61A Lecture 21

Announcements

Binary Trees

Binary Tree Class

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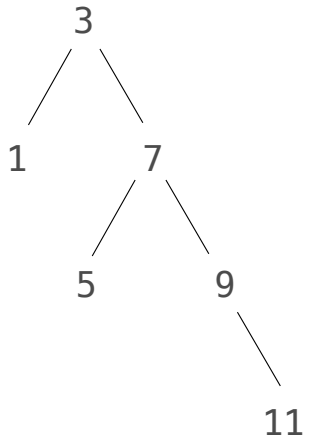

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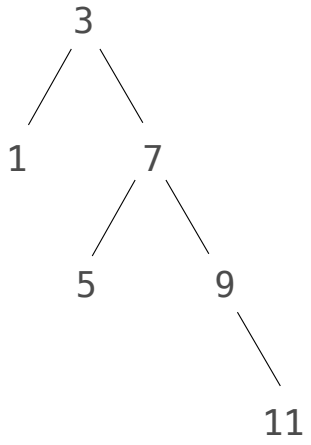

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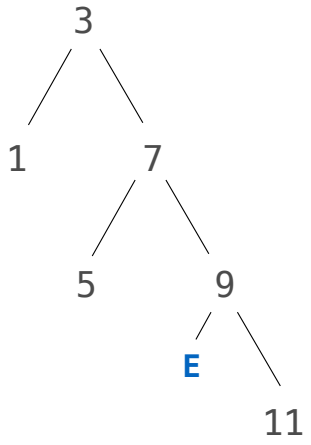

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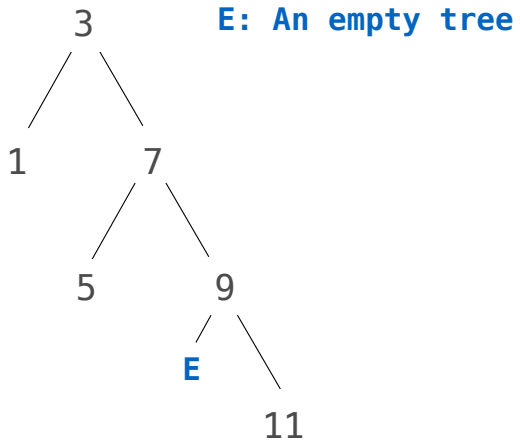

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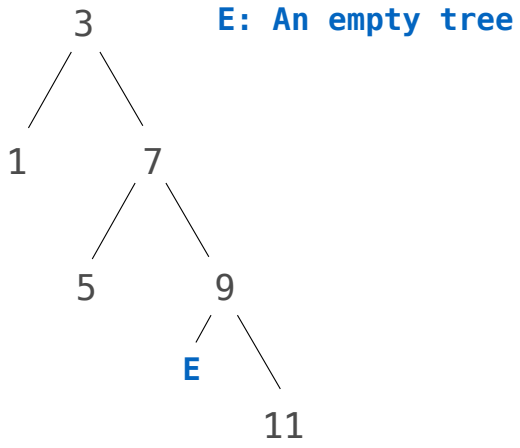

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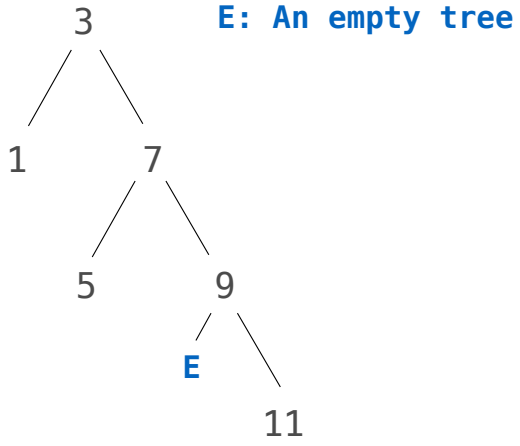

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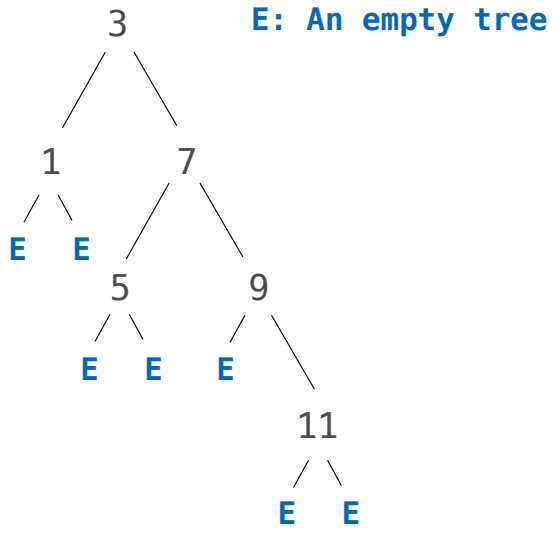


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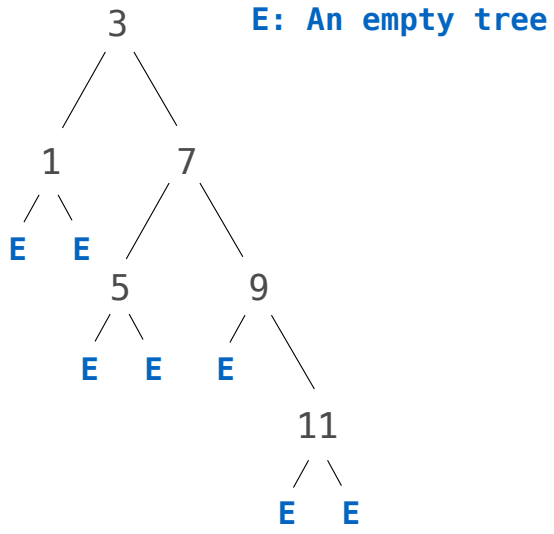

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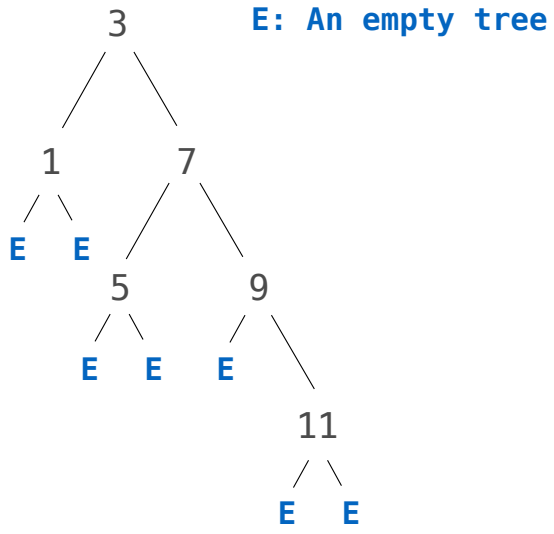

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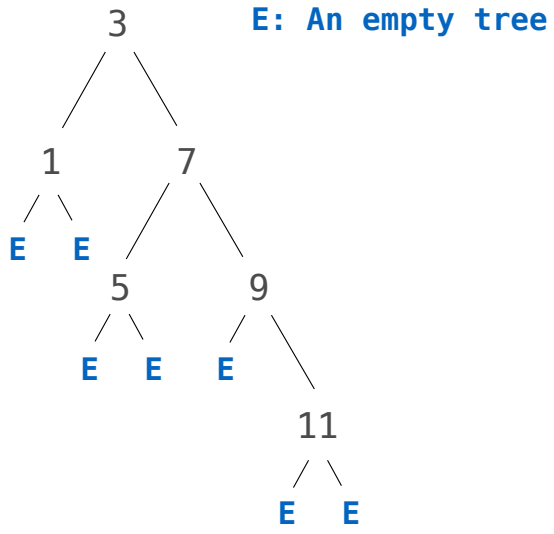

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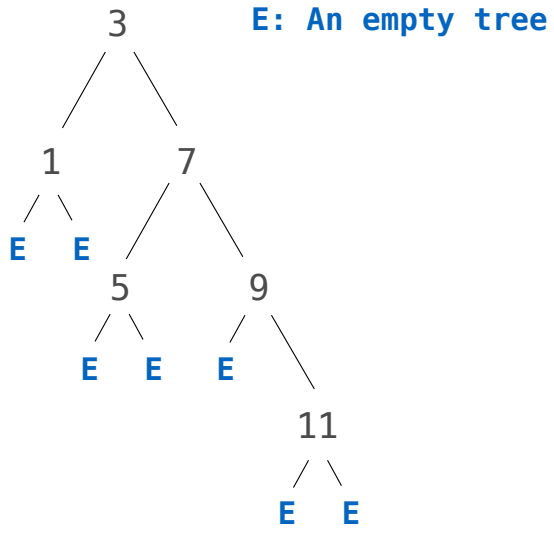

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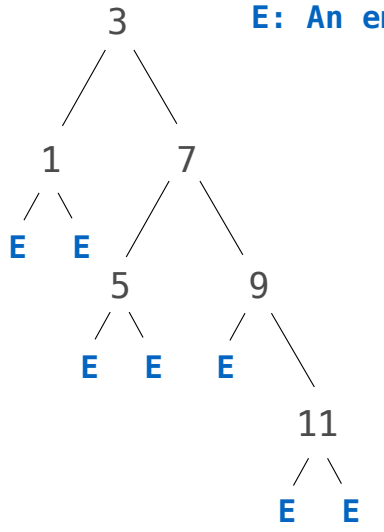

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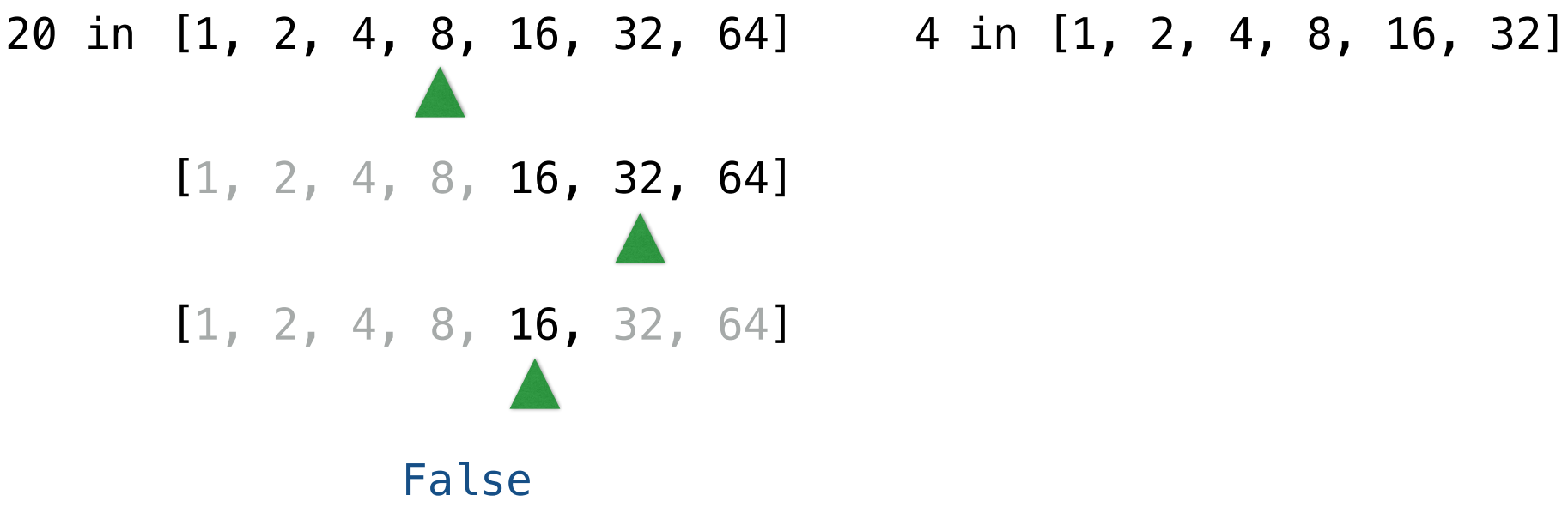

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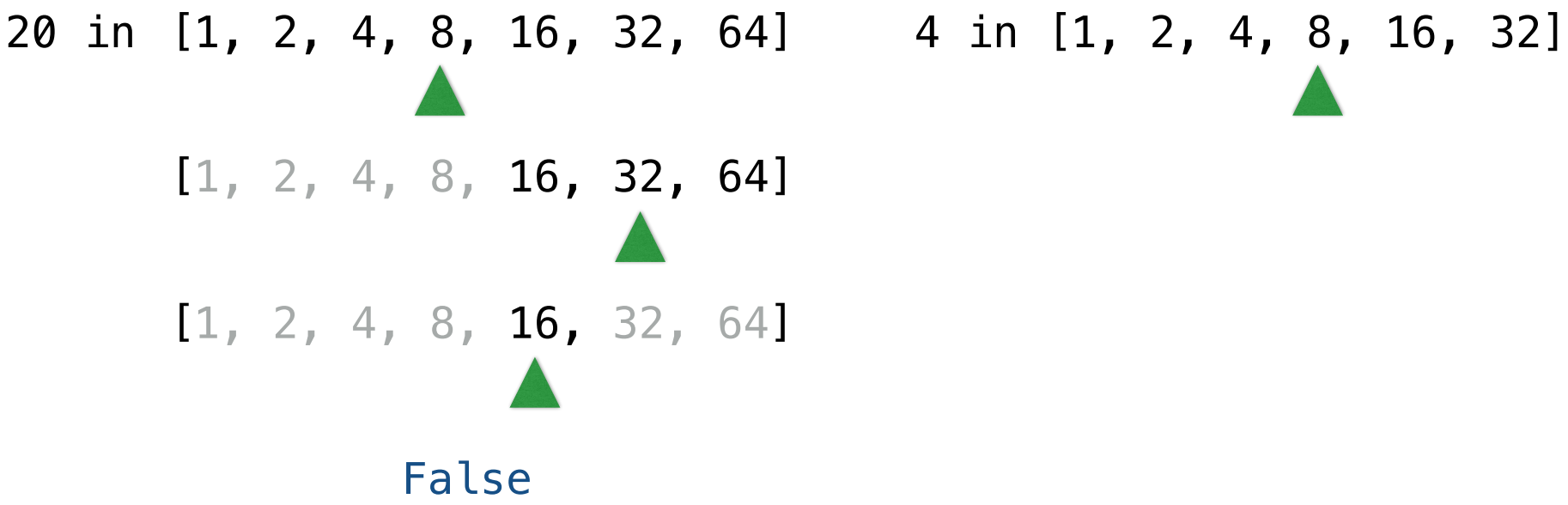

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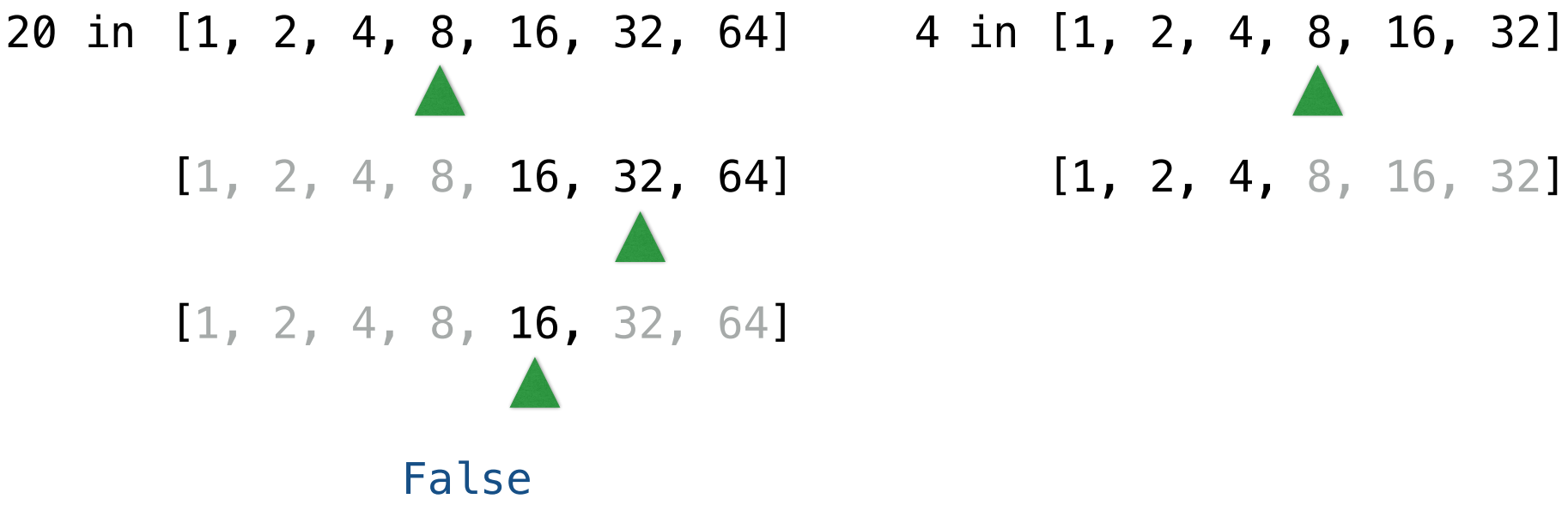

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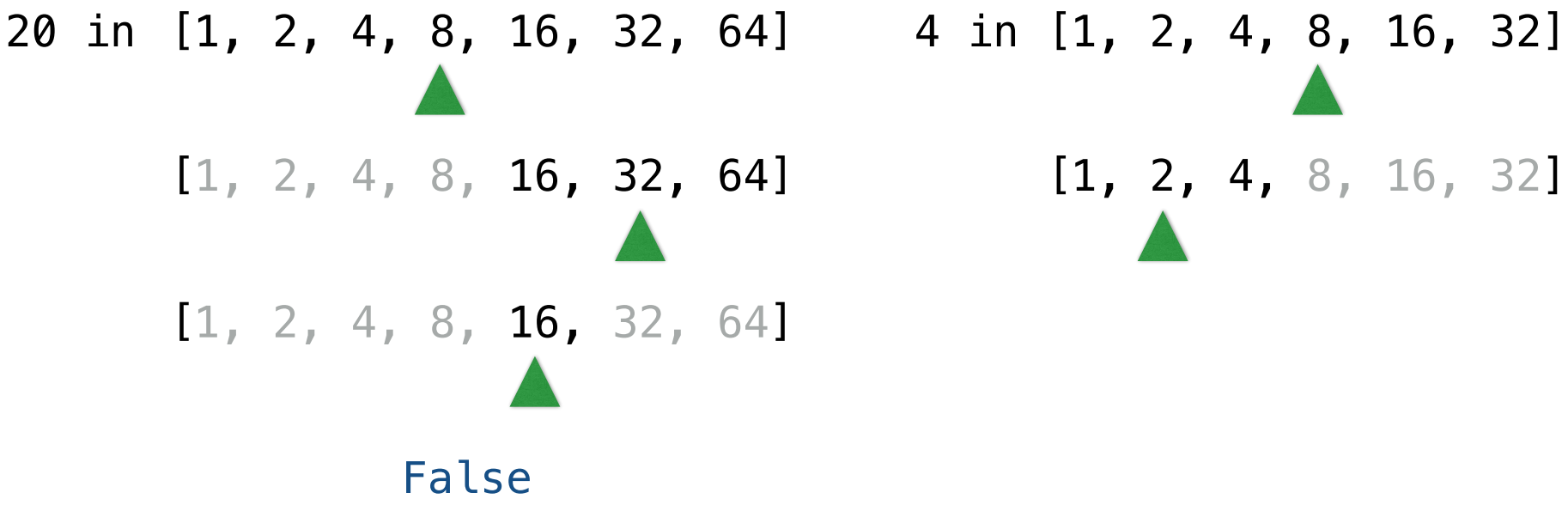

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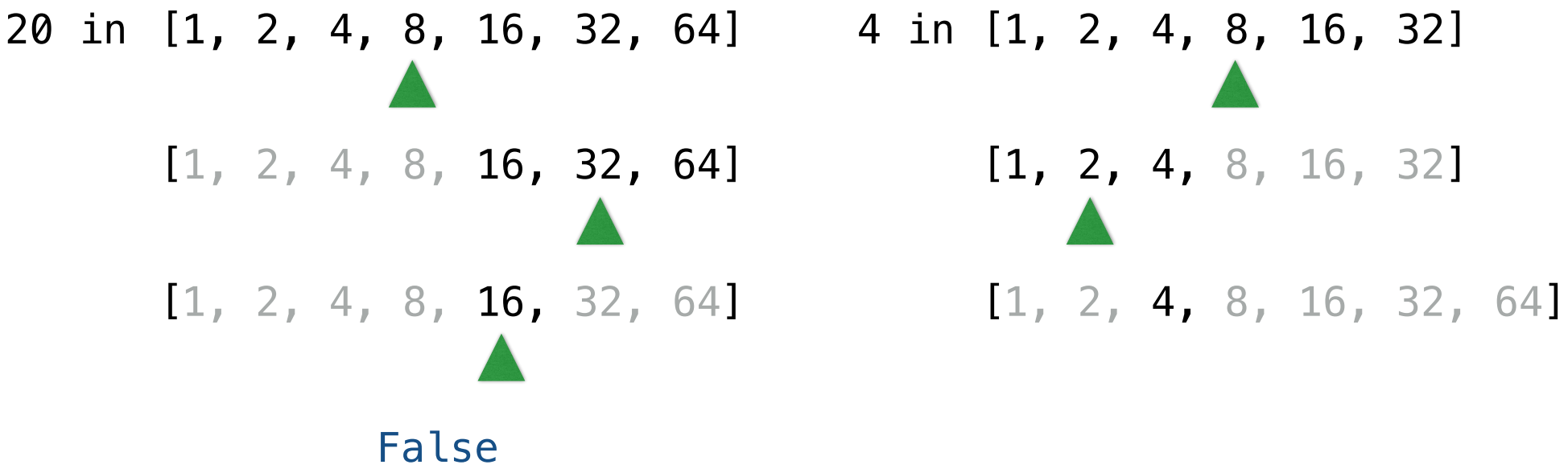

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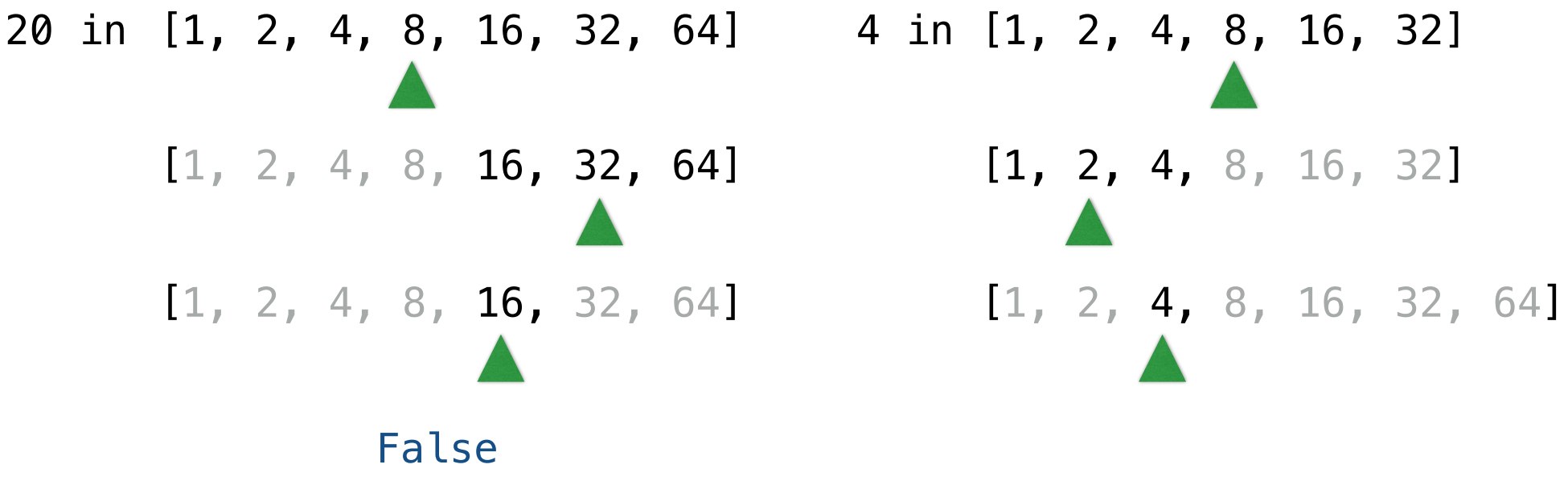

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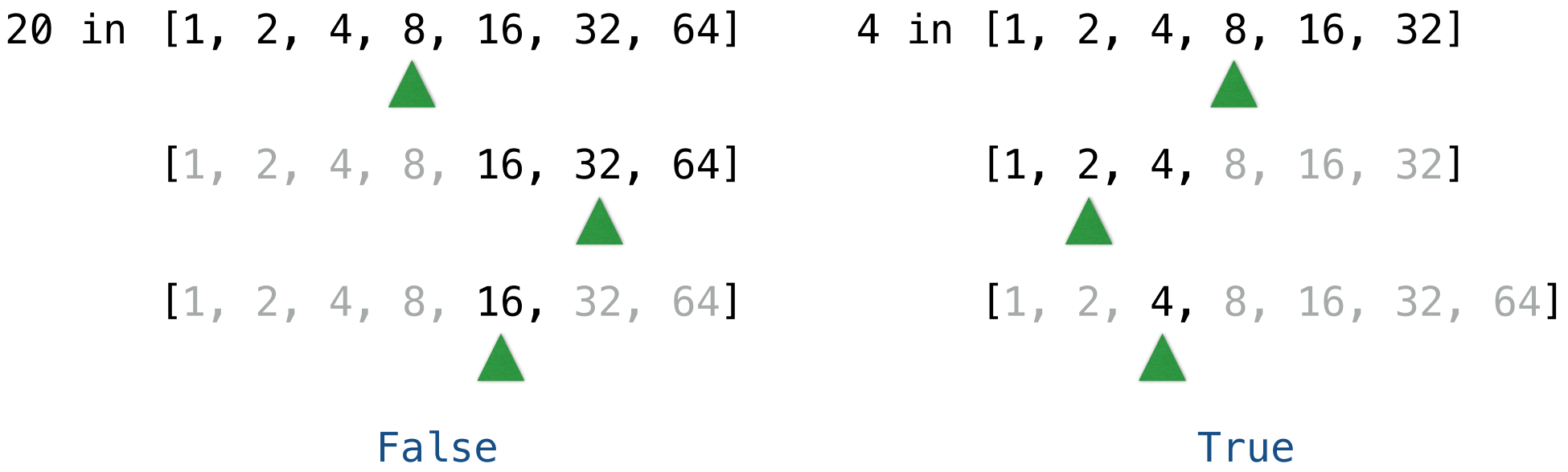

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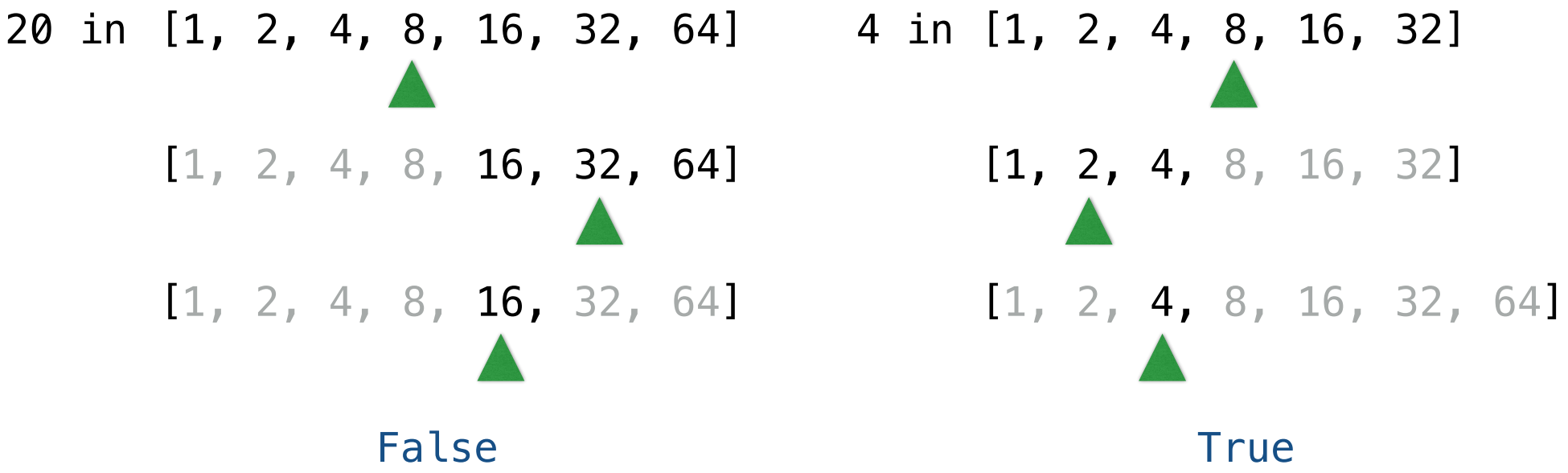

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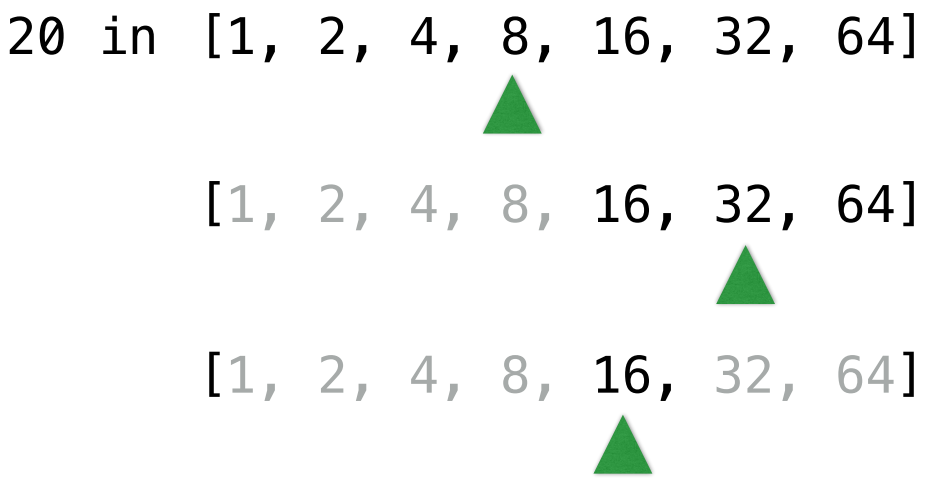

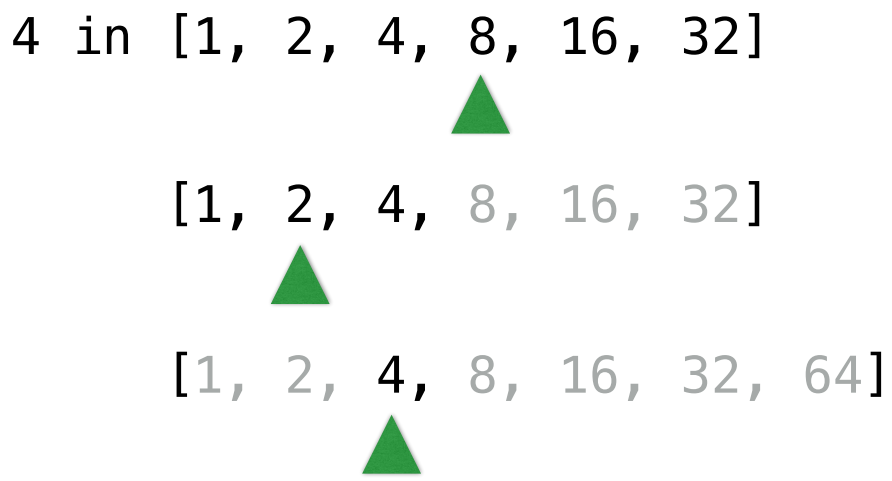

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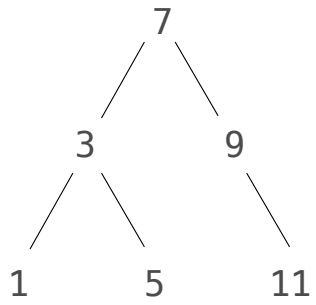

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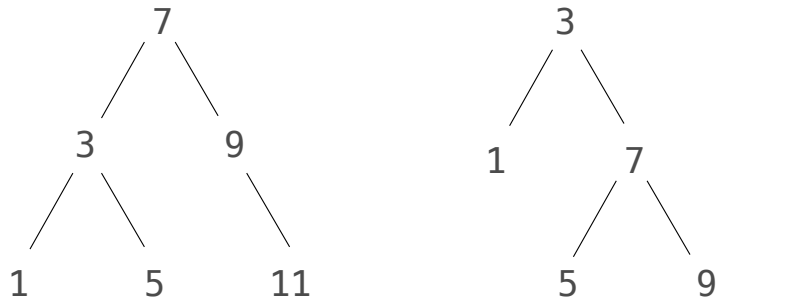

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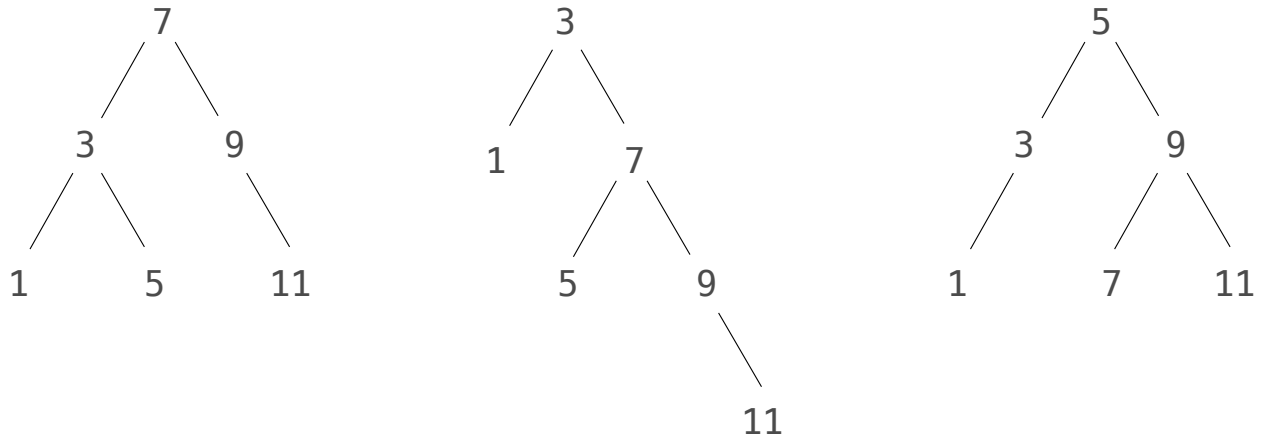

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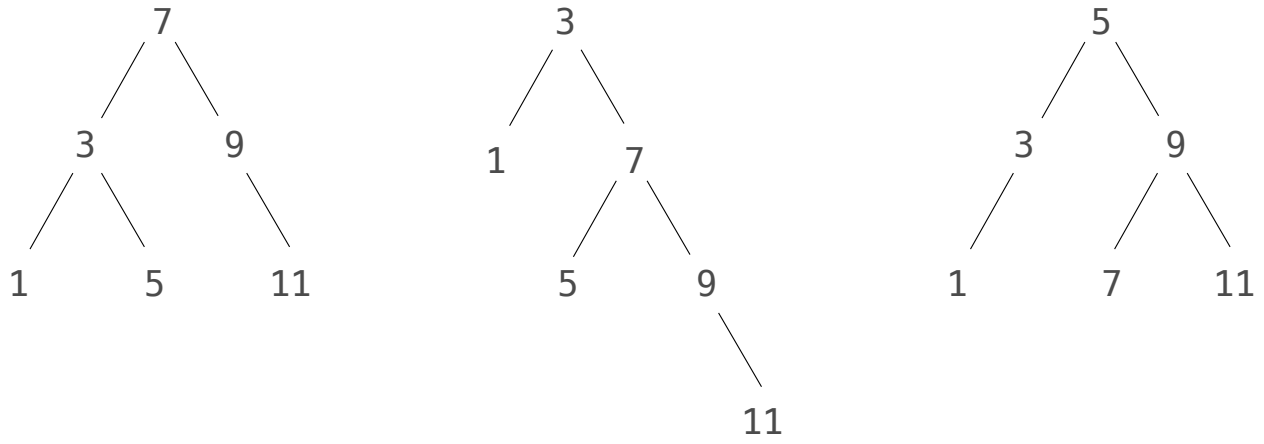

(Demo)

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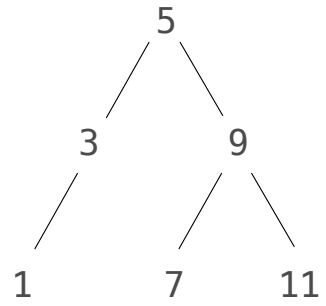

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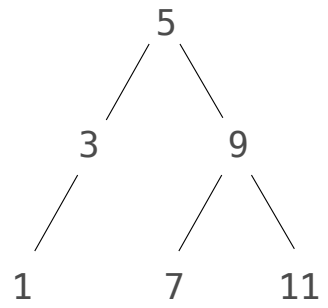

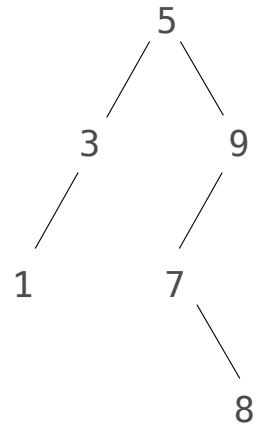

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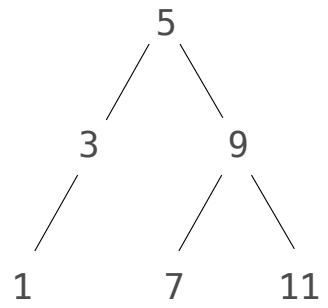

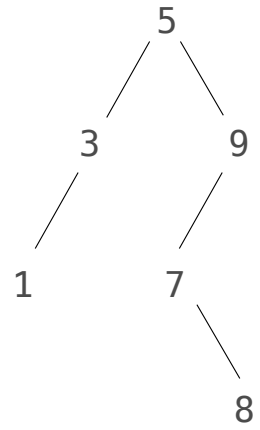

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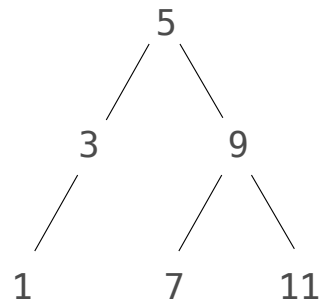

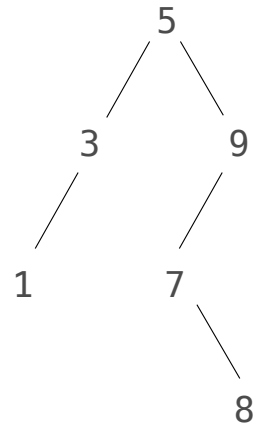

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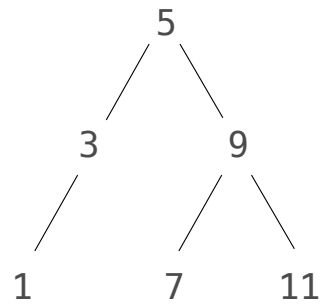

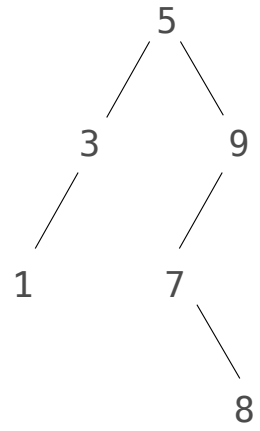

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What's the second largest element in a binary search tree?

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    if t.is_leaf():
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    elif _____________________:
        return t.root
    elif _____________________:
        return _____________
    else:
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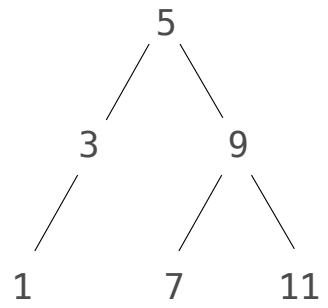

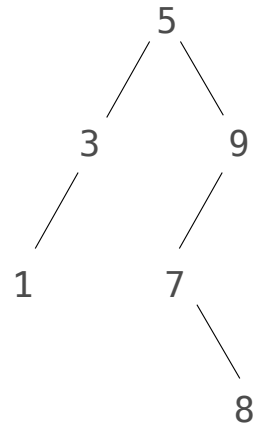

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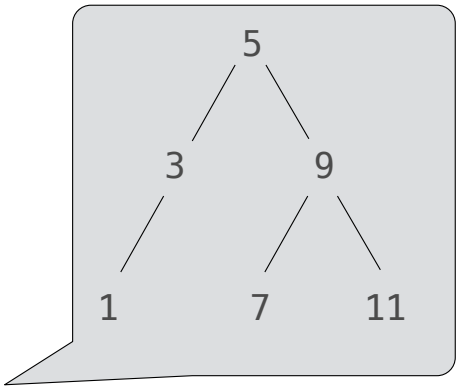

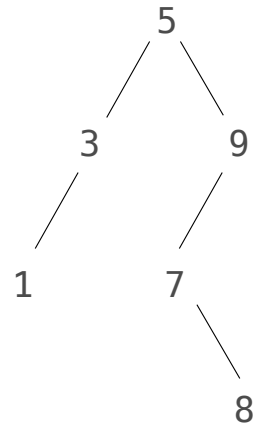

Discussion Questions

What's the largest element in a binary search tree?

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def largest(t):
    if t.right is BTree.empty:
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    else:
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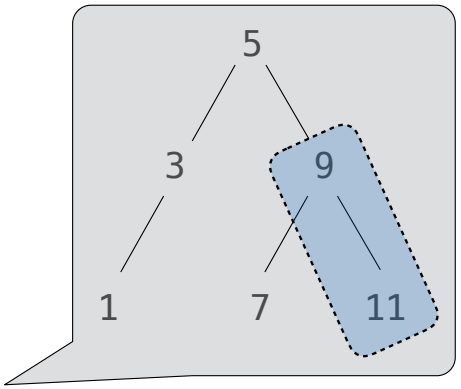

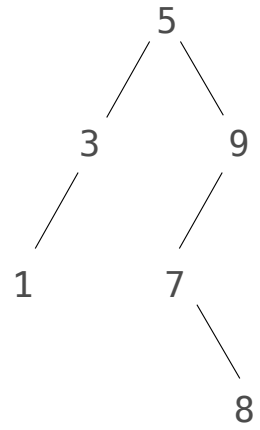

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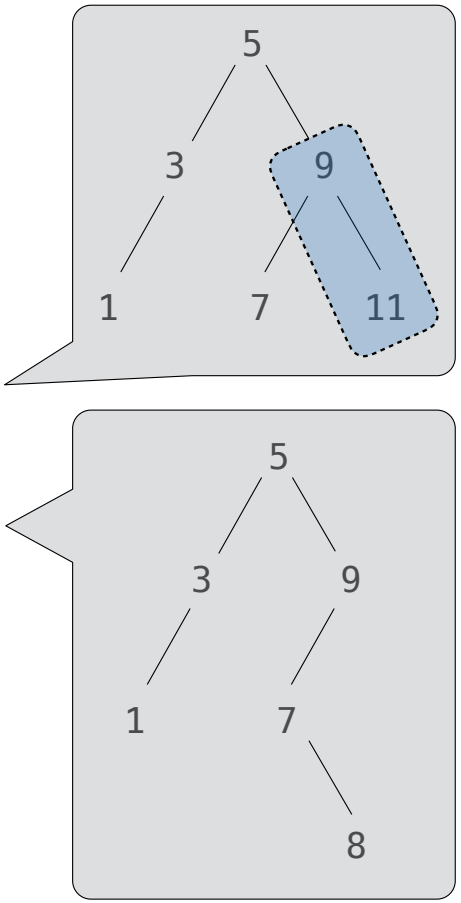

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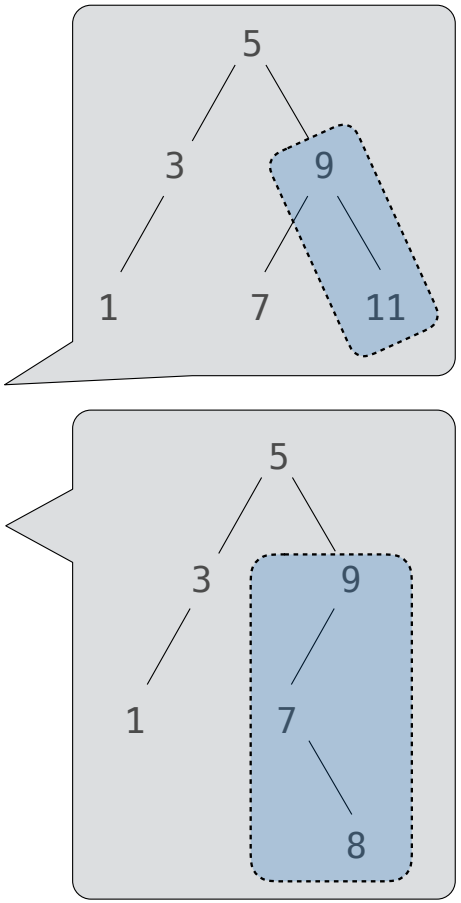

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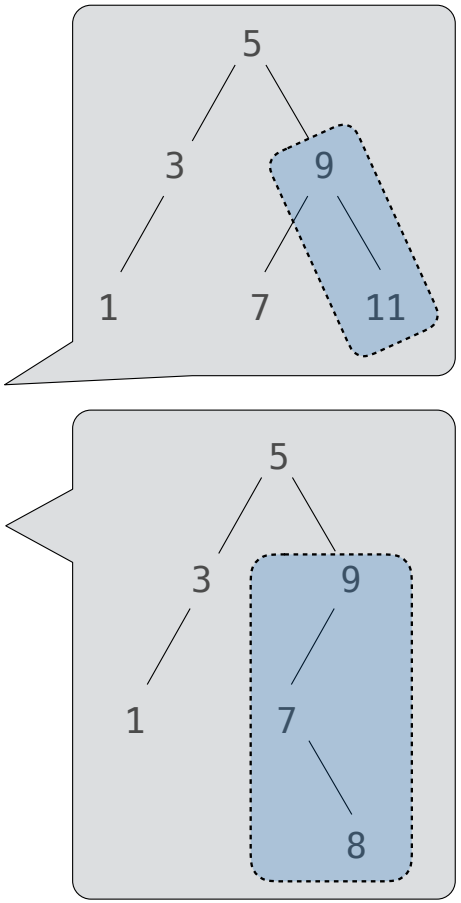

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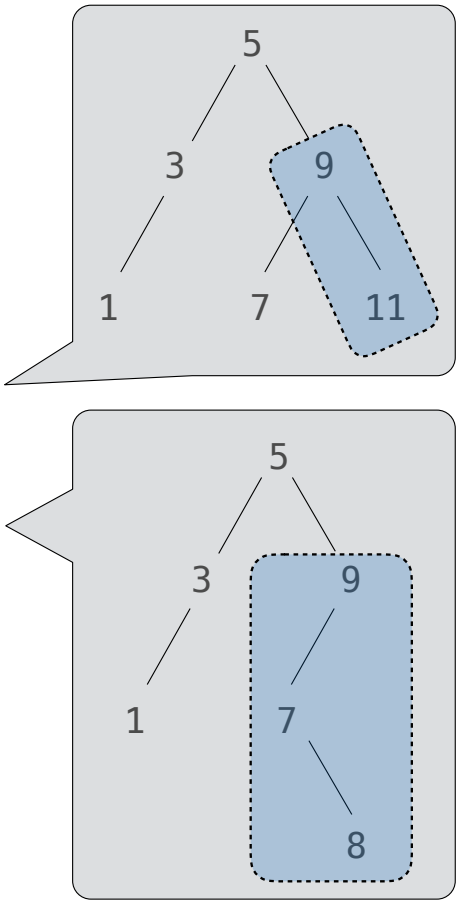

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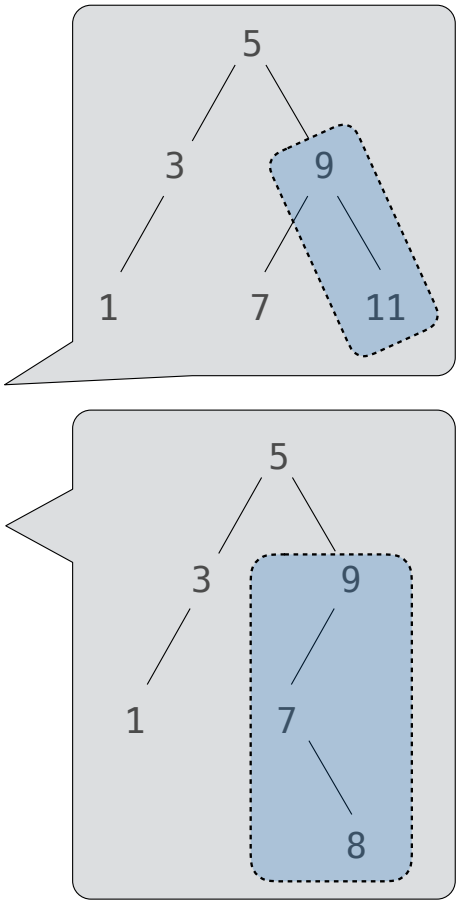

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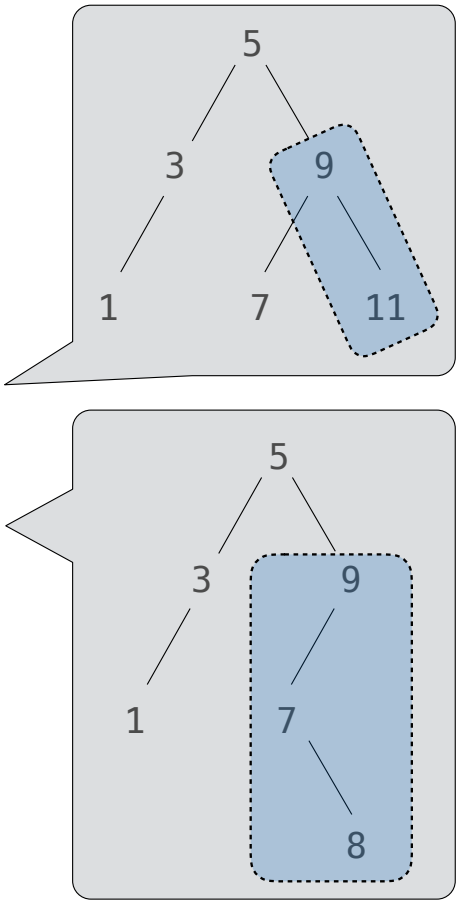

Sets as Binary Search Trees

Membership in Binary Search Trees

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`contains` traverses the tree

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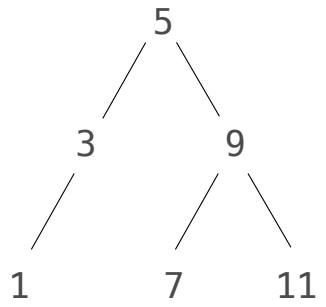

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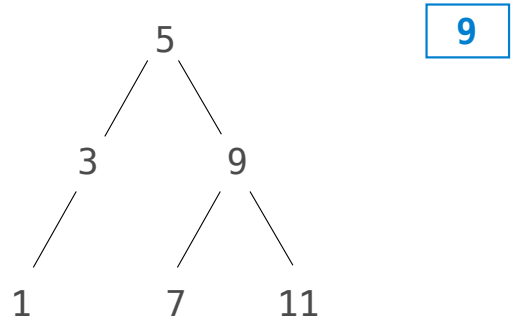

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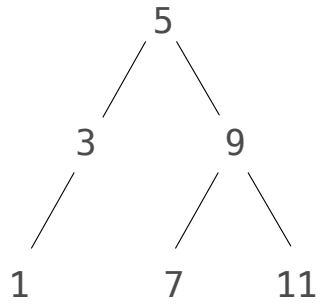
def contains(s, v):
```

9



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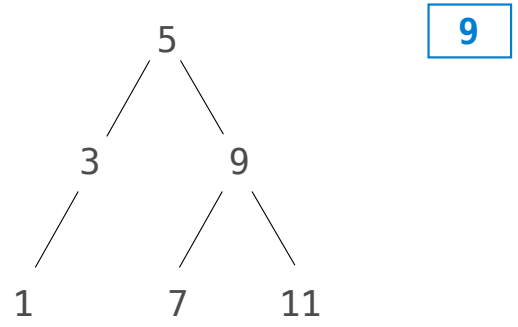

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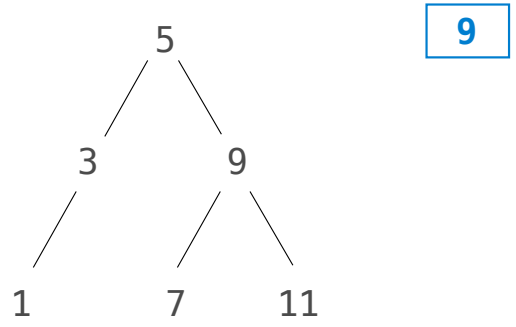

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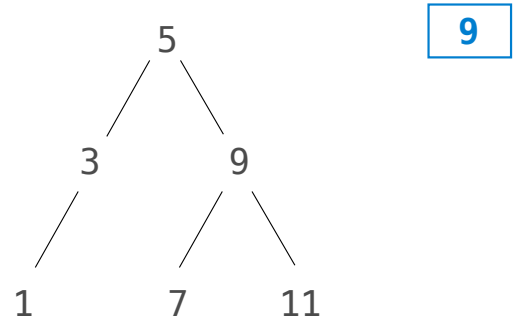

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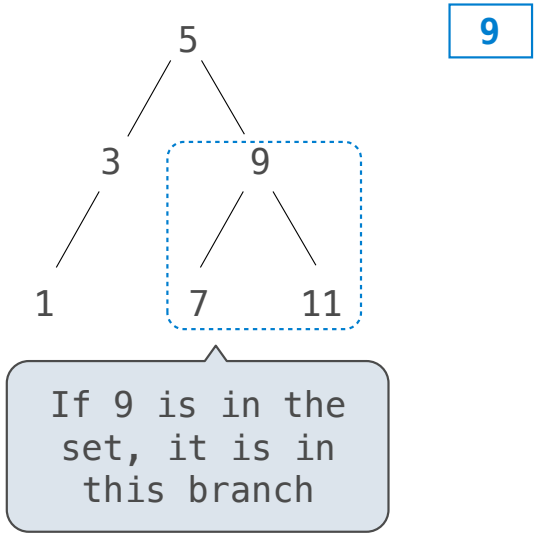

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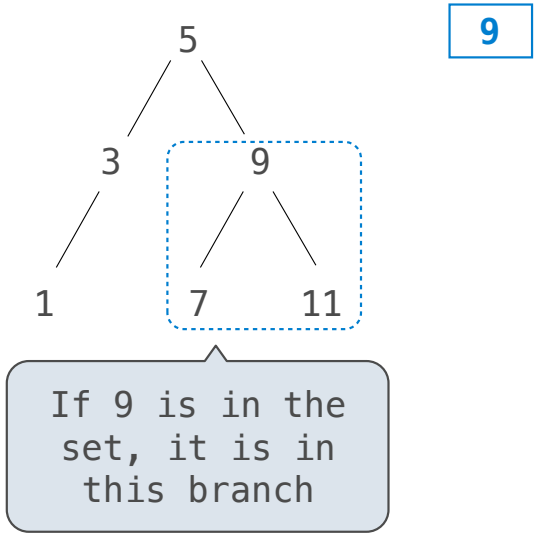

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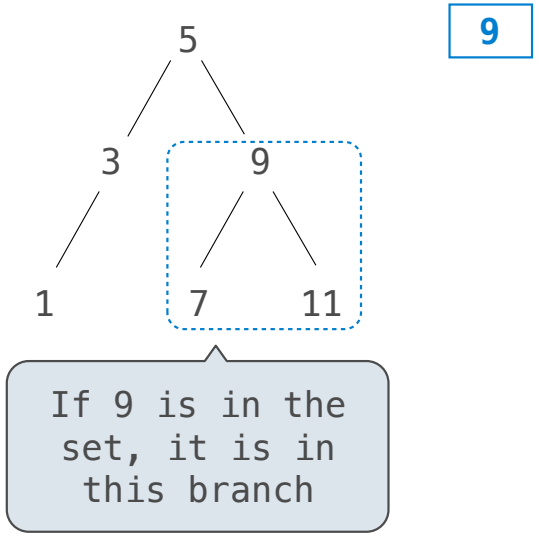

Order of growth?

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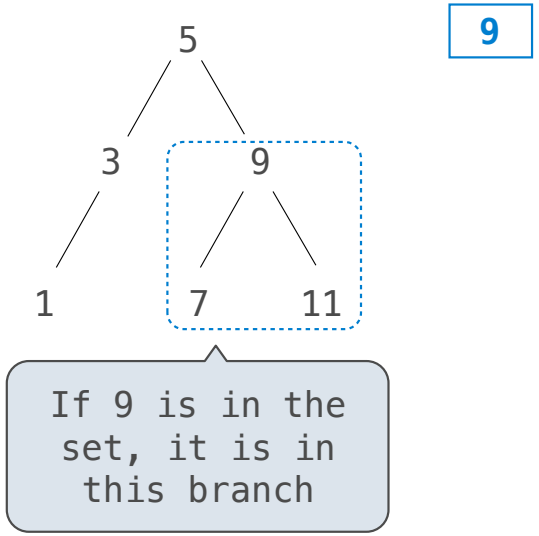

Order of growth? $\Theta(h)$ on average

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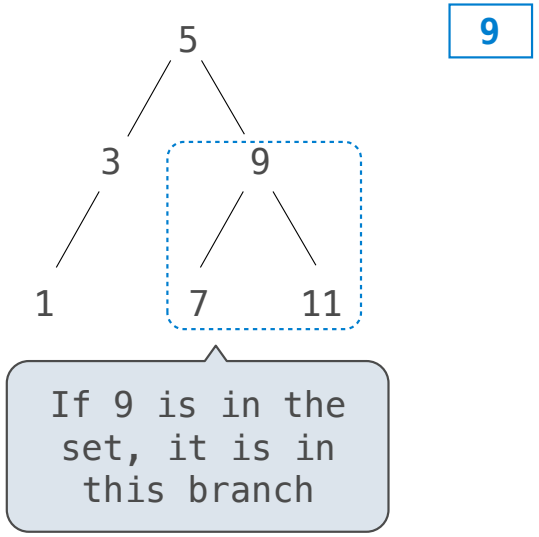

Order of growth? $\Theta(h)$ on average $\Theta(\log n)$ on average for a balanced tree

Adjoining to a Tree Set

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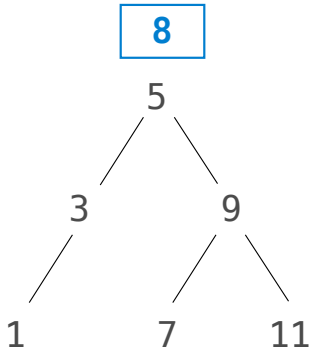

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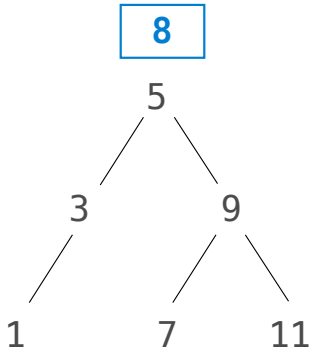

Right!

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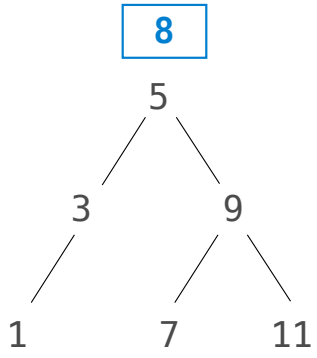

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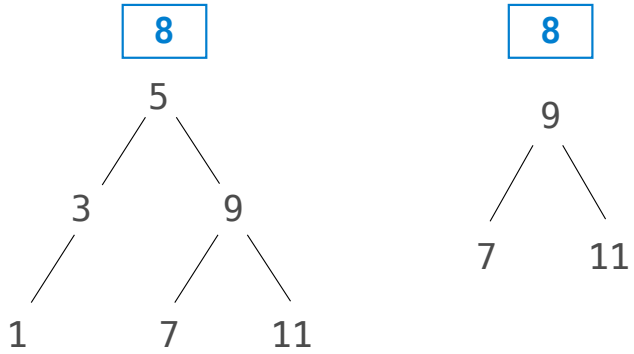

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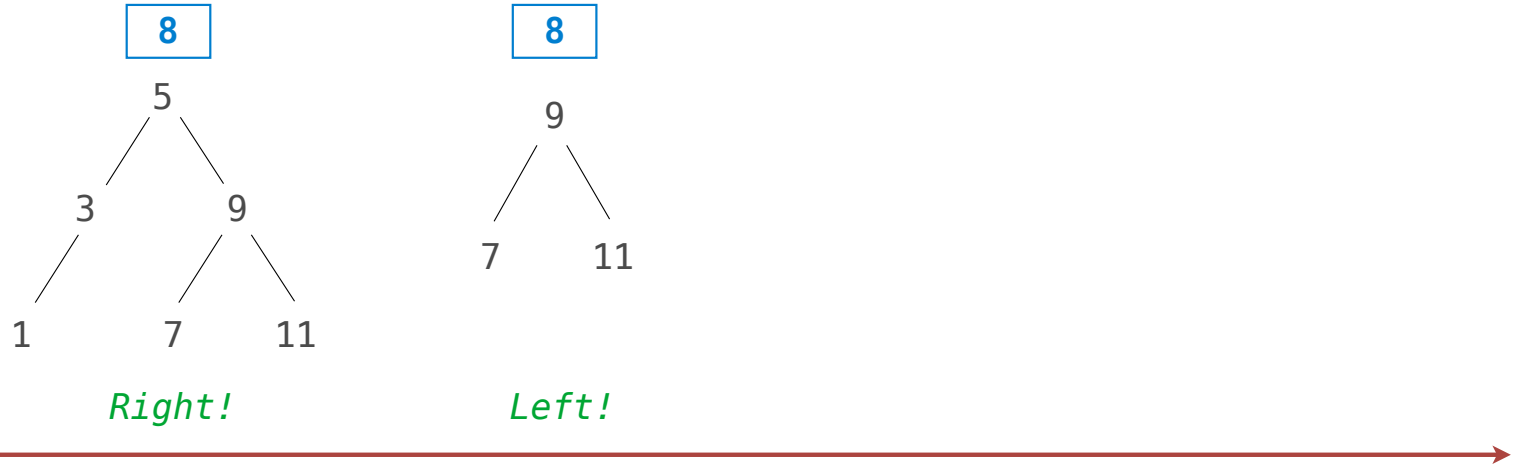

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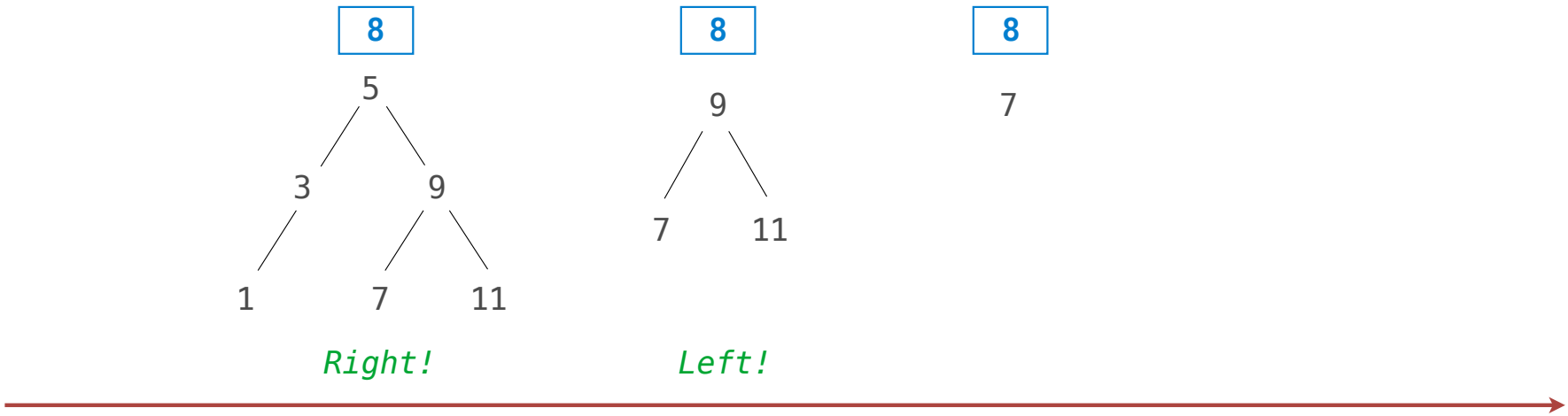

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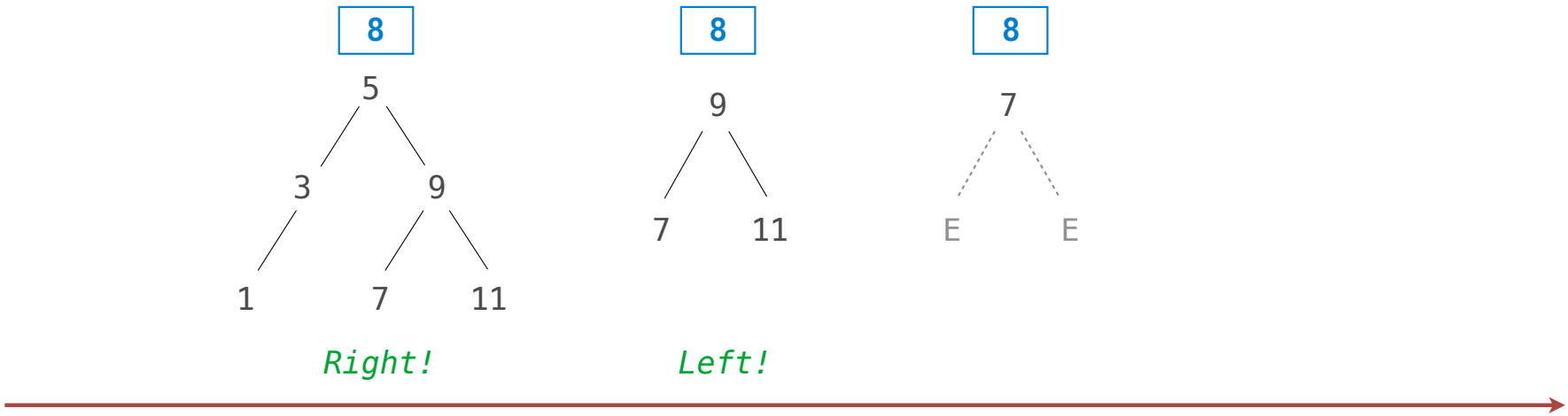

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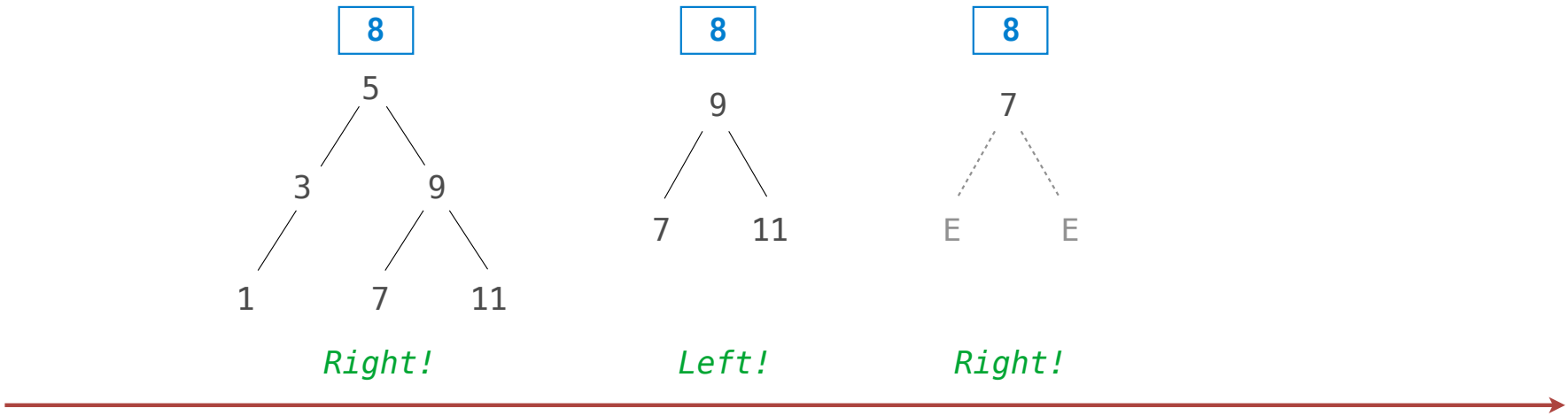

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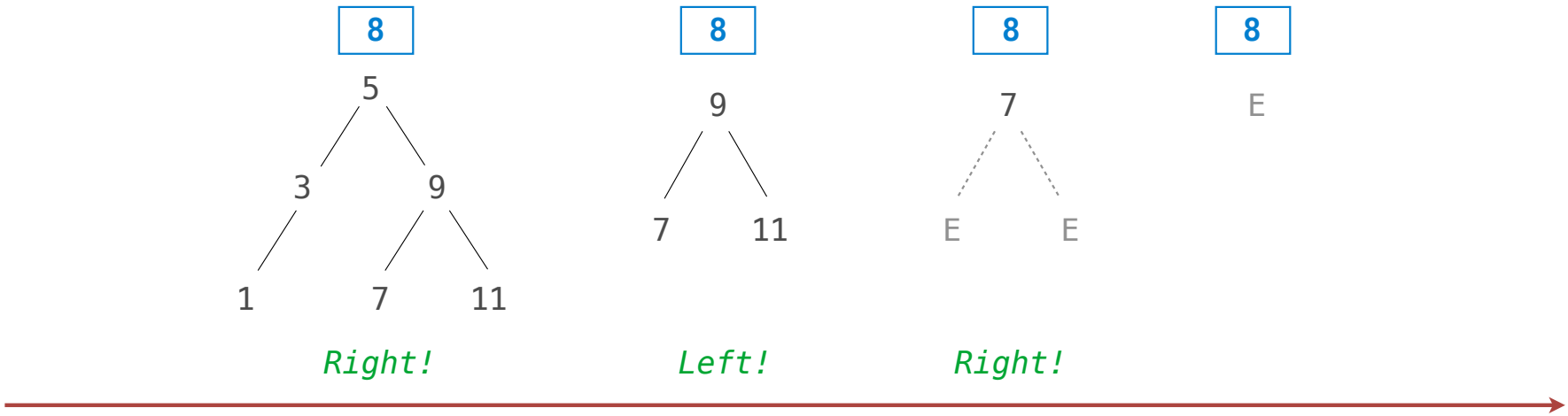

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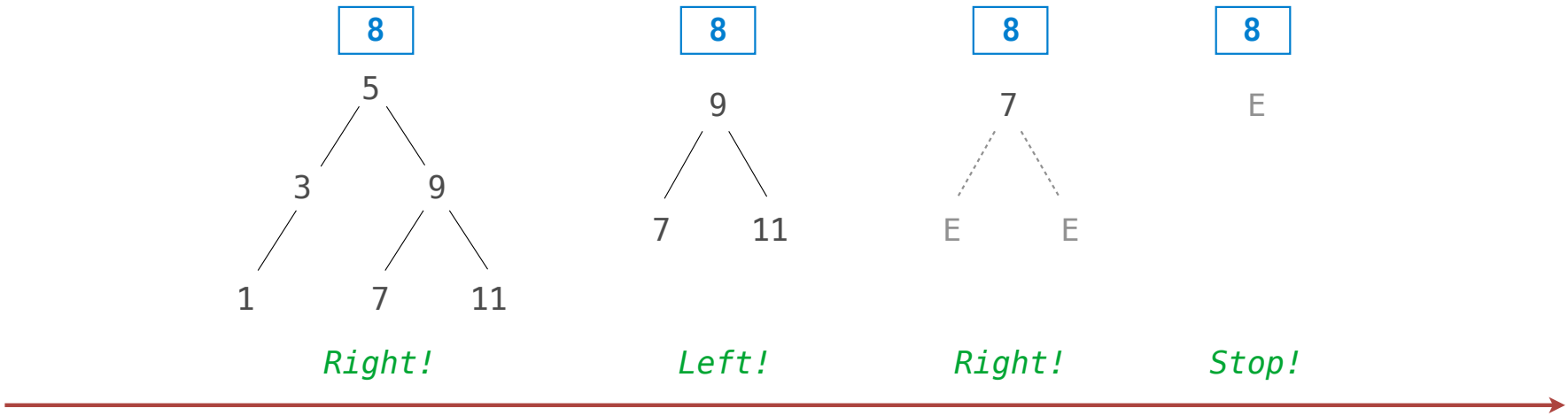

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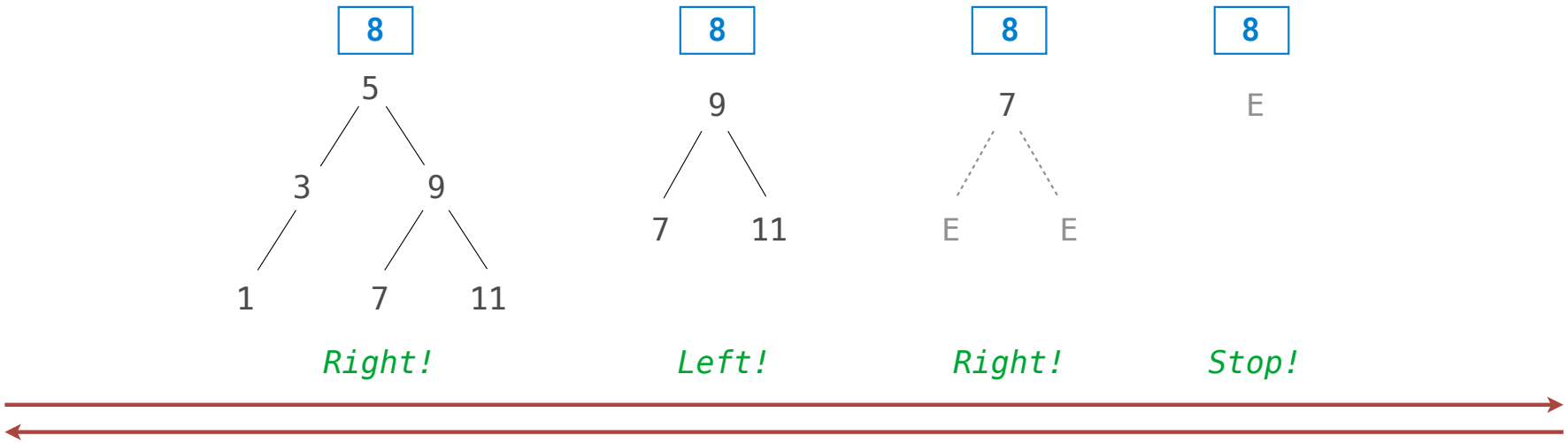

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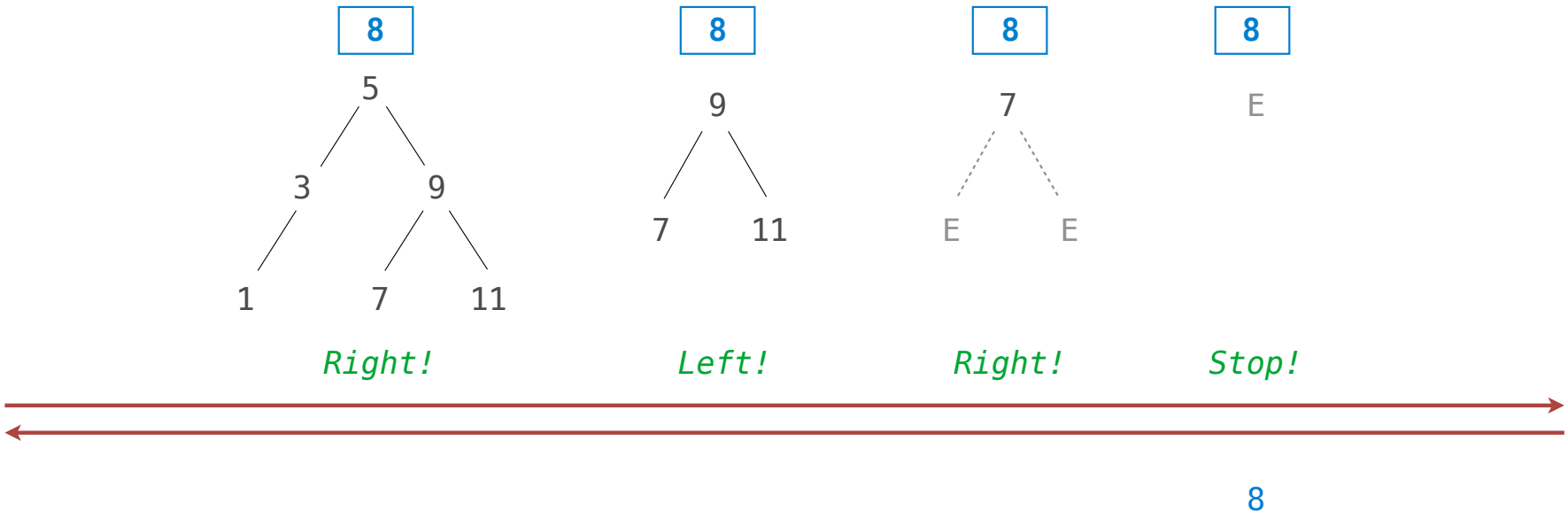

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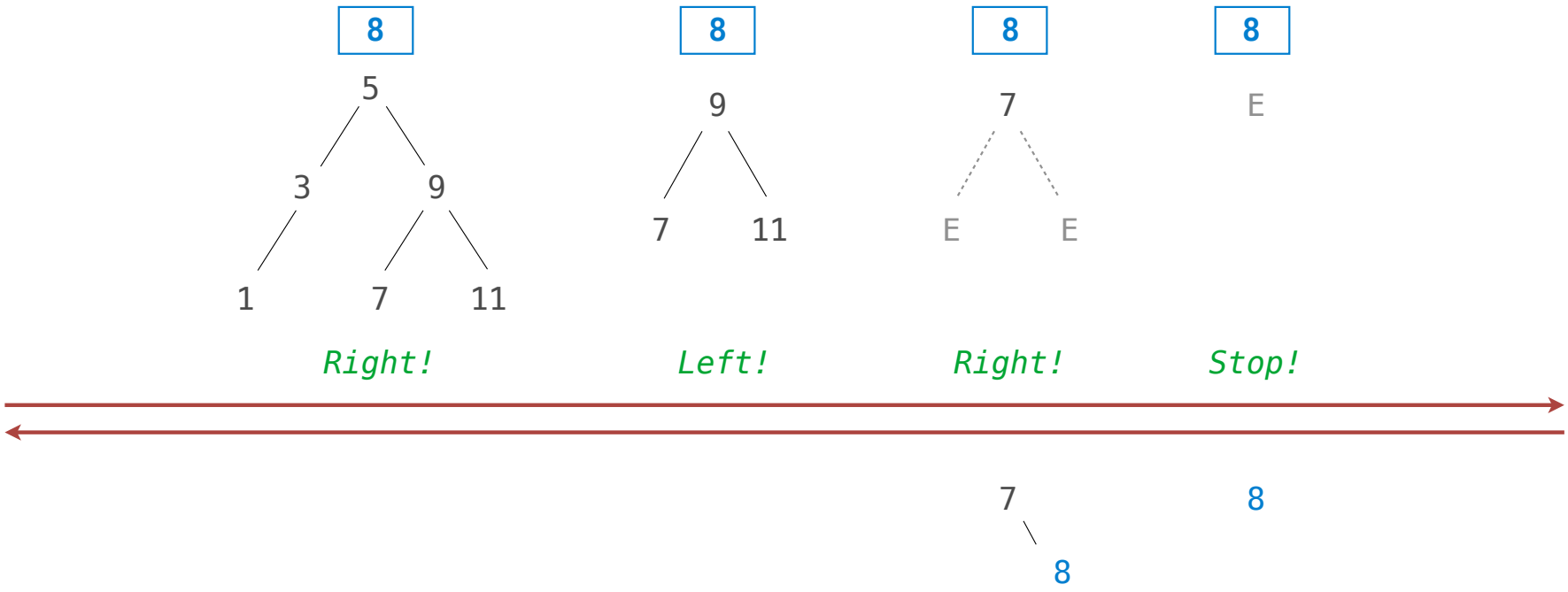

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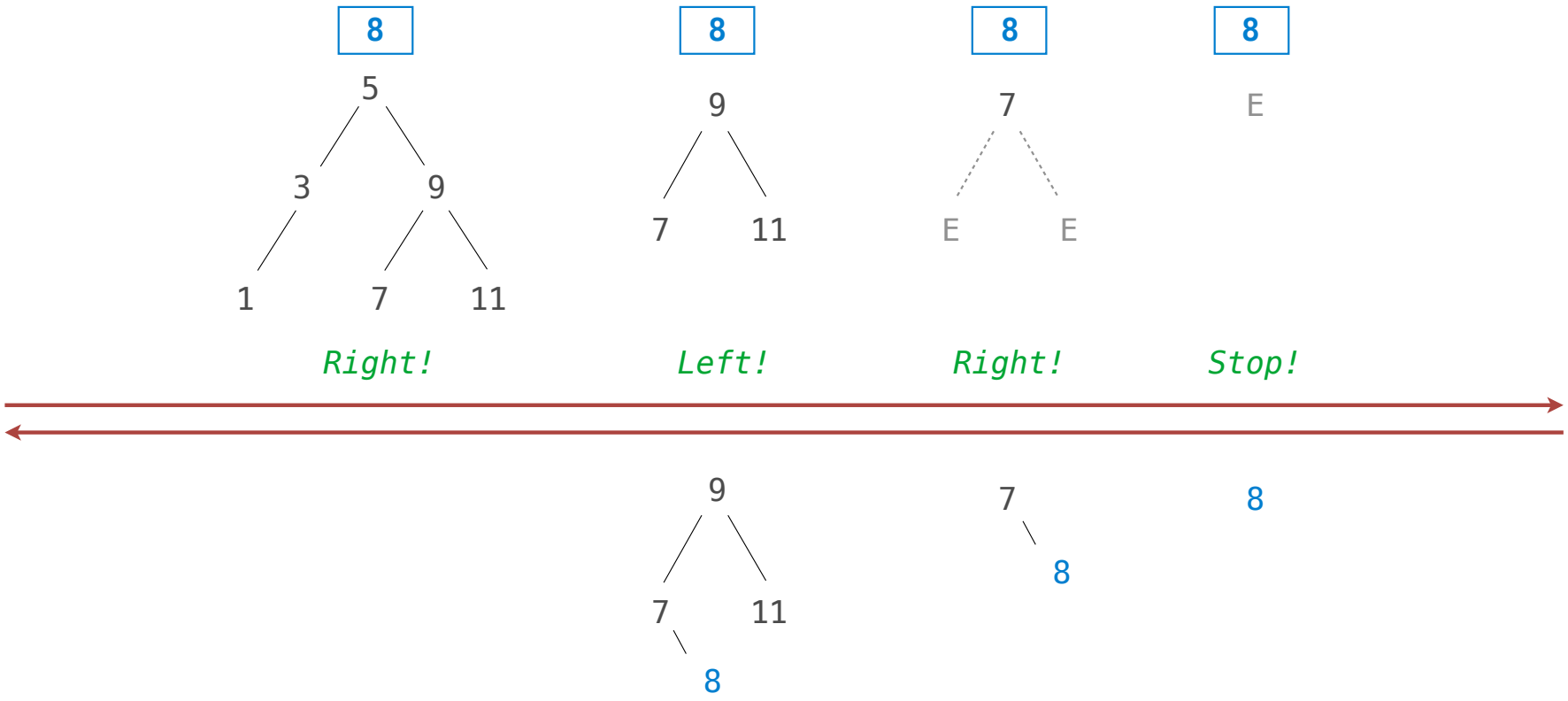

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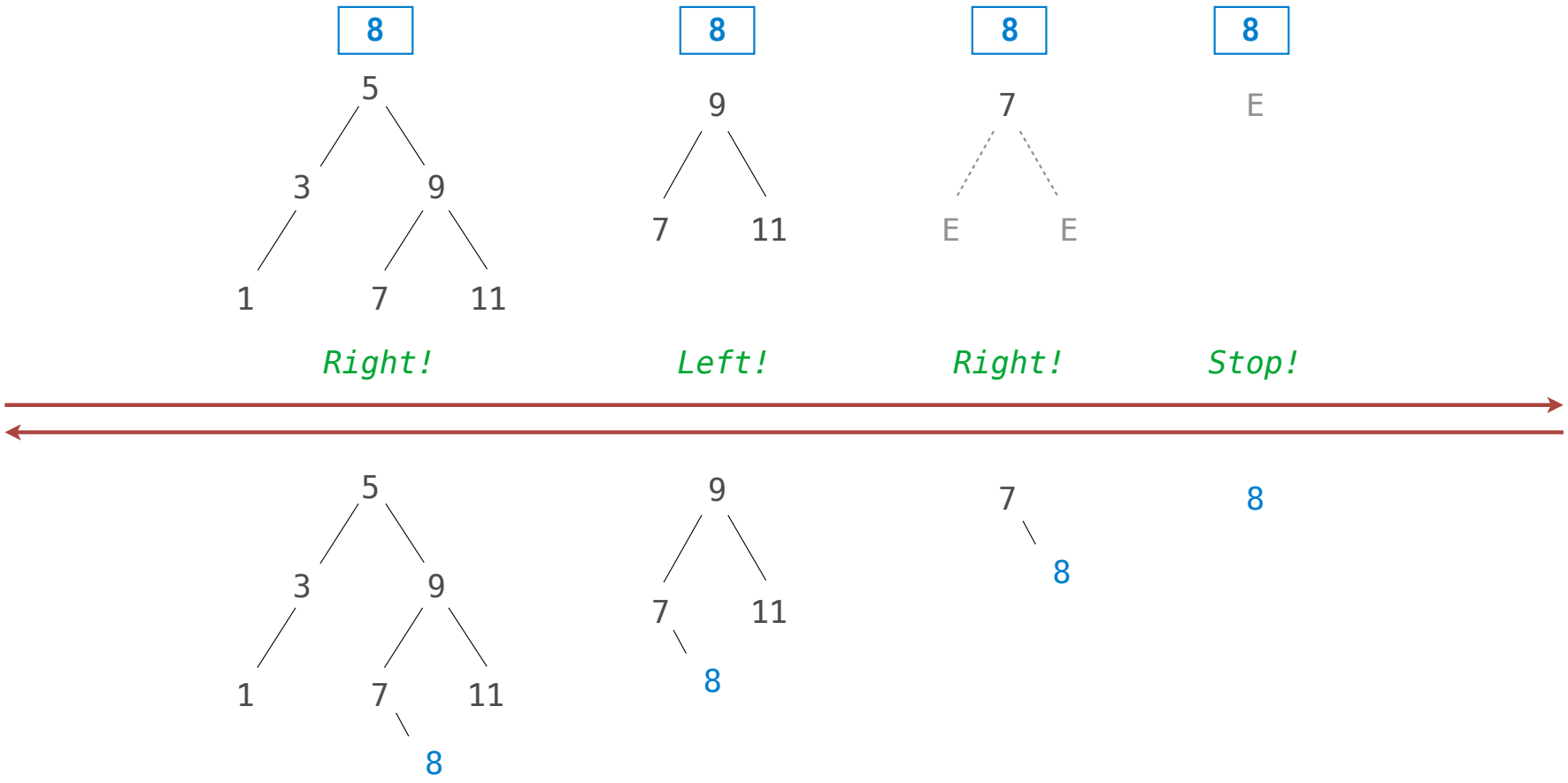

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