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**COMPUTER SCIENCE 61A**October 27, 2016

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**1 Introduction**

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In the next part of the course, we will be working with the **Scheme** programming language. In addition to learning how to write Scheme programs, we will eventually write a Scheme interpreter in Project 4!

Scheme is a dialect of the **Lisp** programming language, a language dating back to 1958. The popularity of Scheme within the programming language community stems from its simplicity – in fact, previous versions of CS 61A were taught in the Scheme language.

**2 Primitives**

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Scheme has a set of *atomic* primitive expressions. Atomic means that these expressions cannot be divided up.

```
scheme> 123
123
scheme> 123.123
123.123
scheme> #t
True
scheme> #f
False
```

Unlike in Python, the only primitive in Scheme that is a false value is `#f` and its equivalents, `false` and `False`. The `define` special form defines variables and procedures

by binding a value to a variable, just like the assignment statement in Python. When a variable is defined, the `define` special form returns a symbol of its name. A procedure is what we call a function in Scheme!

The syntax to define a variable and procedure are:

- `(define <variable name> <value>)`
- `(define (<function name> <parameters>) <function body>)`

```
scm> (define a 3)                ; a = 3
a
scm> a
3
scm> (define (foo x) x)          ; procedure named foo
foo
scm> (foo a)
3
```

## 2.1 Questions

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1. What would Scheme print?

```
scm> (define a 1)
```

```
scm> a
```

```
scm> (define b a)
```

```
scm> b
```

```
scm> (define c 'a)
```

```
scm> c
```

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## 3 Call Expressions

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Scheme call expressions follow prefix notation, where an operator is followed by zero or more operand subexpressions. Operators may be symbols, such as `+` and `*` or more complex expressions, as long as they evaluate to procedure values.

```
scm> (- 1 1)           ; 1 - 1
0
scm> (/ 8 4 2)         ; 8 / 4 / 2
1
scm> (* (+ 1 2) (+ 1 2)) ; (1 + 2) * (1 + 2)
9
```

To call a function in Scheme, you first need a set of parentheses. Inside the parentheses, you specify a function, then the arguments (remember the spaces!).

Evaluating a Scheme function call works just like Python:

1. Evaluate the operator (the first expression after the `()`, then evaluate each of the operands.
2. Apply the operator to those evaluated operands.

When you evaluate `(+ 1 2)`, you evaluate the `+` symbol, which is bound to a built-in addition function. Then, you evaluate `1` and `2`, which are primitives. Finally, you apply the addition function to `1` and `2`.

Some important built-in functions you'll want to know are:

- `+`, `-`, `*`, `/`
- `equal?`, `=`, `>`, `>=`, `<`, `<=`

### 3.1 Questions

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1. What would Scheme print?

```
scm> (+ 1)

scm> (* 3)

scm> (+ (* 3 3) (* 4 4))

scm> (define a (define b 3))

scm> a

scm> b
```

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## 4 Special Forms

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There are certain expressions that look like function calls, but *don't* follow the rule for order of evaluation. These are called *special forms*. You've already seen one — `define`, where the first argument, the variable name, doesn't actually get evaluated to a value.

### 4.1 If Statements

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Another common special form is the **if** form. An **if** expression looks like:

```
(if <condition> <then> <else>)
```

where `<condition>`, `<then>` and `<else>` are expressions. First, `<condition>` is evaluated. If it evaluates to `#t`, then `<then>` is evaluated. Otherwise, `<else>` is evaluated. Remember that only `False` and `#f` are false-y values; everything else is truth-y.

```
scm> (if (< 4 5) 1 2)
1
scm> (if #f (/ 1 0) 42)
42
```

### 4.2 Boolean Operators

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Scheme also has boolean operators **and**, **or**, and **not** like in Python! In addition, **and** and **or** are also special forms because they are short-circuiting operators.

```
scm> (and 1 2 3)
3
scm> (or 1 2 3)
1
scm> (or True (/ 1 0))
True
scm> (and False (/1 0))
False
scm> (not 3)
False
scm> (not True)
False
```

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### 4.3 Questions

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1. What does Scheme print?

```
scm> (if (or #t (/ 1 0)) 1 (/ 1 0))
```

```
scm> (if (> 4 3) (+ 1 2 3 4) (+ 3 4 (* 3 2)))
```

```
scm> ((if (< 4 3) + -) 4 100)
```

```
scm> (if 0 1 2)
```

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### 4.4 Lambdas and Defining Functions

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Scheme has lambdas too! The syntax is

```
(lambda (<PARAMETERS>) <EXPR>)
```

Like in Python, lambdas are function values. Also like in Python, when a lambda expression is called in Scheme, a new frame is created where the parameters are bound to the arguments passed in. Then, <EXPR> is evaluated in this new frame. Note that <EXPR> is not evaluated until the lambda function is called.

```
scm> (define x 3)
```

```
x
```

```
scm> (define y 4)
```

```
y
```

```
scm> ((lambda (x y) (+ x y)) 6 7)
```

```
13
```

Like in Python, lambda functions are also values! So you can do this to define functions:

```
scm> (define square (lambda (x) (* x x)))
```

```
square
```

```
scm> (square 4)
```

```
16
```

When you do (define (<FUNCTION NAME> <PARAMETERS>) <EXPR>), Scheme will automatically transform it to (define <FUNCTION NAME> (lambda (<PARAMETERS>) <EXPR>)). In this way, lambdas are more central to Scheme than they are to Python.

## 4.5 Let

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There is also a special form based around `lambda`: `let`. The structure of `let` is as follows:

```
(let ( (<SYMBOL1> <EXPR1>)
      ...
      (<SYMBOLN> <EXPRN>) )
  <BODY> )
```

This special form is really just equivalent to:

```
( (lambda (<SYMBOL1> ... <SYMBOLN>) <BODY>) <EXPR1> ... <EXPRN>)
```

`let` effectively binds symbols to expressions, then runs the body of the `let` form. This can be useful if you need to reuse a value multiple times, or if you want to make your code more readable.

For example, we can use the approximation  $\sin(x) \approx x$  (which is true for small  $x$ ) and the trigonometric identity  $\sin(x) = 3 \sin(x/3) - 4 \sin^3(x/3)$  to approximate  $\sin(x)$  for any  $x$ .

```
(define (sin x)
  (if (< x 0.000001)
      x
      (let ( (recursive-step (sin (/ x 3))) )
          (- (* 3 recursive-step)
              (* 4 (expt recursive-step 3))))))
```

## 4.6 Questions

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1. Write a function that calculates factorial. (Note we have not seen any iteration yet.)

```
(define (factorial x)
```

```
)
```

2. Write a function that calculates the  $n^{\text{th}}$  Fibonacci number.

```
(define (fib n)
```

```
  (if (< n 2)
```

```
      1
```

```
)
```

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## 5 Pairs and Lists

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To construct a (linked) list in Scheme, you can use the constructor `cons` (which takes two arguments). `nil` represents the empty list. If you have a linked list in Scheme, you can use selector `car` to get the first element and selector `cdr` to get the rest of the list. (`car` and `cdr` don't stand for anything anymore, but if you want the history go to [http://en.wikipedia.org/wiki/CAR\\_and\\_CDR](http://en.wikipedia.org/wiki/CAR_and_CDR).)

```
scm> nil
()
scm> (null? nil)
#t
scm> (cons 2 nil)
(2)
scm> (cons 3 (cons 2 nil))
(3 2)
scm> (define a (cons 3 (cons 2 nil)))
a
scm> (car a)
3
scm> (cdr a)
(2)
scm> (car (cdr a))
2
scm> (define (len a)
      (if (null? a)
          0
          (+ 1 (len (cdr a)))))
len
scm> (len a)
2
```

If a list is a "good looking" list, like the ones above where the second element is always a linked list, we call it a **well-formed list**. Interestingly, in Scheme, the second element does not have to be a linked list. You can give something else instead, but `cons` always takes exactly 2 arguments. These lists are called **malformed list**. The difference is a dot:

```
scm> (cons 2 3)
(2 . 3)
scm> (cons 2 (cons 3 nil))
(2 3)
scm> (cdr (cons 2 3))
3
scm> (cdr (cons 2 (cons 3 nil)))
(3)
```

In general, the rule for displaying a pair is as follows: use the dot to separate the `car` and `cdr` fields of a pair, but if the dot is immediately followed by an open parenthesis, then remove the dot and the parenthesis pair. Thus, `(0 . (1 . 2))` becomes `(0 1 . 2)`

There are many useful operations and shorthands on lists. One of them is `list` special form `list` takes zero or more arguments and returns a list of its arguments. Each argument is in the `car` field of each list element. It behaves the same as quoting a list, which also creates the list.

```
scm> (list 1 2 3)
(1 2 3)
scm> '(1 2 3)
(1 2 3)
scm> (car '(1 2 3))
1
scm> (equal? '(1 2 3) (list 1 2 3))
#t
scm> '(1 . (2 3))
(1 2 3)
scm> '(define (square x) (* x x))
(define (square x) (* x x))
```



1. Define a function that takes 2 lists and concatenates them together. Notice that simply calling `(cons a b)` would not work because it will create a deep list. Instead, think recursively!

```
(define (concat a b)
```

```
)  
  
scm> (concat '(1 2 3) '(2 3 4))  
(1 2 3 2 3 4)
```

2. Define `replicate`, which takes an element `x` and a non-negative integer `n`, and returns a list with `x` repeated `n` times.

```
(define (replicate x n)
```

```
)  
  
scm> (replicate 5 3)  
(5 5 5)
```

3. A **run-length encoding** is a method of compressing a sequence of letters. The list (a a a b a a a a) can be compressed to ((a 3) (b 1) (a 4)), where the compressed version of the sequence keeps track of how many letters appear consecutively.

Write a Scheme function that takes a compressed sequence and expands it into the original sequence. *Hint*: try to use functions you defined earlier in this worksheet.

**(define** (uncompress s)

```
)  
scm> (uncompress '((a 1) (b 2) (c 3)))  
(a b b c c c)
```

4. Define `deep-apply`, which takes a nested list and applies a given procedure to every element. `deep-apply` should return a nested list with the same structure as the input list, but with each element replaced by the result of applying the given procedure to that element. Use the built-in `list?` procedure to detect whether a value is a list. The procedure `map` has been defined for you.

```
(define (map fn lst)
  (if (null? lst)
      nil
      (cons (fn (car lst)) (map fn (cdr lst)))))
(define (deep-apply fn nested-list)
```

```
)
```

```
scm> (deep-apply (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-apply (lambda (x) (* x x)) '(1 ((4) 5) 9))
(1 ((16) 25) 81)
scm> (deep-apply (lambda (x) (* x x)) 2)
4
```

## 6 Extra Questions

1. Fill in the following to complete an abstract tree data type:

```
(define (make-tree root branches) (cons root branches))
```

```
(define (root tree) _____ )
```

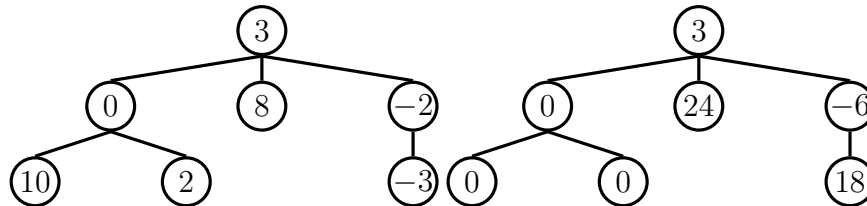
```
(define (branches tree) _____ )
```

2. Using the abstract data type above, write a function that sums up the entries of a tree, assuming that the entries are all numbers. Hint: you may want to use the map function you defined above, as well as an additional helper function.

```
(define (tree-sum tree)
```

```
)
```

3. Using the abstract data type above, write a Scheme function that creates a new tree where the entries are the product of the entries along the path to the root in the original tree. Hint: you may want to write helper functions.



```
(define (path-product-tree t)
```