

## 61A Lecture 7

## Announcements

### Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person
- Slight rule changes
- Your score is the number of entries against which you win more than 50.00001% of the time
- Strategies are time-limited
- All strategies must be deterministic, pure functions of the players' scores
- All winning entries will receive extra credit
- The real prize: honor and glory
- See website for detailed rules

#### Fall 2011 Winners

Kaylee Mann  
Yan Duan & Ziming Li  
Brian Prike & Zhenghao Qian  
Parker Schuh & Robert Chatham

#### Fall 2012 Winners

Chenyang Yuan  
Joseph Hui

#### Fall 2013 Winners

Paul Bramsen  
Sam Kumar & Kangsik Lee  
Kevin Chen

#### Fall 2014 Winners

Alan Tong & Elaine Zhao  
Zhenyang Zhang  
Adam Robert Villaflor & Joany Gao  
Zhen Qin & Dian Chen  
Zizheng Tai & Yihe Li

[cs61a.org/proj/hog\\_contest](http://cs61a.org/proj/hog_contest)

### Hog Contest Winners

#### Spring 2015 Winners

Sinho Chewi & Alexander Nguyen Tran  
Zhaoxi Li  
Stella Tao and Yao Ge

#### Fall 2015 Winners

Micah Carroll & Vasilis Oikonomou  
Matthew Wu  
Anthony Yeung and Alexander Dai

#### Spring 2016 Winners

Michael McDonald and Tianrui Chen  
Andrei Kassiantchouk  
Benjamin Krieges

#### Spring 2017 Winners

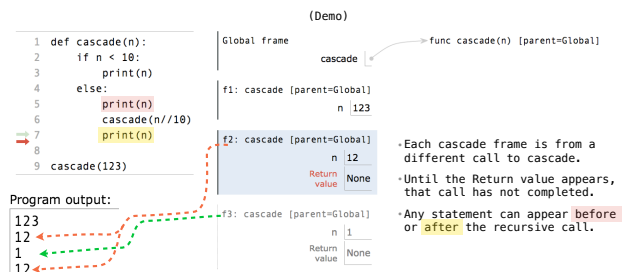
Cindy Jin and Sunjoon Lee  
Anny Patino and Christian Vasquez  
Asana Choudhury and Jenna Wen  
Michelle Lee and Nicholas Chew

#### Fall 2017 Winners

Your name could be here FOREVER!

## Order of Recursive Calls

### The Cascade Function



Interactive Diagram

### Two Definitions of Cascade

(Demo)

```
def cascade(n):
  if n < 10:
    print(n)
  else:
    print(n)
    cascade(n//10)
    print(n)

def cascade(n):
  print(n)
  if n >= 10:
    cascade(n//10)
  print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

### Example: Inverse Cascade

## Inverse Cascade

Write a function that prints an inverse cascade:

```

1           def inverse_cascade(n):
12          grow(n)
123         print(n)
1234        shrink(n)
123        def f_then_g(f, g, n):
12          if n:
1           f(n)
            g(n)

grow = lambda n: f_then_g(
shrink = lambda n: f_then_g(
    
```

## Tree Recursion

## Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

```

n: 0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465
    
```

```

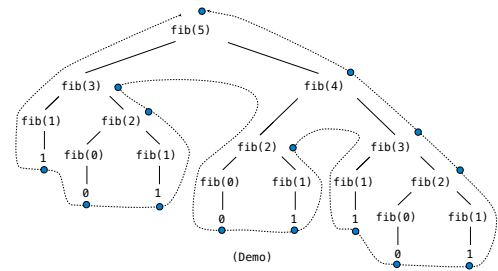
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
    
```



<http://en.wikipedia.org/wiki/File:Fibonacci.jpg>

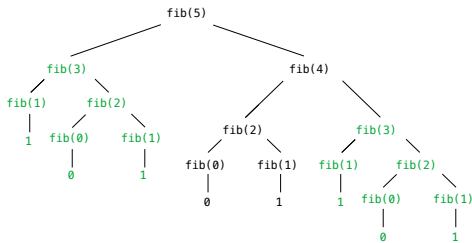
## A Tree-Recursive Process

The computational process of fib evolves into a tree structure



## Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times



(We will speed up this computation dramatically in a few weeks by remembering results)

## Example: Counting Partitions

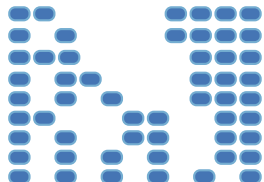
## Counting Partitions

The number of partitions of a positive integer  $n$ , using parts up to size  $m$ , is the number of ways in which  $n$  can be expressed as the sum of positive integer parts up to  $m$  in increasing order.

count\_partitions(6, 4)

```

2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
2 + 2 + 2 = 6
1 + 1 + 2 + 2 = 6
1 + 1 + 1 + 1 + 2 = 6
1 + 1 + 1 + 1 + 1 + 1 = 6
    
```

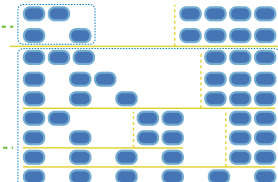


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count\_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count\_partitions(2, 4)
  - count\_partitions(6, 3)
- Tree recursion often involves exploring different choices.



## Counting Partitions

The number of partitions of a positive integer  $n$ , using parts up to size  $m$ , is the number of ways in which  $n$  can be expressed as the sum of positive integer parts up to  $m$  in increasing order.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

·Recursive decomposition: finding simpler instances of the problem.

·Explore two possibilities:

- Use at least one 4
- Don't use any 4

·Solve two simpler problems:

- count\_partitions(2, 4) → with\_m = count\_partitions(n-m, m)
- count\_partitions(6, 3) → without\_m = count\_partitions(n, m-1)

·Tree recursion often involves exploring different choices.

(Demo)

[Interactive Diagram](#)