61A Lecture 25

Announcements

Pairs Review

In the late 1950s, computer scientists used confusing names
• cons: Two-argument procedure that creates a pair

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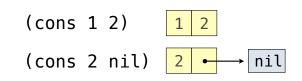


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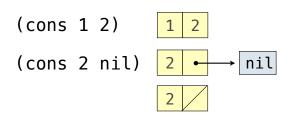
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- car: Procedure that returns the first element of a pair
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- nil: The empty list



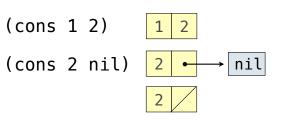
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(cons 2 nil)

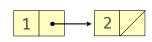
(cons 1 2) 1 2 $(cons 2 nil) 2 \rightarrow nil$ $2 \rightarrow 2$

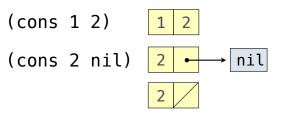
2

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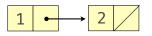


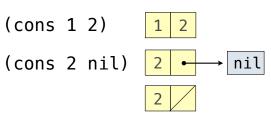


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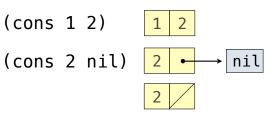




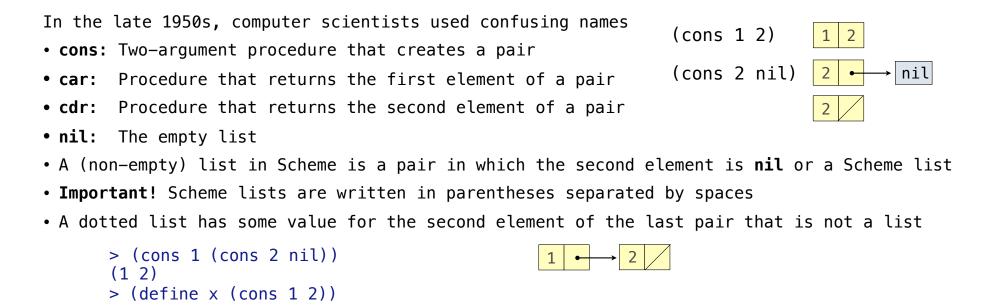
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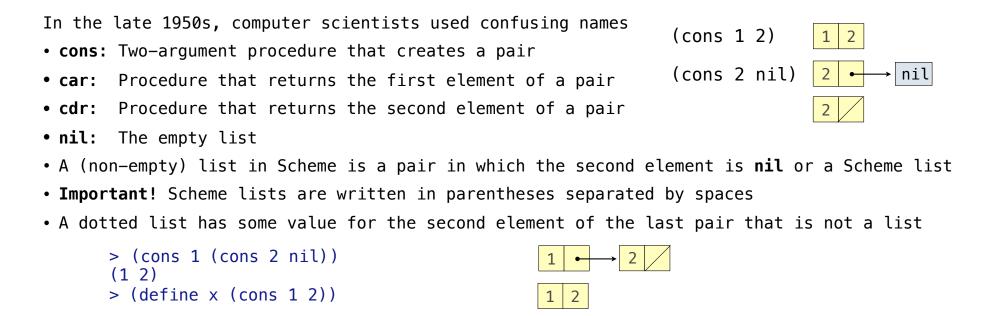
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> (cons 1 (cons 2 nil))
(1 2)
```

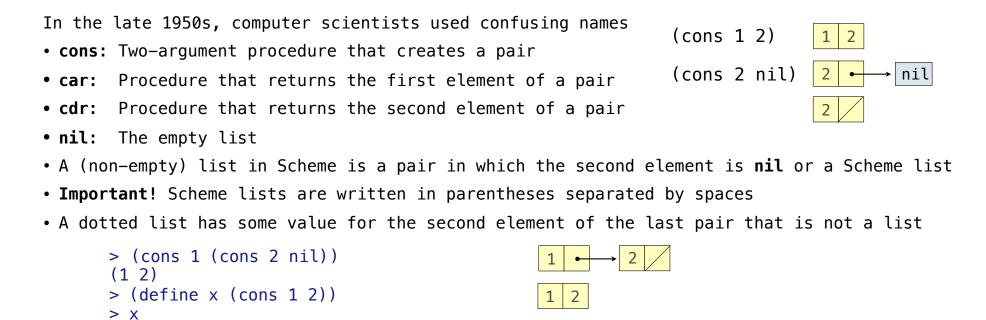
```
1 → 2 /
```

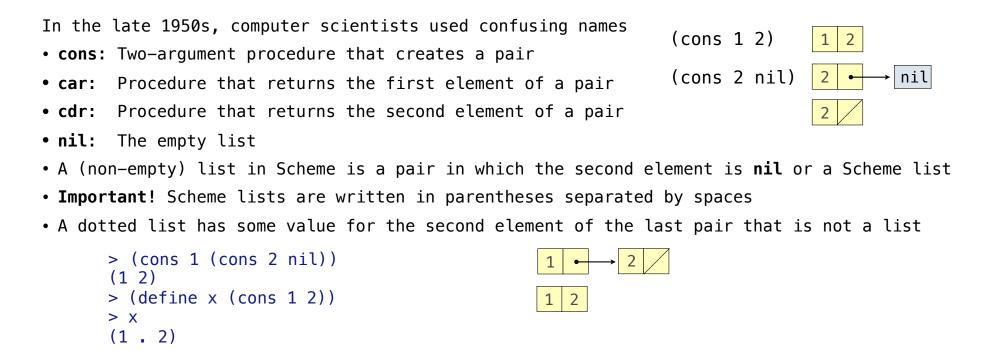


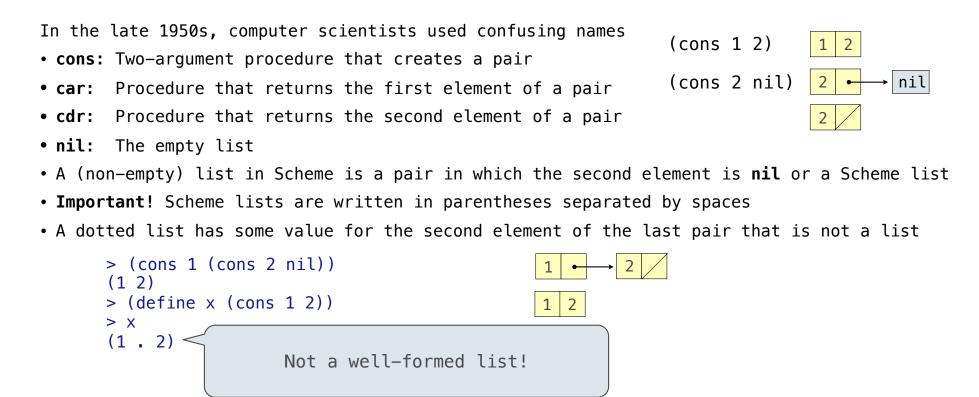
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• A (non-empty) list in Scheme is a pair in which the second element is nil or a Scheme list			
 Important! Scheme lists are written in parentheses separated by spaces 			
• A dotted list has some value for the second element of the last pair that is not a list			
	> (cons 1 (cons 2 nil)) $1 \leftrightarrow 4$	2	

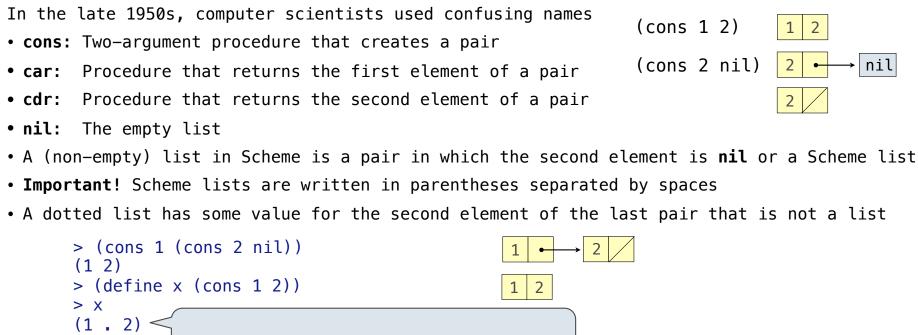


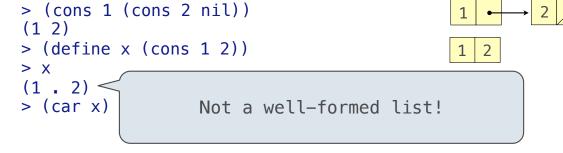






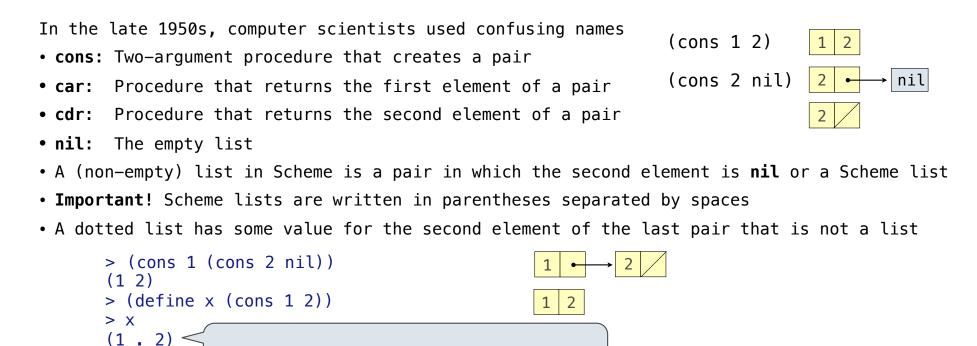






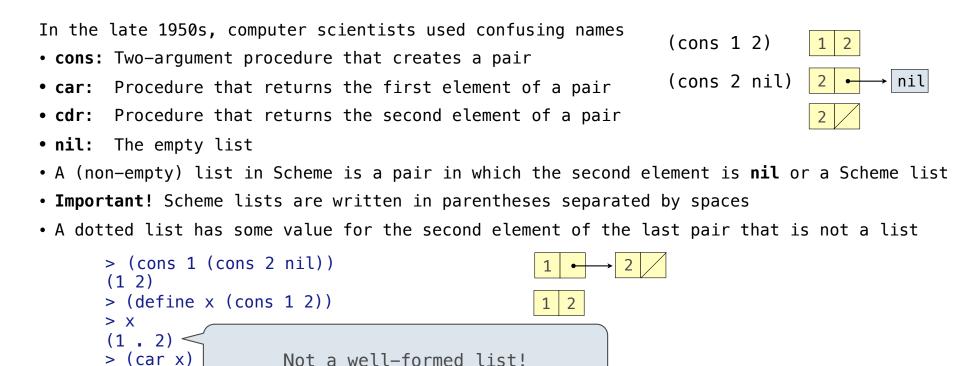
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1



Not a well-formed list!

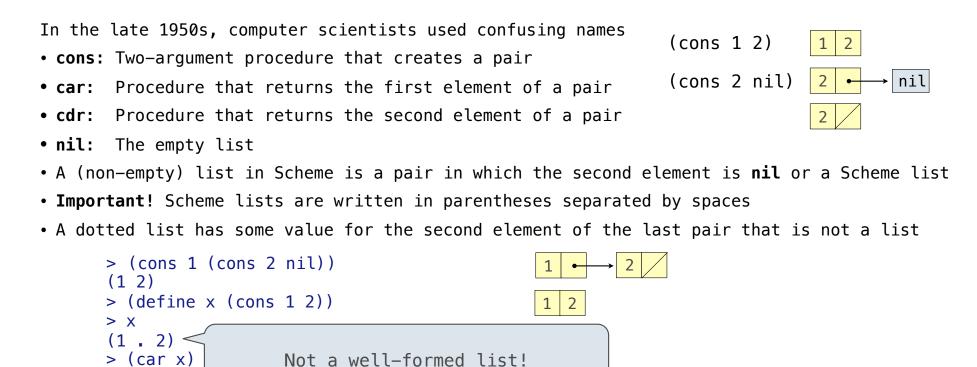
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1

2

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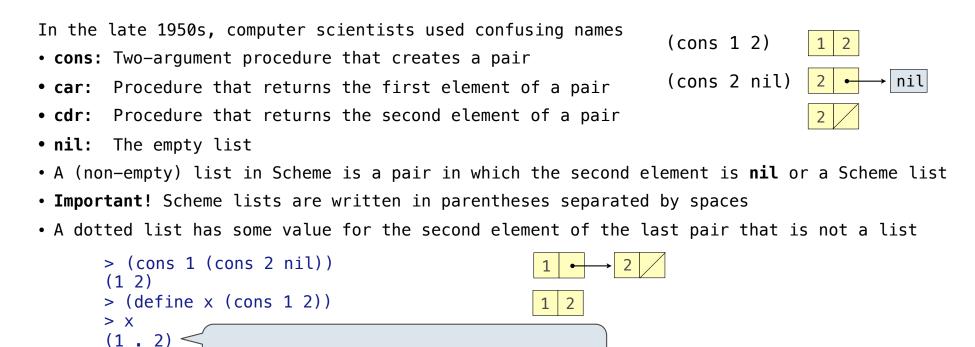


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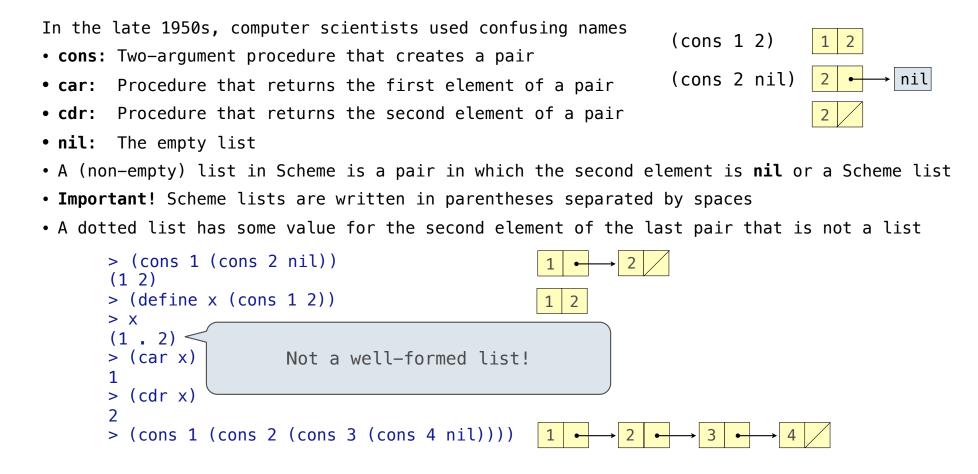
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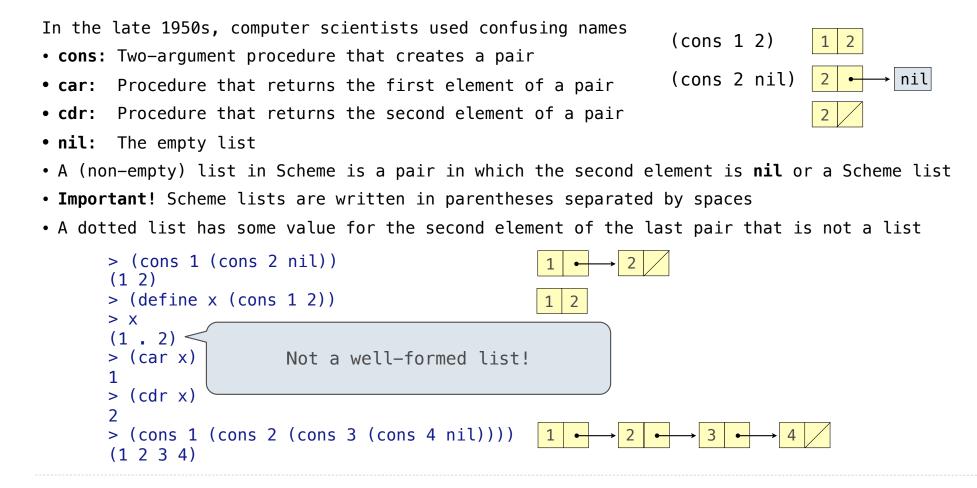
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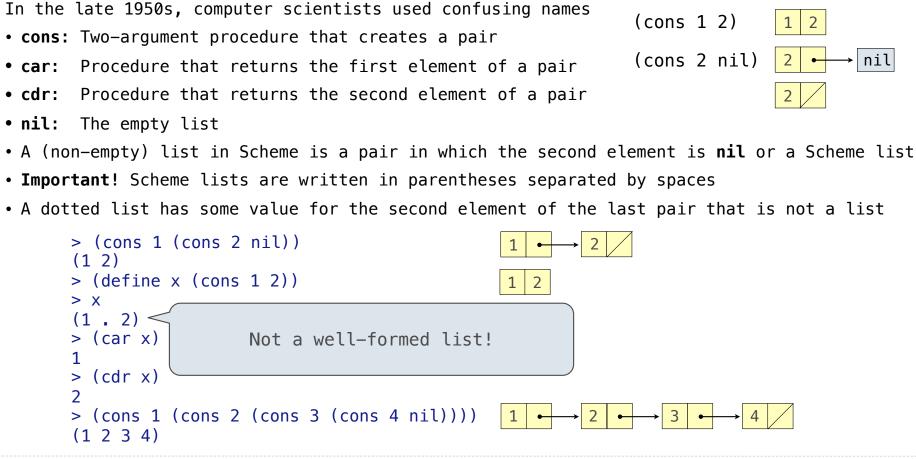


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> (cons 1 (cons 2 (cons 3 (cons 4 nil))))







Exceptions

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Today's Topic: Handling Errors

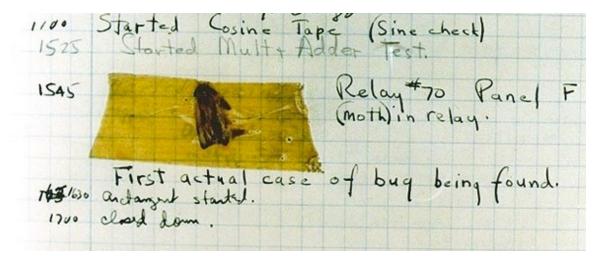
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Grace Hopper's Notebook, 1947, Moth found in a Mark II Computer

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If f calls g and g calls h, exceptions can shift control from h to f without waiting for g to return.

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If **f** calls **g** and **g** calls **h**, exceptions can shift control from **h** to **f** without waiting for **g** to return.

(Exception handling tends to be slow.)

Raising Exceptions

Assert statements raise an exception of type AssertionError

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Example: Reduce

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def reduce(f, s, initial):
"""Combine elements of s pairwise using f, starting with initial.
E.g., reduce(mul, [2, 4, 8], 1) is equivalent to mul(mul(mul(1, 2), 4), 8).
>>> reduce(mul, [2, 4, 8], 1)
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                                                                                     2
                                                                  pow
  a sequence of values that can be the second argument
initial is ...
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                                                                            2
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    """Combine elements of s pairwise using f, starting with initial.
    E.g., reduce(mul, [2, 4, 8], 1) is equivalent to mul(mul(mul(1, 2), 4), 8).
    >>> reduce(mul, [2, 4, 8], 1)
    64
    .....
f is ...
                                                                            4
  a two-argument function
s is ...
                                                                            2
                                                                                     2
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3

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                                                                            64
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                                                                                         3
                                                                            4
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                                                                            2
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    64
                                                                        16,777,216
    .....
                                                                            64
                                                             pow
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                                                                                         3
                                                                            4
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                                             (Demo)
```

Sierpinski's Triangle

(Demo)