61A Lecture 20

Announcements

Sets

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One more built-in Python container type

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>>> s
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>>> s.union({1, 5})
{1, 2, 3, 4, 5}
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{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
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(Demo)

Implementing Sets

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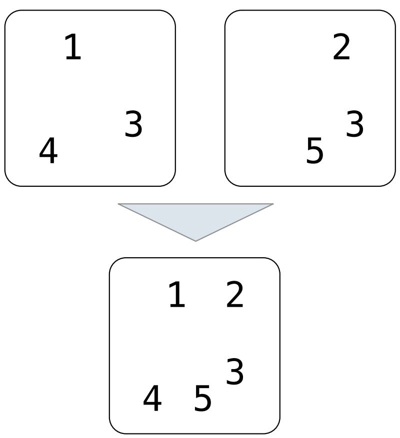

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What we should be able to do with a set:

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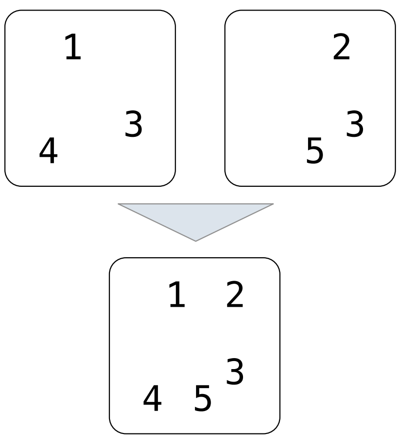

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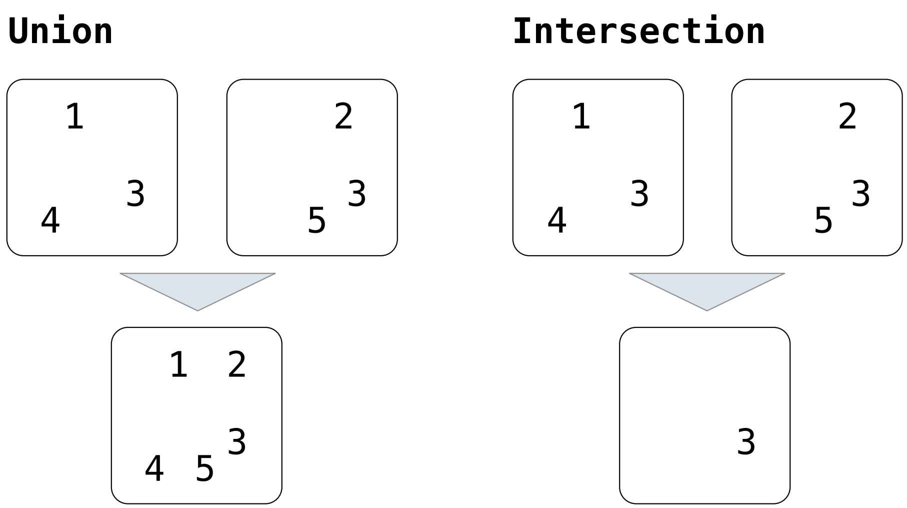

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- **Membership testing:** Is a value an element of a set?
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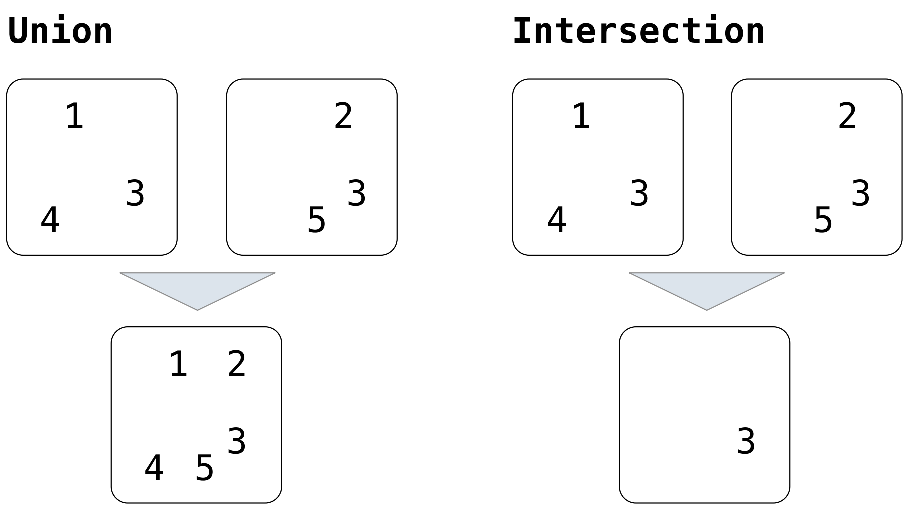

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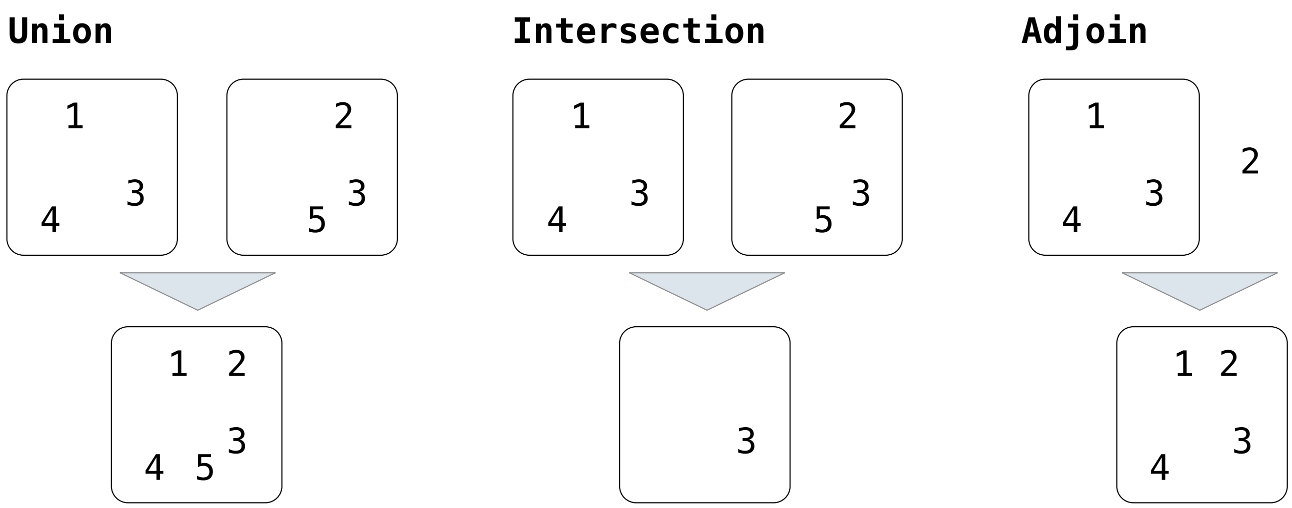

Sets as Linked Lists

Sets as Unordered Sequences

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Assuming v either does not appear in s ***or*** *appears in a uniformly distributed random location*

(Demo)

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def adjoin(s, v):
    if contains(s, v):
        return s
    else:
        return Link(v, s)
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def adjoin(s, v):
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def intersect(set1, set2):
    in_set2 = lambda v: contains(set2, v)
    return filter_link(in_set2, set1)
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def union(set1, set2):
    not_in_set2 = lambda v: not contains(set2, v)
    set1_not_set2 = filter_link(not_in_set2, set1)
    return extend_link(set1_not_set2, set2)
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Parts of the program that...	Assume that sets are...	Using...
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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, contains, adjoin, intersect, union

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Parts of the program that...	Assume that sets are...	Using...
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Different parts of a program may make different assumptions about data

Searching an Ordered List

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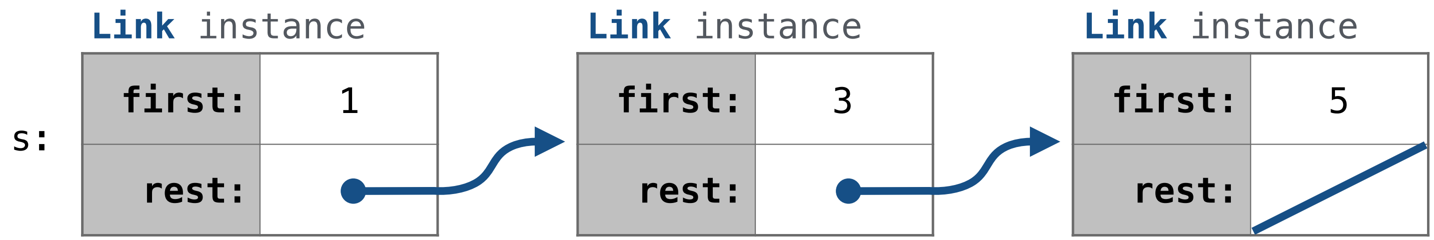

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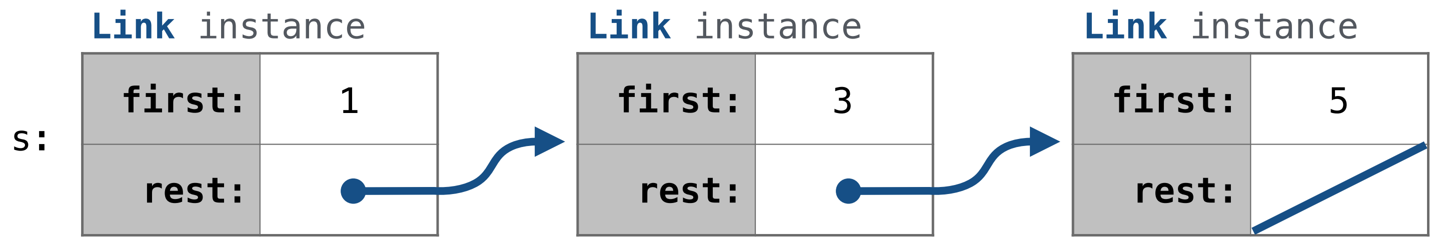
Operation **Time order of growth**



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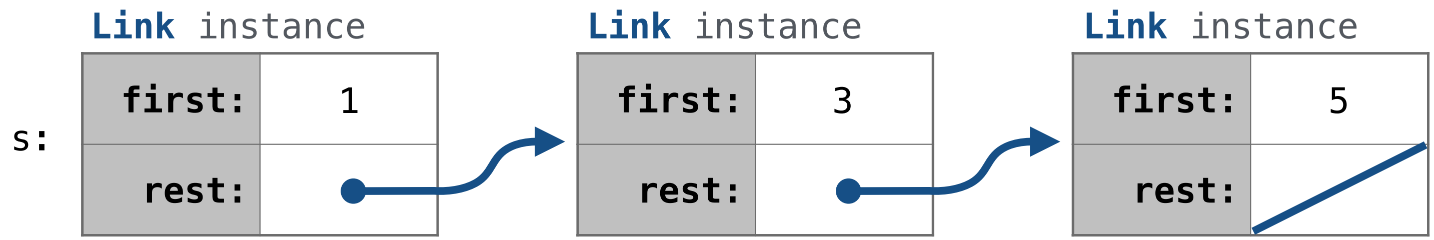
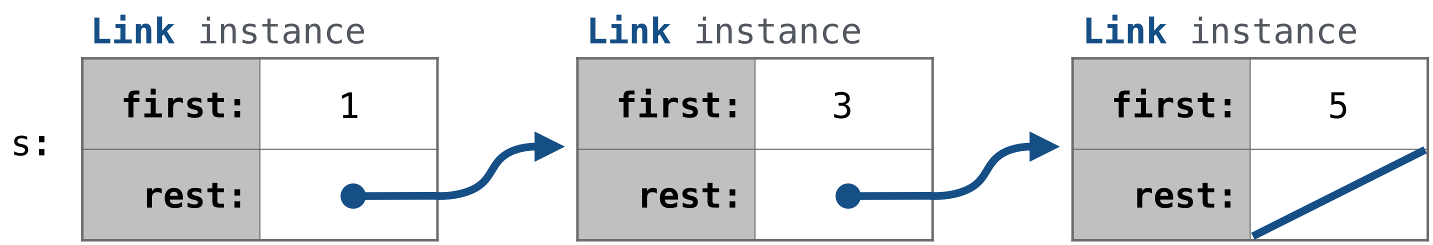
contains	



Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
```

Operation	Time order of growth
contains	



Searching an Ordered List

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>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
True
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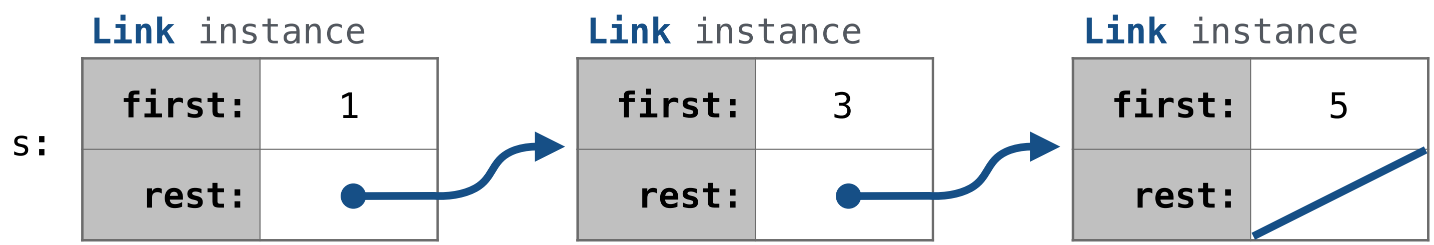
Operation	Time order of growth
contains	



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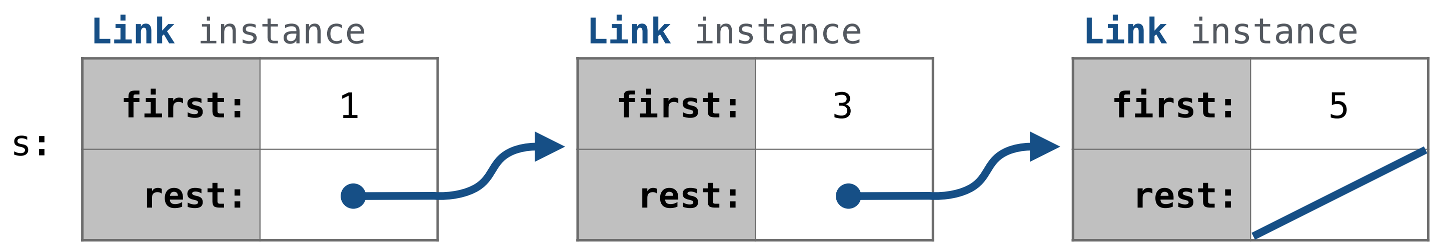
Operation	Time order of growth
contains	



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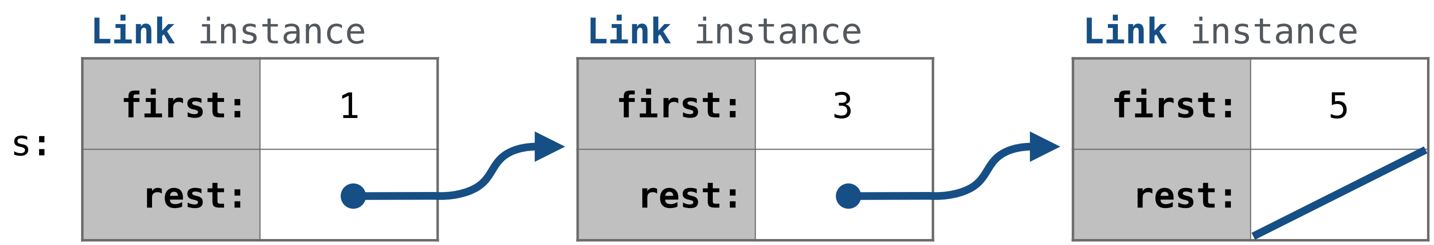

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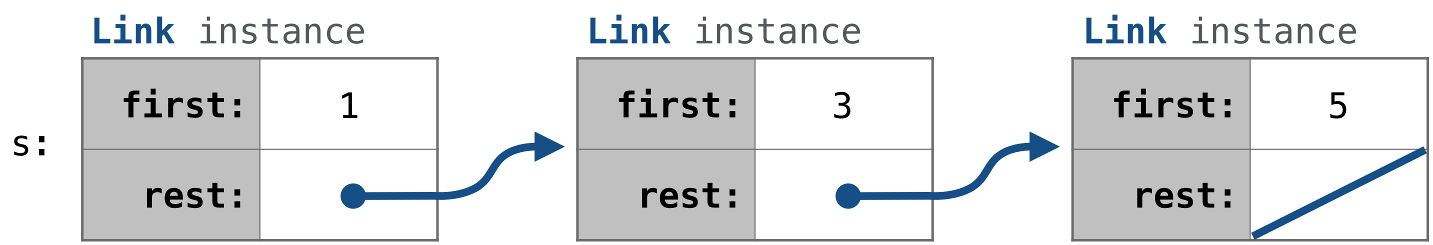
Operation	Time order of growth
contains	$\Theta(n)$



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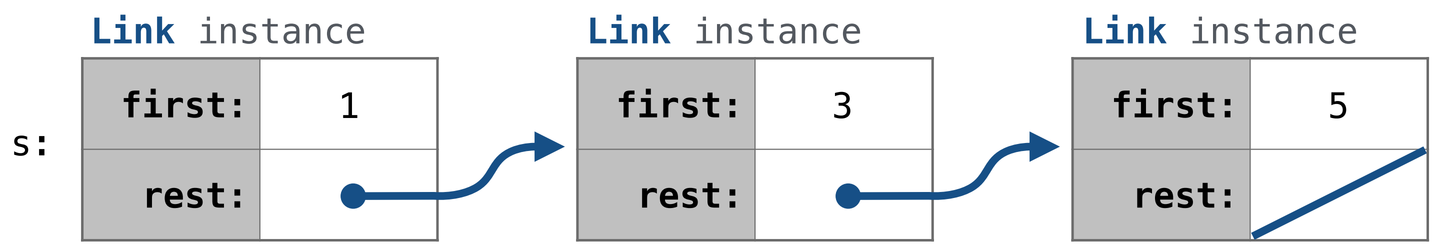

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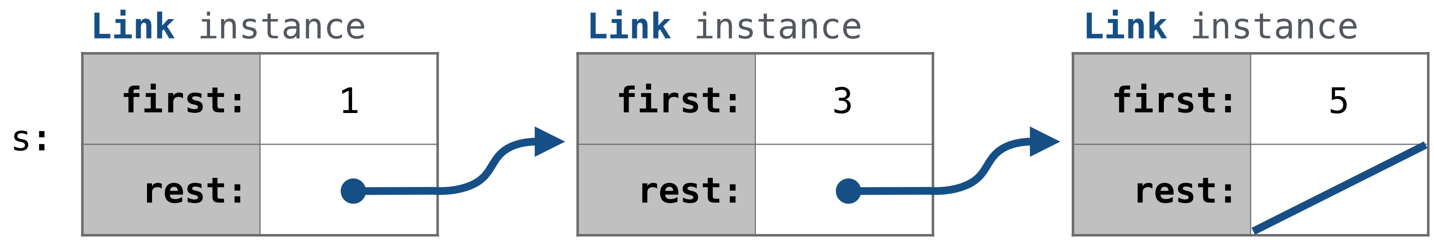

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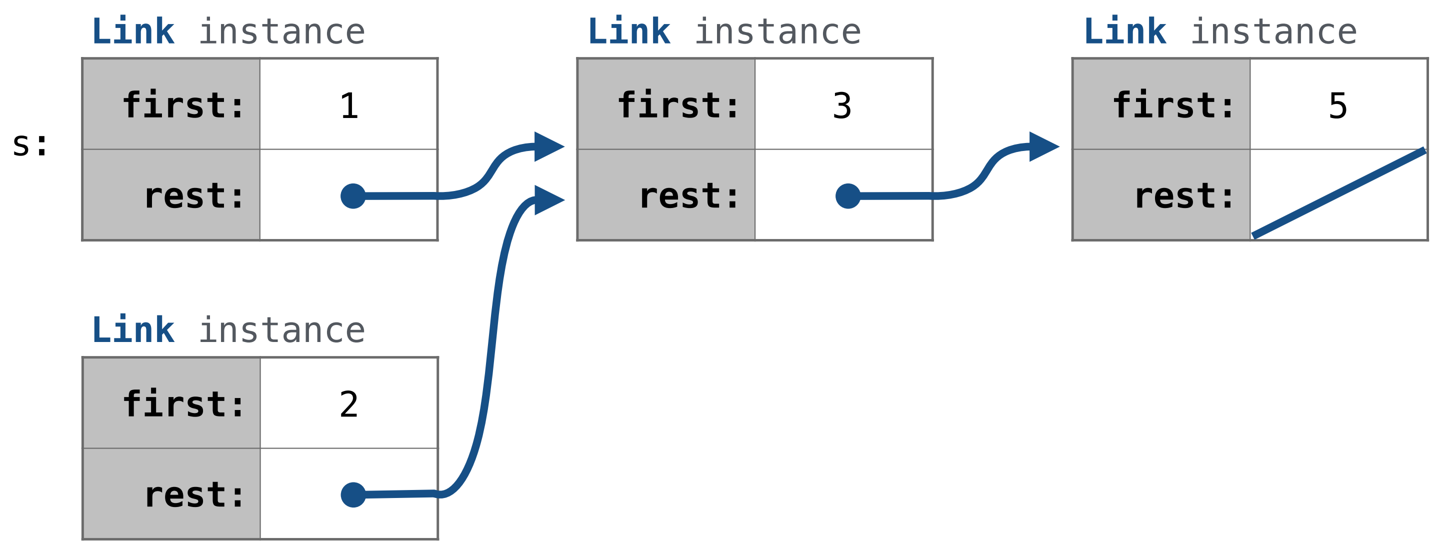

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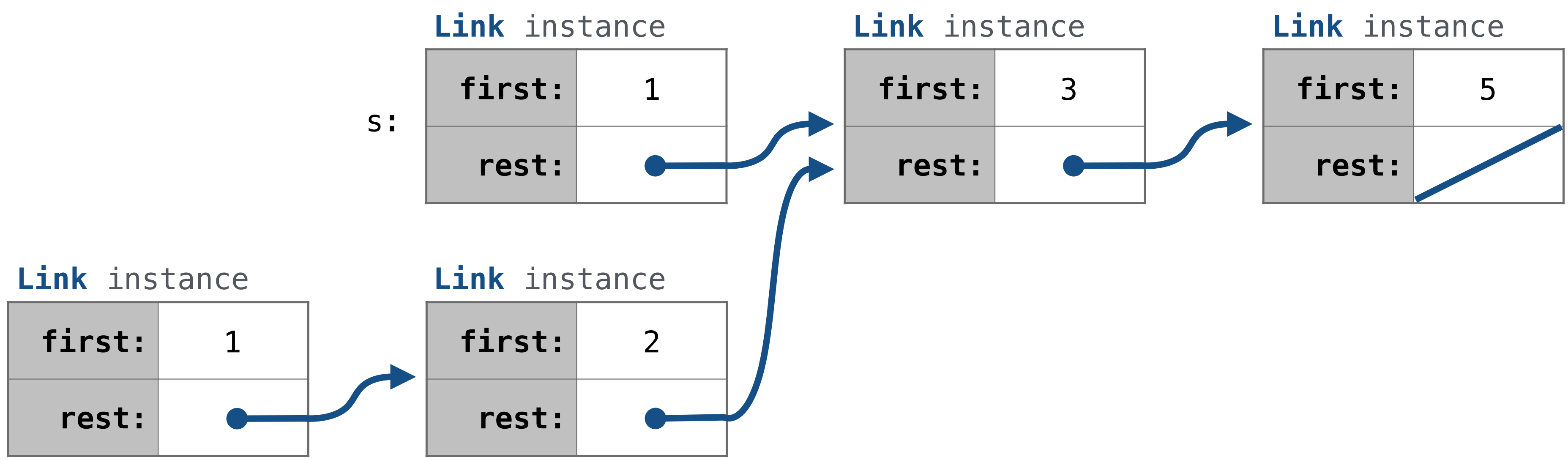

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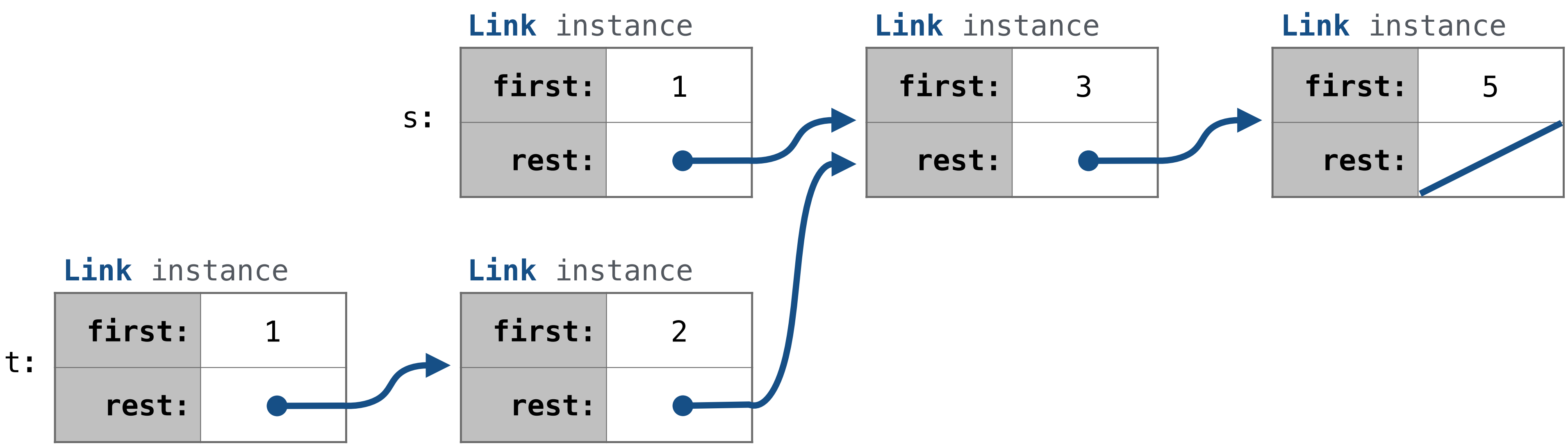

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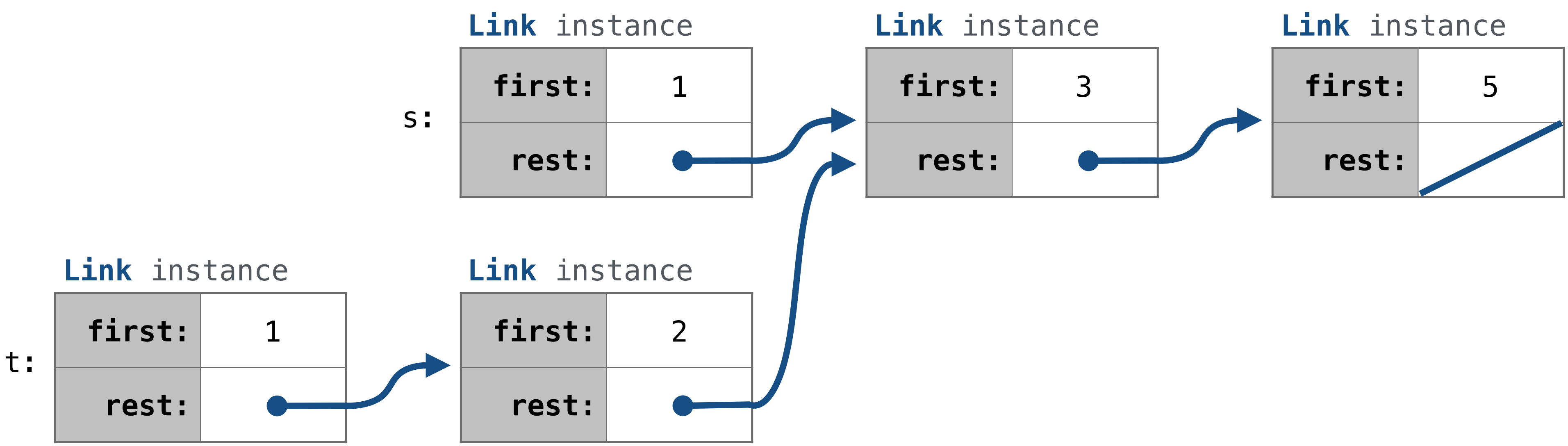

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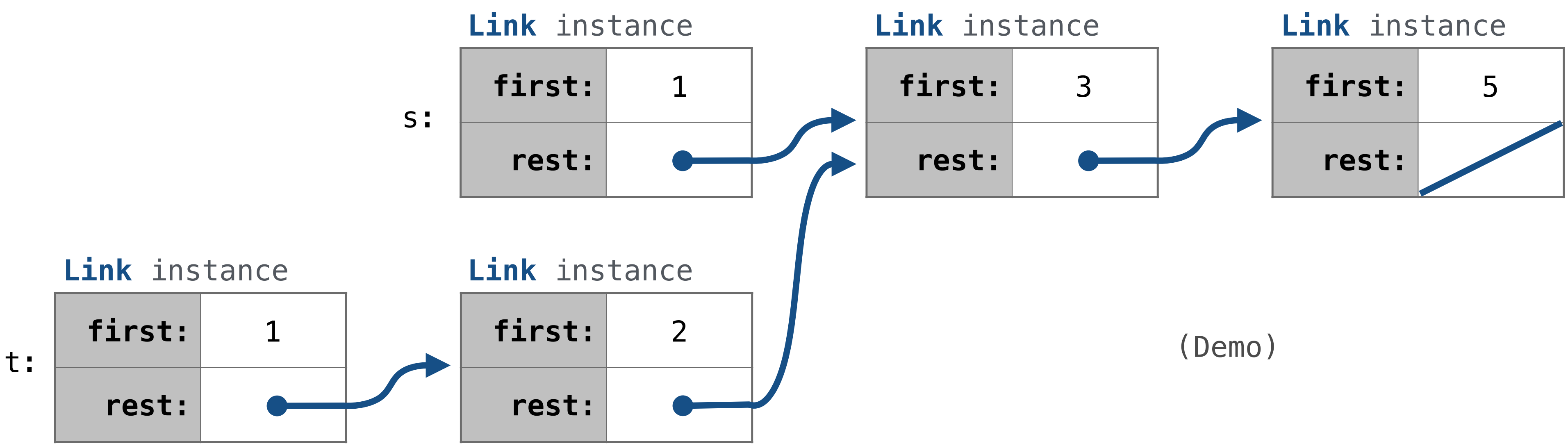
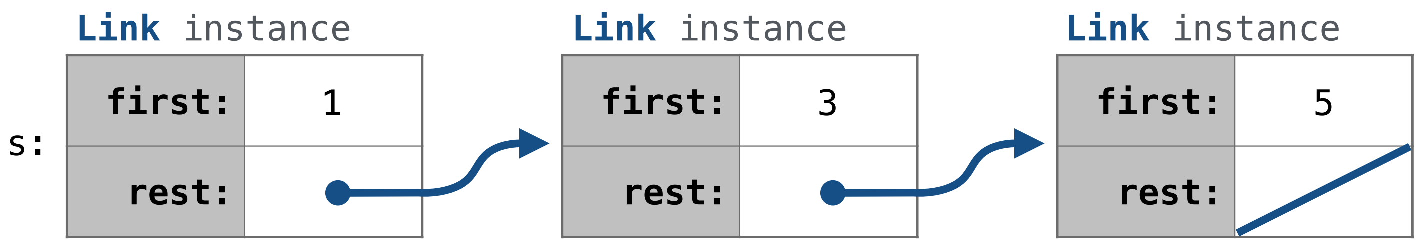


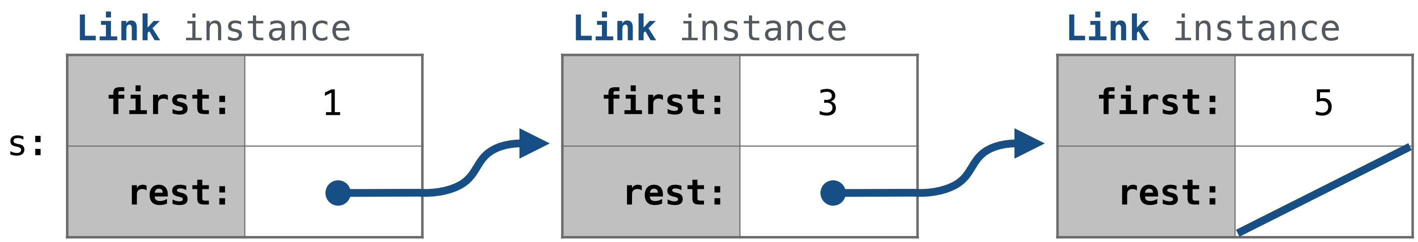
(Demo)

Set Mutation

Adding to an Ordered List

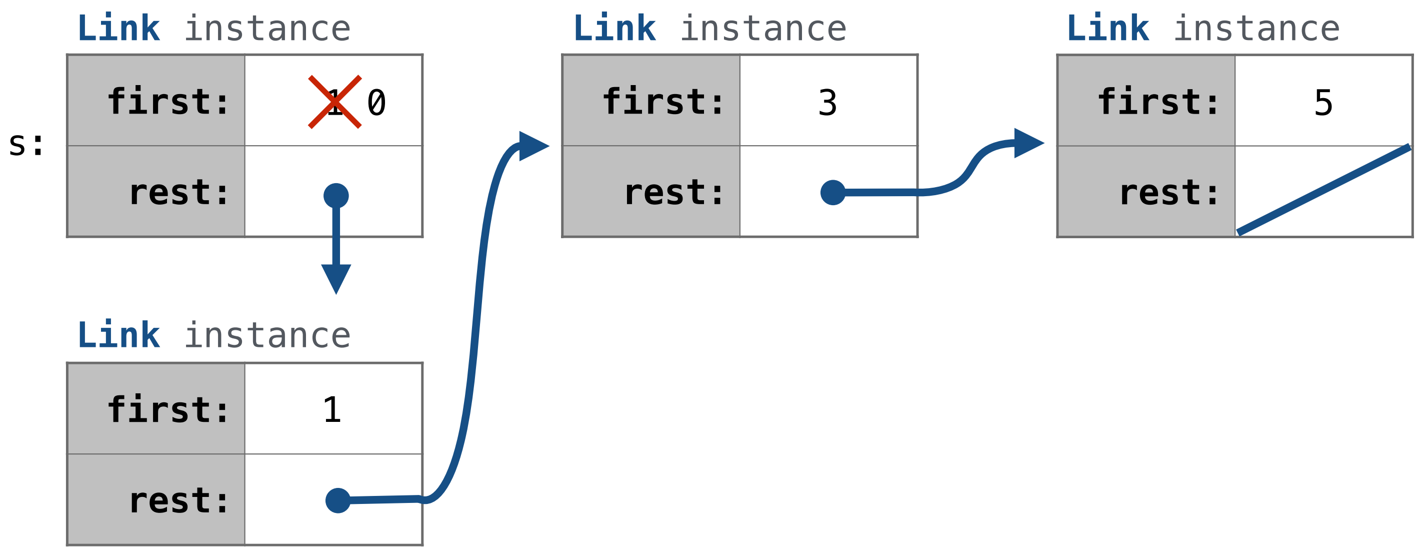


Adding to an Ordered List

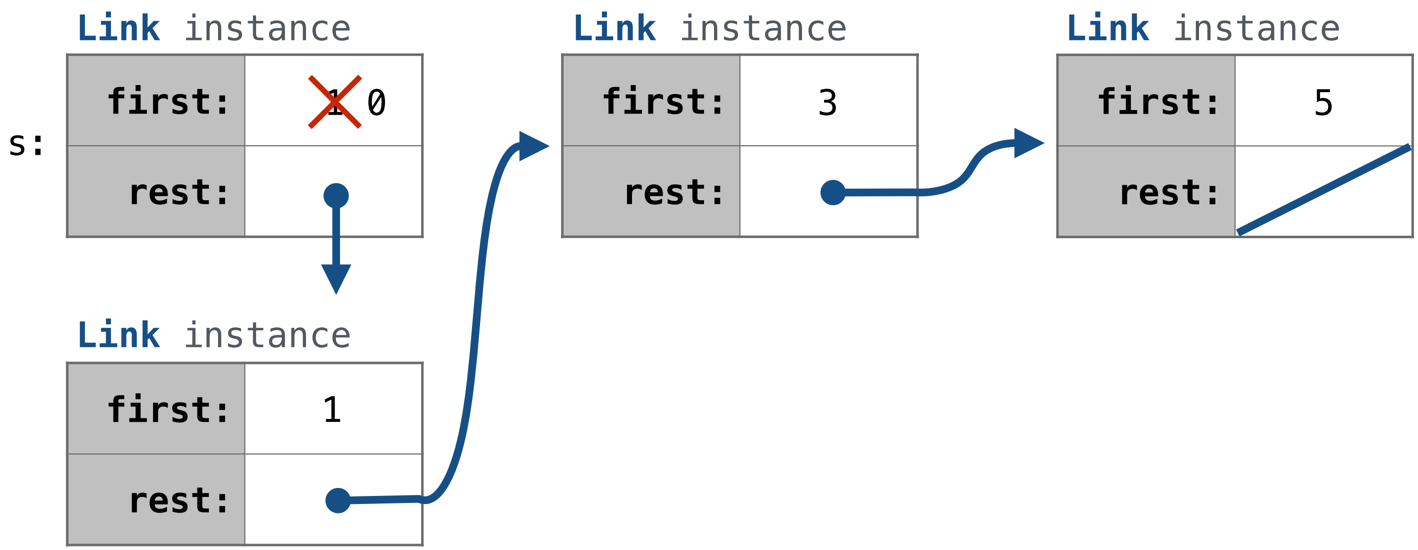



```
add(s, 0)
```

Adding to an Ordered List

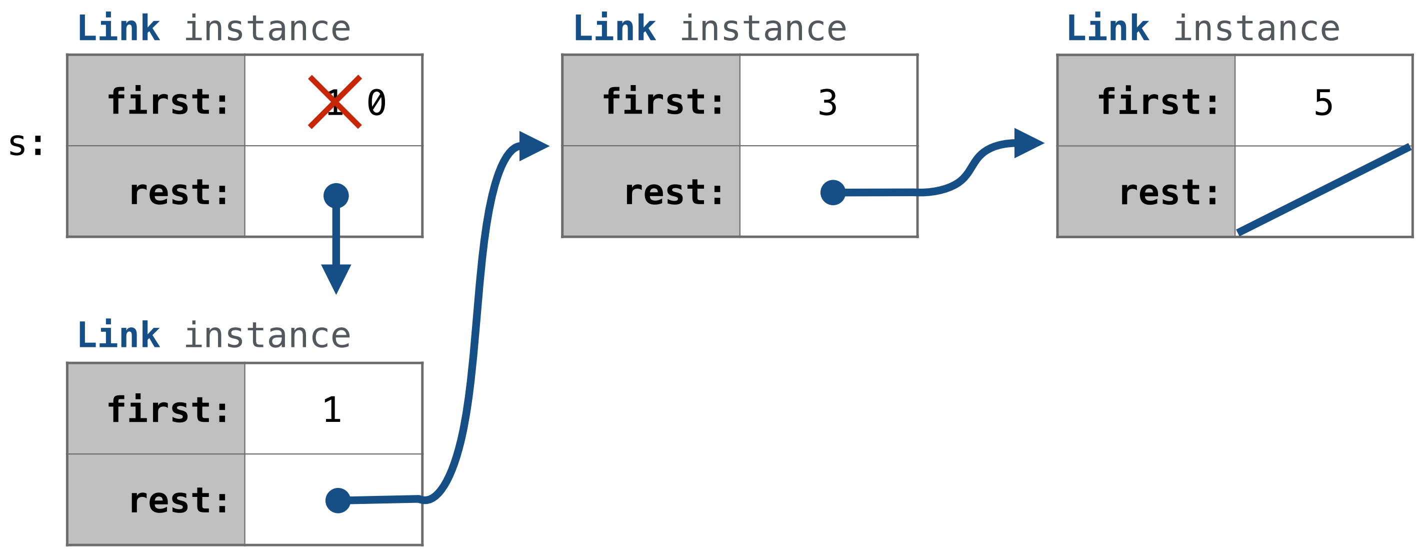


Adding to an Ordered List



add(s, 3)

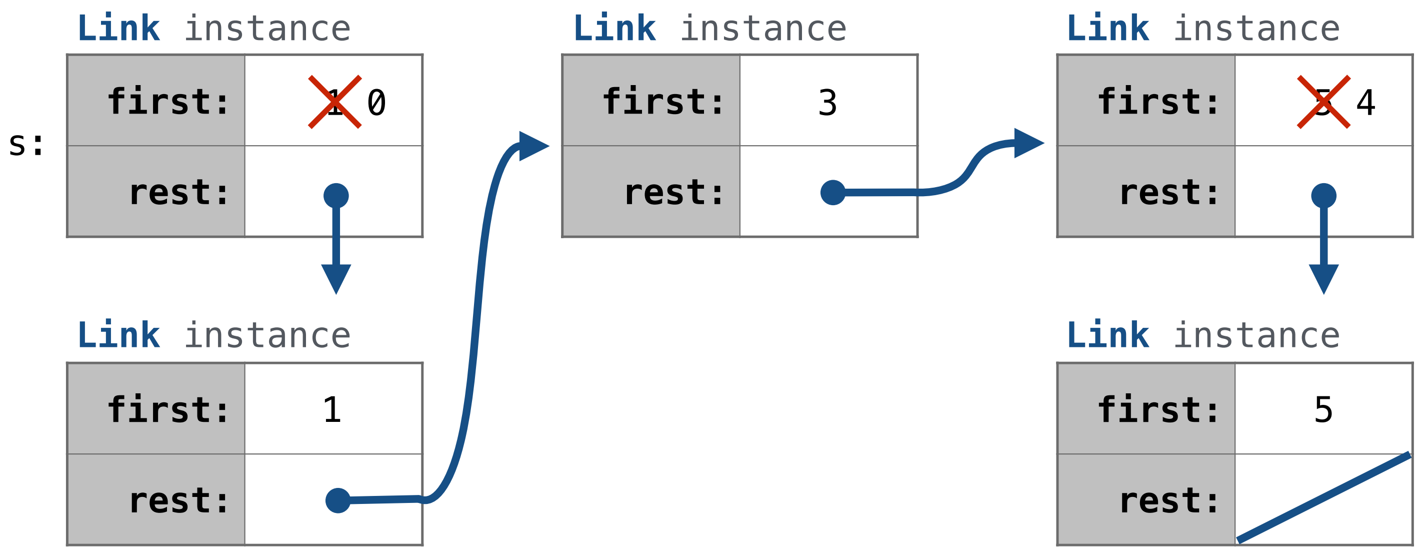
Adding to an Ordered List



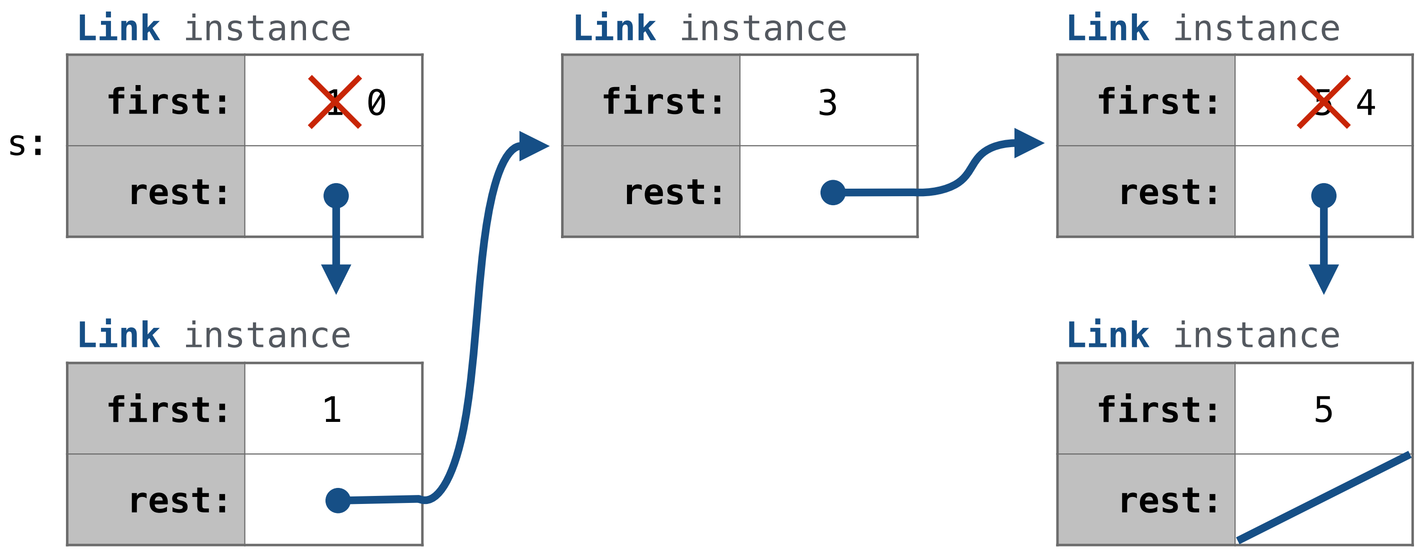

```
add(s, 3)

add(s, 4)
```

Adding to an Ordered List

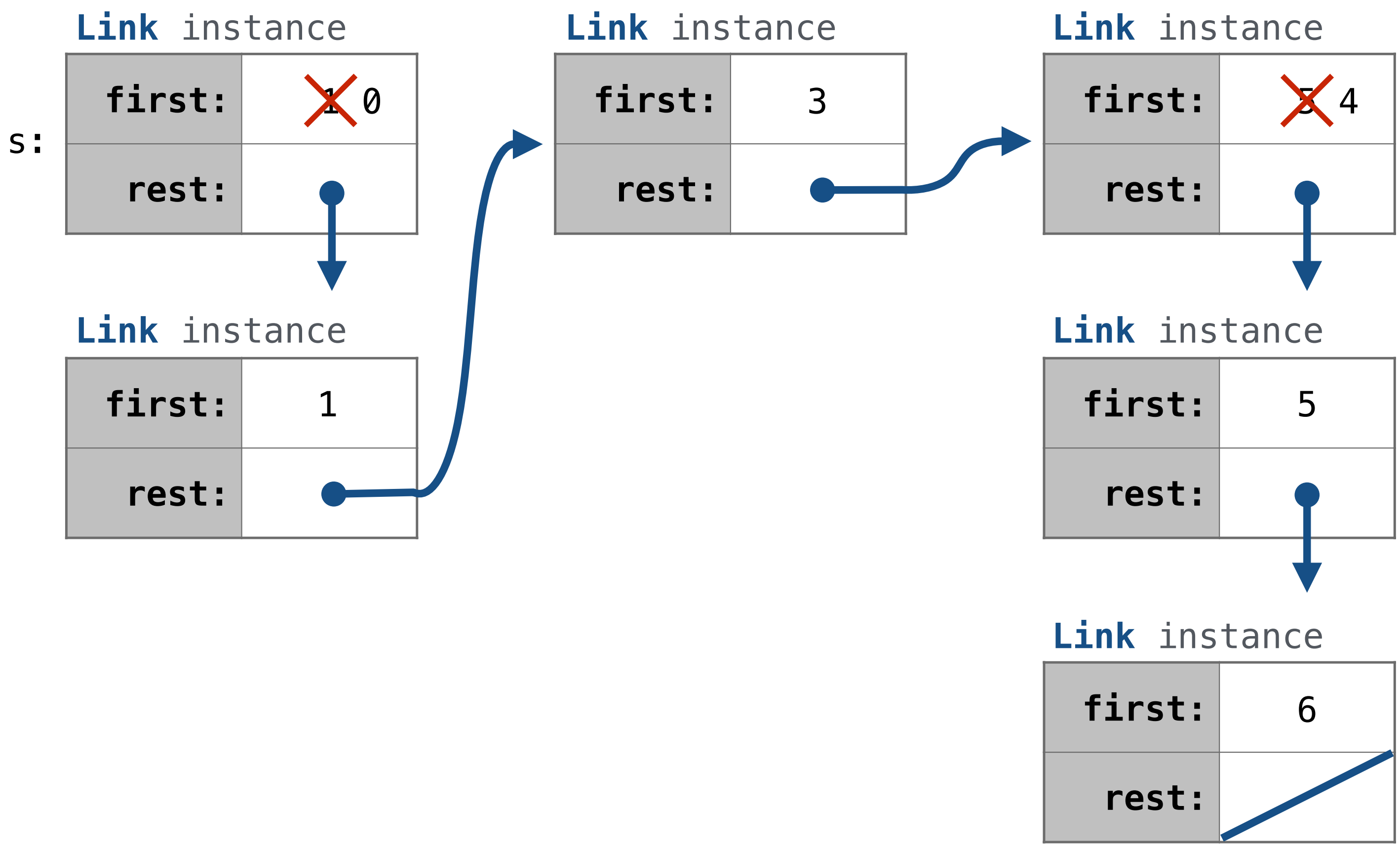


Adding to an Ordered List

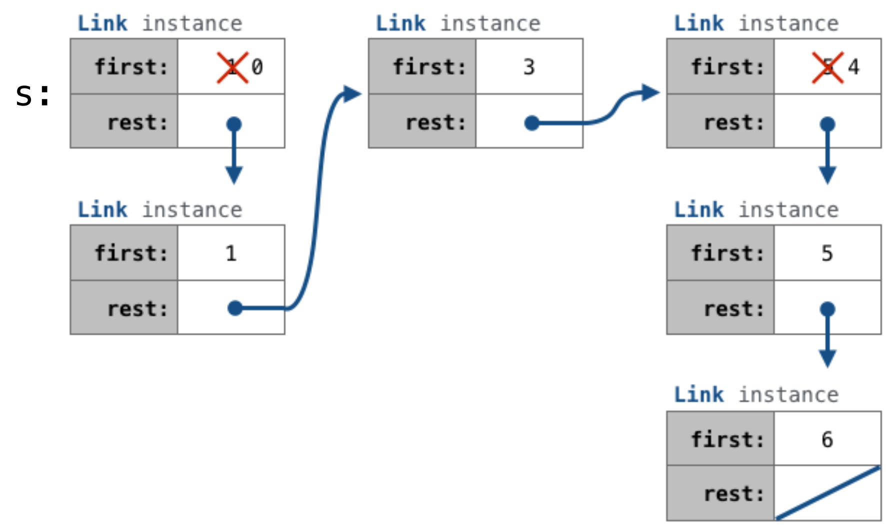


add(s, 6)

Adding to an Ordered List

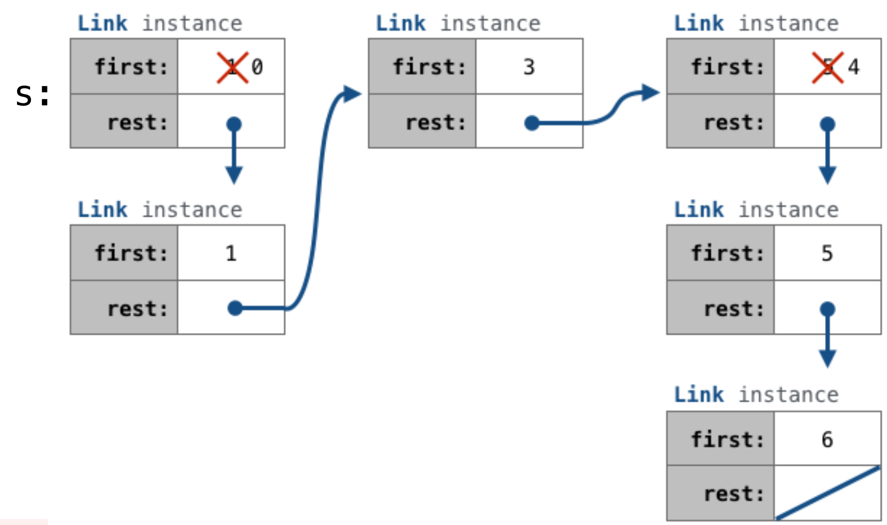


Adding to an Ordered List



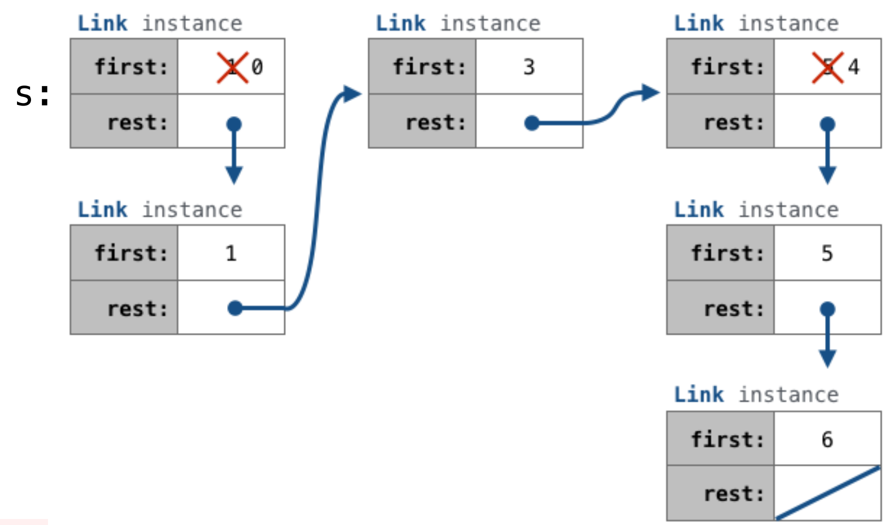
Adding to an Ordered List

```
def add(s, v):
```



Adding to an Ordered List

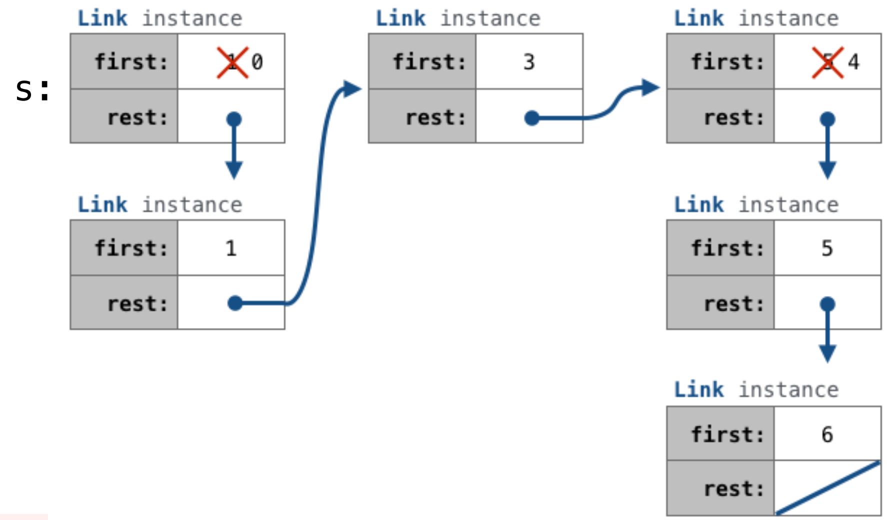
```
def add(s, v):
    """Add v to a set s and return s.
```



Adding to an Ordered List

```
def add(s, v):
    """Add v to a set s and return s.

    >>> s = Link(1, Link(3, Link(5)))
```



Adding to an Ordered List

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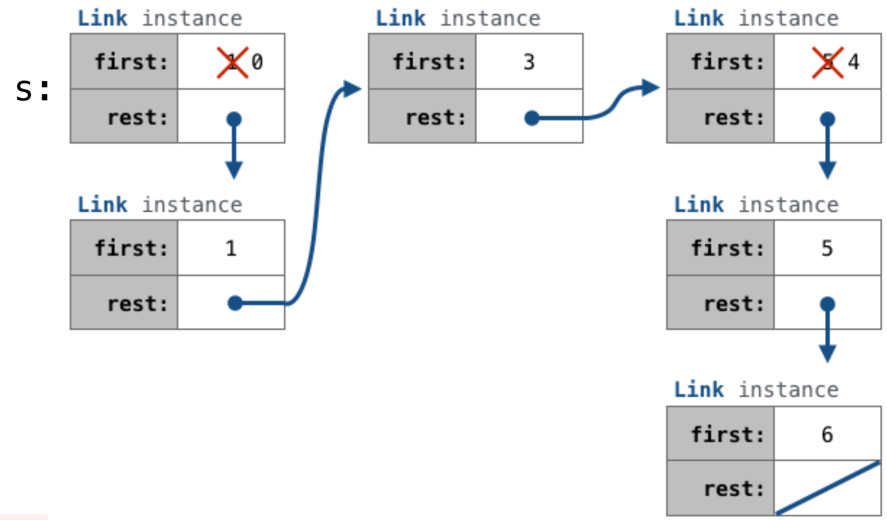
```



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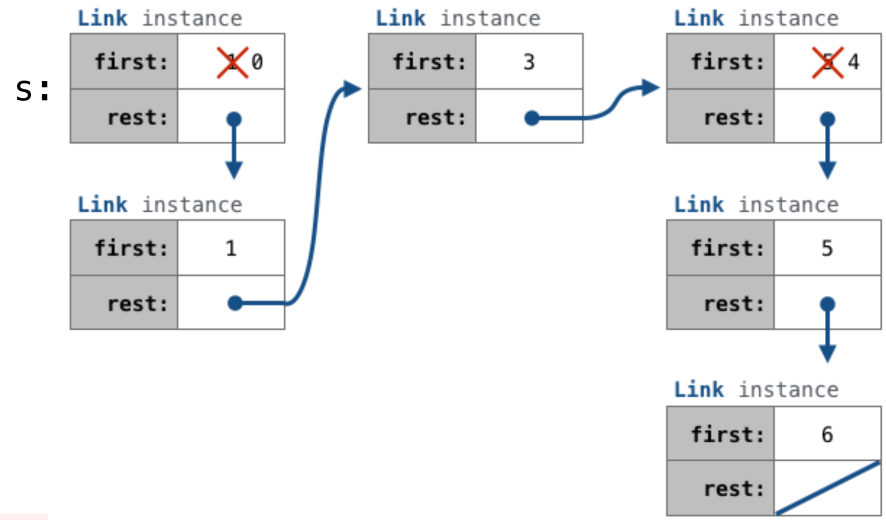
```



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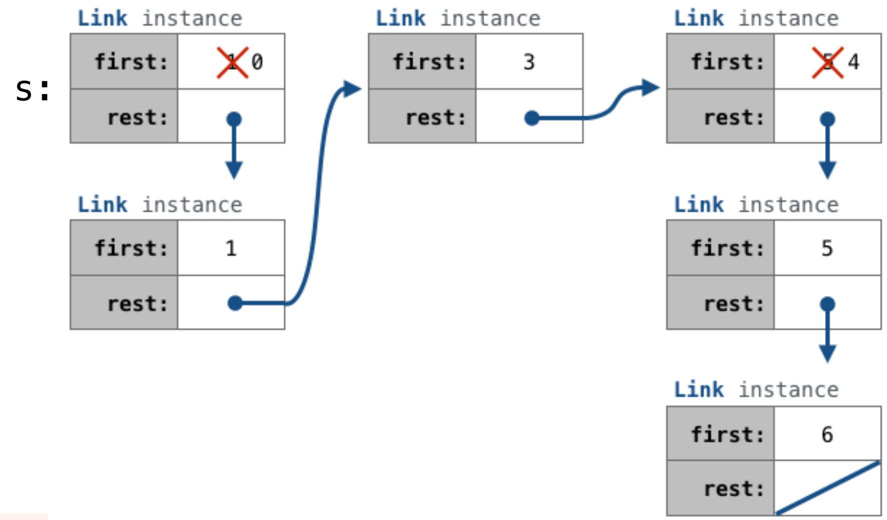
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
```



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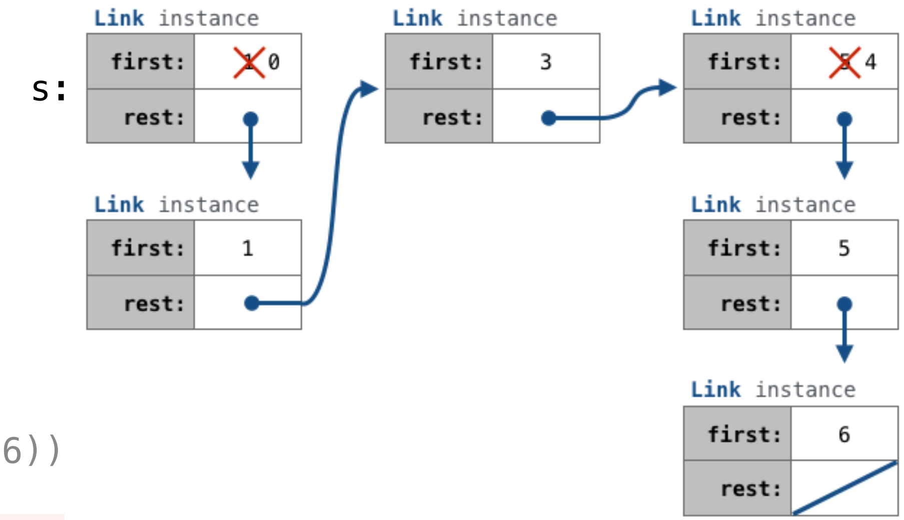
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """
```



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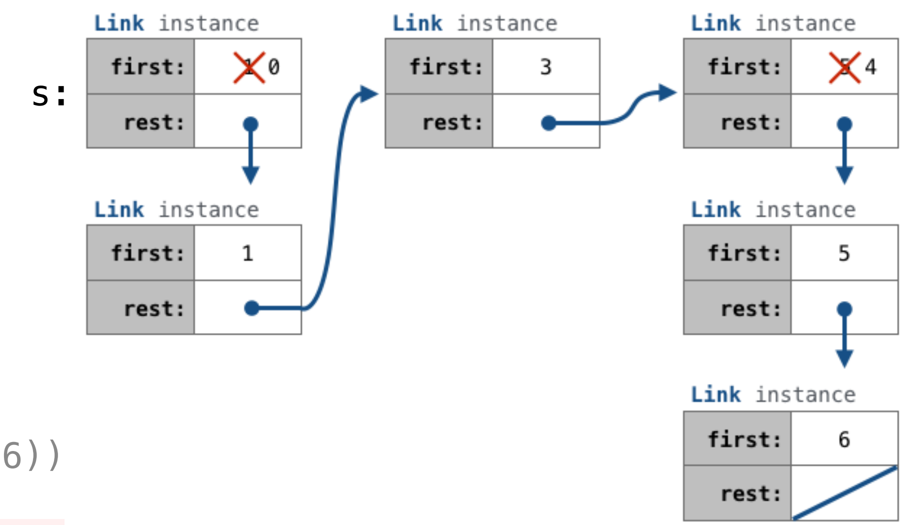
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """
    assert not empty(s), "Cannot add to an empty set."
```



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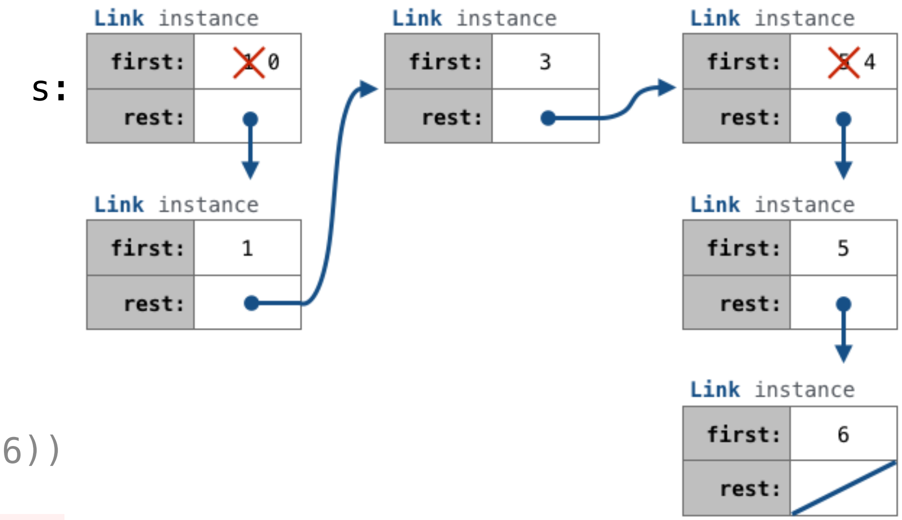
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """


    if s.first > v:
        s.first, s.rest = __________________________ , ______________________________
```



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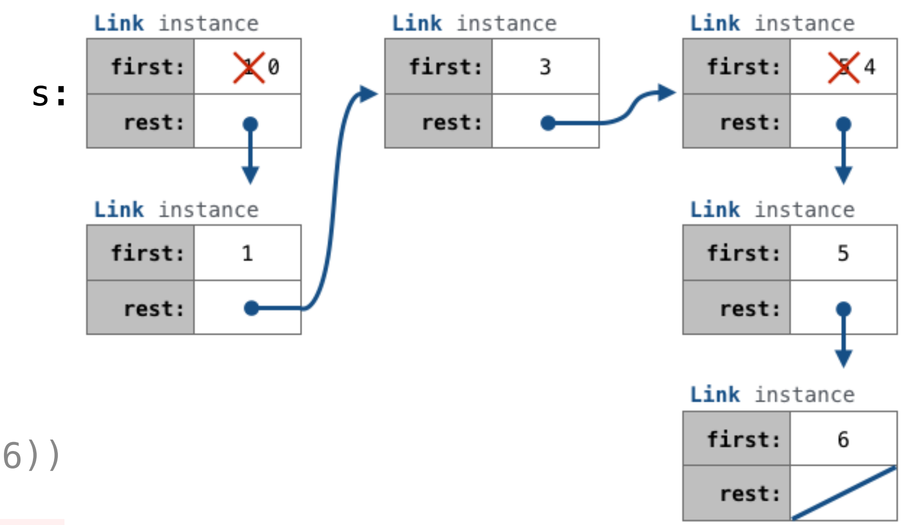
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """

    if s.first > v:
        s.first, s.rest = __________________________ , _____________________________
    elif s.first < v and empty(s.rest):
        s.rest = ___________________________________________________________________
```



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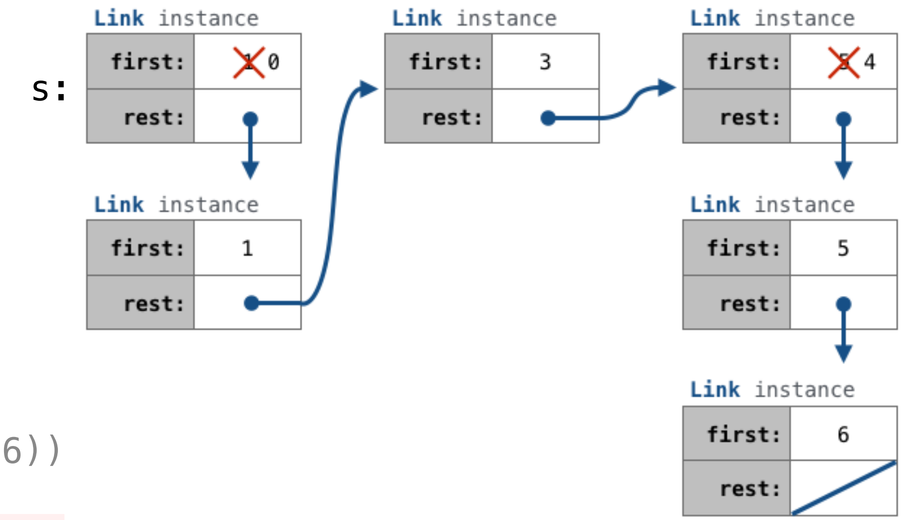

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        s.rest = ___________________________________________________
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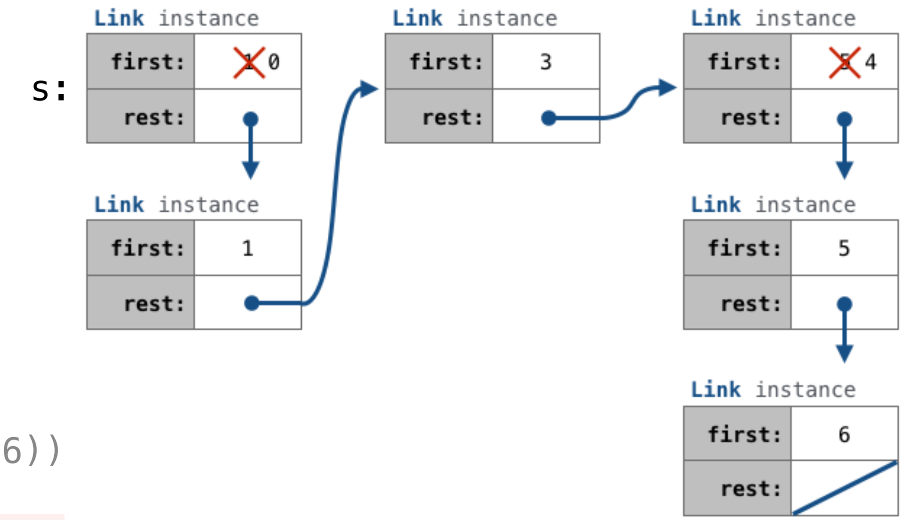
```



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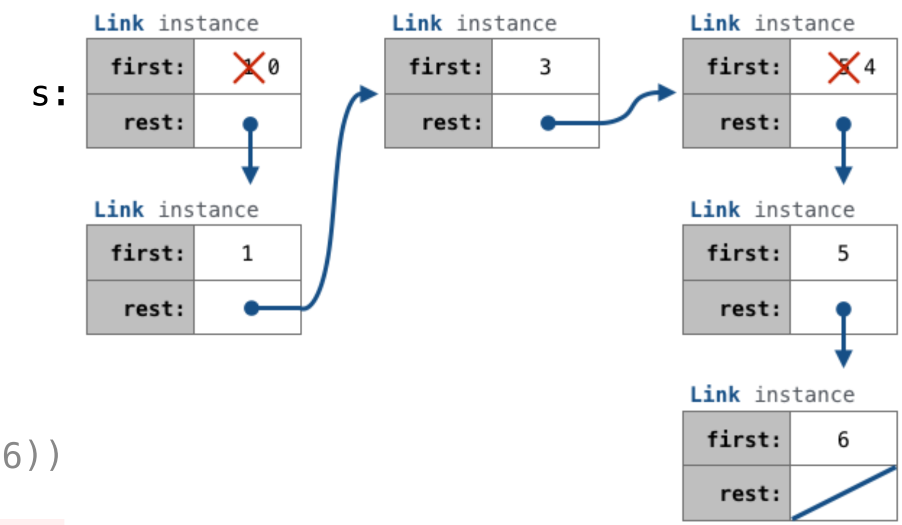
```



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    """

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    elif s.first < v and empty(s.rest):
        s.rest = ____________________________________________
    elif s.first < v:
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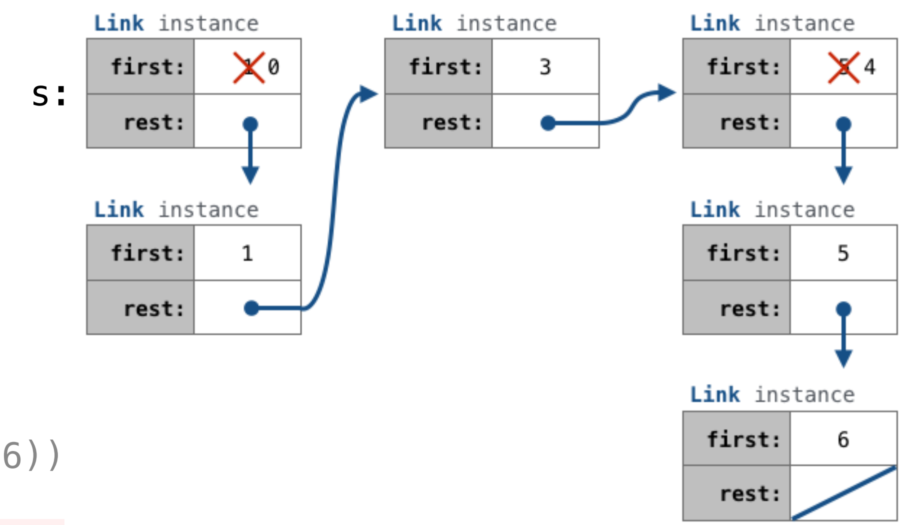

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    """

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        s.first, s.rest = _____________v_____________ , _______Link(s.first, s.rest)_______
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        ________________________________________________________________________
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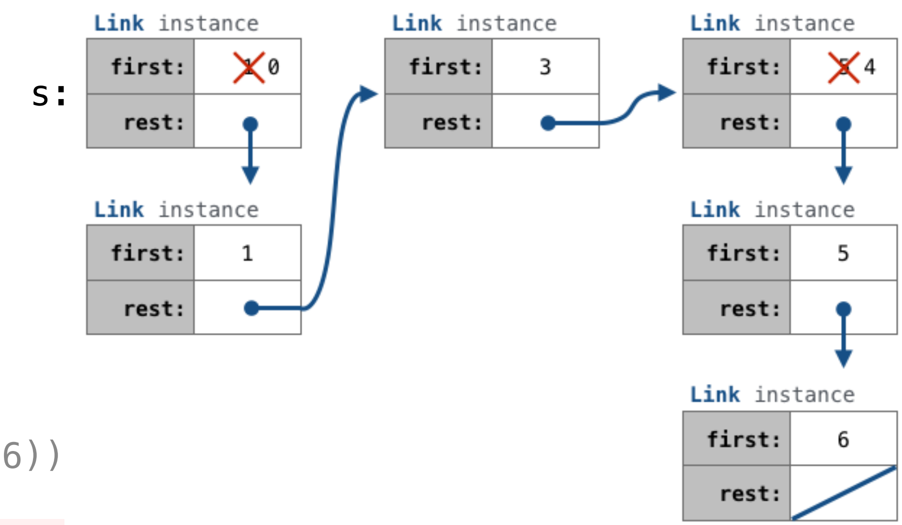
```



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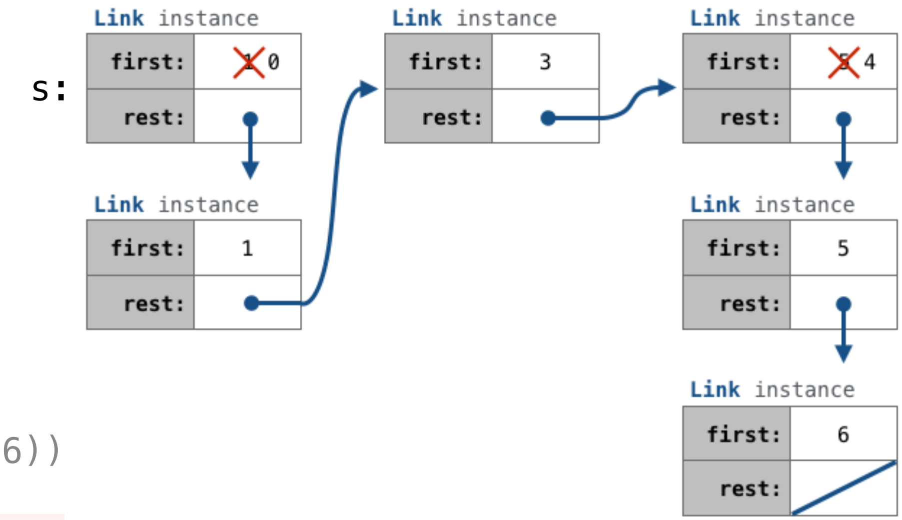
    elif s.first < v and empty(s.rest):
        s.rest = ______Link(v, s.rest)______
    elif s.first < v:
        ______add(s.rest, v)______
    return s
```



Set Operations

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Intersecting Ordered Linked Lists

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```
def intersect(set1, set2):
```

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
        return Link.empty
```

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
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    else:
```

Intersecting Ordered Linked Lists

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def intersect(set1, set2):
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        e1, e2 = set1.first, set2.first
```

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    if empty(set1) or empty(set2):
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    else:
        e1, e2 = set1.first, set2.first
        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
```

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        e1, e2 = set1.first, set2.first
        if e1 == e2:
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        elif e1 < e2:
            return intersect(set1.rest, set2)
```

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Order of growth?

Intersecting Ordered Linked Lists

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Order of growth? $\Theta(n)$

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```

Order of growth? $\Theta(n)$ (Demo)