

## 61A Lecture 7

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## Announcements

## Hog Contest Rules

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- Up to two people submit one entry;  
Max of one entry per person
- Slight rule changes
- Your score is the number of entries  
against which you win more than  
50.00001% of the time
- Strategies are time-limited
- All strategies must be deterministic,  
pure functions of the players' scores
- All winning entries will receive  
extra credit
- The real prize: honor and glory
- See website for detailed rules

### **Fall 2011 Winners**

Kaylee Mann  
Yan Duan & Ziming Li  
Brian Prike & Zhenghao Qian  
Parker Schuh & Robert Chatham

### **Fall 2012 Winners**

Chenyang Yuan  
Joseph Hui

### **Fall 2013 Winners**

Paul Bramsen  
Sam Kumar & Kangsik Lee  
Kevin Chen

### **Fall 2014 Winners**

Alan Tong & Elaine Zhao  
Zhenyang Zhang  
Adam Robert Villaflor & Joany Gao  
Zhen Qin & Dian Chen  
Zizheng Tai & Yihe Li

## Hog Contest Winners

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### **Spring 2015 Winners**

Sinho Chewi & Alexander Nguyen Tran  
Zhaoxi Li  
Stella Tao and Yao Ge

### **Fall 2015 Winners**

Micah Carroll & Vasilis Oikonomou  
Matthew Wu  
Anthony Yeung and Alexander Dai

### **Spring 2016 Winners**


Michael McDonald and Tianrui Chen  
Andrei Kassiantchouk  
Benjamin Krieges

### **Spring 2017 Winners**

Cindy Jin and Sunjoon Lee  
Anny Patino and Christian Vasquez  
Asana Choudhury and Jenna Wen  
Michelle Lee and Nicholas Chew

### **Fall 2017 Winners**

Your name could be here FOREVER!



## Order of Recursive Calls

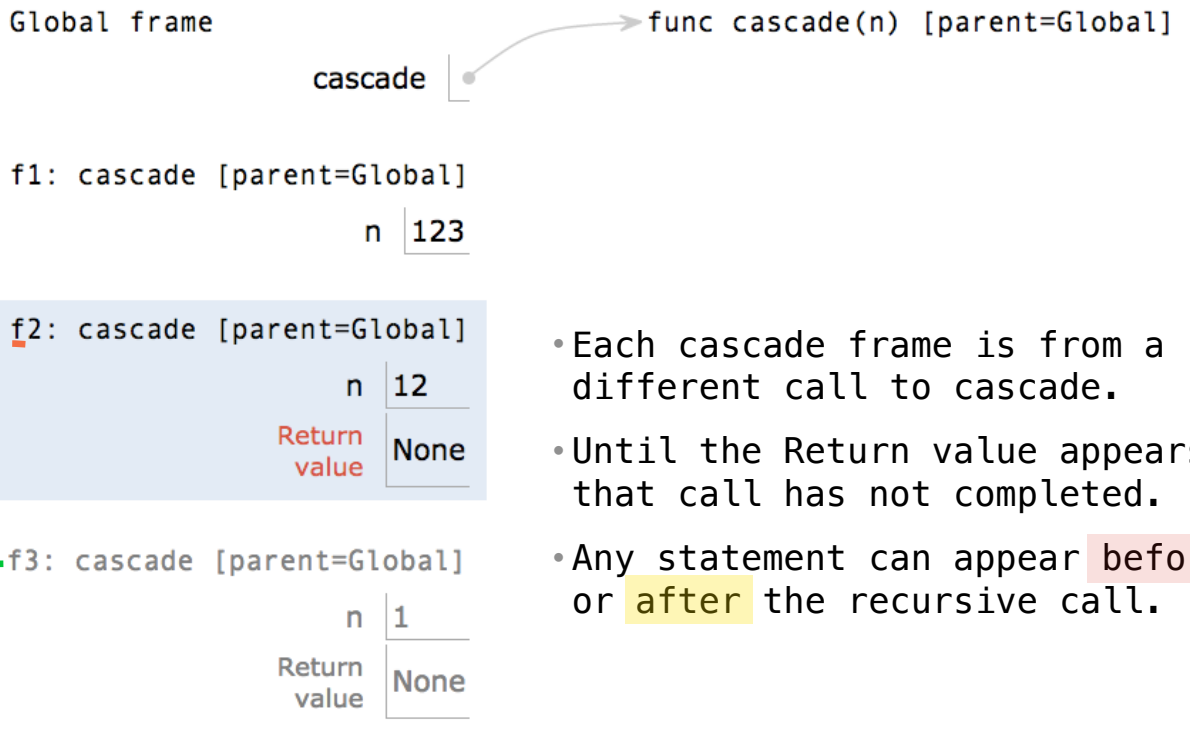
# The Cascade Function

(Demo)

```
1 def cascade(n):  
2     if n < 10:  
3         print(n)  
4     else:  
5         print(n)  
6         cascade(n//10)  
7         print(n)  
8  
9 cascade(123)
```

Program output:

```
123  
12  
1  
12
```



- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram

## Two Definitions of Cascade

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(Demo)

```
def cascade(n):  
    if n < 10:  
        print(n)  
    else:  
        print(n)  
        cascade(n//10)  
        print(n)
```

```
def cascade(n):  
    print(n)  
    if n >= 10:  
        cascade(n//10)  
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

## Example: Inverse Cascade



## Inverse Cascade

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Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```
def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

```
grow = lambda n: f_then_g(
shrink = lambda n: f_then_g(
```

## Tree Recursion

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

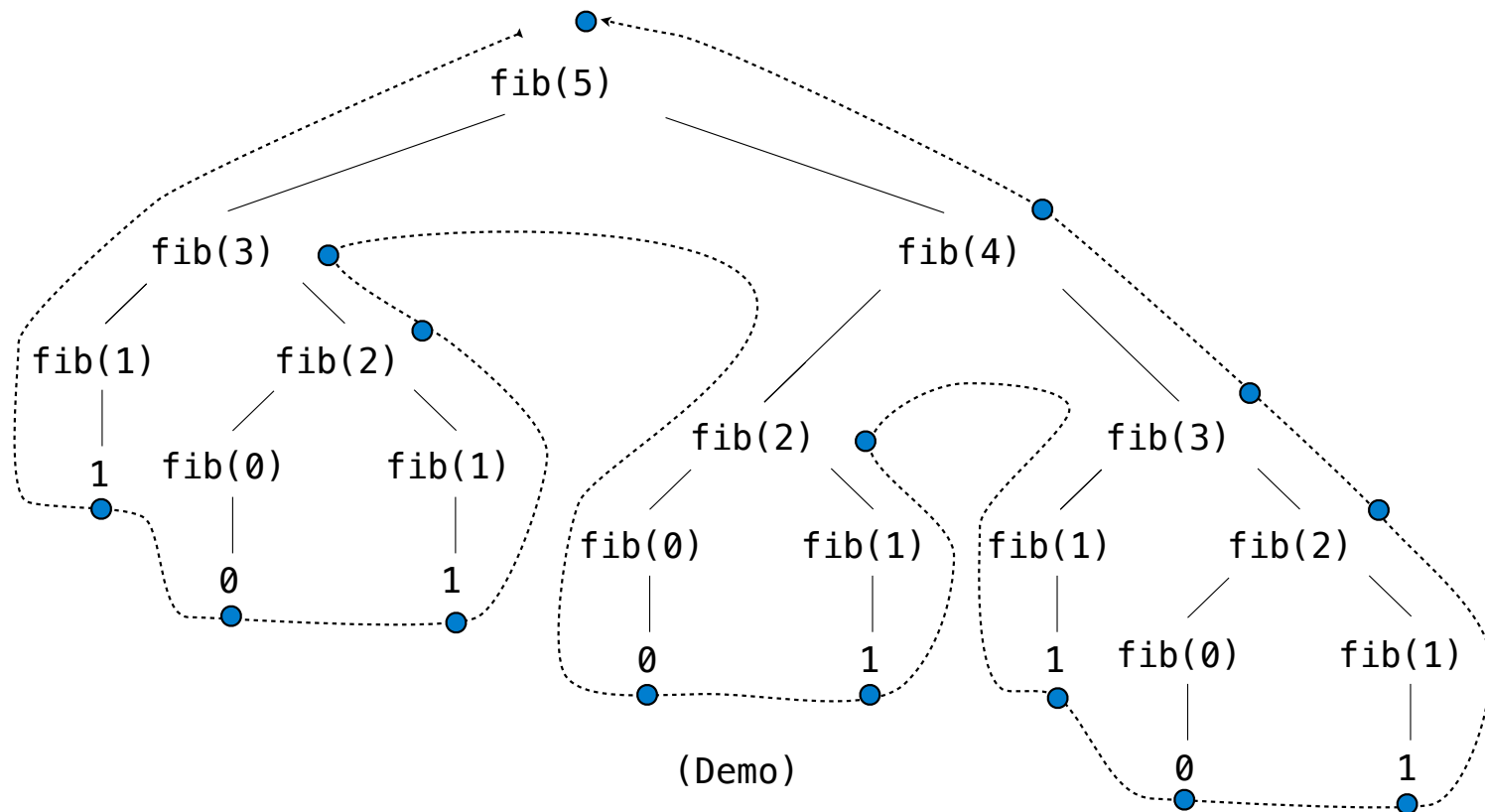
<b>n:</b>	0, 1, 2, 3, 4, 5, 6, 7, 8,	...	35
<b>fib(n):</b>	0, 1, 1, 2, 3, 5, 8, 13, 21,	...	9,227,465

```
def fib(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fib(n-2) + fib(n-1)
```



## A Tree-Recursive Process

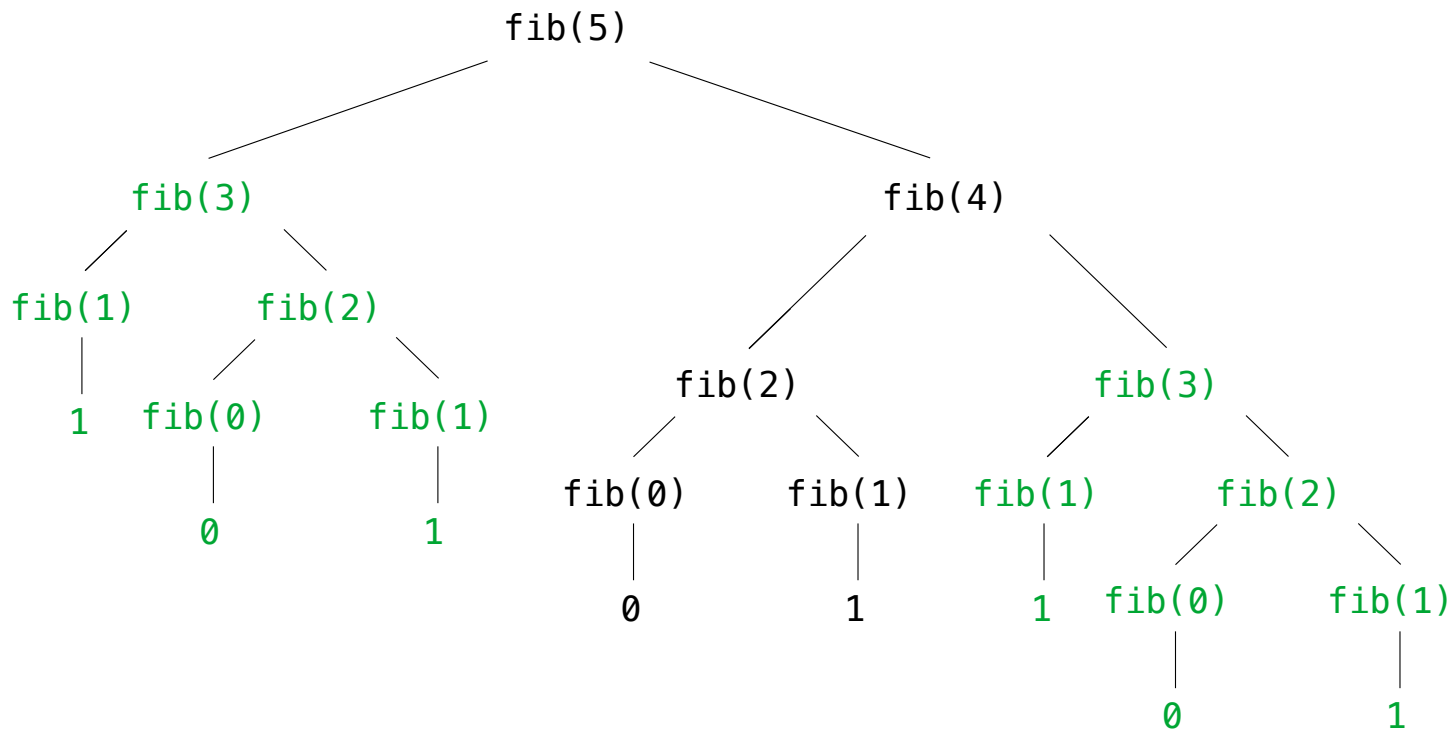
The computational process of fib evolves into a tree structure



## Repetition in Tree-Recursive Computation

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This process is highly repetitive; fib is called on the same argument multiple times



(We will speed up this computation dramatically in a few weeks by remembering results)

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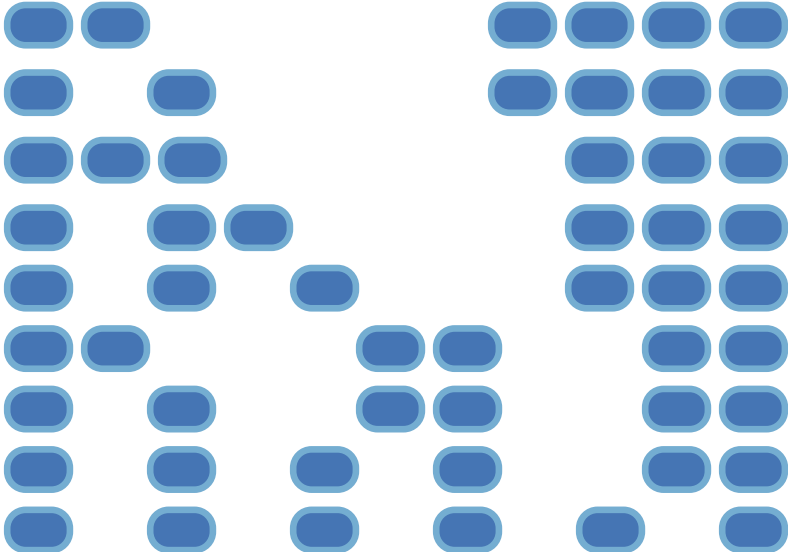
## Example: Counting Partitions

# Counting Partitions

The number of partitions of a positive integer  $n$ , using parts up to size  $m$ , is the number of ways in which  $n$  can be expressed as the sum of positive integer parts up to  $m$  in increasing order.

```
count_partitions(6, 4)
```

- $2 + 4 = 6$
- $1 + 1 + 4 = 6$
- $3 + 3 = 6$
- $1 + 2 + 3 = 6$
- $1 + 1 + 1 + 3 = 6$
- $2 + 2 + 2 = 6$
- $1 + 1 + 2 + 2 = 6$
- $1 + 1 + 1 + 1 + 2 = 6$
- $1 + 1 + 1 + 1 + 1 + 1 = 6$

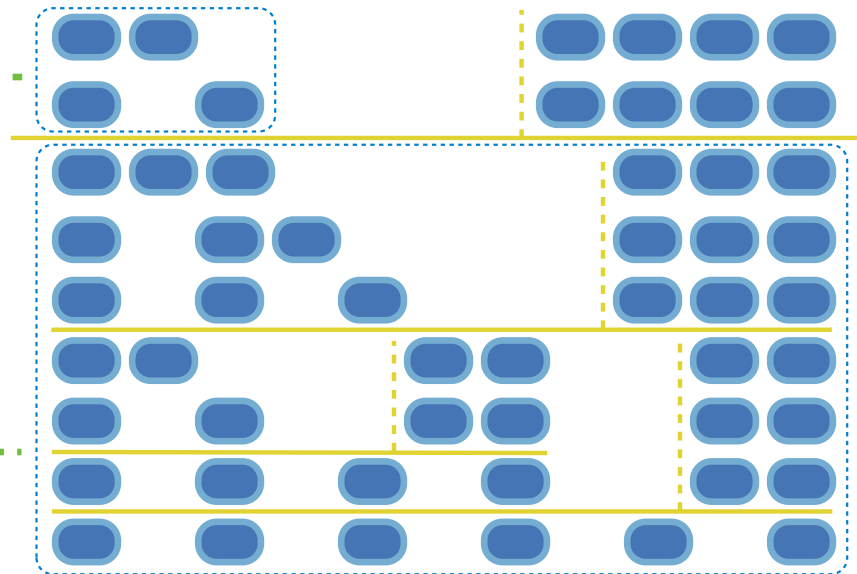


## Counting Partitions

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`count_partitions(6, 4)`

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - `count_partitions(2, 4)`
  - `count_partitions(6, 3)`
- Tree recursion often involves exploring different choices.





## Counting Partitions

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- Recursive decomposition: finding simpler instances of the problem.
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  - `count_partitions(6, 3)`
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):  
    if n == 0:  
        return 1  
    elif n < 0:  
        return 0  
    elif m == 0:  
        return 0  
    else:  
        with_m = count_partitions(n-m, m)  
        without_m = count_partitions(n, m-1)  
        return with_m + without_m
```

(Demo)

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[Interactive Diagram](#)