

61A Lecture 30

Announcements

Efficient Sequence Processing

Sequence Operations

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Map, filter, and reduce express sequence manipulation using compact expressions

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Example: Sum all primes in an interval from **a** (inclusive) to **b** (exclusive)

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range iterator

next: 1
end: 6

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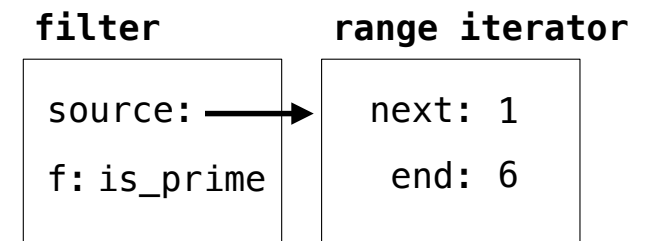
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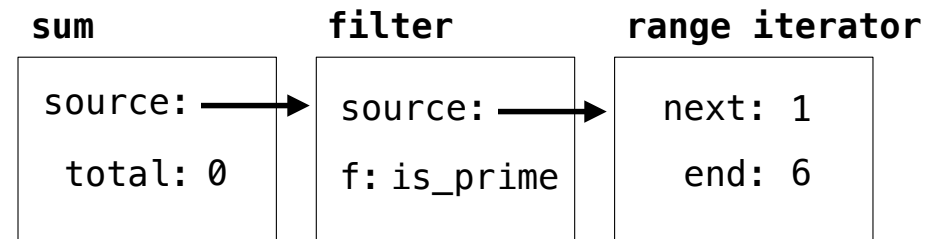
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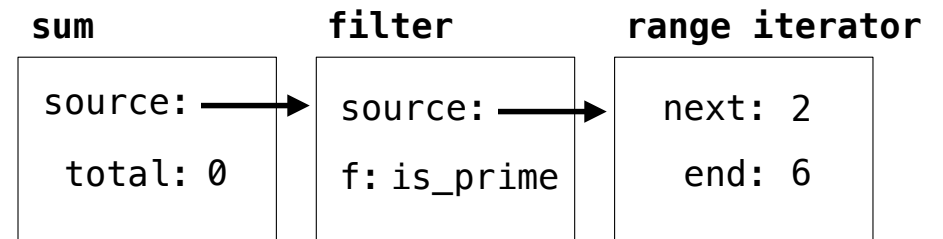
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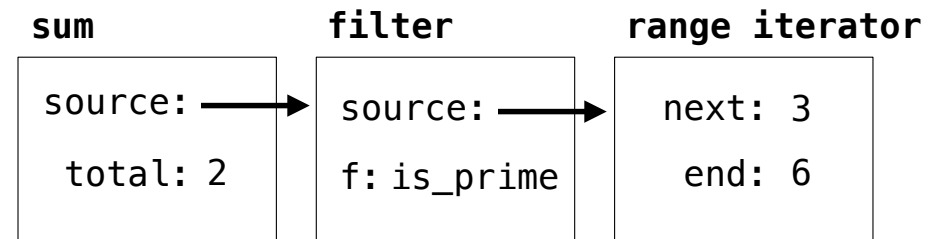
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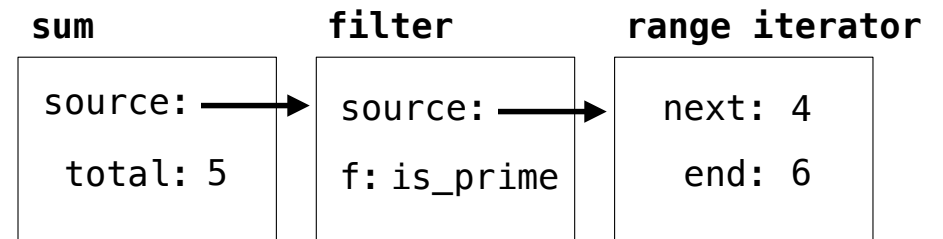
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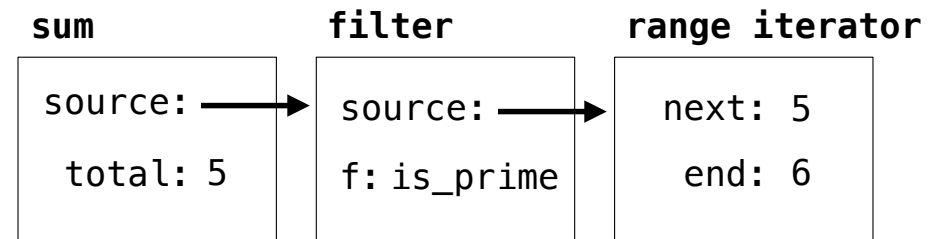
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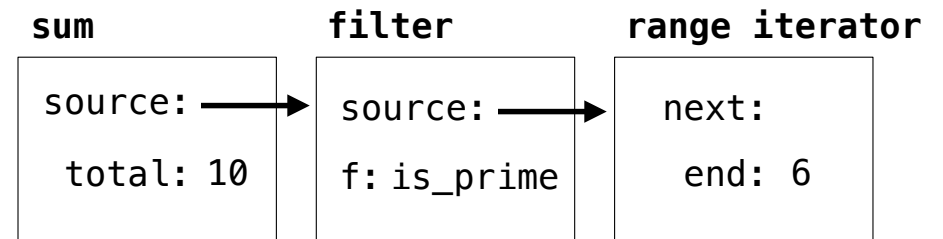
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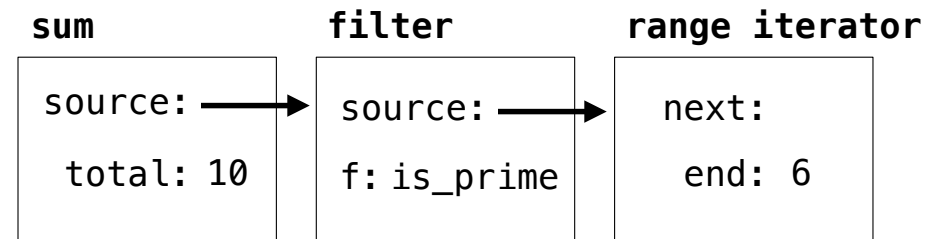
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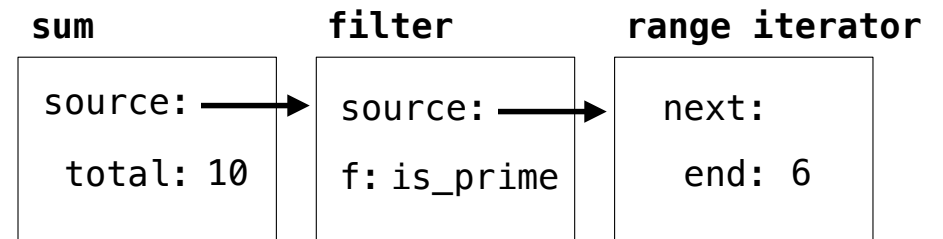
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(Demo)

Streams

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(Demo)

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Stream Processing

(Demo)

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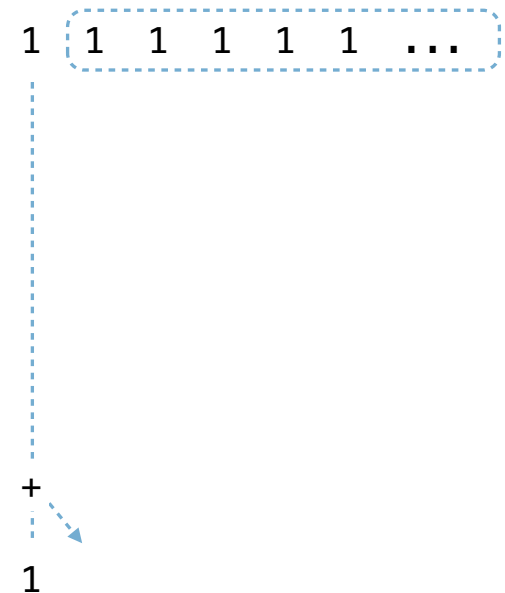
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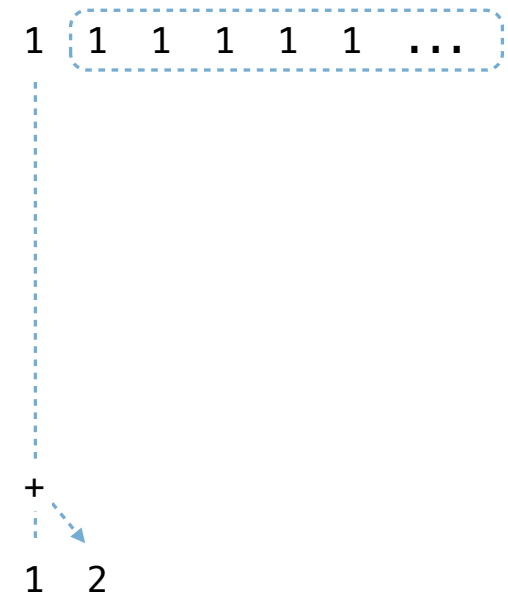
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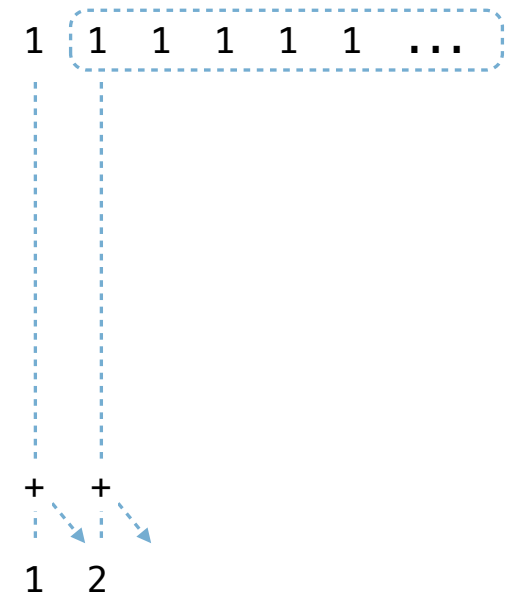
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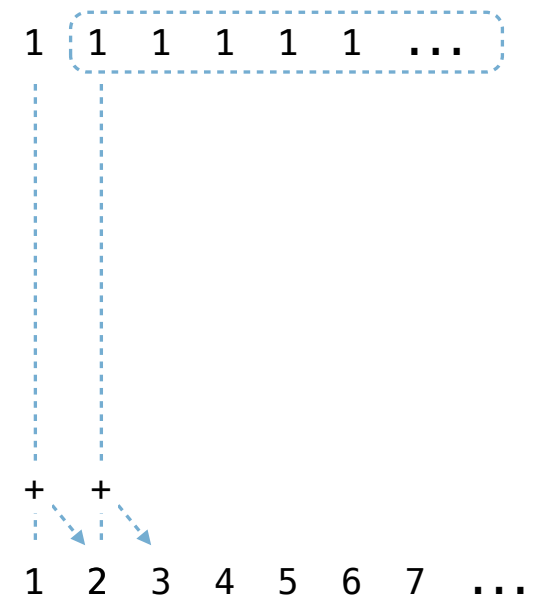
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What's (prefix a 8)? (_ _ _ _ _ _ _ _)

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```

```
(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                         (f (cdr-stream s)))))
```

What's (prefix a 8)? (_ _ _ _ _ _ _ _)

Example: Repeats

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(define (f s) (cons-stream (car s)
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What's (prefix a 8)? (1 2 3 _ _ _ _ _)

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What's (prefix a 8)? (1 2 3 1 2 3 1 2)

What's (prefix (f a) 8)? (1 1)

What's (prefix (g a) 8)? ()

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Higher-Order Stream Functions

Higher-Order Functions on Streams

Implementations are identical,
but change cons to cons-stream
and change cdr to cdr-stream

(Demo)

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but change cons to cons-stream
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```
(define (map f s)
  (if (null? s)
      nil
      (cons (f (car s))
            (map f
                (cdr s))))))
```

```
(define (filter f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons (car s)
                (filter f (cdr s)))
          (filter f (cdr s)))))
```

```
(define (reduce f s start)
  (if (null? s)
      start
      (reduce f
              (cdr s)
              (f start (car s)))))
```

(Demo)

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(Demo)

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Implementations are identical,
but change cons to cons-stream
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```
(define (map-stream f s)
  (if (null? s)
      nil
      (cons-stream (f (car s))
                    (map-stream f
                                (cdr-stream s))))))
```

```
(define (filter-stream f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons-stream (car s)
                        (filter-stream f (cdr-stream s)))
          (filter-stream f (cdr-stream s)))))
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(define (reduce-stream f s start)
  (if (null? s)
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(Demo)

A Stream of Primes

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