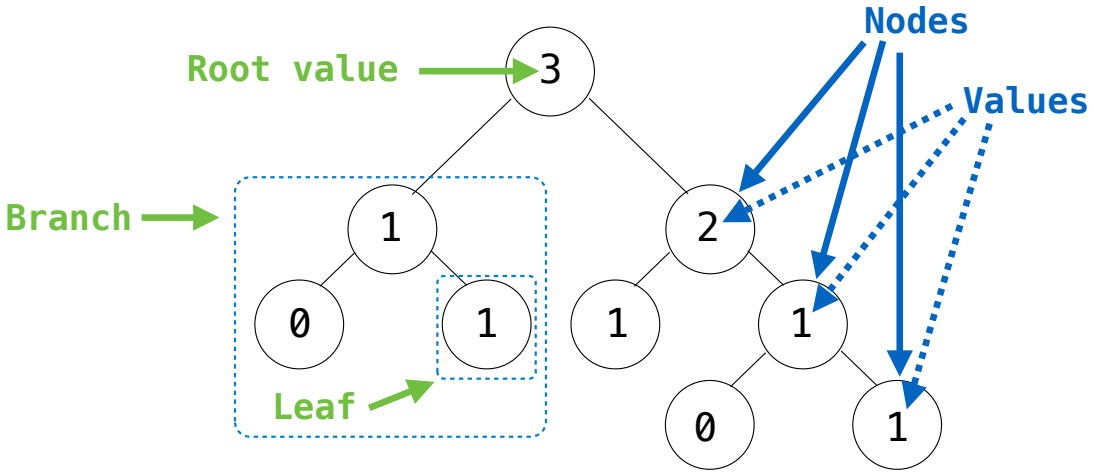


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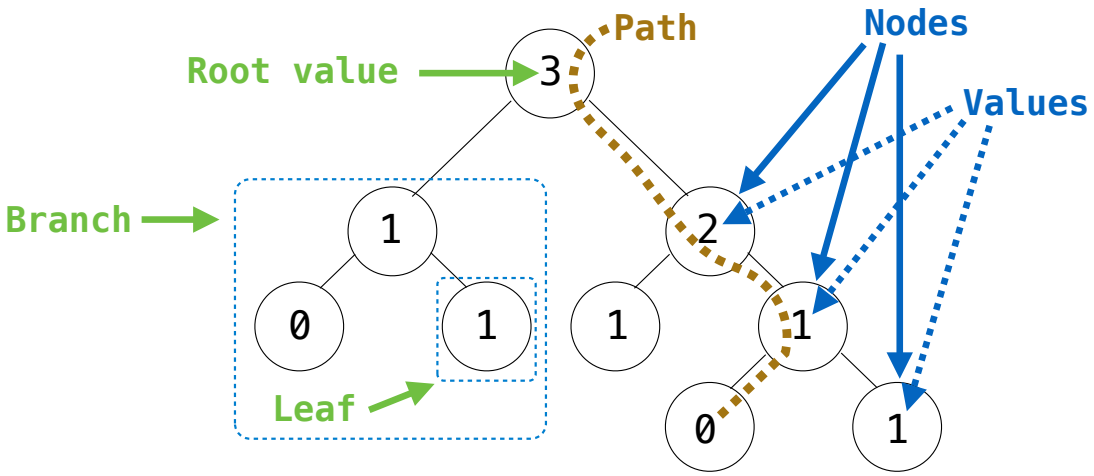


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def tree(root, branches=[]):
    for branch in branches:
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    return [root] + list(branches)
def root(tree):
    return tree[0]
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(Demo)

# Tree Mutation

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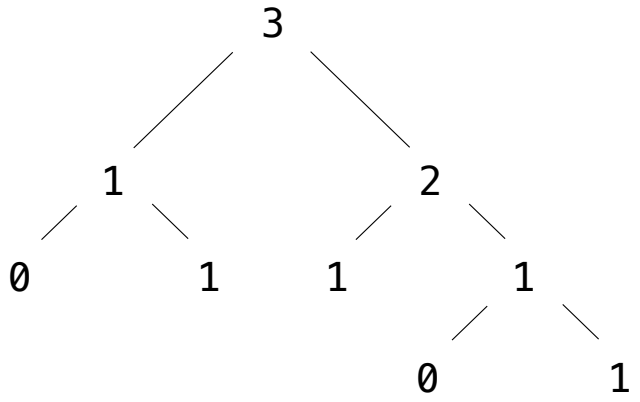

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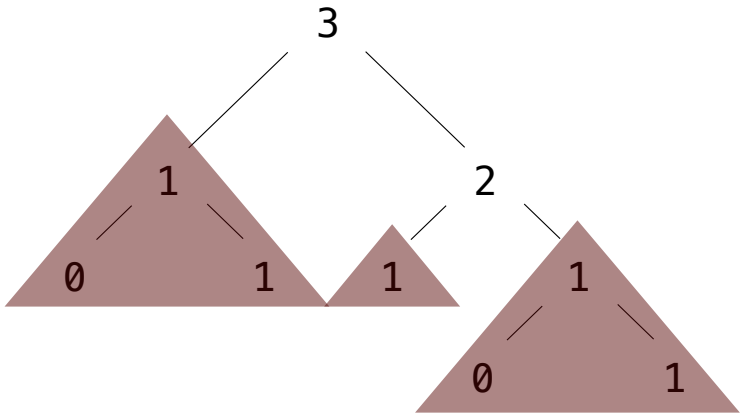

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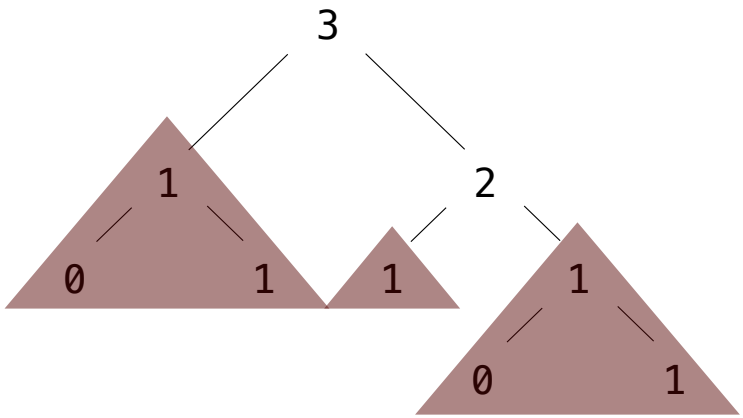


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def prune(t, n):
    """Prune sub-trees whose root value is n."""
    t.branches = [_____________ for b in t.branches if ____________________]
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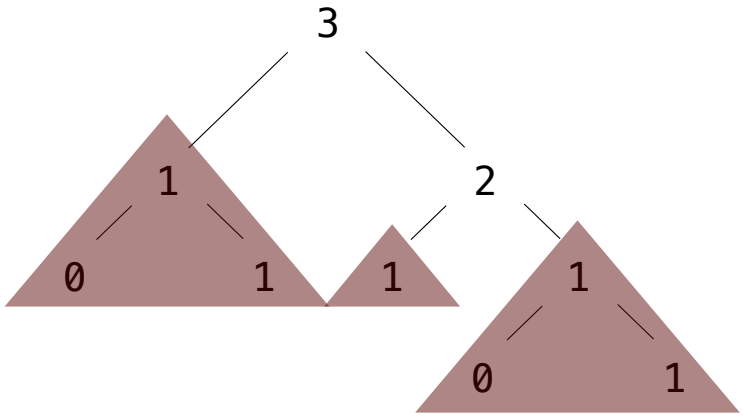


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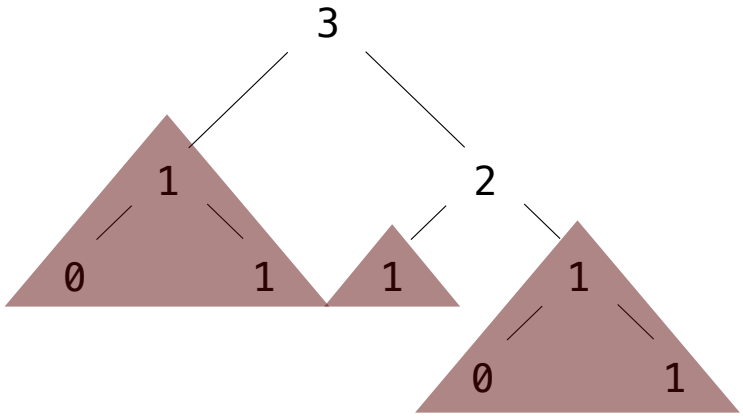


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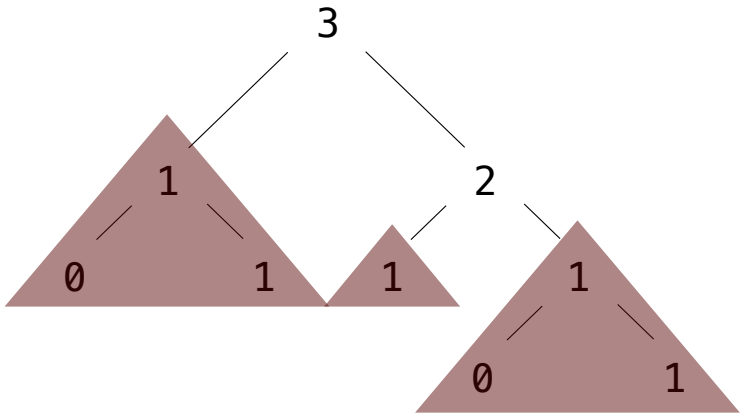


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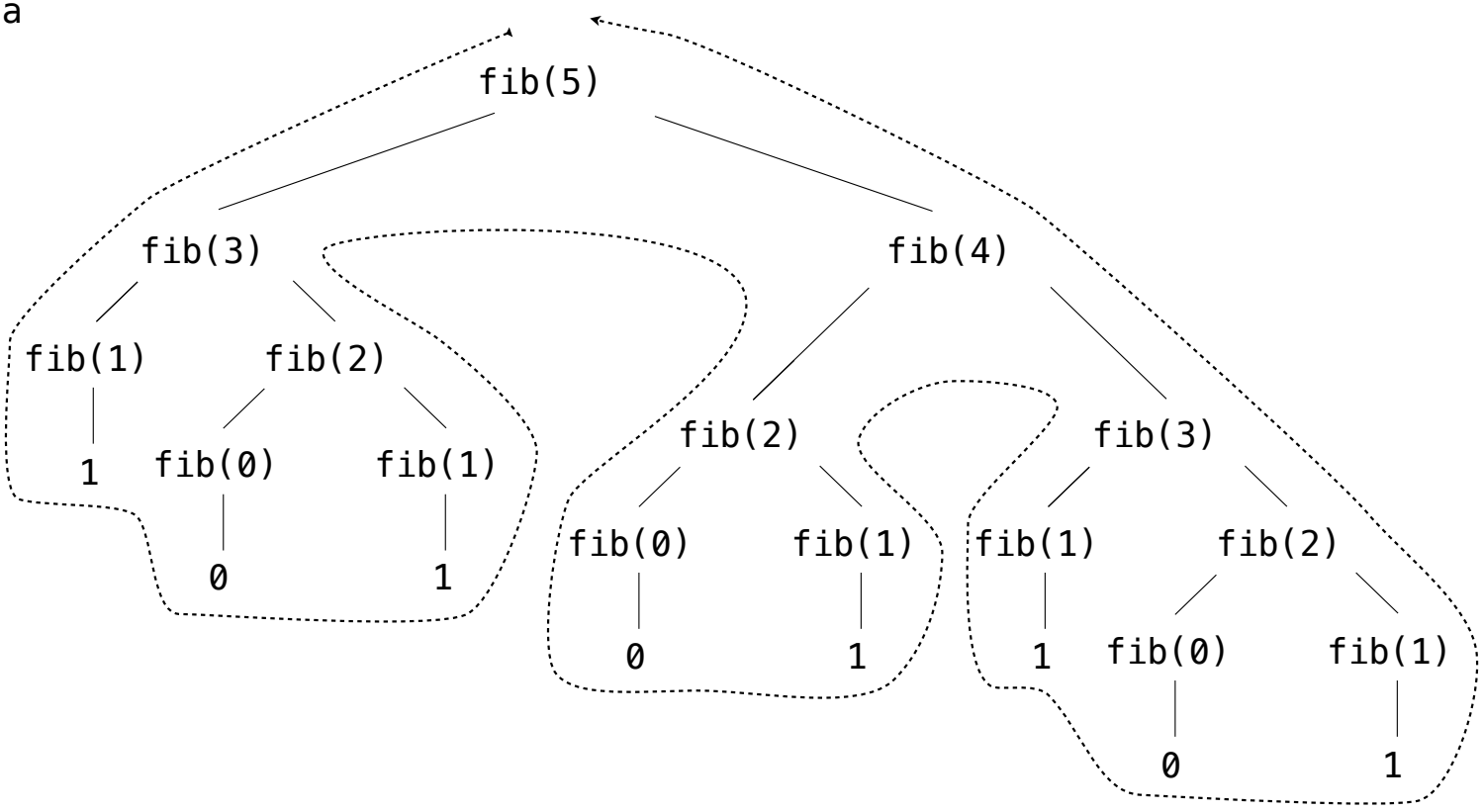

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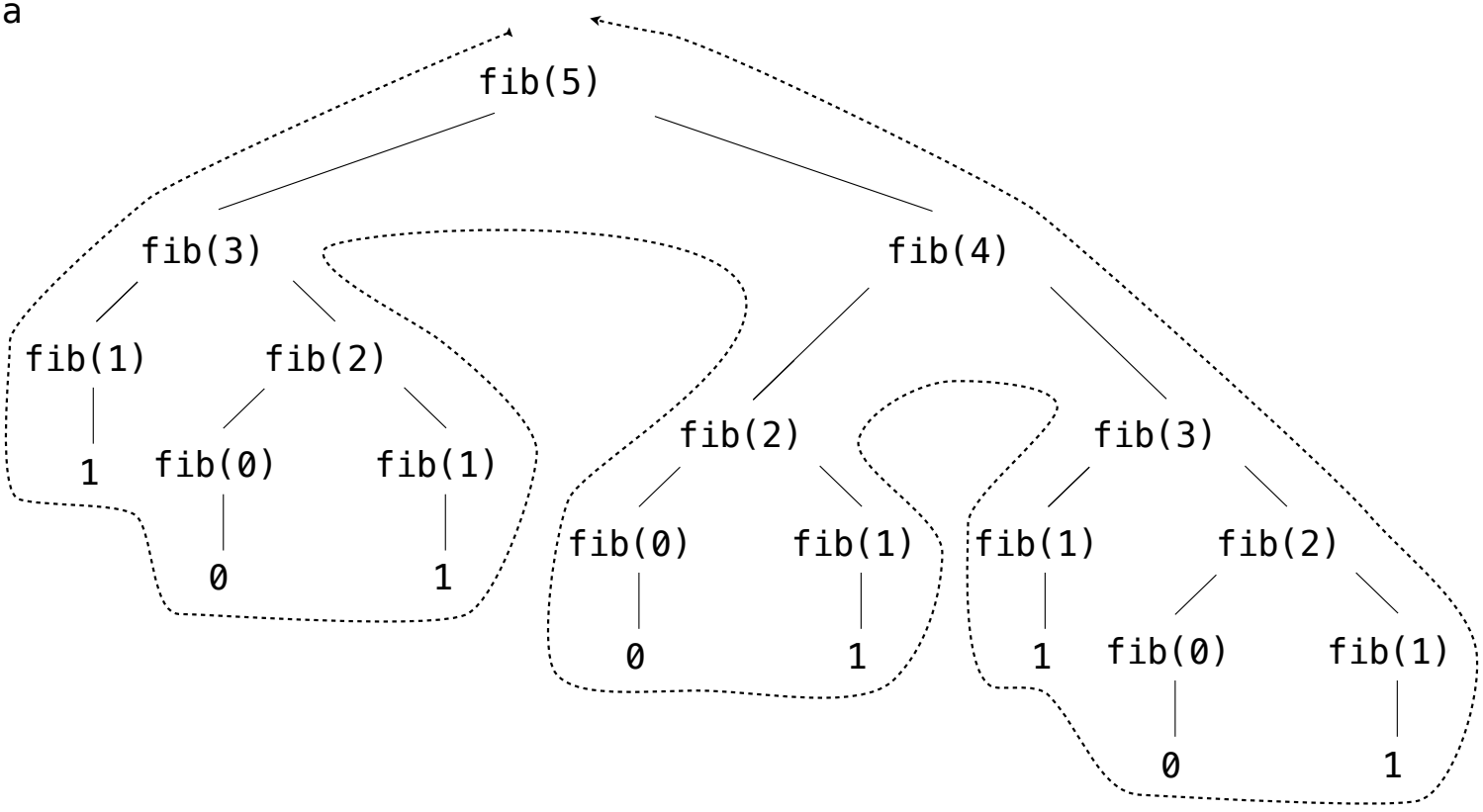

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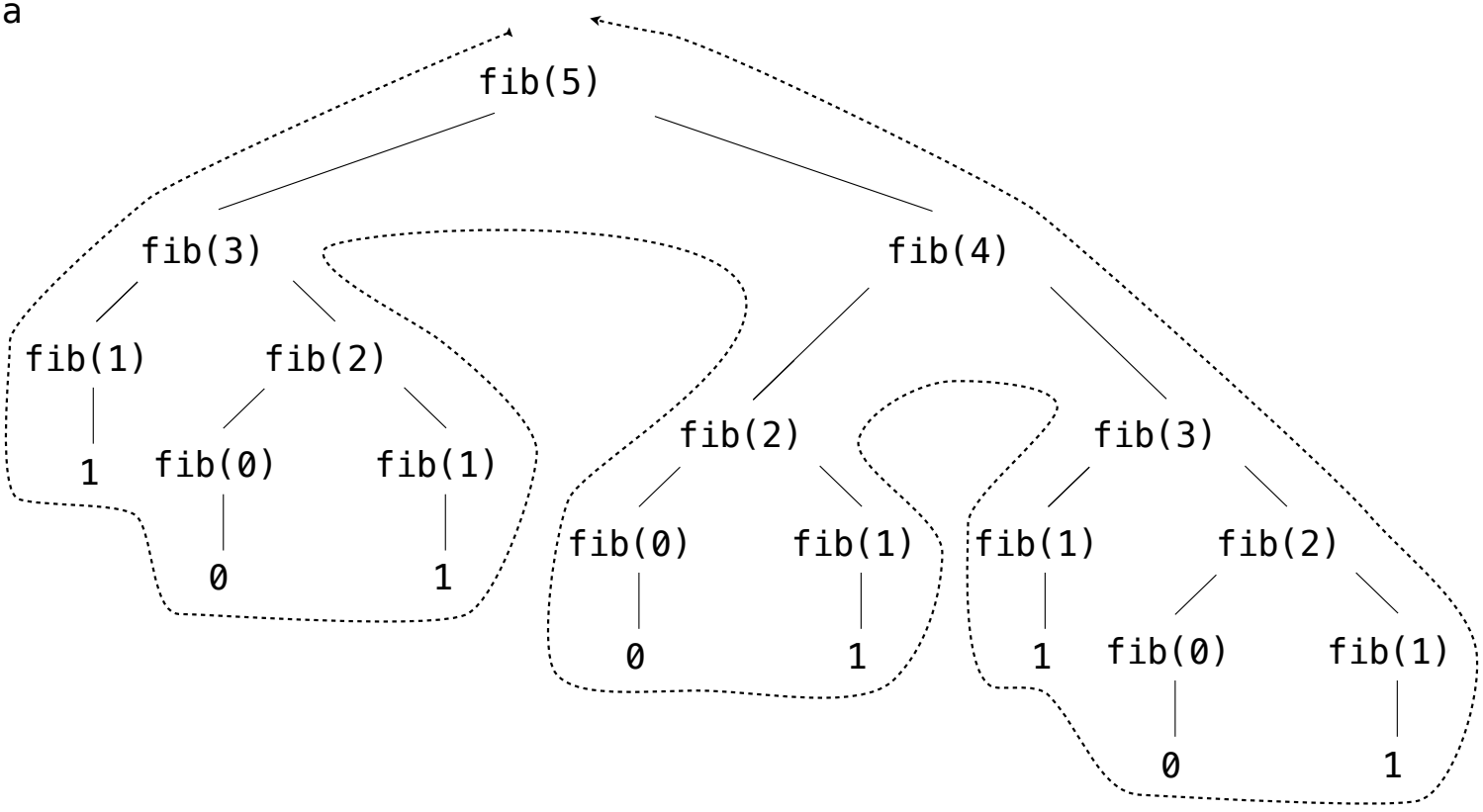

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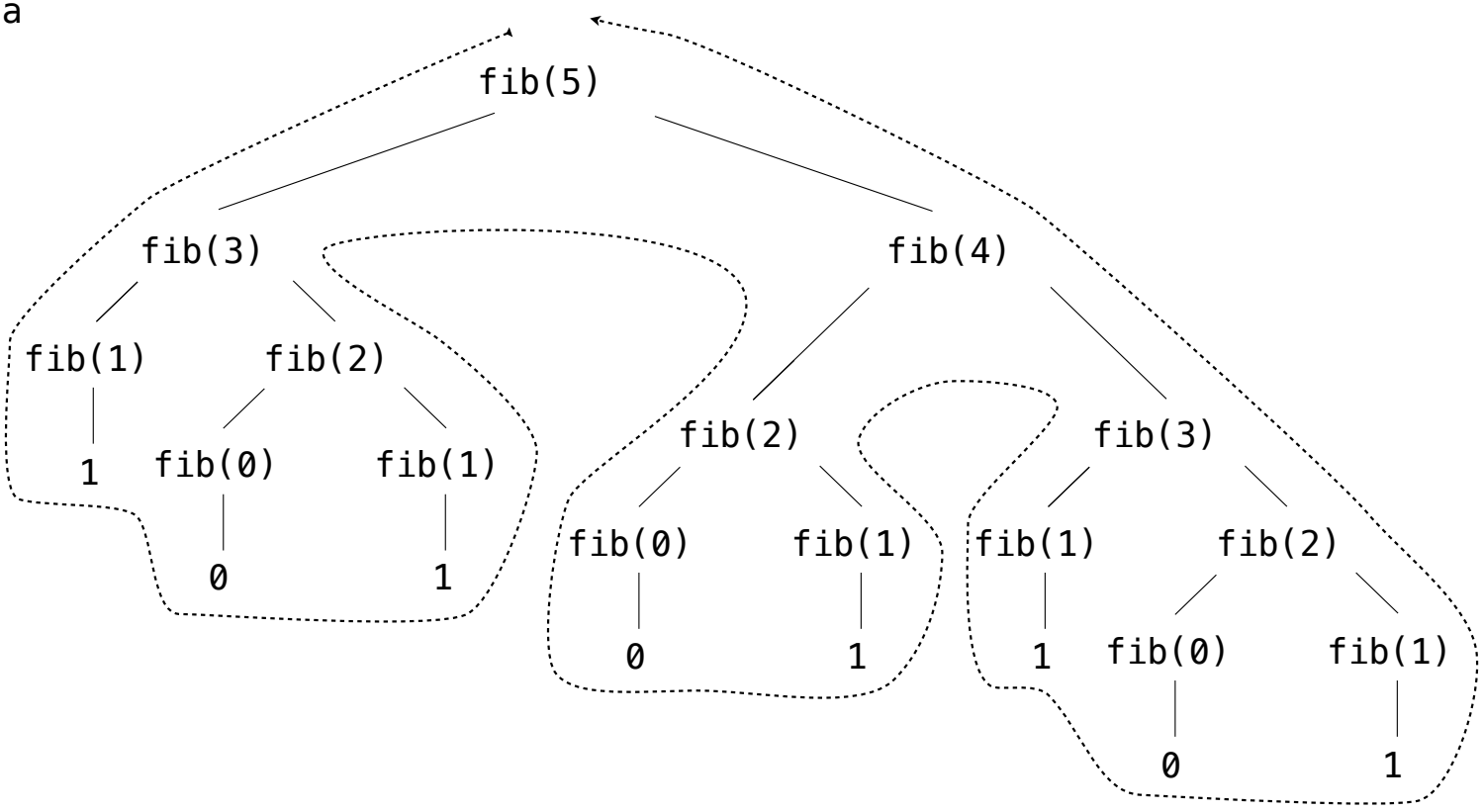

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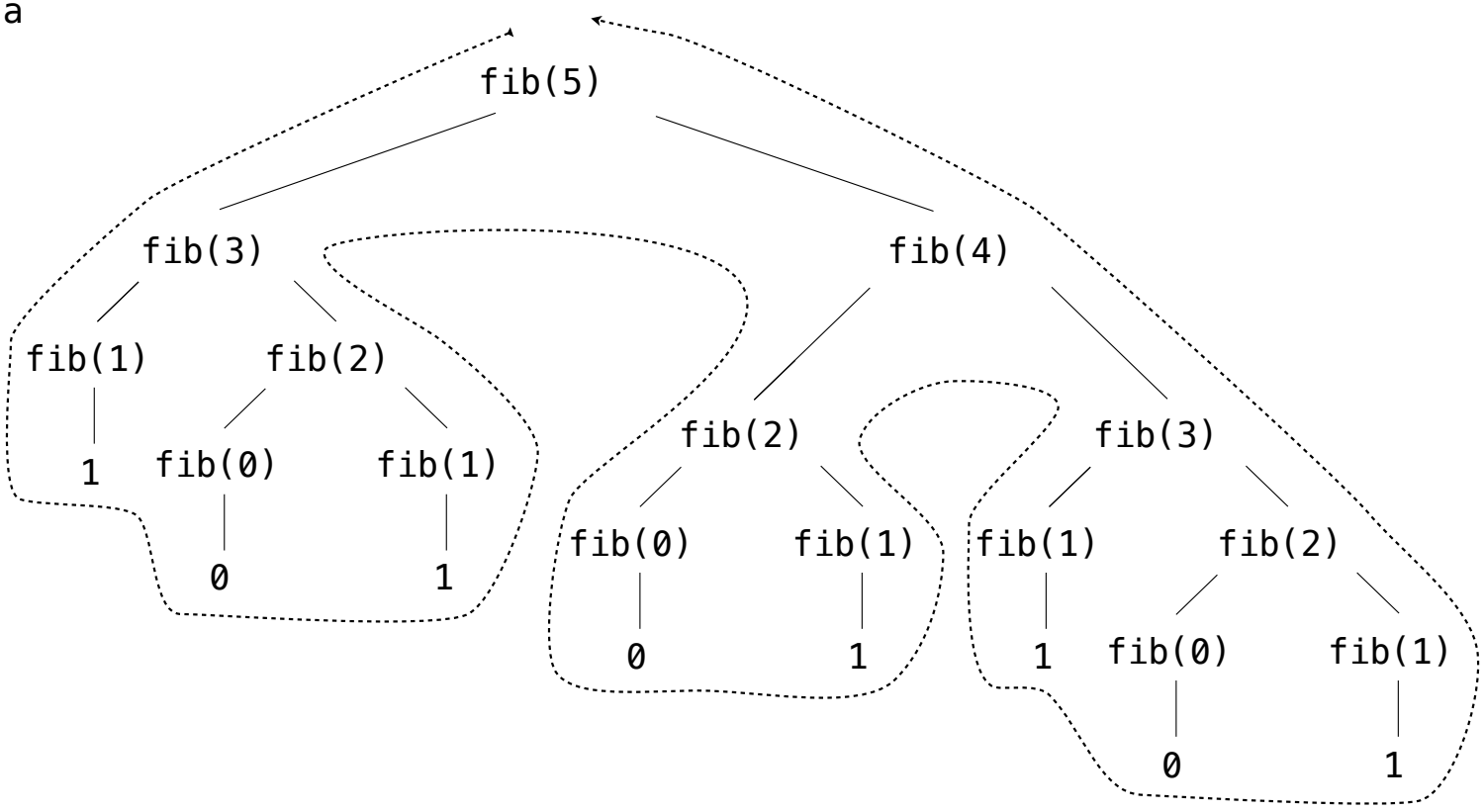

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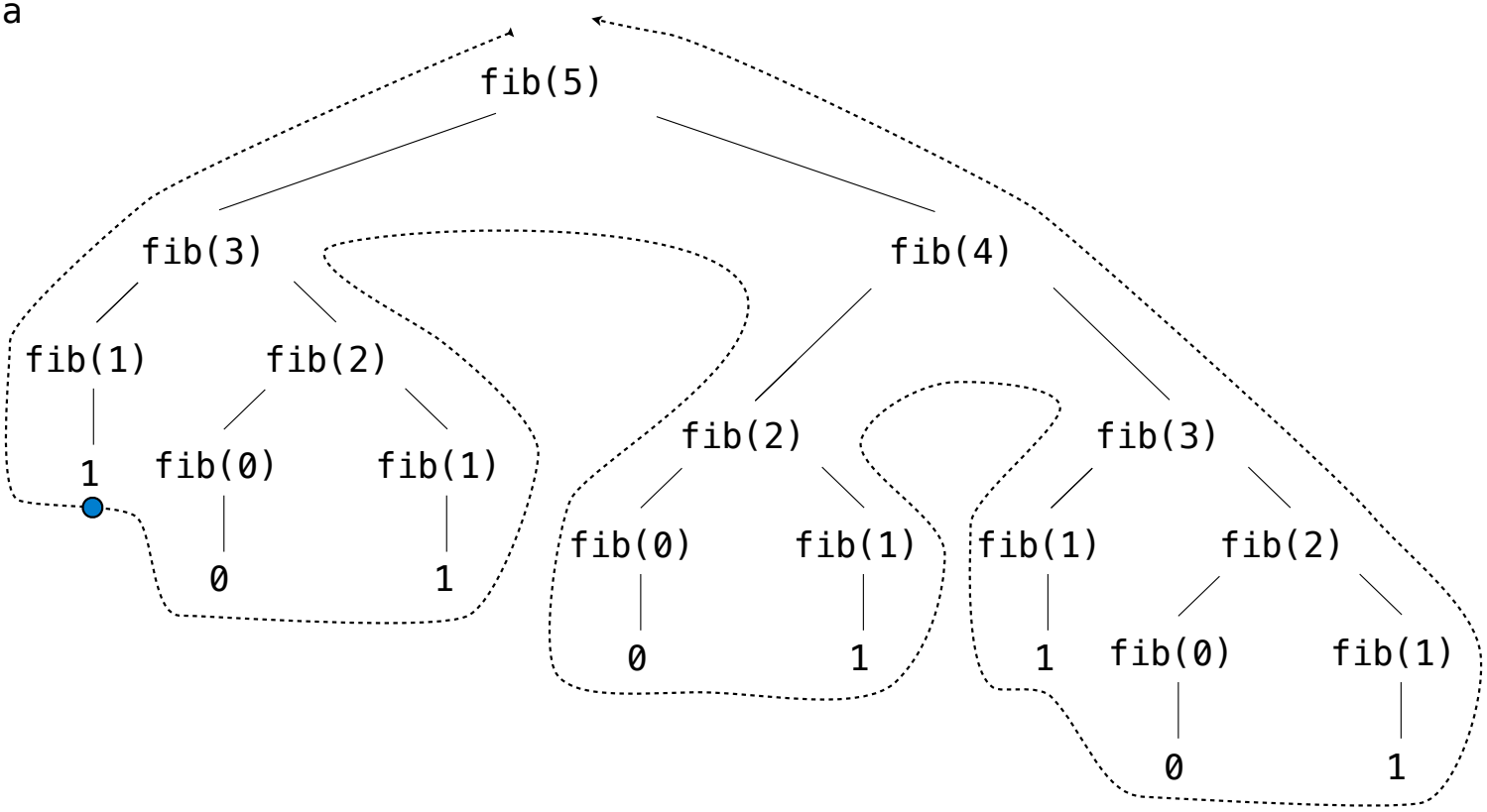

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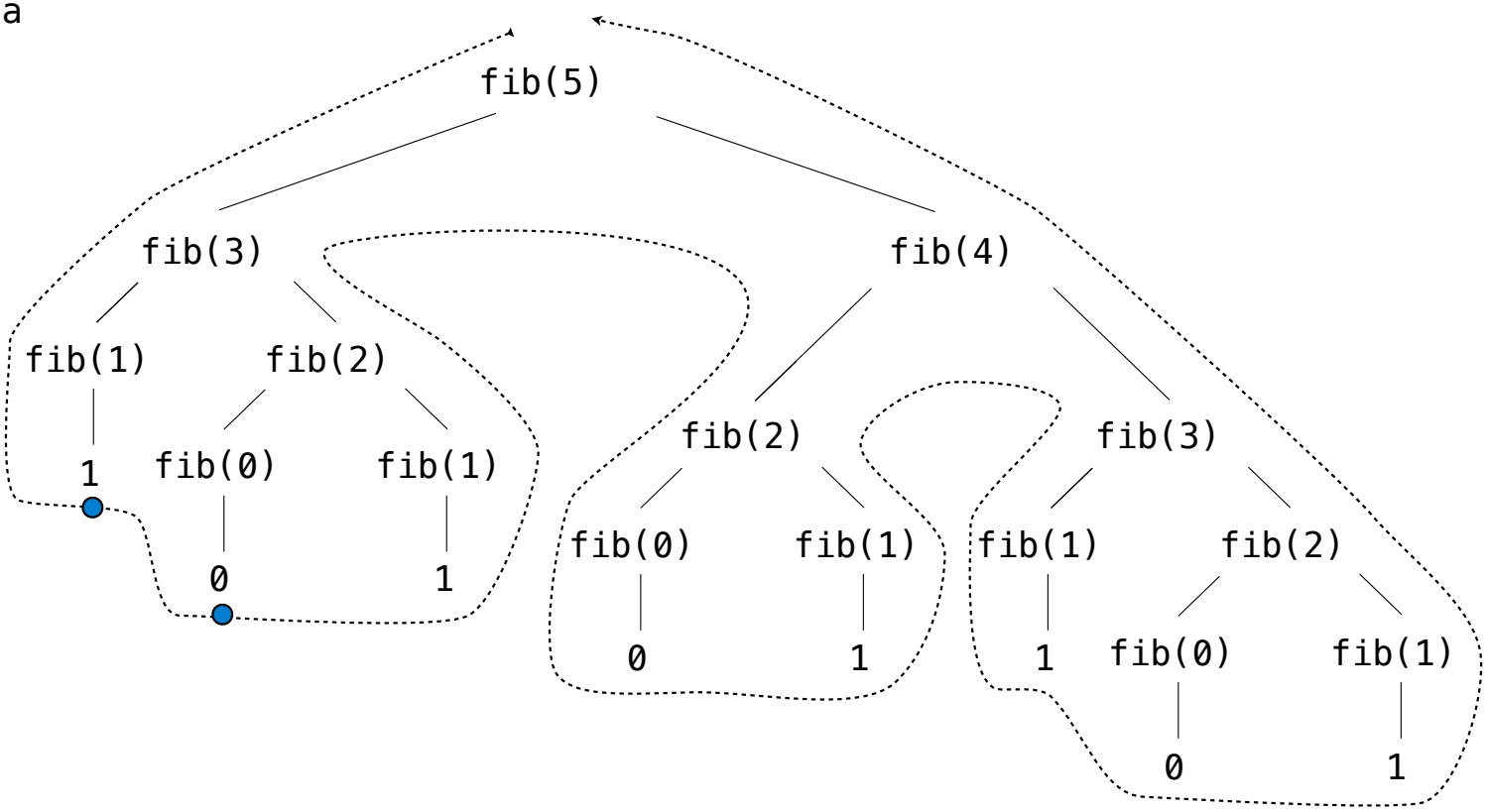

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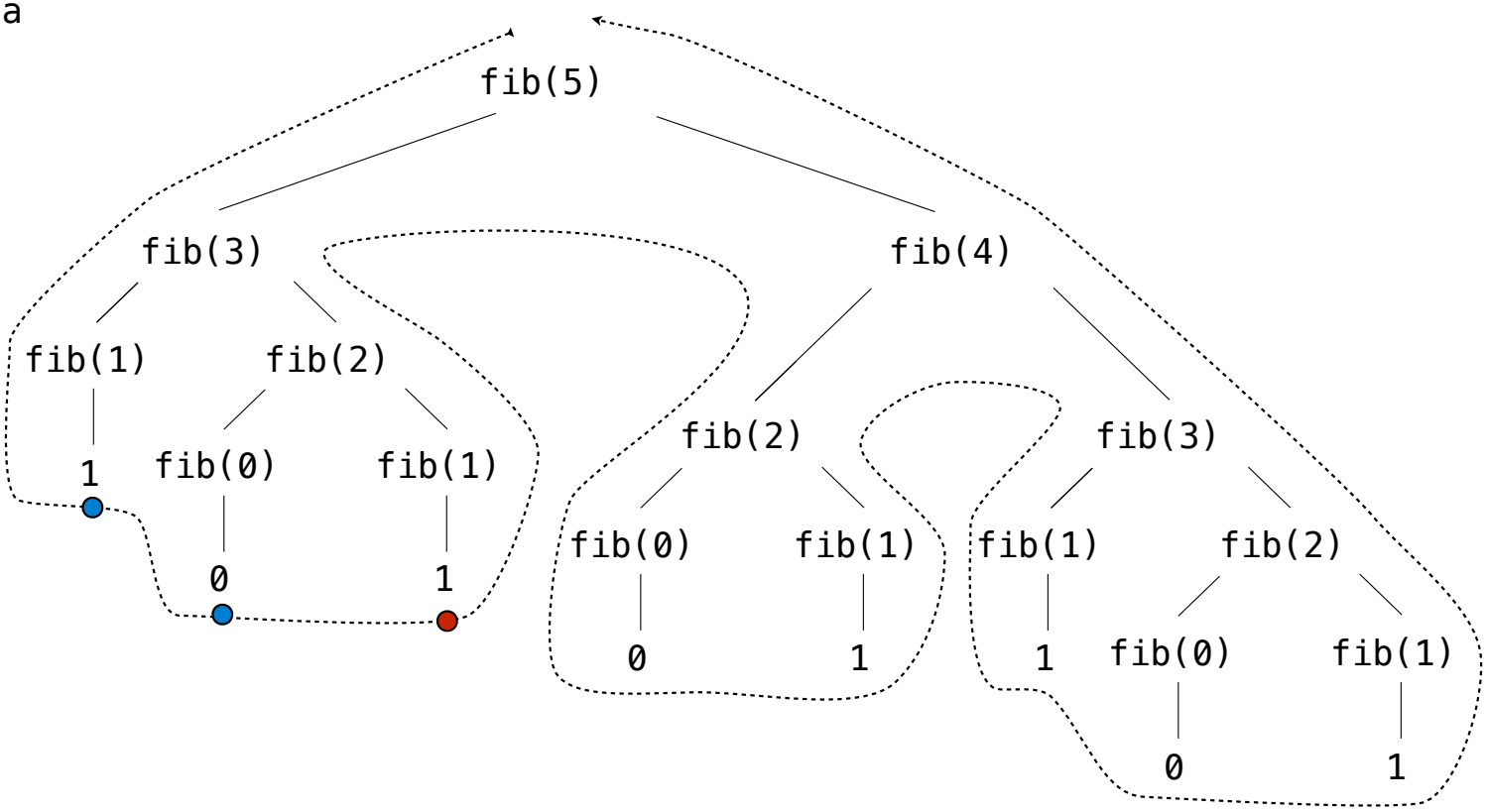

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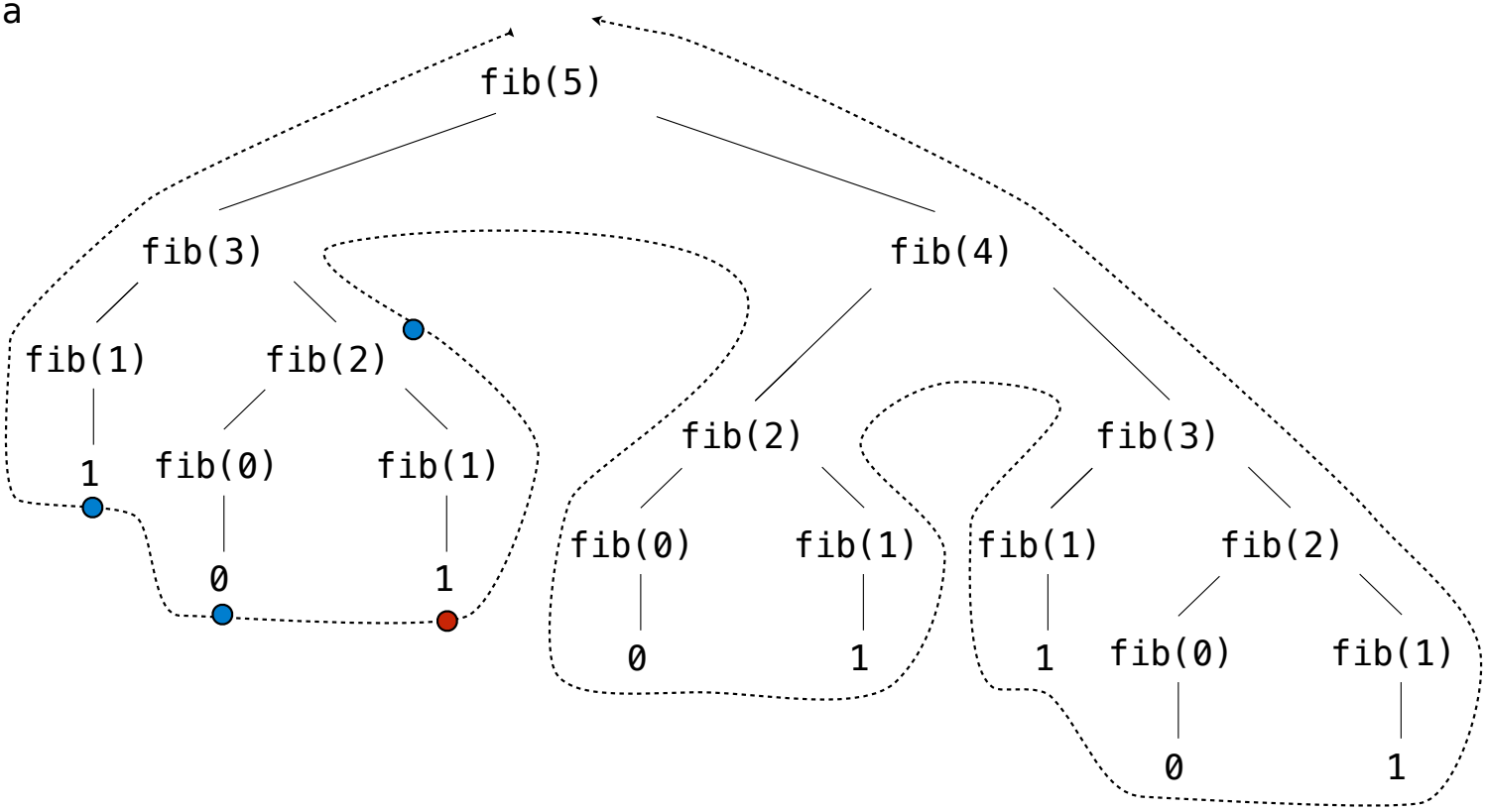

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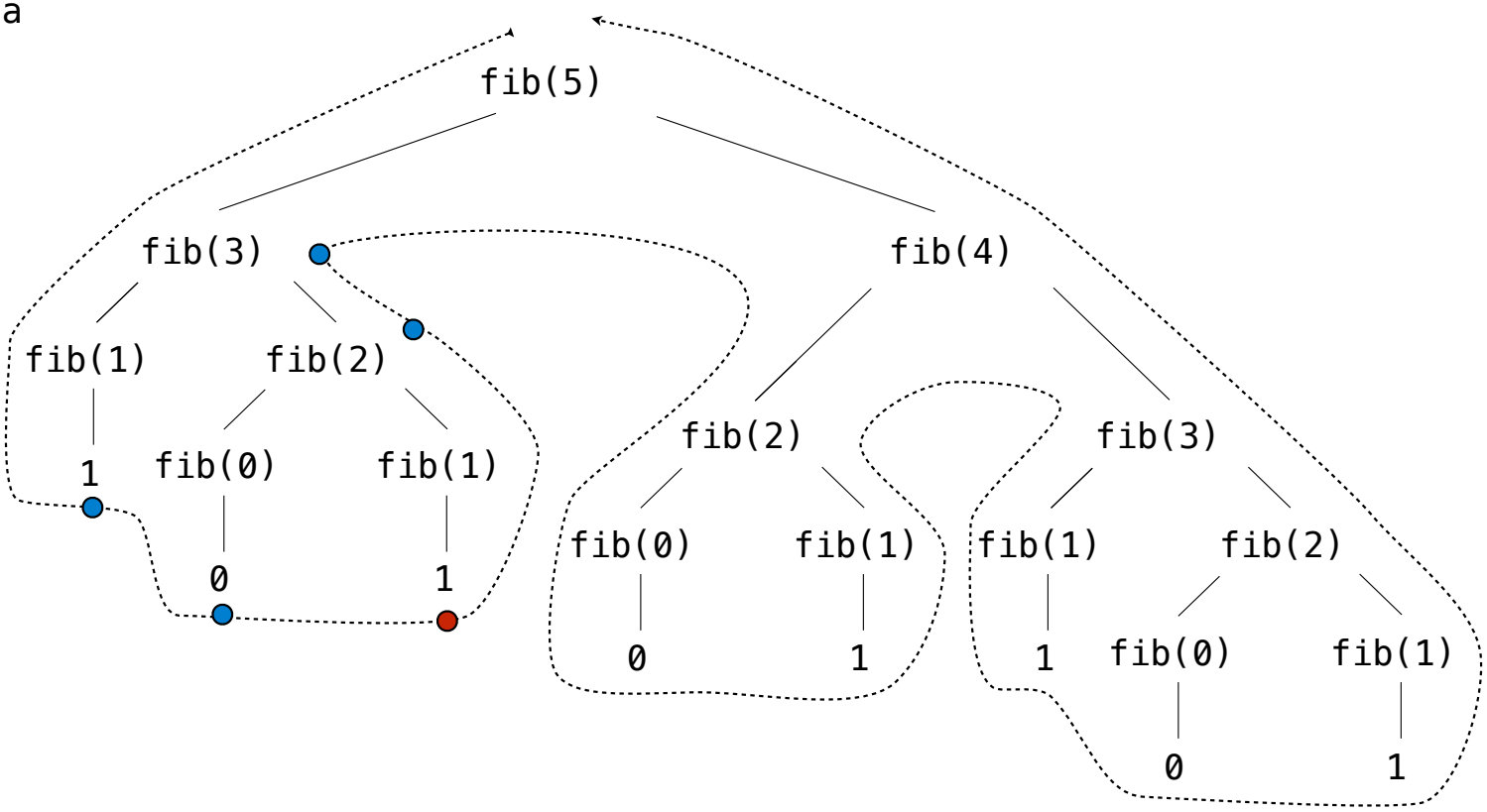

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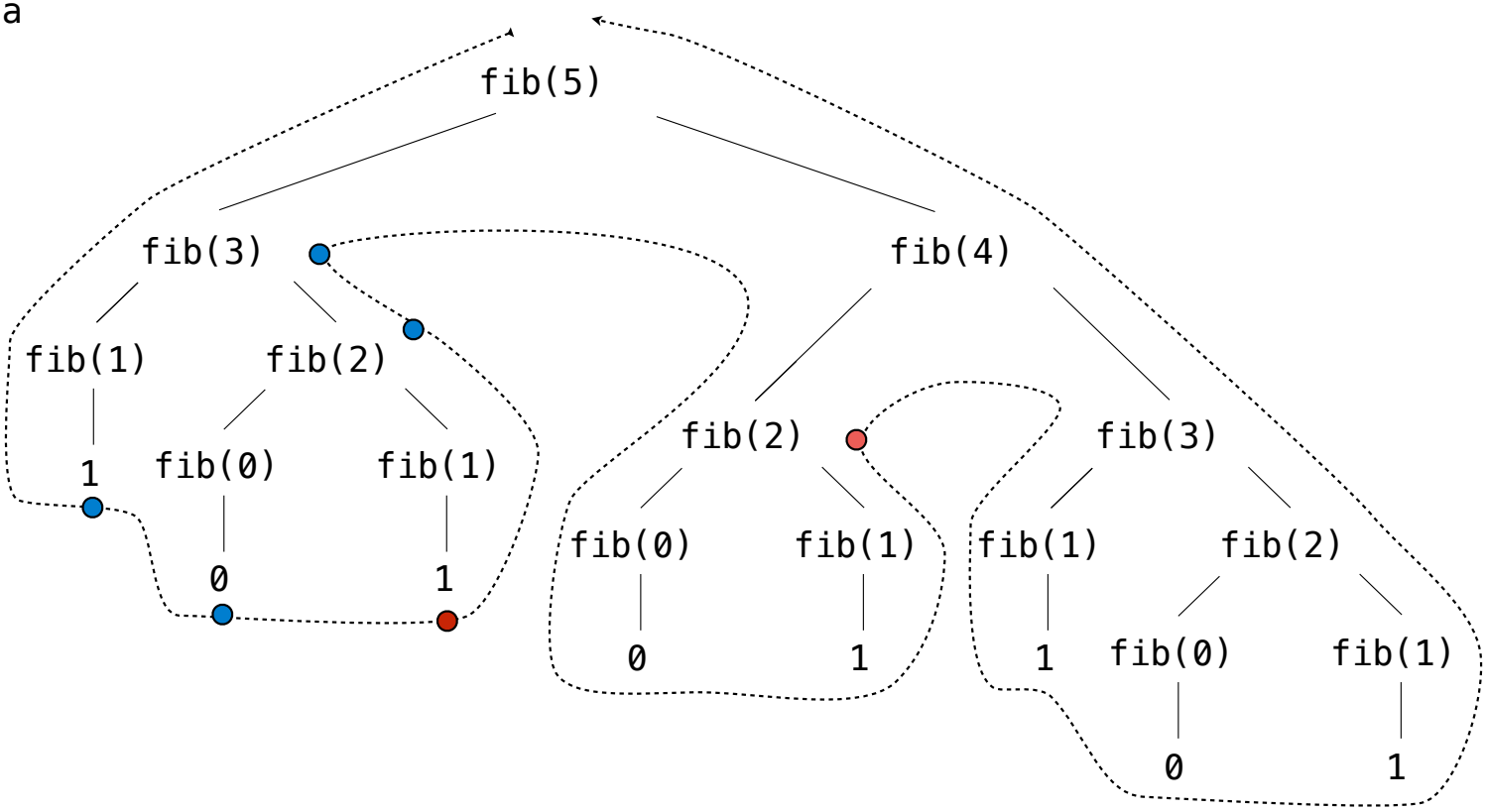

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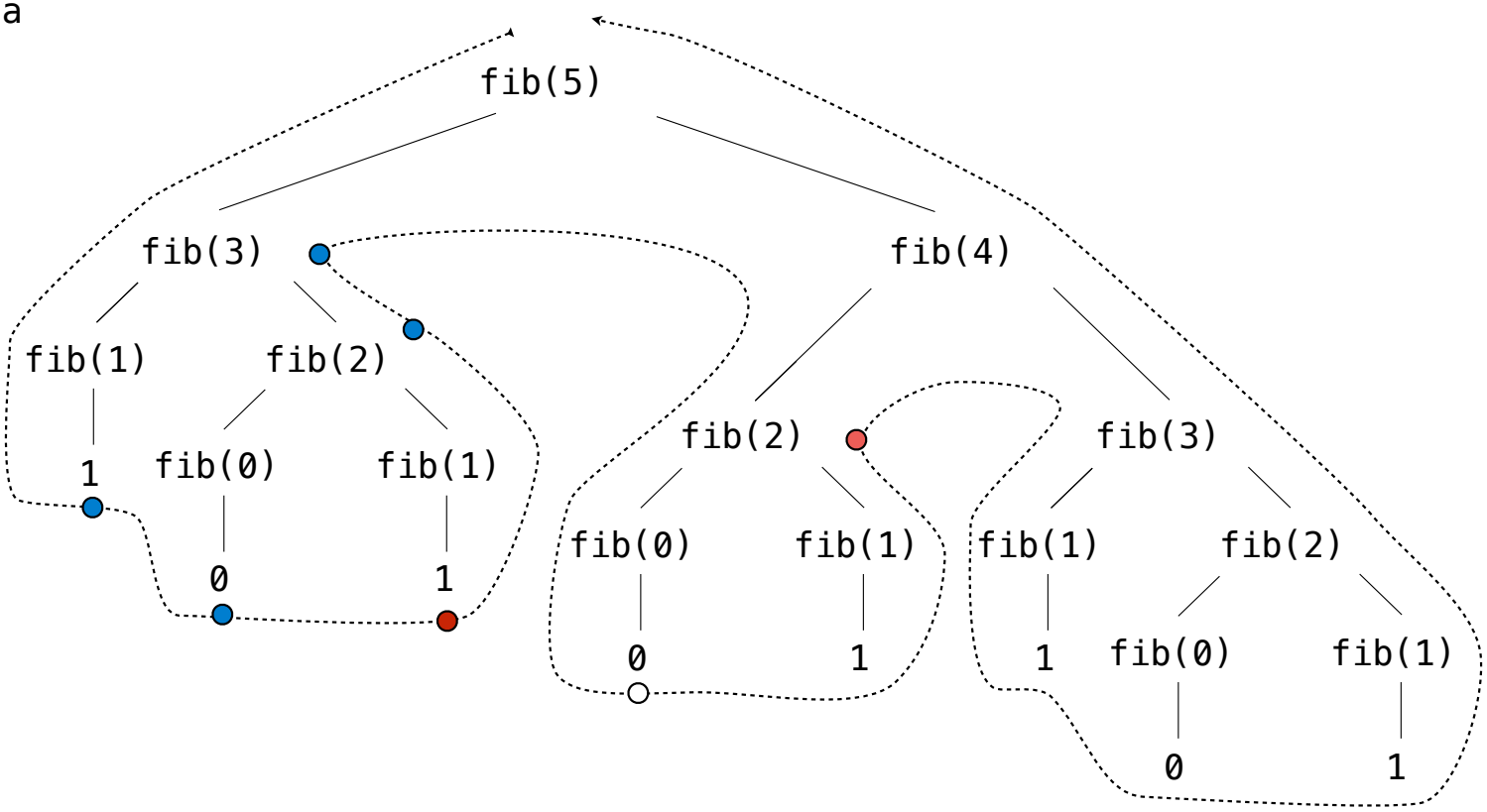

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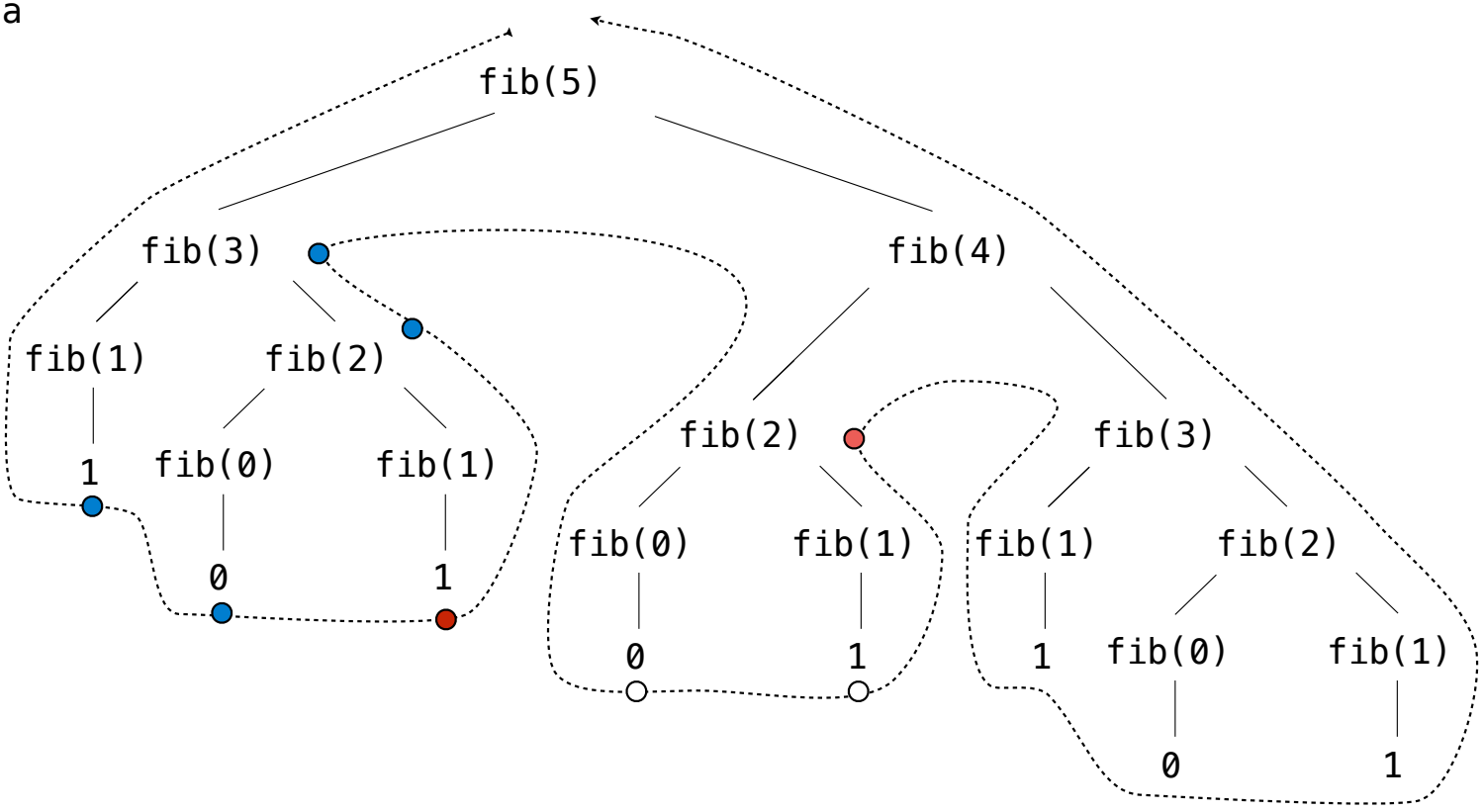

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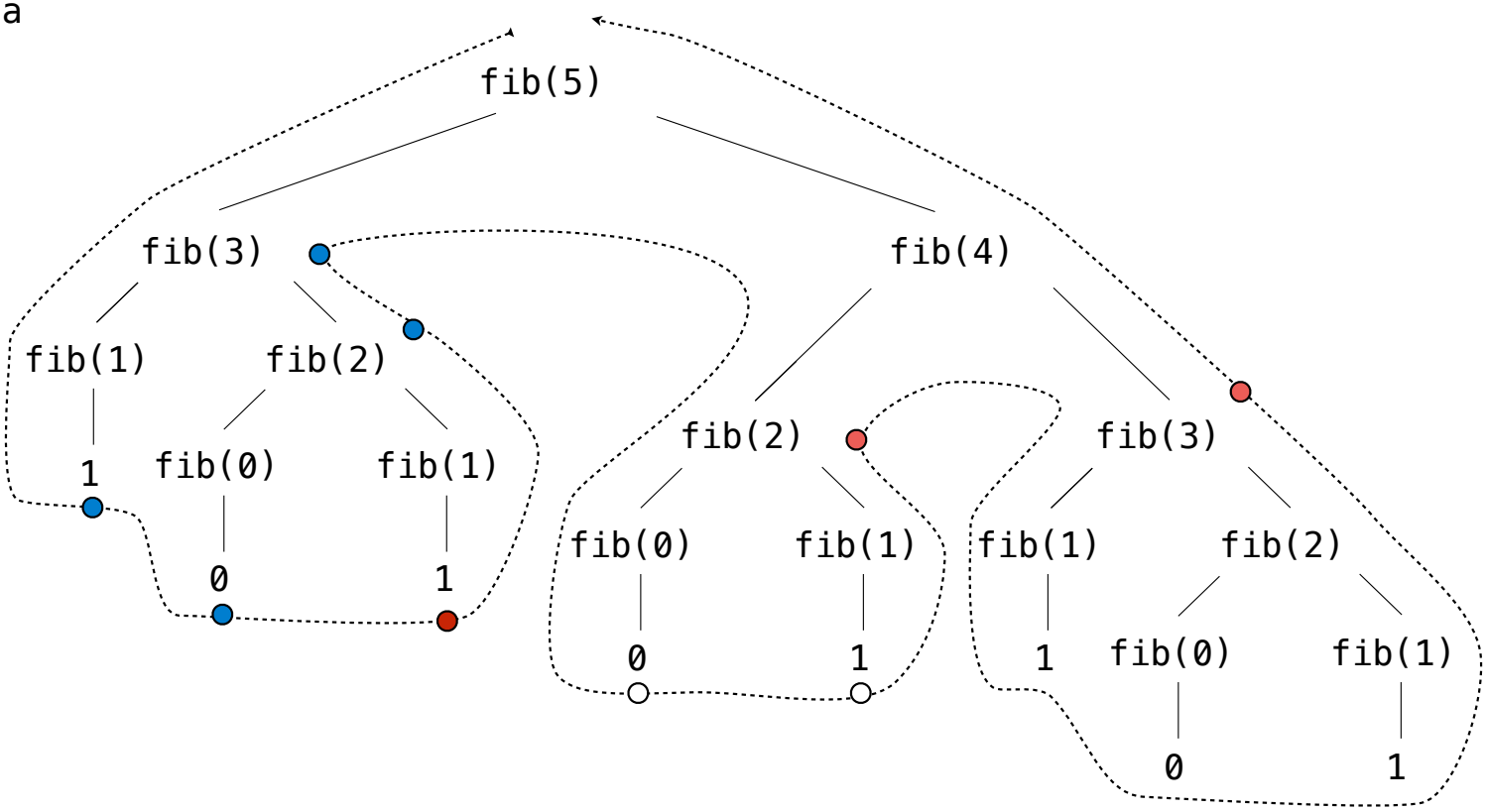

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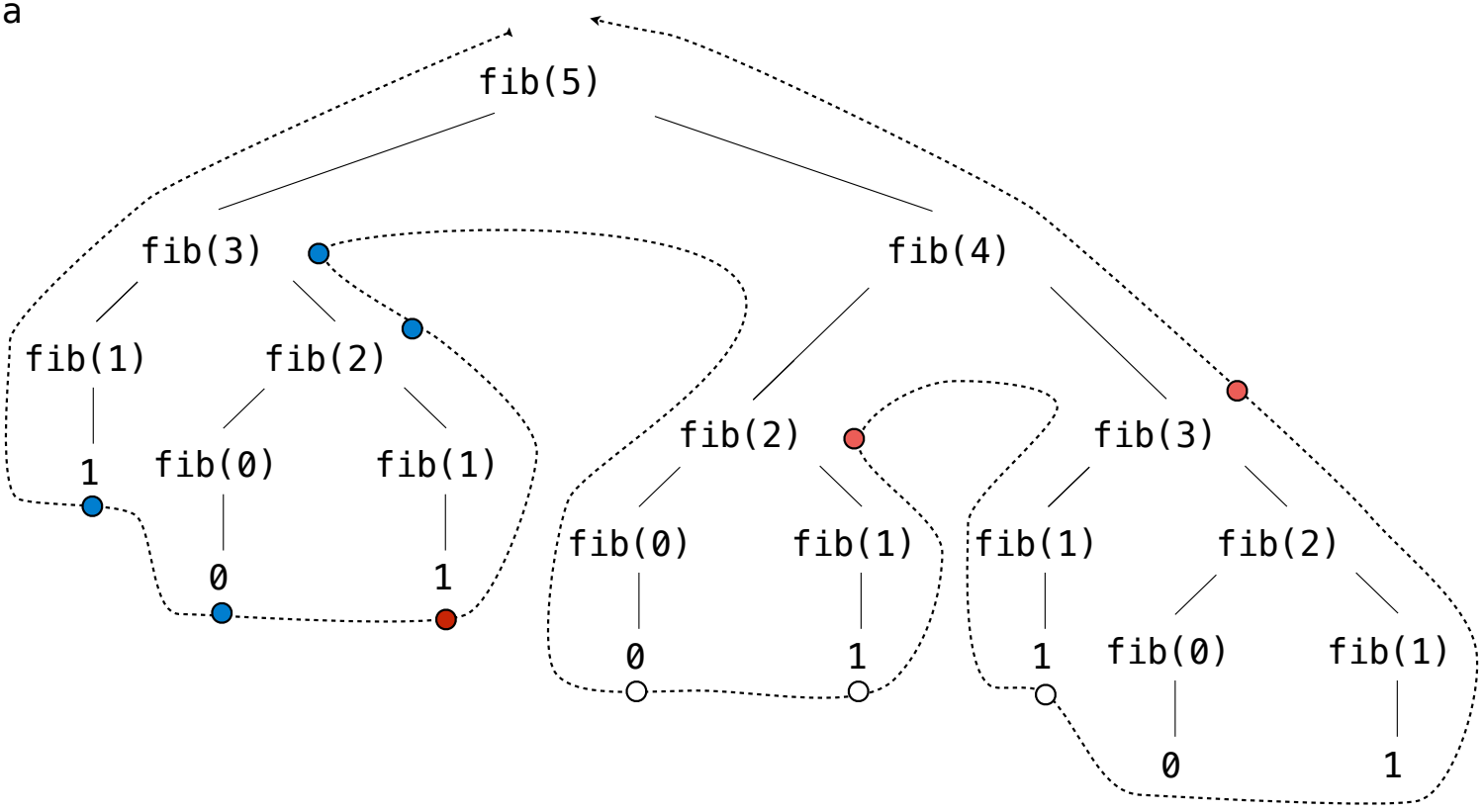

# Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing

**Memoization:**

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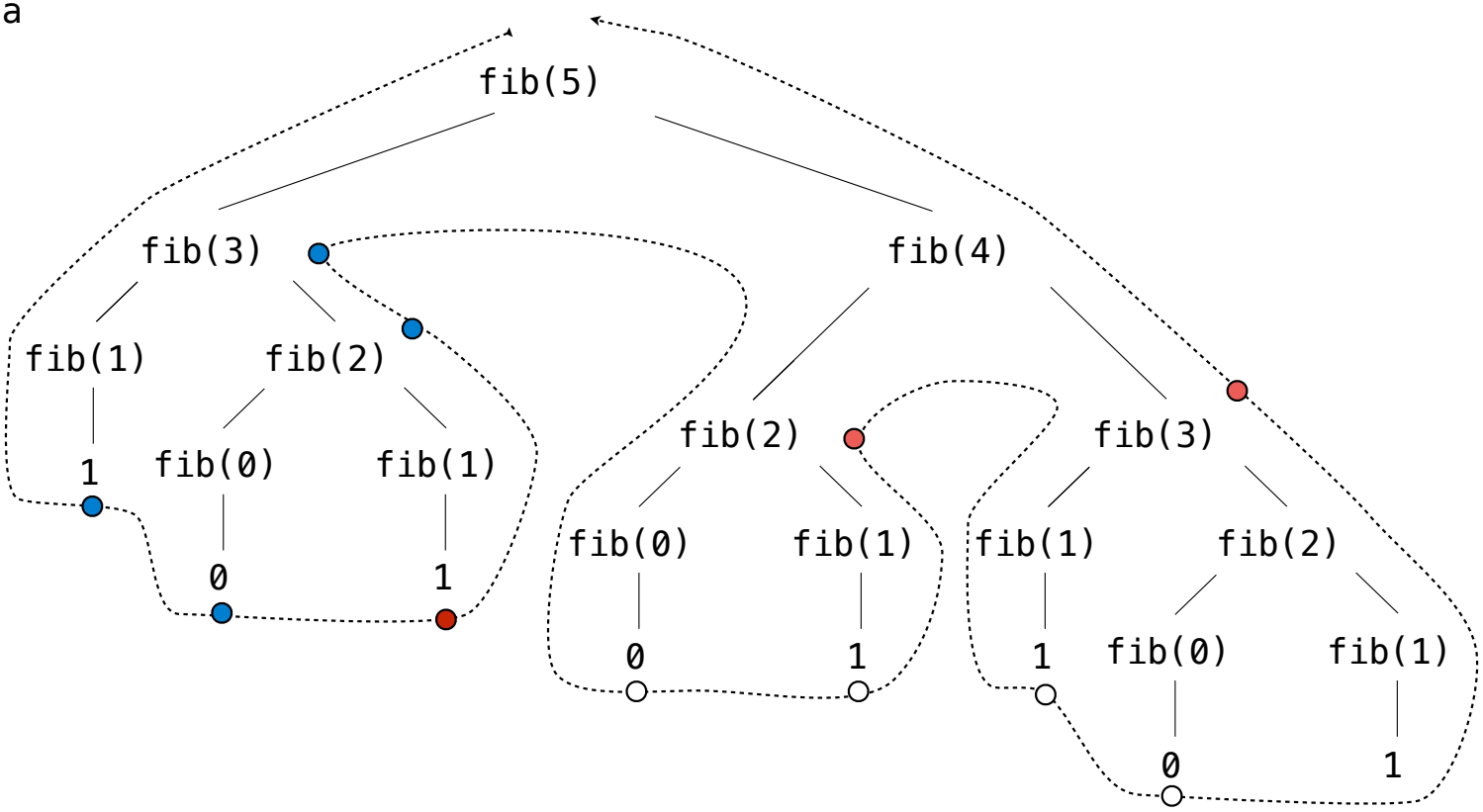

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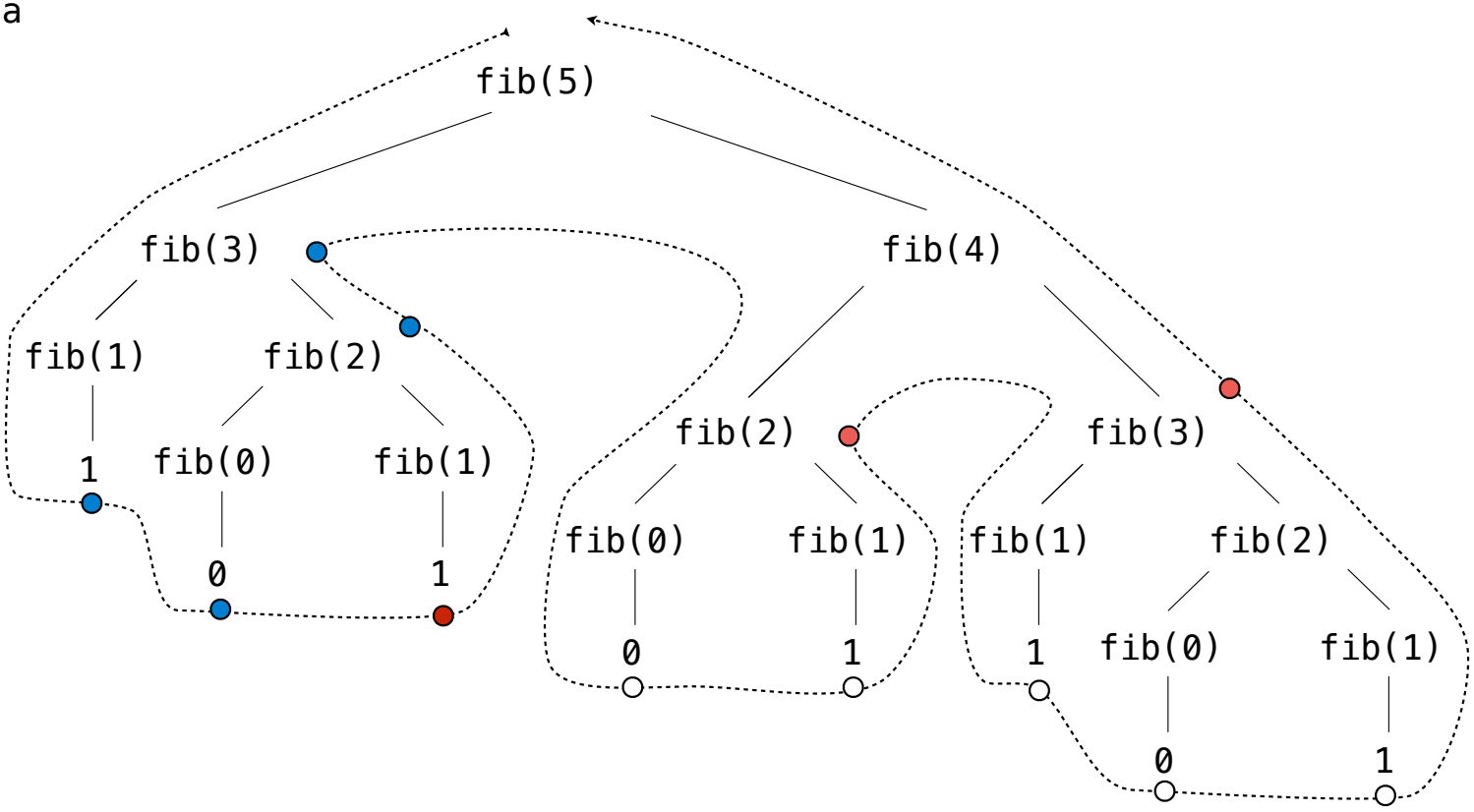

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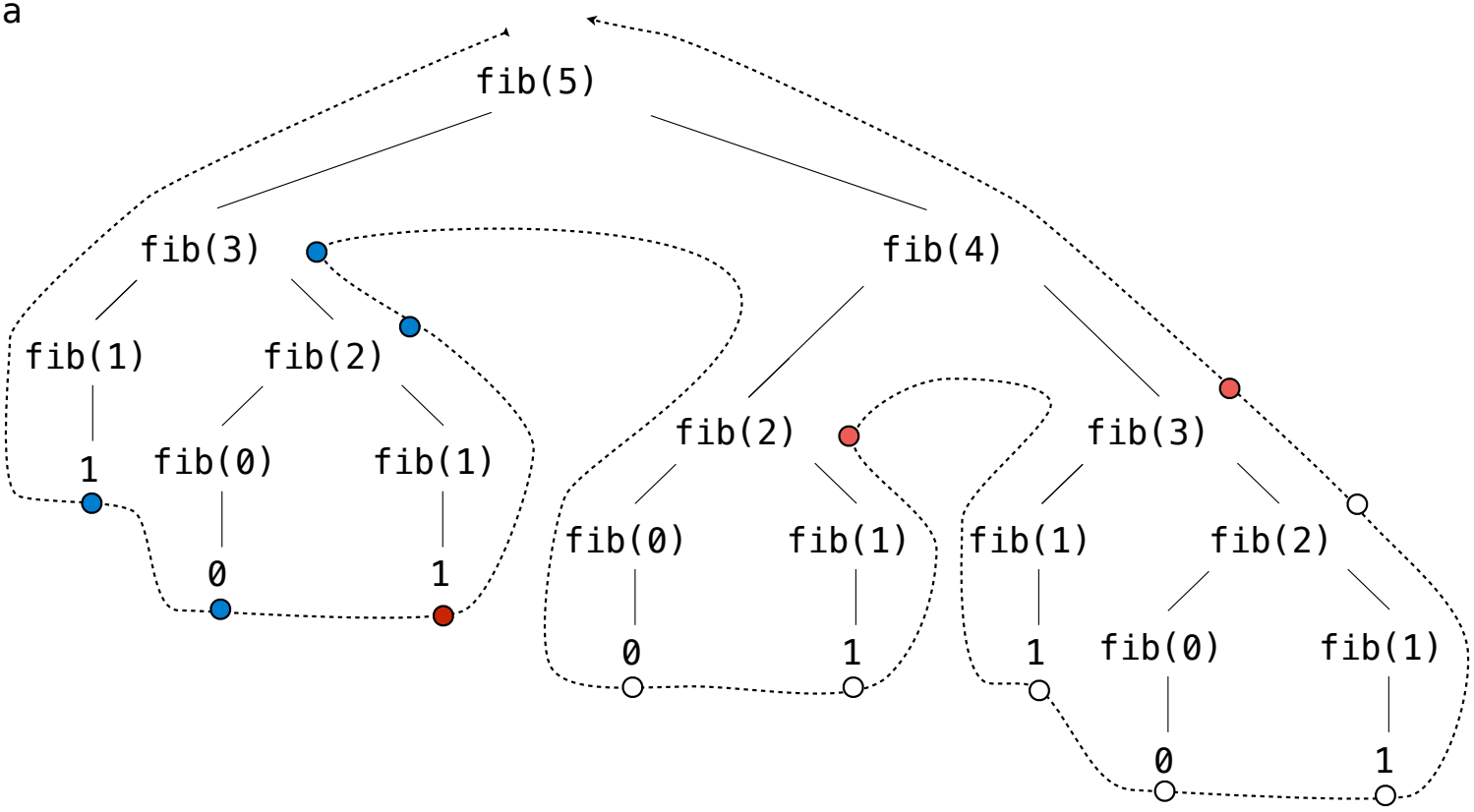

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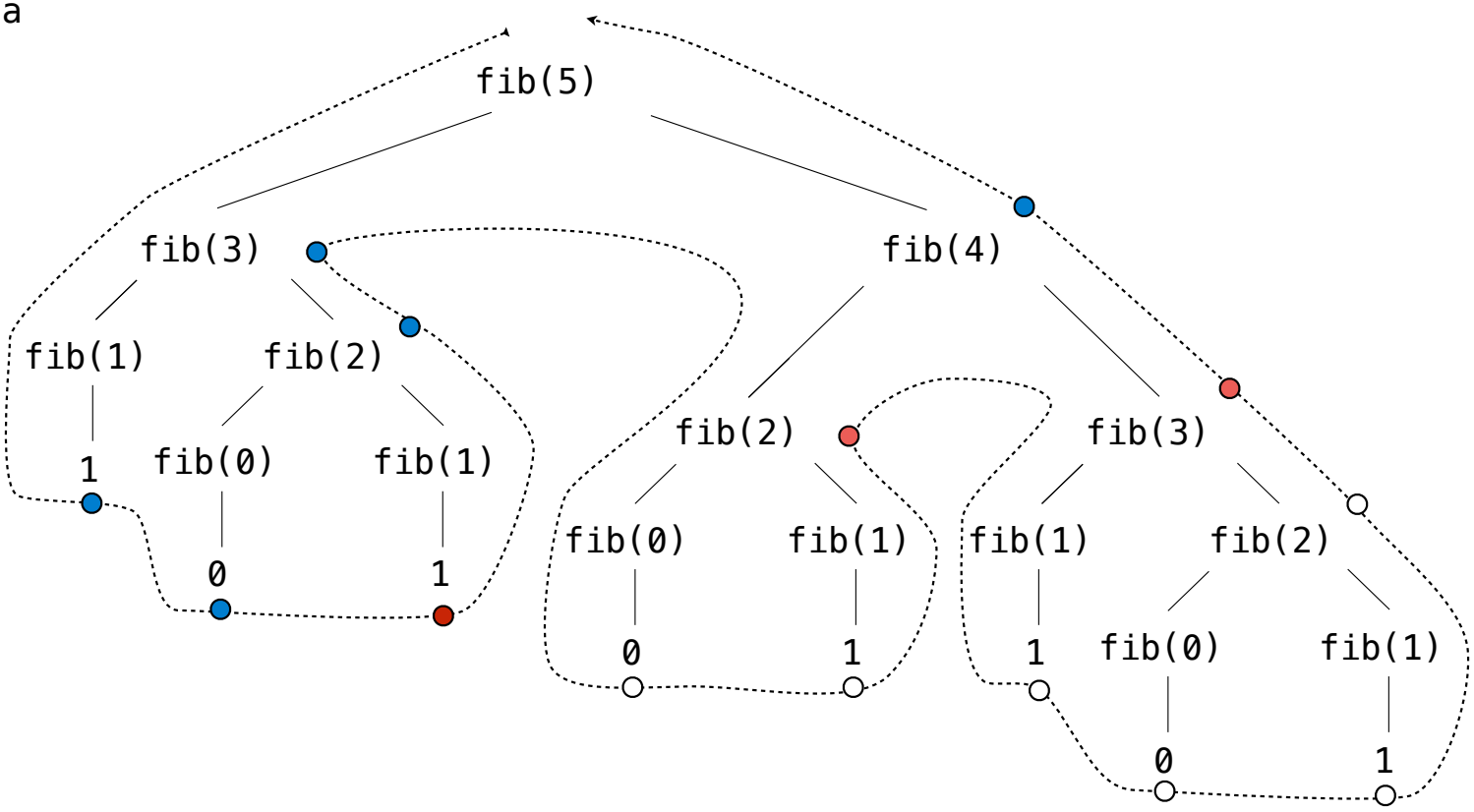

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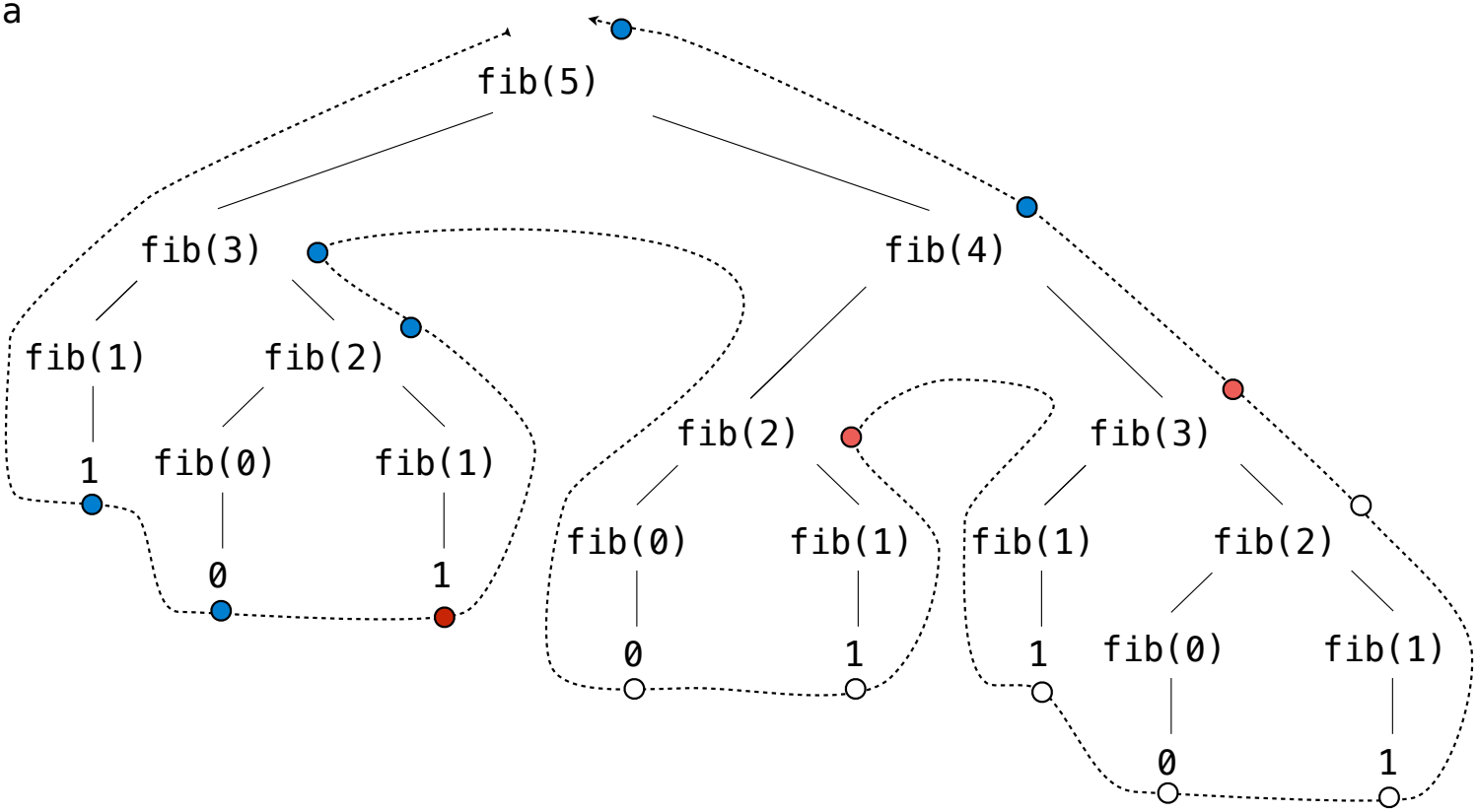

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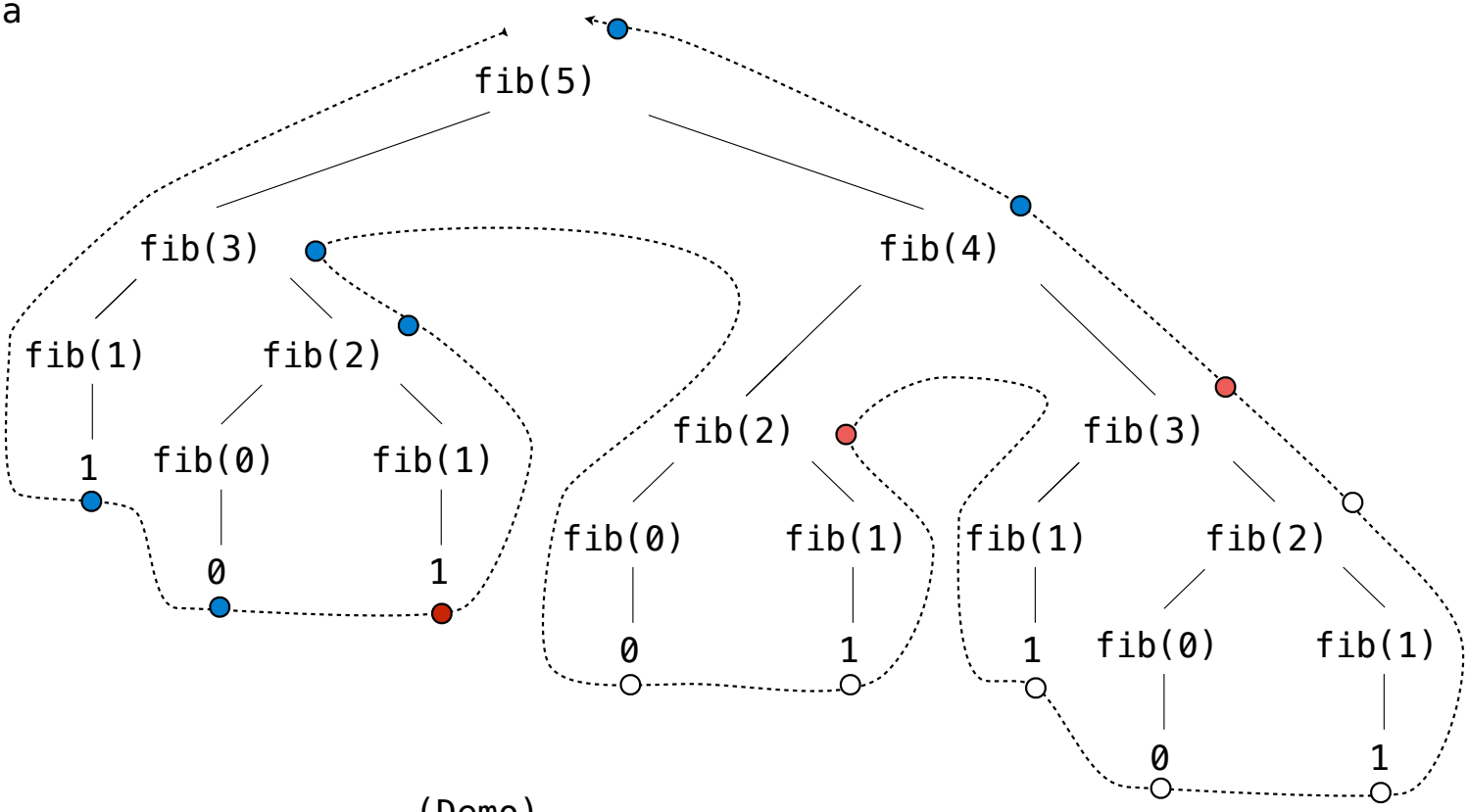


(Demo)

# Hailstone Trees

# Hailstone Trees

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```
Pick a positive integer n as the start
```

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Pick a positive integer n as the start

If n is even, divide it by 2
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Continue this process until n is 1

(Demo)

1

2

# Hailstone Trees

Pick a positive integer n as the start

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If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

# Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

# Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

16

# Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

16

32

# Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

16

32

64

# Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

16

32

64

128

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Pick a positive integer n as the start

If n is even, divide it by 2

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Continue this process until n is 1

(Demo)

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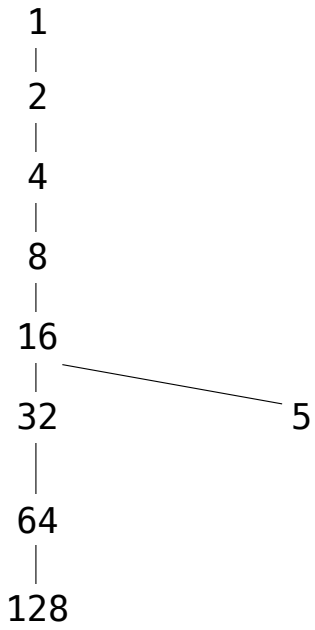

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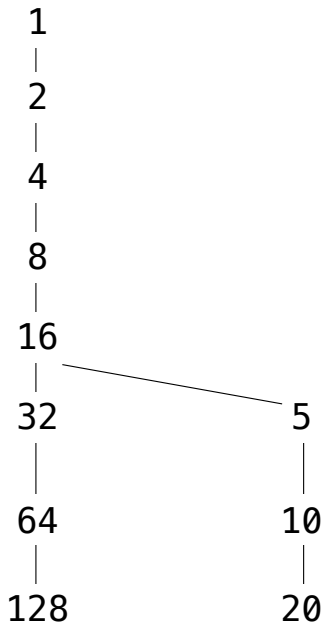

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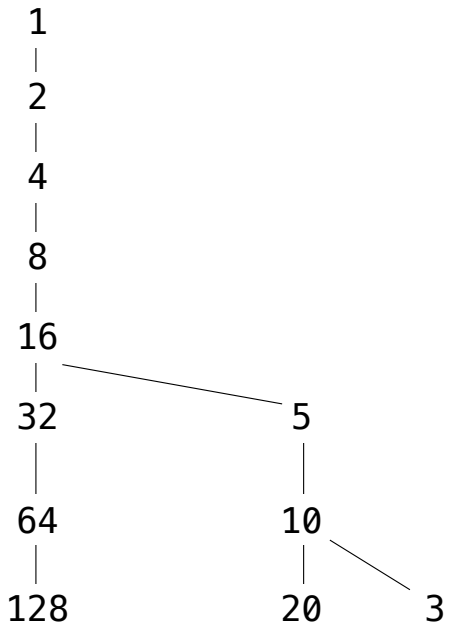

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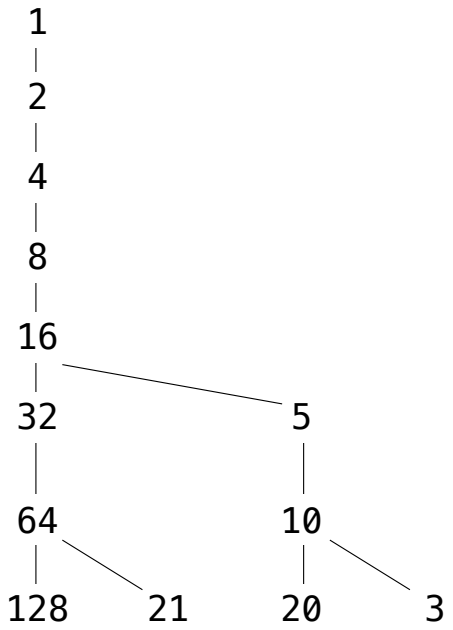

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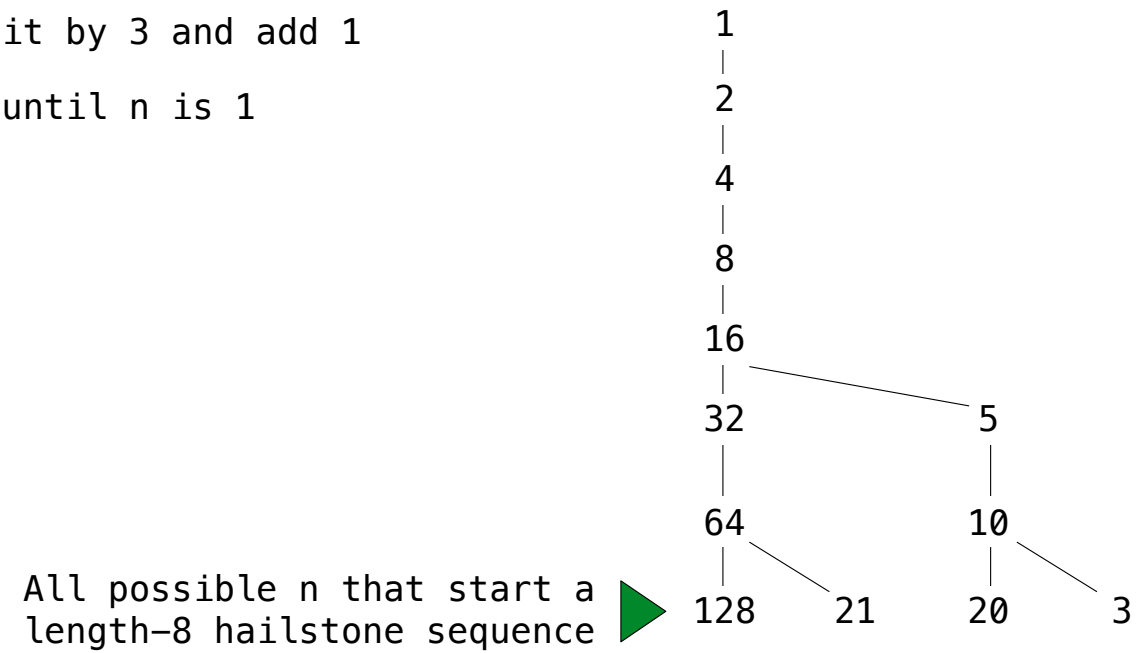

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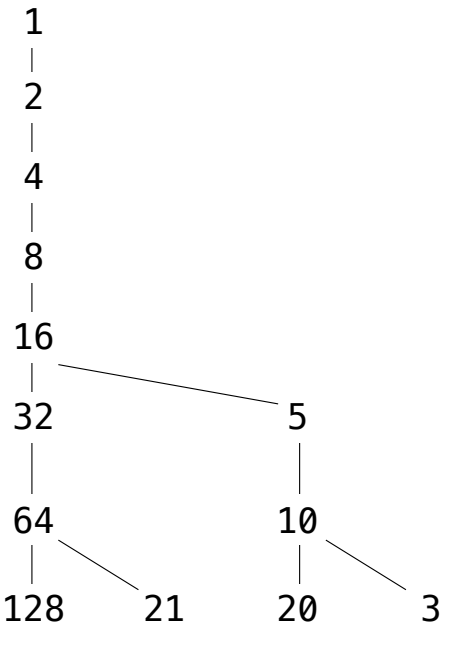
(Demo)

```
def hailstone_tree(k, n=1):
    """Return a Tree in which the paths from the
    leaves to the root are all possible hailstone
    sequences of length k ending in n."""
```

All possible n that start a length-8 hailstone sequence



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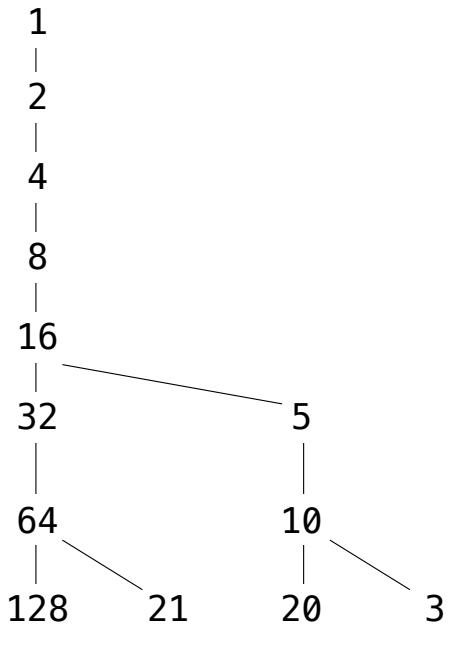


(Demo)